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# GLONASS phase bias estimation and its PPP ambiguity resolution using homogeneous receivers

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Abstract Integer ambiguity resolution (IAR) appreciably improves the position accuracy and shortens the convergence time of precise point positioning (PPP). However, while many studies are limited to GPS, there is a need to investigate the performance of GLONASS PPP ambiguity resolution. Unfortunately, because of the frequency-division multiple-access strategy of GLONASS, GLONASS PPP IAR faces two obstacles. First, simultaneously observed satellites operate at different wavelengths. Second and most importantly, distinct inter-frequency bias (IFB) exists between different satellites. For the former, we adopt an undifferenced method for uncalibrated phase delay (UPD) estimation and proposed an undifferenced PPP IAR strategy. We select a set of homogeneous receivers with identical receiver IFB to perform UPD estimation and PPP IAR. The code and carrier phase IFBs can be absorbed by satellite wide-lane and narrow-lane UPDs, respectively, which is in turn consistent with PPP IAR using the same type of receivers. In order to verify the method, we used 50 stations to generate satellite UPDs and another 12 stations selected as users to perform PPP IAR. We found that the GLONASS satellite UPDs are stable in time and space and can be

& Weiwei Song weiweisong\_whu@163.com estimated with high accuracy and reliability. After applying UPD correction, 91 % of wide-lane ambiguities and 99 % of narrow-lane ambiguities are within  $(-0.15, +0.15)$  cycles of the nearest integer. After ambiguity resolution, the 2-hour static PPP accuracy improves from (0.66, 1.42, 1.55) cm to (0.38, 0.39, 1.39) cm for the north, east, and up components, respectively.

Keywords GLONASS · Precise point positioning · Ambiguity resolution - Uncalibrated phase delay - Interfrequency bias

# Introduction

Precise point positioning (PPP) has proven to be a powerful tool that provides globally high precision in a number of applications, such as geophysics and meteorology (Zumberge et al. [1997;](#page-10-0) Kouba and Heroux [2001](#page-9-0)). Integer carrier phase ambiguity resolution (IAR) is the key to improve the precision of PPP with observations made over a short period, especially for the east component (Teunissen et al. [1997](#page-10-0); Geng et al. [2009\)](#page-9-0). In original PPP, because only one receiver is involved, the uncalibrated phase delay (UPD) is absorbed into the ambiguity, preventing the isolation of the integer nature of ambiguities (Gabor and Nerem [1999](#page-9-0); Collins et al. [2008](#page-9-0); Ge et al. [2008;](#page-9-0) Laurichesse et al. [2009](#page-9-0); Loyer et al. [2012](#page-9-0)). Thus, UPDs must be separated from the precise satellite clock products and applied in PPP to fix the ambiguities.

An ionosphere-free observation is generally conducted in PPP to eliminate the first-order ionospheric delay, and the ambiguities are thus generally decomposed into widelane (WL) and narrow-lane (NL) ambiguities to be fixed sequentially (Ge et al. [2008](#page-9-0)). On the basis of a dense

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reference network, the satellite WL and NL UPDs can be estimated by averaging the fractional parts of all involved float WL and NL ambiguity estimates, which are derived from the Hatch–Melbourne–Wübbena (HMW) (Hatch [1982;](#page-9-0) Melbourne [1985](#page-9-0); Wübbena [1985\)](#page-10-0) combination and the real-valued ionosphere-free ambiguities, respectively.

Gabor and Nerem ([1999\)](#page-9-0) first reported a method for ambiguity resolution at a single station, in which they estimated daily mean WL and NL UPDs with a global network of stations. In order to remove the receiver UPDs, single differencing between satellites (SDBS) is applied in their method. Ge et al. ([2008\)](#page-9-0) proposed a similar method in which the WL UPD estimates do not directly contribute to estimating the NL UPDs. The WL UPDs were estimated every several days, while the NL UPDs were estimated every 15 min. This method was assessed with a global network using daily data, where more than 80 % of the independent SDBS ambiguities were fixed.

Laurichesse et al. [\(2009](#page-9-0)) directly fixed undifferenced ambiguities to integers and used the clock estimates to absorb NL UPDs. Their WL UPD determination is similar to that of Ge et al. [\(2008](#page-9-0)), while the NL UPDs are not determined but assimilated into the clock estimates. Collins et al. [\(2008](#page-9-0)) adopted a similar method and developed a decoupled clock model. In his method, the NL ambiguities were fixed to integers before estimating the satellite clocks, and the code clocks are thus different from carrier phase clocks. Li et al. ([2013\)](#page-9-0) proposed a method of estimating the UPDs of L1 and L2 based on previously generated WL and NL UPDs, which is similar to the method of Ge et al. [\(2008](#page-9-0)) and Laurichesse et al. ([2009\)](#page-9-0).

