

GNSS multi-carrier fast partial ambiguity resolution strategy tested with real BDS/GPS dual- and triple-frequency observations

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Received: 19 August 2013 / Accepted: 14 December 2013 / Published online: 31 December 2013
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Abstract The regional constellation of BeiDou navigation satellite system (BDS) has been officially in operation since December 27, 2012, and real-time kinematic positioning using BDS and GPS multi-frequency observations is feasible. A heavy computational problem arises when resolving ambiguities in the case of multi-system with multi-frequency observations. A multi-carrier fast partial ambiguity resolution strategy is developed with the property that the extra-wide-lane and wide-lane ambiguities in the multi-frequency case can be resolved reliably in advance. Consequently, the technique resolves ambiguities sequentially instead of the usual batch ambiguity resolution (AR) mode so as to improve the computational efficiency of AR significantly. The strategy is demonstrated with real BDS/GPS dual- and triple-frequency observations. The results have shown that the probability of correct AR by the proposed method is comparable to that of the batch AR. Experimentally, the new method is about 2.5 times as fast as the batch AR in the dual-frequency case, 3 times in the mixed dual- and triple-frequency case and 3.5 times in the triple-frequency case.

Keywords Global navigation satellite systems (GNSS) · BeiDou navigation satellite system (BDS) · Multi-frequency · Computational efficiency · Partial ambiguity resolution

Introduction

In addition to the existing GPS and GLONASS, BeiDou navigation satellite system (BDS) has officially announced its regional operation on December 27, 2012, and it is transmitting navigation signals at three L-band frequencies 1,561.098 MHz (B1), 1,268.52 MHz (B3) and 1,207.14 MHz (B2). Therefore, the multi-system multi-frequency RTK positioning is now feasible, which can enhance the geometric strength of the Global navigation satellite systems (GNSS) model, advance the probability of correct ambiguity estimation and further improve the accuracy, availability and reliability of the RTK solution (Feng 2008; Li et al. 2010, 2013; Yang et al. 2011; He et al. 2013).

The key of the RTK positioning is the rapid and reliable determination of integer ambiguities. In the triple-frequency case, the geometry-free three carrier ambiguity resolution (TCAR) method (Forsell et al. 1997) and Cascade Integer Resolution (CIR) (Jung 1999; Hatch et al. 2000) method can resolve ambiguities efficiently. However, these methods are very sensitive to the observation noises (mainly the multipath errors) and the ionospheric delay errors (Vollath et al. 1998; Ji et al. 2007). Hence, the geometry-based ambiguity resolution (AR) by the integer least-squares estimation is still the preferable choice even in the multi-system multi-frequency case, since it can maximize the probability of correct ambiguity estimation (Teunissen 1999). Though the reliable AR in the multi-

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system multi-frequency case is no longer a troublesome issue (Tiberius et al. 2002), the heavy computational problem arises as a result of the vast unknown estimation and high-dimension AR when resolving the integer ambiguities with a geometry-based batch AR model (Vollath 2004; Feng 2008).

It is well known that the computation burden of the geometry-based AR mainly exists at two stages: the float solution estimation (ordinary least-square estimation) and integer ambiguity resolution (integer least-square estimation). Vollath (2004) presented the Factorized Multi-Carrier Ambiguity Resolution (FAMCAR) approach to fast estimate the float ambiguities by separating the geometry and geometry-free information in the floating filter process. Feng (2008) proposed a geometry-based TCAR strategy to reduce the workload of the integer search process by using the ionosphere-reduced combination with minimized total noise in cycles. However, its parameter estimation inevitably suffers from the amplified effects of the carrier-phase noises and multipath errors, since the combined carrier-phase measurements are used instead of the original ones. The geometry-based batch AR under the integer least-square criterion on the level of original ambiguities can be efficiently achieved by the LAMBDA method (Teunissen 1995). Some research has also demonstrated that it is still efficient even in case of high-dimensional ambiguities (Li and Teunissen 2011; Jazaeri et al. 2013). However, the consumed time of batch AR by the LAMBDA method and its improved versions increases quickly with the number of ambiguities (Chang et al. 2005; Jazaeri et al. 2012). As a result, more attention should be paid to the computational efficiency of high-dimension AR in the multi-system multi-frequency case, since it is very crucial for the initialization speed and high output rate of RTK positioning. Furthermore, a more efficient algorithm is always significant to reduce power consumption, save battery power and cut the receiver cost. On the other hand, even when the multi-system multi-frequency observations are used, it is still possible that the full set of ambiguities cannot be fixed reliably due to high noise levels and/or biases in the observations. In those cases, the partial ambiguity resolution (PAR) may be useful, which means that a subset of the ambiguities is selected to be resolved (Teunissen et al. 1999). However, how to identify the optimal ambiguity subset is still an open issue (Teunissen and Verhagen 2009a).

We here presented a multi-carrier fast partial ambiguity resolution strategy to address the computational efficiency problem and partial AR. Then its validity was demonstrated with the real BDS/GPS dual- and triple-frequency data. In this contribution, the methodology of the new method is presented first, followed by three short baseline tests. In the experiments, the success rate and

computational efficiency of single-epoch AR by the new method are evaluated and compared with those of the usual batch AR. Finally, some conclusions and remarks are presented.

