GPS Solut (2014) 18:519–528
DOI 10.1007/s10291-013-0348-2

ORIGINAL ARTICLE

A comparison of three PPP integer ambiguity resolution methods

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Received: 29 January 2013 / Accepted: 17 October 2013 / Published online: 31 October 2013
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Abstract Precise point positioning (PPP) integer ambiguity resolution with a single receiver can be achieved using advanced satellite augmentation corrections. Several PPP integer ambiguity resolution methods have been developed, which include the decoupled clock model, the single-difference between-satellites model, and the integer phase clock model. Although similar positioning performances have been demonstrated, very few efforts have been made to explore the relationship between those methods. Our aim is to compare the three PPP integer ambiguity resolution methods for their equivalence. First, several assumptions made in previous publications are clarified. A comprehensive comparison is then conducted using three criteria: the integer property recovery, the system redundancy, and the necessary corrections through which the equivalence of these three PPP integer ambiguity resolution methods in the user solution is obtained.

Keywords PPP integer ambiguity resolution · Method equivalence · Single-difference between-satellites method · Decoupled clock model · Integer phase clock model

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Introduction

Precise point positioning (PPP) using ionosphere-free code and phase observations (Zumberge et al. 1997) is able to provide centimeter-level positioning accuracy with a single receiver. However, the ambiguity parameter estimated in the conventional PPP model cannot be resolved to the integer value. In fact, the estimated ambiguity parameter is a combination of the integer ambiguity, the receiver biases, and the satellite biases. This means the integer property of the ambiguity parameter is lost. As a result, fixing the integer ambiguity using the conventional PPP model is not feasible.

Following the investigations on integer ambiguity pseudo-fixing (Gao and Shen 2002) and integer ambiguity resolution with simulated data sets (Wang and Gao 2006, 2007), several PPP integer ambiguity resolution methods have been developed and implemented with real data sets in recent years. Ge et al. (2008) proposed a single-difference between-satellites method characterized by eliminating the receiver biases through a single-differencing. The integer property is recovered by sequentially correcting the satellite wide-lane and narrow-lane fractional-cycle biases (FCBs). Collins (2008) developed a method known as the decoupled clock model and proved that the code biases also contributed to the fractional part of phase ambiguities in PPP. By applying the satellite decoupled clock corrections and estimating the receiver decoupled clock parameters, both the undifferenced integer wide-lane and N_1 ambiguities can be directly estimated. Laurichesse et al. (2008) also developed an integer phase clock model featuring different clock terms for code and phase observations. This model utilizes the wide-lane satellite bias (WSB) corrections to resolve the integer wide-lane ambiguity, whereas the integer N_1 ambiguity is directly estimated.

Although similar positioning performances have been demonstrated with these three methods (Collins et al. 2010; Ge et al. 2008; Geng et al. 2009; Laurichesse et al. 2008), very few efforts have been made to explore the relationship between these methods. Geng et al. (2010) compared the single-difference between-satellites method and the integer phase clock model with a focus on how the receiver and satellite biases are isolated from the phase ambiguity in PPP. But, this contribution is based on specific assumptions. For example, the satellite code biases can be absorbed by the code residual in the network solution; the code observations are not employed in the user solution so that the receiver code biases are ignored. As a result, the effects of satellite and receiver code biases on phase ambiguities in PPP are not taken into consideration. In other words, the method equivalence obtained by Geng et al. (2010) is based on those assumptions about the satellite and receiver code biases.

We aim to prove the equivalence of the three PPP integer ambiguity resolution methods without any assumption made in the previous publications. First, these three methods will be explained using the same notation. Then, a comprehensive comparison is carried out in three criteria: the integer property recovery, the system redundancy, and the necessary corrections through which the method equivalence in the user solution can be obtained.

PPP integer ambiguity resolution methods

Three PPP integer ambiguity resolution methods have been described in literatures using different notations and assumptions. This makes it difficult for readers to understand these methods and to make a theoretical comparison. Therefore, these methods are derived again in this section using a consistent notation system. In the following, we will first present the GPS code and carrier phase observation equations and several linear functions of these observations and then describe different PPP models.