In the studies cited above, all current PPP IAR studies are limited to GPS, and there has been little study on GLONASS. With the full operation of GLONASS with 24 satellites since December 8, 2011, and the improved quality of GLONASS orbits and clocks (International GNSS Service (IGS); Dow et al. [2009\)](#page-9-0), there is a great need to assess the performance of GLONASS PPP IAR. However, as a result of the frequency-division multipleaccess (FDMA) strategy, GLONASS PPP IAR is hindered by two main difficulties. First, simultaneously observed satellites have different wavelengths that greatly complicate GLONASS ambiguity resolution (Wang et al. [2001](#page-10-0)). Second and most importantly, distinct inter-frequency bias (IFB) exists between different satellites for both carrier phase and code observations (Pratt et al. [1998;](#page-9-0) Wanninger [2012;](#page-10-0) Al-Shaery et al. [2013\)](#page-9-0). The carrier phase and code IFBs are receiver dependent and must be corrected or the WL and NL fractional parts from different receivers will contradict each other and cause the UPD estimation to fail (Reussner and Wanninger [2011](#page-9-0)). The receiver IFB is caused mainly by four factors (Boriskin and Zyryanov [2008;](#page-9-0) Sleewaegen et al. [2012\)](#page-9-0):

the special design of the antenna analog component, the nominal group-delay variations caused by an imperfect front-end radio-frequency design,

special features of the baseband digital signal processing algorithm, and

individual differences among receivers.

Generally, the GLONASS carrier phase IFB is the same for receivers produced by the same manufacture and can be modeled as a linear function of frequency. With a prior correction model, it can be calibrated nearly exactly (Wanninger [2012](#page-10-0)). However, the code IFB behaves differently in that it can be modeled as a linear function of frequency for some types of receivers (Yamada et al. [2010](#page-10-0); Al-Shaery et al. [2013](#page-9-0)), while such a model cannot be applied for other types (Kozlov et al. [2000](#page-9-0); Yamada et al. [2010\)](#page-10-0). Additionally, the code IFB might be different for receivers from the same manufacture (Chuang et al. [2013\)](#page-9-0). Until now, there has been no general model that can be used to correct the code IFB. This will cause the WL UPD estimation to fail with the HMW combination and subsequently cause the NL UPD estimation to fail. Reussner and Wanninger ([2011\)](#page-9-0) proposed to use a precise ionospheric model to help the estimation of WL UPD based on the WL observable of L1 and L2 carrier phases. However, this method depends on a precise ionospheric delay model that is precise enough to fix the WL ambiguity with a wavelength of about 86 cm. Yi ([2015\)](#page-10-0) proposed an extra-narrow-lane (ENL) method to avoid the thorny GLONASS WL ambiguity resolution, but the shortcoming is that the ENL wavelength has a short wavelength of about 5cm.

In order to resolve GLONASS ambiguities in PPP, we select 62 stations with the same receiver and antenna type to ensure identical carrier phase and code IFBs for each receiver. The IFB-caused bias in the WL and NL ambiguity will then be the same for all involved receivers and can be absorbed into the satellite WL and NL UPD estimates, such that GLONASS PPP IAR can be performed in a straightforward manner. Numerical results are provided to assess GLONASS UPD estimation results and PPP ambiguity fixing results with respect to the fixing percentage and positioning accuracy.

#### Methods

The receiver IFB is not separable from the receiver clock, and we thus define the IFB of each satellite as the difference with respect to satellite R11 (which has a frequency index of zero). The carrier phase and code observations on frequency  $g(g = 1, 2)$  for a particular epoch can then be written as

<span id="page-2-0"></span>
$$
P_{g,i}^{k} = \rho_{i}^{k} + \frac{\mu_{i}^{k}}{f_{g}^{k2}} + b_{g,i}^{k} + F_{g,i}^{k} + e_{g,i}^{k}
$$
  

$$
L_{g,i}^{k} = \rho_{i}^{k} - \frac{\mu_{i}^{k}}{f_{g}^{k2}} + B_{g,i}^{k} + \lambda_{g}^{k} N_{g,i}^{k} + H_{g,i}^{k} + \varepsilon_{g,i}^{k}
$$
 (1)