Methodology

Assuming that the so-called float solution of the multi-frequency GNSS model together with its variance–covariance (vc) matrix is denoted as

$$\begin{bmatrix} \hat{\mathbf{a}} \\ \hat{\mathbf{b}} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \mathbf{Q}_{\hat{\mathbf{a}}} & \mathbf{Q}_{\hat{\mathbf{a}}\hat{\mathbf{b}}} \\ \mathbf{Q}_{\hat{\mathbf{b}}\hat{\mathbf{a}}} & \mathbf{Q}_{\hat{\mathbf{b}}} \end{bmatrix} \quad (1)$$

where $\hat{\mathbf{a}}$ is the float solution of the integer ambiguity vector, including all original DD ambiguities in all frequencies; $\hat{\mathbf{b}}$ is the float solution of the real parameter vector, including the baseline components (coordinates) and possibly troposphere and ionosphere delay parameters. For the widely used LAMBDA method and its various improved versions (Teunissen 1995; Chang et al. 2005; Jazaeri et al. 2012), the integer ambiguities are resolved efficiently in batch mode by the following minimum problem

$$\tilde{\mathbf{z}} = \min_{\mathbf{z} \in \mathbb{Z}} (\hat{\mathbf{z}} - \mathbf{z})^T \mathbf{Q}_{\hat{\mathbf{z}}}^{-1} (\hat{\mathbf{z}} - \mathbf{z}) \quad (2)$$

where $\hat{\mathbf{z}} = \mathbf{Z}\hat{\mathbf{a}}$, $\mathbf{Q}_{\hat{\mathbf{z}}} = \mathbf{Z}\mathbf{Q}_{\hat{\mathbf{a}}}\mathbf{Z}^T$, \mathbf{Z} denote the so-called decorrelation transform matrix, \mathbb{Z} indicates the set of integers. Then the fixed solution of the real parameter vector and its vc matrix are obtained as (Teunissen 1995)

$$\tilde{\mathbf{b}} = \hat{\mathbf{b}} - \mathbf{Q}_{\hat{\mathbf{b}}\hat{\mathbf{z}}}\mathbf{Q}_{\hat{\mathbf{z}}}^{-1} (\hat{\mathbf{z}} - \tilde{\mathbf{z}}) \quad \text{and} \quad \mathbf{Q}_{\tilde{\mathbf{b}}} = \mathbf{Q}_{\hat{\mathbf{b}}} - \mathbf{Q}_{\hat{\mathbf{b}}\hat{\mathbf{z}}}\mathbf{Q}_{\hat{\mathbf{z}}}^{-1}\mathbf{Q}_{\hat{\mathbf{z}}\hat{\mathbf{b}}} \quad (3)$$

where $\mathbf{Q}_{\hat{\mathbf{b}}\hat{\mathbf{z}}} = \mathbf{Q}_{\hat{\mathbf{b}}\hat{\mathbf{a}}}\mathbf{Z}^T$. If we set the decorrelation matrix \mathbf{Z} as $\begin{bmatrix} \mathbf{Z}_1 \\ \mathbf{Z}_2 \end{bmatrix}$, then the minimum problem (2) can be expressed by the following equivalent problem

$$\begin{aligned} \tilde{\mathbf{z}} = \begin{bmatrix} \tilde{\mathbf{z}}_1 \\ \tilde{\mathbf{z}}_2 \end{bmatrix} &= \min_{\mathbf{z}_1, \mathbf{z}_2 \in \mathbb{Z}} \begin{bmatrix} \hat{\mathbf{z}}_1 - \mathbf{z}_1 \\ \hat{\mathbf{z}}_2 - \mathbf{z}_2 \end{bmatrix}^T \begin{bmatrix} \mathbf{Q}_{\hat{\mathbf{z}}_1} & \mathbf{Q}_{\hat{\mathbf{z}}_1\hat{\mathbf{z}}_2} \\ \mathbf{Q}_{\hat{\mathbf{z}}_2\hat{\mathbf{z}}_1} & \mathbf{Q}_{\hat{\mathbf{z}}_2} \end{bmatrix}^{-1} \begin{bmatrix} \hat{\mathbf{z}}_1 - \mathbf{z}_1 \\ \hat{\mathbf{z}}_2 - \mathbf{z}_2 \end{bmatrix} \\ &= \min_{\mathbf{z}_1, \mathbf{z}_2 \in \mathbb{Z}} \left[(\hat{\mathbf{z}}_1 - \mathbf{z}_1)^T \mathbf{Q}_{\hat{\mathbf{z}}_1}^{-1} (\hat{\mathbf{z}}_1 - \mathbf{z}_1) + (\hat{\mathbf{z}}_{2/1} - \mathbf{z}_2)^T \mathbf{Q}_{\hat{\mathbf{z}}_{2/1}}^{-1} (\hat{\mathbf{z}}_{2/1} - \mathbf{z}_2) \right] \end{aligned} \quad (4)$$