GPS observations and linear functions

For the purpose of this study, the GPS code and phase observations at frequency L_i are written as:

$$P_i = \rho + c(dt^{\mathrm{r}} - dt^{\mathrm{s}}) + T + \frac{f_1^2}{f_i^2} I_1 + b_{Pi}^{\mathrm{r}} - b_{P_i}^{\mathrm{s}} + \varepsilon_{P_i} \tag{1}$$

$$L_i = \rho + c(dt^{\mathrm{r}} - dt^{\mathrm{s}}) + T - \frac{f_1^2}{f_i^2} I_1 - \lambda_i N_i + b_{L_i}^{\mathrm{r}} - b_{L_i}^{\mathrm{s}} + \varepsilon_{L_i} \tag{2}$$

where the frequency index i equals 1 and 2. The carrier frequencies are $f_1 = 154 f_0$, $f_2 = 120 f_0$, with $f_0 = 10.23\,\mathrm{MHz}$. The symbol P_i denotes the raw code observation, L_i is the raw phase observation, ρ is the geometric distance between receiver and satellite, c is the speed of light in vacuum, dt^{r} is the receiver clock error, dt^{s} is the satellite clock error, T is the tropospheric delay, I_1 is the first-order ionospheric delay on frequency L_1, λ_i is the wavelength of frequency L_i, N_i is the integer ambiguity, $b_{P_i}^{\mathrm{r}}$ is the receiver code hardware delay (bias), $b_{L_i}^{\mathrm{r}}$ is the receiver phase hardware delay (bias), $b_{P_i}^{\mathrm{s}}$ is the satellite code hardware delay (bias), $b_{L_i}^{\mathrm{s}}$ is the satellite phase hardware delay (bias), ε_{P_i} contains code multipath and code noise, and ε_{L_i} contains phase multipath and phase noise of frequency L_i. The b-terms are often referred to as biases instead of the more narrow designation of hardware delays.

The general linear functions of the observations are $P_{\mathrm{LC}} = \alpha P_1 + \beta P_2$ and $L_{\mathrm{LC}} = \alpha L_1 + \beta L_2$, where α and β are combination coefficients. Using $\alpha_{\mathrm{IF}} = f_1^2/(f_1^2 - f_2^2)$ and $\beta_{\mathrm{IF}} = -f_2^2/(f_1^2 - f_2^2)$, we obtain the ionosphere-free (IF) code function P_{IF} and phase function L_{IF},

$$\begin{aligned} P_{\mathrm{IF}} &= \alpha_{\mathrm{IF}} P_1 + \beta_{\mathrm{IF}} P_2 \\ &= \rho + (cdt^{\mathrm{r}} + b_{P_{\mathrm{IF}}}^{\mathrm{r}}) - (cdt^{\mathrm{s}} + b_{P_{\mathrm{IF}}}^{\mathrm{s}}) + T + \varepsilon_{P_{\mathrm{IF}}} \end{aligned} \tag{3}$$

$$\begin{aligned} L_{\mathrm{IF}} &= \alpha_{\mathrm{IF}} L_1 + \beta_{\mathrm{IF}} L_2 \\ &= \rho + (cdt^{\mathrm{r}} + b_{L_{\mathrm{IF}}}^{\mathrm{r}}) - (cdt^{\mathrm{s}} + b_{L_{\mathrm{IF}}}^{\mathrm{s}}) + T - \lambda_{\mathrm{IF}} N_{\mathrm{IF}} + \varepsilon_{L_{\mathrm{IF}}} \end{aligned} \tag{4}$$

$$cdt_{P_{\mathrm{IF}}}^{\mathrm{r}} = cdt^{\mathrm{r}} + b_{P_{\mathrm{IF}}}^{\mathrm{r}} \tag{5}$$

$$cdt_{P_{\mathrm{IF}}}^{\mathrm{s}} = cdt^{\mathrm{s}} + b_{P_{\mathrm{IF}}}^{\mathrm{s}} \tag{6}$$

$$cdt_{L_{\mathrm{IF}}}^{\mathrm{r}} = cdt^{\mathrm{r}} + b_{L_{\mathrm{IF}}}^{\mathrm{r}} \tag{7}$$

$$cdt_{L_{\mathrm{IF}}}^{\mathrm{s}} = cdt^{\mathrm{s}} + b_{L_{\mathrm{IF}}}^{\mathrm{s}} \tag{8}$$

$$b_{P_{\mathrm{IF}}}^{\mathrm{r}} = \alpha_{\mathrm{IF}} b_{P_1}^{\mathrm{r}} + \beta_{\mathrm{IF}} b_{P_2}^{\mathrm{r}} \tag{9}$$

$$b_{P_{\mathrm{IF}}}^{\mathrm{s}} = \alpha_{\mathrm{IF}} b_{P_1}^{\mathrm{s}} + \beta_{\mathrm{IF}} b_{P_2}^{\mathrm{s}} \tag{10}$$