where  $k$  and  $i$  denote a satellite and a receiver, respectively, and  $P_{g,i}^{k}$  and  $L_{g,i}^{k}$  are code and carrier phase observations with corresponding wavelength  $\lambda_g^k$  and frequency  $f_g^k$ ,  $\rho_i^k$  is the non-dispersive delay that includes geometric delay, tropospheric delay, satellite and receiver clock biases, and any other delay that affects all the observations identically. The quotient  $\mu_i^k / f_g^k$  denotes the slant ionospheric delay,  $N_{g,i}^k$ denotes an integer ambiguity, and  $F_{g,i}^{k}$  and  $H_{g,i}^{k}$  denote the code and carrier phase IFBs with respect to R11. The terms  $b_{g,i}^k$  and  $B_{g,i}^k$  denote the code and carrier phase hardware biases, respectively, with  $b_{g,i}^k = b_{g,i} - b_g^k$  and  $B_{g,i}^k =$  $B_{g,i} - B_g^k$ ; here,  $b_{g,i}$  and  $B_{g,i}$  are for the receiver, whereas  $b_g^k$ and  $B_g^k$  are for the satellite. Finally,  $e_{g,i}^k$  and  $e_{g,i}^k$  denote unmodeled code and carrier phase errors including multipath effects and noise. The phase center correction and the phase windup effect (Wu et al. [1993\)](#page-10-0) must be considered in modeling.

In order to eliminate the first-order ionosphere delay, the well-known ionospheric-free combination is used for PPP (Zumberge et al. [1997\)](#page-10-0):

$$
P_{c,i}^k = \alpha P_{1,i}^k - \beta P_{2,i}^k = \rho_i^k + b_{c,i}^k + F_{c,i}^k + e_{c,i}^k
$$
  
\n
$$
L_{c,i}^k = \alpha L_{1,i}^k - \beta L_{2,i}^k = \rho_i^k + B_{c,i}^k + N_{c,i}^k + H_{c,i}^k + \varepsilon_{c,i}^k
$$
\n(2)

where  $\alpha = f_1^{k2}$  $/(f_1^{k2} - f_2^{k2})$  and  $\beta = f_2^{k2}$  $/(f_1^{k2} - f_2^{k2})$ . The receiver carrier phase IFB and hardware bias are not separable from the related float ambiguity, and we thus group them together:

$$
\tilde{N}_{c,i}^k = B_{c,i}^k + N_{c,i}^k + H_{c,i}^k
$$
\n(3)

Substituting  $(3)$  into  $(2)$ , we get

$$
P_{c,i}^k = \rho_i^k + b_{c,i}^k + F_{c,i}^k + e_{c,i}^k
$$
  
\n
$$
L_{c,i}^k = \rho_i^k + \tilde{N}_{c,i}^k + e_{c,i}^k
$$
\n(4)

The related float ionospheric-free ambiguity is usually expressed as the combination of WL and NL integer ambiguities and their UPDs:

$$
\tilde{N}_{c,i}^k = \frac{f_2^k}{f_1^k + f_2^k} \lambda_w^k N_{w,i}^k + \lambda_n^k (N_{n,i}^k + \phi_{n,i} - \phi_n^k + H_{n,i}^k)
$$
(5)

with

$$
\phi_n^k = \frac{f_1^k}{f_1^k + f_2^k} B_1^k - \frac{f_2^k}{f_1^k + f_2^k} B_2^k
$$
\n(6)

$$
\phi_{n,i} = \frac{f_1^k}{f_1^k + f_2^k} B_{1,i} - \frac{f_2^k}{f_1^k + f_2^k} B_{2,i}
$$
\n(7)

$$
H_{n,i}^k = \frac{f_1^k}{f_1^k + f_2^k} H_{1,i}^k - \frac{f_2^k}{f_1^k + f_2^k} H_{2,i}^k
$$
\n(8)

where  $N_{w,i}^{k}$  and  $N_{n,i}^{k}$  are the WL and NL ambiguities, respectively, having corresponding wavelengths  $\lambda_w^k$  and  $\lambda_n^k$ .  $\phi_{n,i}$  and  $\phi_n^k$  are the receiver and satellite NL UPDs, respectively, and  $H_{n,i}^k$  is the receiver NL IFB.

# Effect of the GLONASS receiver IFB on the UPD estimation and its treatment

The WL ambiguity can be derived with the HMW combination of the carrier phase and code observations as

$$
\tilde{N}_{w,i}^k = \left( \frac{L_{1,i}}{\lambda_1} - \frac{L_{2,i}}{\lambda_2} - \frac{f_1^k P_{1,i}^k + f_2^k P_{2,i}^k}{(f_1^k + f_2^k)\lambda_w^k} \right) \n= N_{w,i}^k + \phi_{w,i} - \phi_w^k + H_{w,i}^k
$$
\n(9)

with

$$
\phi_{w,i} = \left(\frac{B_{1,i}}{\lambda_1} - \frac{B_{2,i}}{\lambda_2} - \frac{f_1^k b_{1,i} + f_2^k b_{2,i}}{(f_1^k + f_2^k)\lambda_w^k}\right) \tag{10}
$$

$$
\phi_w^k = \left(\frac{B_1^k}{\lambda_1} - \frac{B_2^k}{\lambda_2} - \frac{f_1^k b_1^k + f_2^k b_2^k}{(f_1^k + f_2^k)\lambda_w^k}\right) \tag{11}
$$

$$
H_{w,i}^k = \left(\frac{H_{1,i}^k}{\lambda_1} - \frac{H_{2,i}^k}{\lambda_2} - \frac{f_1^k F_{1,i}^k + f_2^k F_{2,i}^k}{(f_1^k + f_2^k)\lambda_w^k}\right)
$$
(12)

where  $\phi_{w,i}$  and  $\phi_w^k$  are the receiver and satellite WL UPDs, respectively, and  $H_{w,i}^k$  is the receiver WL IFB.