where $\hat{\mathbf{z}}_1 = \mathbf{Z}_1\hat{\mathbf{a}}$, $\hat{\mathbf{z}}_2 = \mathbf{Z}_2\hat{\mathbf{a}}$, $\mathbf{Q}_{\hat{\mathbf{z}}_1} = \mathbf{Z}_1\mathbf{Q}_{\hat{\mathbf{a}}}\mathbf{Z}_1^T$, $\mathbf{Q}_{\hat{\mathbf{z}}_2} = \mathbf{Z}_2\mathbf{Q}_{\hat{\mathbf{a}}}\mathbf{Z}_2^T$, $\mathbf{Q}_{\hat{\mathbf{z}}_1\hat{\mathbf{z}}_2} = \mathbf{Q}_{\hat{\mathbf{z}}_2\hat{\mathbf{z}}_1}^T = \mathbf{Z}_1\mathbf{Q}_{\hat{\mathbf{a}}}\mathbf{Z}_2^T$, $\hat{\mathbf{z}}_{2/1} = \hat{\mathbf{z}}_2 - \mathbf{Q}_{\hat{\mathbf{z}}_2\hat{\mathbf{z}}_1}\mathbf{Q}_{\hat{\mathbf{z}}_1}^{-1}(\hat{\mathbf{z}}_1 - \mathbf{z}_1)$, $\mathbf{Q}_{\hat{\mathbf{z}}_{2/1}} = \mathbf{Q}_{\hat{\mathbf{z}}_2} - \mathbf{Q}_{\hat{\mathbf{z}}_2\hat{\mathbf{z}}_1}\mathbf{Q}_{\hat{\mathbf{z}}_1}^{-1}\mathbf{Q}_{\hat{\mathbf{z}}_1\hat{\mathbf{z}}_2}$. It is well known that $\hat{\mathbf{z}}_{2/1}$ and $\mathbf{Q}_{\hat{\mathbf{z}}_{2/1}}$ are the conditional estimate for the ambiguity subset $\hat{\mathbf{z}}_2$ along with the corresponding vc matrix. From the formula (4), we can deduce that:

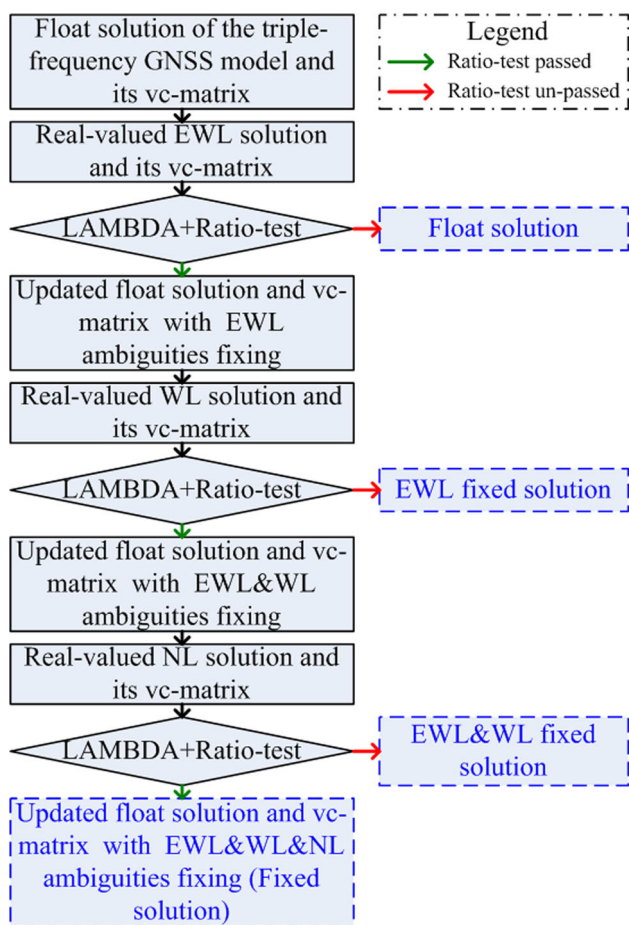


Fig. 1 Flowchart of the triple-carrier fast partial ambiguity resolution strategy (NL Narrow-lane)

(I) If only the ambiguity subset \hat{z}_1 is solved to their integer value, namely discarding the integer constraint $z_2 \in \mathbb{Z}$, the second term on the right-hand side of the formula (4) can be made zero for any z_1 by setting $z_2 = \hat{z}_{2/1}$. Consequently, the full AR problem (4) is replaced by the following partial AR problem

$$\hat{z}_1 = \min_{z_1 \in \mathbb{Z}} (\hat{z}_1 - z_1)^T Q_{\hat{z}_1}^{-1} (\hat{z}_1 - z_1) \quad (5)$$

However, how to identify the optimal ambiguity subset is not yet solved (Teunissen and Verhagen 2009a). Though it is possible to identify the optimal subset by trying all possible subsets with a proper criterion, this approach is computationally heavy, especially for the multi-system multi-frequency case. In the multi-frequency case, it is very easy to resolve the Extra-Wide-Lane (EWL) and Wide-Lane (WL) ambiguities reliably, which suffers relatively low effects from the multipath errors and atmosphere delay biases. Therefore, it is reasonable to select \hat{z}_1 as the subset consisting of all EWL or WL ambiguities.