According to  $(8)$  and  $(12)$ , the receiver WL and NL IFBs depend on both the receivers and satellites involved. The WL and NL IFBs of a particular satellite will be different for each individual receiver, and the original carrier phase IFB can reach 0.73 m for the complete L1 or L2 frequency band (Wanninger [2012](#page-10-0)), while the code IFB can reach 5 m (Yamada et al. [2010](#page-10-0)). The WL and NL IFBs thus vary from  $-0.5$  to  $+0.5$ cycles. This will be illustrated in Fig. [5.](#page-6-0) If the code or carrier phase IFB is not calibrated, neither the UPD determination nor the ambiguity resolution can be achieved (Reussner and Wanninger [2011\)](#page-9-0).

However, if we use receivers whose carrier phase and code IFBs are identical, the WL and NL receiver IFBs for a particular satellite will be the same for all involved receivers and can be absorbed by the satellite UPD estimates. GLONASS PPP ambiguity fixing can therefore be carried out in a straightforward manner. Thus, Eqs. (5) and (9) can be rewritten as

$$
\tilde{N}_{w,i}^k = N_{w,i}^k + \phi_{w,i} - \tilde{\phi}_w^k \tag{13}
$$

$$
\tilde{N}_{c,i}^k = \frac{f_2^k}{f_1^k + f_2^k} \lambda_w^k N_{w,i}^k + \lambda_n^k (N_{n,i}^k + \phi_{n,i} - \tilde{\phi}_n^k)
$$
\n(14)

with

$$
\tilde{\phi}_w^k = \phi_w^k - H_w^k \tag{15}
$$

$$
\tilde{\phi}_n^k = \phi_n^k - H_n^k \tag{16}
$$

Note that the WL and NL receiver IFBs are identical for all receivers of satellite  $k$ , and the subscript  $i$  is thus dropped in (15) and (16).

With the above reformulation, the receiver IFB has no effect on UPD estimation or PPP ambiguity resolution. According to [\(13](#page-2-0)) and (14), GLONASS undifferenced WL and NL UPDs can be generated with the method proposed by Collins et al. [\(2008](#page-9-0)) or that proposed by Laurichesse et al. [\(2009](#page-9-0)).

It is noted that the WL UPD is only used for the WL ambiguity fixing and not to derive the NL ambiguities or to reconstruct the fixed ionospheric-free ambiguities. The WL IFB for a specific satellite might differ slightly among the involved receivers; however, if it does not affect the WL ambiguity fixing, we can neglect this difference. The NL IFB is only composed of the carrier phase IFB, which is identical for all receivers from the same manufacture (Wanninger [2012](#page-10-0)), and we thus ensure that the NL IFB can be absorbed entirely into the satellite NL UPD and that it will not affect PPP ambiguity resolution or precision.

#### Undifferenced PPP ambiguity resolution

A new undifferenced PPP ambiguity resolution scheme is presented in this subsection. With the WL and NL UPD products, the integer nature of PPP ambiguity can be retrieved and then fixed. The rounding strategy (Dong and Bock [1989](#page-9-0); Ge et al. [2008\)](#page-9-0) is adopted to fix the WL ambiguities, while the LAMBDA (Teunissen [1995\)](#page-10-0) method is applied to search for the NL ambiguities with strong correlation.

#### Fixing WL ambiguity

The float WL ambiguity estimate is corrected with the WL satellite UPD product to remove the satellite UPD. The solution reads

$$
\hat{N}_{w,i}^k = \left\langle L_{w,i}^k - P_{w,i}^k - \phi_w^k \right\rangle \tag{17}
$$

and its variance is

$$
\sigma_{\hat{N}_{w,i}^{k}}^{2} = \frac{\left\langle (L_{w,i}^{k} - P_{w,i}^{k} - \phi_{w}^{k} - \hat{N}_{w,i}^{k})^{2} \right\rangle}{R^{k}} + \sigma_{\phi_{w}^{k}}^{2}
$$
(18)

where  $\langle \cdot \rangle$  represents averaging over all involved ambiguities and  $R^k$  is the number of epochs used for averaging.

However, a common receiver WL UPD still exists in each of the WL ambiguities. We can get the receiver WL UPD estimate  $\phi_{w,i}$  by averaging the fractional parts of involved WL ambiguities, which reads

$$
\phi_{w,i} = \left\langle \hat{N}^k_{w,i} - [\hat{N}^k_{w,i}] \right\rangle \tag{19}
$$

and its variance is

$$
\sigma_{\phi_{w,i}}^2 = \frac{\left\langle (\hat{N}_{w,i}^k - [\hat{N}_{w,i}^k] - \phi_{w,i})^2 \right\rangle}{R_i^k}
$$
(20)

where  $[\cdot]$  represents rounding to the nearest integer and  $R_i^k$ denotes the number of ambiguities pertinent to receiver i.The above receiver WL UPD is used to further correct the WL ambiguity estimates and thus retrieve their integer properties:

$$
N_{w,i}^{k} = \hat{N}_{w,i}^{k} - \phi_{w,i}
$$
 (21)

The variance is

$$
\sigma_{N_{w,i}^k}^2 = \sigma_{\hat{N}_{w,i}^k}^2 + \sigma_{\phi_{w,i}}^2 \tag{22}
$$

With the WL ambiguity estimate and variance, the fixing decision can be made according to either the fixing probability calculated by Dong and Bock ([1989](#page-9-0)) or that calculated by Blewitt ([1989\)](#page-9-0).

It is noted that the receiver WL UPD is only needed for fixing the WL ambiguity, rather than for deriving the NL ambiguity, and there is thus no need to estimate it precisely. Furthermore, if the receiver WL UPD is biased, the fixed values for all the WL ambiguities of this receiver will be shifted by a common value. Such a common WL shift will result in a common change in the corresponding NL ambiguities, which can be absorbed by the receiver clock parameter.

#### Fixing NL ambiguity

Having fixed the WL ambiguity, the corresponding float NL ambiguity can be obtained following the principle of (14):

$$
\hat{N}_{n,i}^k = N_{n,i}^k + \phi_{n,i} - \tilde{\phi}_n^k \tag{23}
$$

Its variance depends on the variance of unit-weight and the inversed normal matrix.

The satellite NL UPD product is then applied to remove the satellite UPDs. However, the receiver NL UPD still exists in each NL ambiguity, destroying its integer nature.

Additionally, because the code observations are assigned a much lower weight than the carrier phase observations, when searching the integer ambiguities with the LAMBDA method, the NL ambiguities will be linearly dependent on the receiver clock. In other words, if we denote the correct ambiguity by  $N_0$  and the corresponding clock parameter byc<sub>0</sub>, then with *m* as an integer, any of the parameter set

$$
\{N_0 + m(1 \cdots 1)^T, \quad c_0 - \lambda_n m\}, \quad m \in \mathbb{Z} \tag{24}
$$

is identical to each other.

In order to separate the receiver UPD and receiver clock from the NL ambiguities, the ambiguity  $N$  with the highest satellite elevation is chosen and fixed to its nearest integer  $N<sub>I</sub>$  as the reference. The reference ambiguity can be expressed as an artificial observation:

$$
N = N_I, \quad P_I \in \infty \tag{25}
$$

where  $P_I$  is the weight of this artificial observation.

After applying the ambiguity reference, the receiver NL UPD is assimilated into the receiver clock, keeping the integer nature of all NL ambiguities and removing the linear dependence between the receiver clock and NL ambiguities. It is noted that if the reference ambiguity is biased, which is inevitable because of the poor precision of the float ambiguity, the fixed values for all ambiguities will be shifted by a common integer value. However, with hourly data, the float NL ambiguity should have the precision of a few cycles, and such a small common shift can be absorbed by the receiver clock parameter and will not affect the integer ambiguity resolution or position results.

In this study, the well-known ratio test is used to validate the ambiguity resolution (Han [1997\)](#page-9-0). The ratio is generally defined as the ratio of the second minimum quadratic form of the residuals to the minimum quadratic form of the residuals. It is considered an index of the reliability of ambiguity resolution. In this study, the criterion for the ratio test is selected as 3, which is generally deemed as conservative in ambiguity validation.

## Data processing

The positioning and navigation data analyst (PANDA) software, originally developed at Wuhan University 10 years ago, was used to process GLONASS data. PANDA is a versatile and fundamental platform for scientific studies in China (Liu and Ge [2003;](#page-9-0) Shi et al. [2008](#page-9-0)). The proposed GLONASS UPD estimation approach and undifferenced PPP ambiguity resolution were implemented in PANDA.