(II) It is possible to resolve the ambiguity subsets \hat{z}_1 and \hat{z}_2 sequentially when the corresponding success rate

is very close to that of the batch AR, namely the subset \hat{z}_1 is resolved by the minimum problem (5) first, and then the subset \hat{z}_2 is resolved by the following minimum problem

$$\tilde{z}_2 = \min_{z_2 \in \mathbb{Z}} (\hat{z}_{2/1} - z_2)^T Q_{\hat{z}_{2/1}}^{-1} (\hat{z}_{2/1} - z_2) \quad (6)$$

where $\hat{z}_{2/1} = \hat{z}_2 - Q_{\hat{z}_2} Q_{\hat{z}_1}^{-1} (\hat{z}_1 - \tilde{z}_1)$. When $\hat{z} \sim N(z, Q_{\hat{z}})$, it is well known that resolving ambiguity subsets \hat{z}_1 and \hat{z}_2 in batch model by the formulae (2) or (4) can maximize the probability of correct integer estimation (Teunissen 1999). Hence, the success rate of resolving the subsets \hat{z}_1 and \hat{z}_2 sequentially by the formulae (5) and (6) will be lower than that of resolving them in batch model by (2) or (4). From the formulae (4–6), it is evident that if the integer solution \tilde{z}_1 from (5) is identical with the integer subset \tilde{z}_1 from (4), the integer solution \tilde{z}_2 from (6) will be exactly identical with the integer subset \tilde{z}_2 . Therefore, if the success rate of resolving the ambiguity subset \hat{z}_1 by the minimum problem (5) is sufficiently close to 1, for instance when the subset \hat{z}_1 consists of all EWL or WL ambiguities, the sequential AR success rate will be very close to that of the batch AR, namely the maximum success rate. Besides, the sequential AR is computationally more efficient than the batch AR in virtue of lower dimension of the ambiguity vector, and this merit can be exploited to improve the efficiency of AR in the multi-system multi-frequency case.

Considering (4–6), we presented a multi-carrier fast partial ambiguity resolution (MCFPAR) strategy, taking the triple-frequency case as example, whose flowchart is given in Fig. 1.

From Fig. 1, we know that the outputs of the MCFPAR method in the triple-frequency case are four types of solutions: the float solution, the EWL-fixed solution, the EWL&WL-fixed solution and the fixed solution. In this study, the EWL ambiguities are composed of the two closest carriers, for instance, L2 and L5 for GPS, B3 and B2 for BDS; the WL ambiguities are composed of the two other carriers, namely L1 and L2 for GPS, B1 and B3 for BDS; and the last NL ambiguities are the L1 and B1 ambiguities for GPS and BDS, respectively. Though the MCFPAR procedure is similar to integer bootstrapping method (Teunissen 1998, 1999), it resolves each ambiguity subsets by combining the LAMBDA method and the ratio test (Euler and Schaffrin 1991) rather than the simple integer rounding as the integer bootstrapping. Therefore, the MCFPAR method also benefits from the various improvements to the LAMBDA method and the better validation procedure, for example, the fixed failure rate ratio test (Teunissen 2004, 2005; Teunissen and Verhagen 2009b; Verhagen and Teunissen 2013), so as to improve its efficiency and reliability accordingly.

Experimental validation

A single-epoch multi-system RTK model based on the original multi-frequency double-differenced (DD) observations was adopted to obtain the float ambiguity vector and its variance–covariance matrix in each epoch (Li et al. 2013). Then the presented MCFPAR strategy was compared with the usual batch AR procedure during the AR processing. The MLAMBDA algorithm (Chang et al. 2005) implemented with C language was applied to solve real-valued ambiguities to their integer values in both two schemes, and the solved integer ambiguities were validated by the ratio test with the fixed threshold value of 2. The success rate (the probability of correct fixing) and the failure rate (the probability of incorrect fixing) were computed to compare the AR performance of two schemes. The reliable integer-valued ambiguities determined by using all data were used to decide whether the single-epoch fixed ambiguities have been resolved correctly. The consumed CPU time of AR processing (all the steps from float to fixed solution) in each epoch was also evaluated to compare the computational efficiency of two schemes. The cut-off elevation of 10° was set in all data processing. Both the DD tropospheric and ionospheric delays were assumed to be small enough to be ignored in this study since the

longest baseline is only 8 km. All results presented in this section were performed on a PC, 2.8 GHz with 3.25 GB memory running Windows XP professional.

Three BDS/GPS dual- and triple-frequency data sets were collected with the receivers produced by Beijing Unicore Communications Inc. and Shanghai Compass Navigation Co. Ltd. The details of the three data sets are given in Table 1. The success and failure rate of single-epoch AR are presented in Tables 2, 3 and 4. The consumed CPU time of AR processing in each epoch is plotted in Figs. 2, 3, 4 and 5, and their average and 95 % percentile values are given in Table 5.