The data were collected by the crustal movement observation network of China (CMONOC), which was established for geodetic purposes early in 2000. Currently, the CMONOC comprises approximately 260 stations all equipped with a Trimble NetR8 geodetic dual-frequency GPS/GLONASS receiver and a TRM59800.00 antenna and SCIS radome. We selected 62 stations with the same receiver firmware version, whose C1 and P2 code IFBs are the same, to conduct the UPD estimation and PPP ambiguity resolution. In order to simulate a system providing a PPP service, 50 stations were used as reference stations for the generation of UPDs and the other 12 stations were used as user stations for the PPP ambiguity fixing with UPD products. The distribution of stations is shown in Fig. 1.

Observations made on DOY 1–30, 2012, at sampling intervals of 30 s were used. Because there is no IGS final clock product for GLONASS, the final orbit and clock products of the European Space Agency (ESA) IGS analysis center were used. The elevation cutoff angle was set at  $8^\circ$ , and the carrier phase and code observations were used with elevation-dependent weighting. We applied absolute antenna phase centers, phase windup corrections and station displacement models proposed by the IERS Conventions 2003 (McCarthy and Petit [2003\)](#page-9-0). Receiver clocks were estimated epoch by epoch, zenith tropospheric delays (ZTDs) were estimated as constant every 60 min, and the coordinates of the reference stations were fixed to values estimated from the weekly network solution in post-processing mode using PANDA. Because there is no accurate C1–P1 DCB product available for GLONASS satellites, we used C1 and P2 code observations for all stations.

In order to assess the benefits of ambiguity resolution, we conducted 2-hourly static PPP for the 12 user stations and the 50 reference stations on DOY 1–30, 2012. The same processing approach as that of the reference stations was used, except that the station coordinates were



Fig. 1 Distribution of stations in the CMONOC network used in the present study

estimated without any a priori constraints. To assess the precision and accuracy improvement and thus judge the efficiency of ambiguity fixing, 2-hourly static PPP ambiguity resolution results were compared with the PANDA weekly network solutions. We assumed that the ambiguities can be fixed to the correct integers with daily observations, and the hourly ambiguities were then compared with the daily "truth" to check their correctness.

# UPD estimation results and discussion

In this section, the WL and NL fractional parts of different stations are analyzed and the estimated GLONASS WL and NL UPDs are validated by applying them back to the reference stations. The behavior of WL and NL UPDs is also investigated.

#### WL UPD results

Figure 2 gives the estimated WL UPDs for DOY 1, 2012. The error bars show the standard deviation of the fractional parts of the ambiguities, which is an indicator of the precision of the estimated WL UPDs. The standard deviation is smaller than 0.1 cycles for all satellites and is 0.07 cycles on average. This high formal precision reflects the good consistency of the WL ambiguity fractional parts among the receivers. It is confirmed that, using the proposed strategy, the effects of the code and carrier phase IFBs on the WL UPD estimation can be eliminated. The estimated UPDs have a precision that is good enough for fixing the WL ambiguities.

In order to check the fixing efficiency and validate the UPD estimates, we applied the UPDs back to the 50 reference stations. The fixing results are shown in Fig. 3. For the round-off criterion of 0.15 cycles, the fixing percentage exceeds 75 % for each satellite and averages 91 % over all satellites. For the four satellites with frequency numbers



Fig. 2 Estimated WL UPDs (red) and number of used ambiguities (blue) for all satellites on DOY 1, 2012



Fig. 3 Percentage distribution of the fractional parts of all UPDcorrected WL ambiguities within  $-0.25$  to 0.25 (red) and  $-0.15$  to  $0.15$  (*gray*), and satellite frequency (*blue*)

-7 and 6, the fixing percentage is a little lower than for the other satellites; however, under the criterion of 0.25 cycles (which is usually used as the round-off criterion in WL ambiguity fixing), the fixing percentage exceeds 89 % for each satellite and averages 98 % over all satellites. It can be concluded that the receiver code and carrier phase IFBs are identical for each selected receiver and do not affect the WL UPD estimation or WL ambiguity resolution.

Figure 4 shows the fractional parts of WL ambiguities for different types of receivers after applying WL UPDs. The top panel shows the case for three receivers of different types; the lower subplot shows the case for three receivers with the same configuration as the reference stations. It is found that when the station has the same hardware configuration as the reference stations, the UPDcorrected WL fractional parts are all within  $\pm 0.2$  cycles; however, when the receivers are from different manufactures, the fractional parts fall within  $\pm 0.5$  cycles, making it impossible to fix WL ambiguities.