From Table 2, we know that the average success rate of single-epoch WL AR for three data sets are 97.89, 97.01 and 100 % for GPS, BDS and BDS/GPS, respectively, and the corresponding failure rates are 0.05, 0.008 and 0 %. Therefore, even though in the dual-frequency case, the success rate of the WL AR is sufficiently high to satisfy the prerequisite of the presented MCFPAR method, especially when combining the BDS and GPS.

It is shown in Table 3 that for GPS, the average success rates of three data sets are 98.9 % by the batch AR scheme and 97.78 % by the MCFPAR scheme, and the relevant failure rates are 0 and 0.001 %. For BDS, the average success rates are 96.19 and 96.54 %, and the relevant

Table 1 Detail of the BDS/GPS dual- and triple-frequency data sets

Data set	Date	Length (km)	Interval (s)	Total number of epochs	Frequencies	Manufacturer
A	2012-09-02	8.2	5	17,280	L1/2 + B1/2	Beijing Unicore
B	2013-04-07 ~ 09	6.0	5	33,696	L1/2 + B1/2	Beijing Unicore
C	2013-04-07	6.0	1	54,557	L1/2 + B1/2/3	Shanghai ComNav

Table 2 Success and failure (in parenthesis) percentage of single-epoch WL AR in the dual-frequency case

	GPS (L1/2)	BDS (B1/2)	BDS/GPS (L1/2 + B1/2)
A	98.79 (0.006)	92.43 (0.023)	100 (0)
B	98.41 (0.033)	99.03 (0)	100 (0)
C	96.46 (0.11)	99.58 (0)	100 (0)
Average	97.89 (0.05)	97.01 (0.008)	100 (0)

Table 3 Success and failure (in parenthesis) percentage of single-epoch full AR in the dual-frequency case

	GPS (L1/2)		BDS (B1/2)		BDS/GPS (L1/2 + B1/2)	
	Batch AR	MCFPAR	Batch AR	MCFPAR	Batch AR	MCFPAR
A	98.21 (0)	98.53 (0)	90.20 (0)	91.22 (0.017)	94.11 (0)	99.01 (0)
B	99.20 (0)	98.39 (0)	99.67 (0)	99.02 (0)	99.23 (0)	99.99 (0)
C	99.29 (0)	96.46 (0.002)	98.70 (0)	99.58 (0)	99.79 (0)	100 (0)
Average	98.9 (0)	97.78 (0.001)	96.19 (0)	96.54 (0.006)	97.71 (0)	99.67 (0)

Table 4 Success and failure (in parenthesis) percentage of single-epoch AR in the triple-frequency case (data set C)

	BDS (B1/2/3)		BDS/GPS (L1/2 + B1/2/3)	
	Batch AR	MCFPAR	Batch AR	MCFPAR
EWL	–	100 (0)	–	100 (0)
WL	–	100 (0)	–	100 (0)
Full	99.90 (0)	100 (0)	98.02 (0)	100 (0)

failure rates are 0 and 0.006 %. For BDS/GPS, the average success rates are 97.71 and 99.67 %, respectively, and both failure rates are 0 %. Consequently, it is evident that single-epoch AR by the MCFPAR method is comparable to that by the batch AR scheme in the sense of AR success rate, in particular, under multi-GNSS condition, such as the combined BDS and GPS. In addition, the success rates of full AR by the MCFPAR scheme shown in Table 3 are smaller than those of the corresponding WL AR as shown in Table 2, and the difference between these two success rates is the percentage of WL-fixed solutions as a result of the MCFPAR scheme. It can be seen that the difference is very small, since the NL (L1/B1) AR is very reliable when the WL ambiguities have been resolved correctly beforehand. Though the incorrect WL ambiguities result necessarily in incorrect NL ambiguities, the resulting incorrect NL integer ambiguities can barely pass the ratio test. As a result, the failure rate of full AR by the MCFPAR scheme shown in Table 3 is very low and even lower than those of WL AR as shown in Table 2.

From Table 4, it is known that the success rates of EWL and WL AR are 100 %, and their relevant failure rates are

0 %. The success rates of full AR by the batch AR scheme are 99.90 and 98.02 % for triple-frequency BDS and mixed dual- and triple-frequency BDS/GPS, and both are 100 % by the MCFPAR scheme. Accordingly, it is evident that AR by the MCFPAR scheme will be more reliable in the triple-frequency case.