Fig. 4 Fractional parts of WL ambiguities after UPD correction. The different colors in the bottom panel represent three different Trimble receivers

<span id="page-6-0"></span>Figure 5 gives the WL UPD time series of four typical satellites and the daily variations of the whole constellation from DOY 1–14, 2012. The estimates for the same satellite on different days agree with each other, with the rootmean-square (RMS) differences being better than 0.04 cycles. WL UPDs are therefore stable over time and can be predicted for real-time applications with an update time interval of 1 day or even longer. Satellite R02 revealed a large jump on DOY 6; on that day, "changes in spacecraft" status detected'' were reported on the GLONASS We bsite [\(http://www.glonass-center.ru/en/archive/](http://www.glonass-center.ru/en/archive/)). Subsequently (i.e., on DOY 6, 23, 32, 34, 39 and onwards in 2012), a jump in the satellite WL UPD was found every time there was a change in the spacecraft status. We therefore believe this jump is caused by a change in the spacecraft status.

# NL UPD results

After fixing the WL ambiguity, the NL UPDs can be determined using the same strategy as that used for WL UPDs. The satellites R06, R08, R13, R14, R15, and R19, whose frequency numbers range from  $-7$  to  $+6$ , are selected as typical examples. The estimated NL UPDs are presented in the top panel of Fig. 6, arranged according to the epoch time in 30-s intervals. It is seen that, in general, the fractional parts are not constant but change up to 0.2 cycles in a few hours, yet they can still be tabulated at 5- or 10-min intervals.

In order to further investigate the consistency of different receivers, residuals of the NL ambiguities for the six satellites are shown in the lower panel of Fig. 6. Note that even though the frequency number of these satellites varies from  $-7$  to  $+6$ , which will cause larger carrier phase and



Fig. 5 Stability of the daily WL UPDs for the whole constellation; different colors represent different satellites



Fig. 6 Time series of NL UPDs (top) and the residuals of NL ambiguities (bottom) from the network. Different colors represent different satellites

code IFBs, the fractional parts for different receivers agree with an RMS of better than 0.03 cycles. This implies that the IFB does not affect the NL UPD estimation.

Furthermore, the NL UPDs are applied back to the reference network and Fig. 7 shows the distribution of the fractional parts of all NL ambiguities with fixed WL ambiguities. Generally, more than 91 % of ambiguities are within 0.10 cycles of an integer for each satellite and an average of 97 % of ambiguities are within 0.10 cycles of an integer over all satellites. Meanwhile, more than 97 % of ambiguities are within 0.15 cycles of an integer for each satellite and an average of 99 % of ambiguities are within 0.15 cycles of an integer over all satellites. Note that no frequency-dependent trend of the percentage is observed.



Fig. 7 Percentage distribution of the fractional parts of all NL ambiguities with fixed WL ambiguities after applying the NL UPDs within  $-0.15$  to 0.15 (red) and  $-0.10$  to 0.10 (gray), and satellite frequency (blue)

### <span id="page-7-0"></span>PPP ambiguity resolution results

In order to assess the performance of the GLONASS PPP ambiguity resolution, 12 of the 62 selected stations that were not used in the UPD estimation were used as user stations. Float PPP and PPP with IAR using 2 h of data from DOY 1–30, 2012, were carried out in parallel to demonstrate the contribution of IAR. There were thus generally 360 2-hourly solutions for each station if there was no data loss. Solutions with data of  $\lt 100$  min were removed. The position results were then compared with the weekly solution in post-processing mode using PANDA.

We first processed daily static PPP ambiguity resolution and attempted to fix ambiguities with a tracking arc of over 30 min by rounding (i.e., using a fixing criterion of 0.25 cycles). Figure 8 shows the mean daily fixing percentage for each station; the minimum is 97.2 %, and the mean over all stations is 98.8 %. Figure 9 shows the corresponding RMS positioning error for each station; all values are below 1.6, 1.7, and 4.7 mm for the north, east, and up directions, respectively. According to the previous statistics, we can be sure that the daily ambiguity is correctly fixed with a high percentage and the fixed ambiguity can be used as the ''truth'' in assessing the correctness of hourly ambiguity fixing.

Figure 10 shows the averaged fixing percentage and averaged ratio value of NL LAMBDA ambiguity resolution. The fixing percentage is the number of fixed sessions over the total number of sessions. The fixing percentage of each station exceeds 93.8 %, and the mean fixing rate over all stations is 98.1 %. Moreover, the ratios exceed 13.5 for all stations and averages 21.7, which is much larger than the chosen criterion of 3.