In Figs. 2–4, it is shown that the consumed CPU time of single-epoch AR is correlative with the dimension of the float ambiguity vector; the larger the dimension the longer the time. For the dual-frequency GPS or BDS case, the maximal dimensions of the float ambiguity vector are 22 (Fig. 3, middle), and it is 40 for the dual-frequency BDS/GPS case (Fig. 3, bottom), 27 for the triple-frequency BDS case and 45 for mixed dual- and triple-frequency BDS/GPS case seen in Fig. 4. Except for a very few epochs, the consumed CPU time of single-epoch AR by the MCFPAR scheme is significantly smaller than that by the batch AR scheme. Three independent runs for both schemes are performed to demonstrate that these exceptional epochs are not caused by the algorithms themselves. The results of GPS dual-frequency AR for the data set A are plotted in Fig. 5. It is shown that these exceptional epochs are different in 3 independent runs for both schemes. Therefore, we can infer that these exceptional epochs may result from the fact that the CPU resource cannot be occupied exclusively for the AR process since it runs on a multitask operating system. Table 5 shows that on average over three independent runs, the MCFPAR scheme is about 2.5 times as fast as the batch AR scheme in the dual-frequency case, 3 times in the mixed dual- and triple-frequency case and 3.5 times in the triple-frequency case.

Table 5 Average and 95 % percentile value of single-epoch AR consumed time (Unit: ms)

Tests	Average value			95 % percentile value			
	Batch AR	MCFPAR	Improved	Batch AR	MCFPAR	Improved	
A	L1/2	0.262	0.107	2.45	0.406	0.159	2.55
	B1/2	0.253	0.098	2.58	0.378	0.136	2.78
	L1/2 + B1/2	1.156	0.470	2.46	1.543	0.621	2.48
B	L1/2	0.254	0.104	2.44	0.398	0.159	2.50
	B1/2	0.358	0.133	2.69	0.565	0.209	2.70
	L1/2 + B1/2	1.316	0.559	2.35	1.852	0.782	2.37
C	L1/2	0.254	0.104	2.44	0.443	0.168	2.64
	B1/2	0.326	0.119	2.74	0.473	0.161	2.94
	L1/2 + B1/2	1.220	0.500	2.44	1.769	0.732	2.42
	L1/2 + B1/2/3	1.896	0.649	2.92	2.831	0.928	3.05
	B1/2/3	0.670	0.199	3.37	0.968	0.272	3.56