Figure 11 shows the over-session averaged RMS positioning errors of the north, east, and up components for both real-valued and fixed solutions at each station. The mean RMS positioning error improves appreciably from



Fig. 8 Fixing percentage of daily static PPP for each user station



Fig. 9 Position RMS of ambiguity-fixed daily static PPP for each station in north (circle), east (diamond), and up (square) directions



Fig. 10 Averaged fixing percentage (gray) and NL LAMBDA fixing ratio (red) over all sessions for each station



Fig. 11 RMS positioning errors in the north, east, and up directions for real-valued and fixed solutions compared with PANDA weekly solutions

(0.59, 1.30, 1.45) centimeters to (0.32, 0.33, 1.33) centimeters in north, east, and up directions, respectively. After ambiguity fixing, the horizontal accuracies of all stations are below sub-centimeter level. The east accuracy has the greatest improvement of 72.9 %, while the north accuracy improves by 44.9 %; the up accuracies of all stations are better than 1.8 cm and have a mean improvement of 6.8 %. Since the satellites move more quickly in the northward direction than in the eastward direction, for a short period of 2 h, the east direction has stronger correlation with the ambiguities and has a larger improvement. The smaller improvement in the vertical direction compared with the horizontal directions is due to the smaller correlation with the ambiguities (Blewitt [1989\)](#page-9-0).

Figure [11](#page-7-0) shows that, for two stations, the accuracy of the up component is reduced when compared with the ambiguity-float solutions. However, the accuracy of the north and east components improved for all 12 stations. We believe that this degradation of the up component is mainly due to the poor estimation of ZTD parameters. In order to verify this conjecture, we constrained the hourly ZTD to the accurate values derived from the daily ambiguity-fixed solution; as shown in Fig. 12, the 2-hourly ambiguity-fixed RMS for the up component improved appreciable to be less than 0.7 cm for each station, having an average value of 0.6 cm. Geng et al. ([2009\)](#page-9-0) also noticed this phenomenon in GPS hourly PPP ambiguity resolution. It is noted that this approach is not feasible for real-time processing because precise ZTDs are not available.

Ambiguity fixing was also conducted for all 50 reference stations to obtain real-valued and fixed solutions. Similar results were obtained; the average fixing percentage and LAMBDA ratio are 98.3 and 23.2 %, respectively, while the position RMS improved from (0.66, 1.42, 1.55) cm to (0.38, 0.39, 1.39) cm, as shown in Fig. 13.



Fig. 12 RMS positioning errors in north (square), east (diamond), and up (circle), for fixed solutions with a tropospheric constraint



Fig. 13 Averaged RMS positioning errors for float solutions (gray) and fixed solutions (red) for the 50 reference stations

After ambiguity fixing, for all 50 reference stations, the RMS positioning errors improve for the east and north components; meanwhile, the RMS positioning errors of the up component improve only for 36 stations (69.2 % of stations). If the hourly ZTD is not estimated in the fixed solutions but fixed to the accurate values derived from daily estimates, the up component improves for all 50 stations.

These statistics demonstrate that the GLONASS PPP ambiguity resolution applied in this study appreciably improves the accuracy of 2-hourly static position estimates, especially the horizontal estimates.

# **Conclusions**

The presented study solved the GLONASS PPP ambiguity fixing problem. A network of 62 stations with homogeneous GLONASS code and carrier phase IFBs was used to conduct GLONASS UPD estimation and PPP ambiguity resolution. Fifty stations were used to generate satellite UPDs and the other 12 to perform PPP ambiguity resolution. As GLONASS satellites operate at distinct wavelengths, differencing between satellites should be avoided, and an approach for estimating undifferenced UPDs was adopted and a strategy for undifferenced PPP ambiguity resolution was proposed.

It was demonstrated that, using the proposed strategy, the effect of the code and carrier phase IFBs can be eliminated such that they do not affect the WL or NL UPD estimation. When applying the UPDs back to the reference stations, the WL ambiguities can be fixed with a mean percentage of 97 % under the criterion of 0.25 cycles and with a mean percentage of 99 % under the criterion of 0.15 cycles for the NL ambiguities. The estimated UPDs have a precision that is good enough for fixing the WL and NL

<span id="page-9-0"></span>ambiguities. This study also demonstrated that with only 2-hour GLONASS observations, PPP ambiguity resolution can improve the positioning accuracy appreciably from (0.66, 1.42, 1.55) cm to (0.38, 0.39, 1.39) cm in the north, east, and up directions, respectively.

The presented method is only applicable to receivers with similar code inter-frequency bias, which might limit its application to existing global navigation satellite system (GNSS) networks. For engineering applications, however, it is possible to select homogeneous receivers in advance and thus fulfill the ambiguity resolution requirement. Commercial companies providing precise positioning services can also use homogeneous receivers to establish their reference GNSS networks and to provide positioning services with the same type of receivers.

In order to ensure that GLONASS PPP ambiguity resolution is not limited to certain receiver types, we recommend that receiver manufacturers force the GLONASS code and carrier phase IFBs to be identical for receivers of the same type, so as to establish an IFB calibration model for each type of receiver using zero baselines. It is expected to fix the ambiguity of GPS+GLONASS observations in real-time kinematic PPP and thus shorten the convergence time and improve the position accuracy, which is the next goal of the authors.

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