Fig. 2 Consumed CPU time of single-epoch AR in each epoch for the data set A

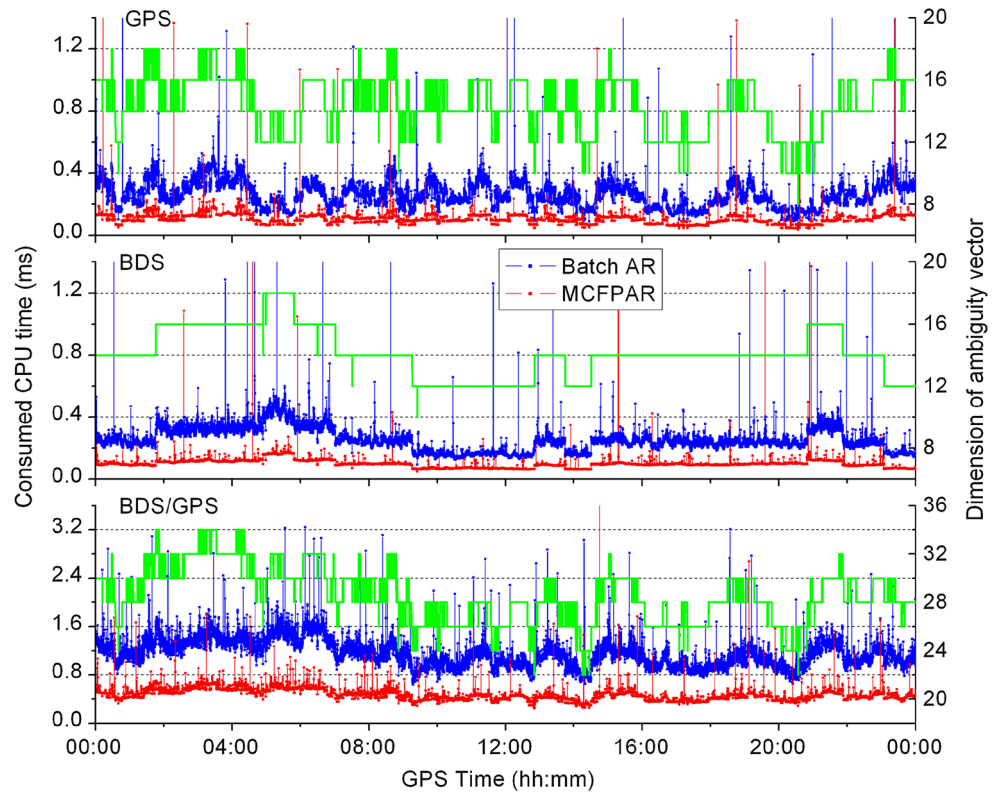


Fig. 3 Consumed CPU time of single-epoch AR in each epoch for the data set B

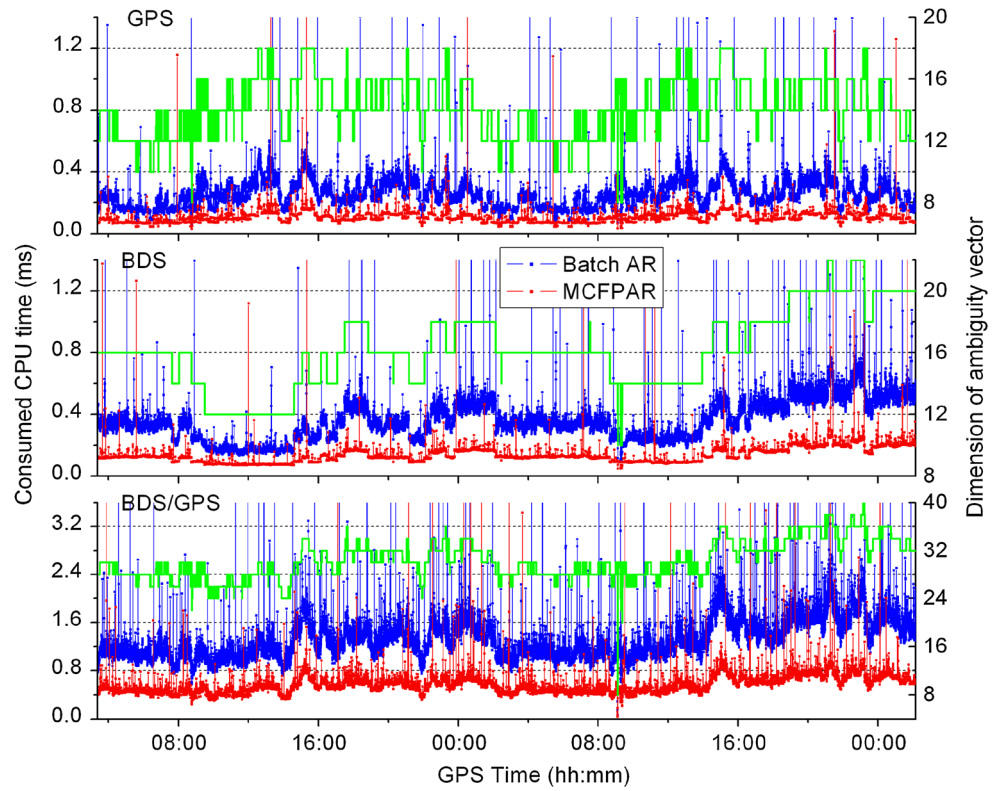
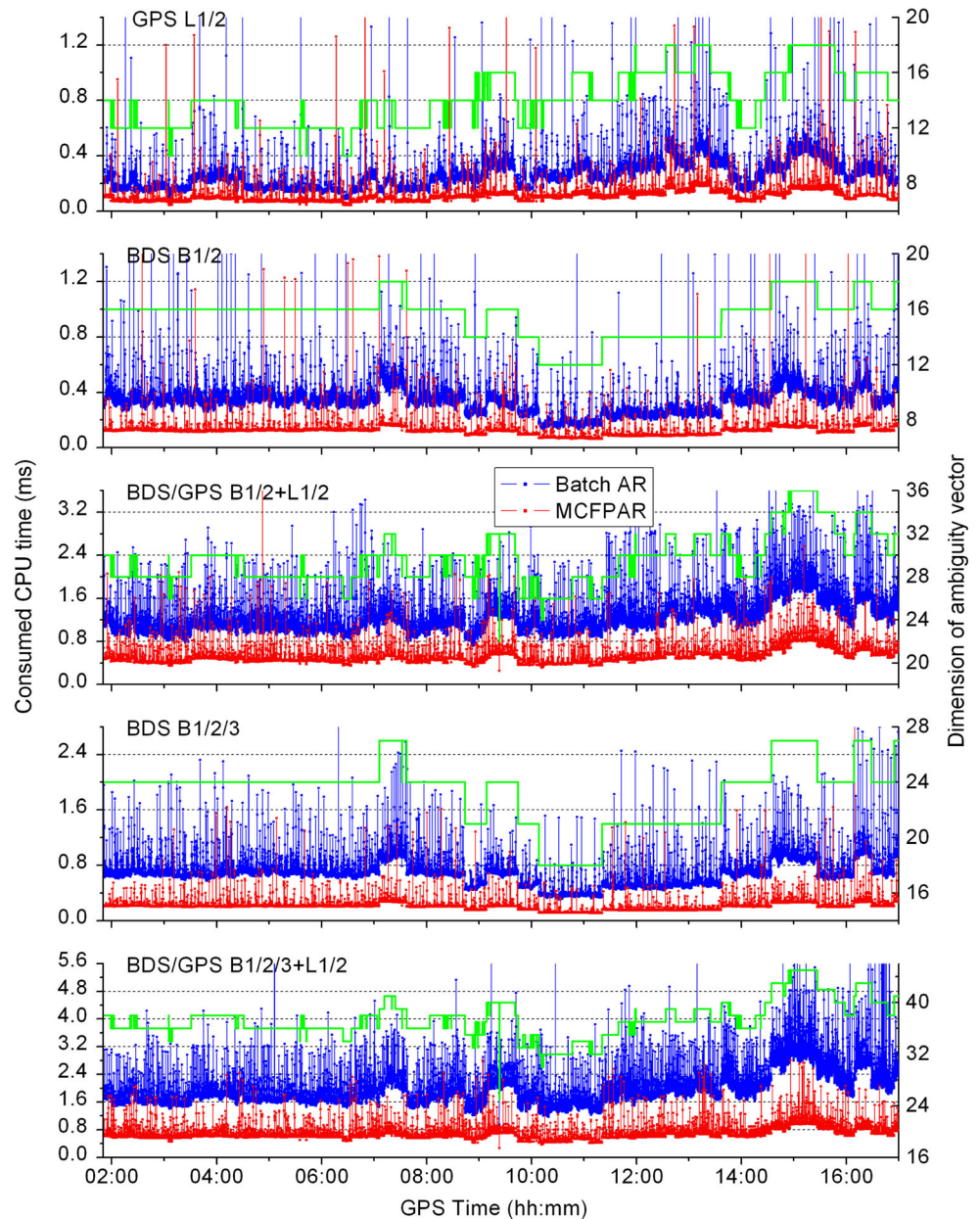


Fig. 4 Consumed CPU time of single-epoch AR in each epoch for the data set C



Conclusions and remarks

The RTK positioning using multi-system multi-frequency observations can enhance the geometric strength of the GNSS model and improve the probability of correct ambiguity estimation. However, it also brings the heavy computational problem due to the resulting high-dimension AR. We presented a multi-carrier fast partial ambiguity resolution strategy to address this problem, and its validity was demonstrated with real BDS/GPS dual- and triple-frequency observations.

From the test results, it can be concluded that even in the dual-frequency case, the success rate of the WL AR is sufficiently high to satisfy the prerequisite of the MCFPAR method, especially when combining the BDS and GPS. As a whole, the success rate of single-epoch full AR by the MCFPAR method is comparable to the optimal one obtained by the usual batch AR procedure. More significantly, the numerical tests have shown that the MCFPAR method is about 2.5 times as fast as the batch AR for the dual-frequency case, 3 times for the mixed dual- and triple-frequency case and 3.5 times for

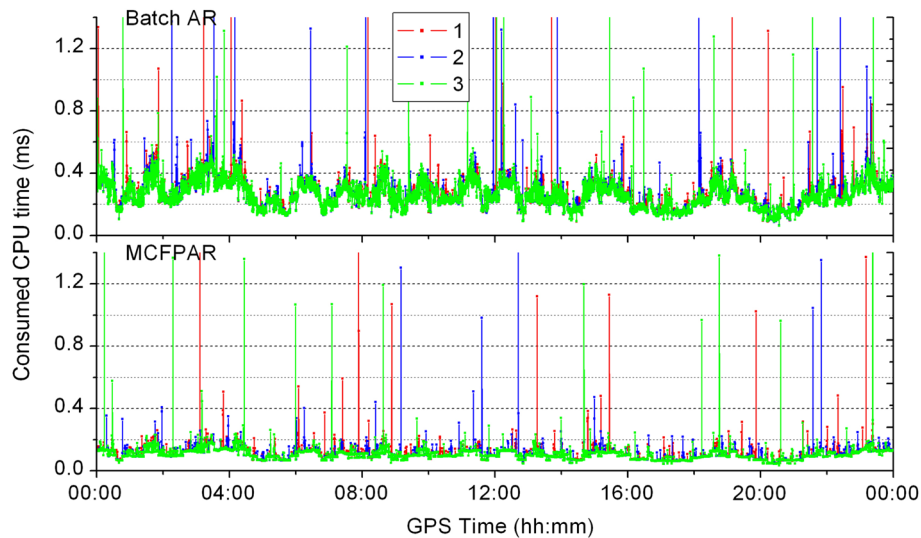


Fig. 5 Consumed CPU time of single-epoch GPS dual-frequency AR for the data set A with 3 independent runs

the triple-frequency case. Furthermore, the MCFPAR method also benefits from various improvements to the LAMBDA method and the better validation procedure so as to improve its efficiency and reliability accordingly.

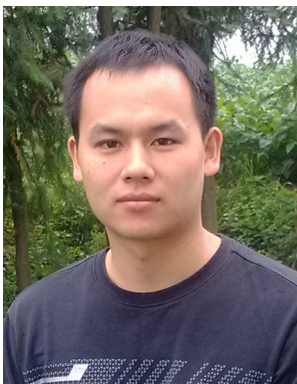
Further research into the performance of the presented algorithms for the long baseline GNSS model with estimating atmosphere bias parameter is ongoing, since the adaptive PAR capability of the MCFPAR method may be especially helpful to reduce the time to first fix when the underlying GNSS model lacks sufficient strength.

Acknowledgments This work is funded by the National Natural Science Funds of China (Grant Nos. 41020144004; 41374019; 41104022), the National “863 Program” of China (Grant No: 2013AA122501) and the 3rd China Satellite Navigation Conference (Grant No. CSNC2012-QY-3). The authors also appreciate two anonymous reviewers for their valuable comments and improvements to this manuscript.

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