$$b_{L_{\mathrm{IF}}}^{\mathrm{r}} = \alpha_{\mathrm{IF}} b_{L_1}^{\mathrm{r}} + \beta_{\mathrm{IF}} b_{L_2}^{\mathrm{r}} \tag{11}$$

$$b_{L_{\mathrm{IF}}}^{\mathrm{s}} = \alpha_{\mathrm{IF}} b_{L_1}^{\mathrm{s}} + \beta_{\mathrm{IF}} b_{L_2}^{\mathrm{s}} \tag{12}$$

$$\lambda_{\mathrm{IF}} = \frac{2cf_0}{f_1^2 - f_2^2} \tag{13}$$

$$N_{\mathrm{IF}} = 17N_1 + 60N_{\mathrm{WL}} \tag{14}$$

Regarding the terms defined in (5) to (8), the following terminology is found in the literature: $cdt_{P_{\mathrm{IF}}}^{\mathrm{r}}$ (receiver code clock error), $cdt_{P_{\mathrm{IF}}}^{\mathrm{s}}$ (satellite code clock error), $cdt_{L_{\mathrm{IF}}}^{\mathrm{r}}$ (receiver phase clock error), and $cdt_{L_{\mathrm{IF}}}^{\mathrm{s}}$ (satellite phase clock error). These "clock" terms are a function of the actual receiver clock error dt^{r}, satellite clock error dt^{s}, receiver code and phase biases $b_{P_{\mathrm{IF}}}^{\mathrm{r}}, b_{L_{\mathrm{IF}}}^{\mathrm{r}}$, and satellite code and phase biases $b_{P_{IF}}^{s}, b_{L_{IF}}^{s}$. The casual reader might mistakenly think that there are two receiver clock errors and two satellite clock errors. This is not the case since the signals at the receiver and at the satellite are generated by a single receiver clock and a single satellite clock, respectively. Equations (13) and (14) follow straightforwardly from the definition of P_{IF} and L_{IF}. The ionosphere-free wavelength is $\lambda_{\mathrm{IF}} = 6.3\,\mathrm{mm}$, and N_{IF} is called the ionosphere-free ambiguity. The wide-lane ambiguity N_{WL} equals $N_1 - N_2$.

The wide-lane (WL) phase combination L_{WL} with coefficients $\alpha_{\mathrm{WL}} = f_1/(f_1 - f_2)$ and $\beta_{\mathrm{WL}} = -f_2/(f_1 - f_2)$ is

$$\begin{aligned} L_{\mathrm{WL}} &= \alpha_{\mathrm{WL}} L_1 + \beta_{\mathrm{WL}} L_2 \\ &= \rho + c(dt^{\mathrm{r}} - dt^{\mathrm{s}}) + T + \frac{f_1^2}{f_2^2} I_1 - \lambda_{\mathrm{WL}} N_{\mathrm{WL}} \\ &\quad + (\alpha_{\mathrm{WL}} b^{\mathrm{r}}_{L_1} + \beta_{\mathrm{WL}} b^{\mathrm{r}}_{L_2}) - (\alpha_{\mathrm{WL}} b^{\mathrm{s}}_{L_1} + \beta_{\mathrm{WL}} b^{\mathrm{s}}_{L_2}) + \varepsilon_{L_{\mathrm{WL}}} \end{aligned} \tag{15}$$

Similarly, the narrow-lane (NL) code combination P_{NL} is

$$\begin{aligned} P_{\mathrm{NL}} &= \alpha_{\mathrm{NL}} P_1 + \beta_{\mathrm{NL}} P_2 \\ &= \rho + c(dt^{\mathrm{r}} - dt^{\mathrm{s}}) + T + \frac{f_1^2}{f_2^2} I_1 + (\alpha_{\mathrm{NL}} b^{\mathrm{r}}_{P_1} + \beta_{\mathrm{NL}} b^{\mathrm{r}}_{P_2}) \\ &\quad - (\alpha_{\mathrm{NL}} b^{\mathrm{s}}_{P_1} + \beta_{\mathrm{NL}} b^{\mathrm{s}}_{P_2}) + \varepsilon_{P_{\mathrm{NL}}} \end{aligned} \tag{16}$$

with $\alpha_{\mathrm{NL}} = f_1/(f_1 + f_2)$ and $\beta_{\mathrm{NL}} = f_2/(f_1 + f_2)$.

Two additional ionosphere-free functions are needed. The first function is the difference in the carrier phase wide-lane function and the pseudo-range narrow-lane functions, which was first proposed by Hatch (1982). This linear combination was also mentioned in Melbourne (1985) and Wübbena (1985) and called the Melbourne–Wübbena (MW) function in most literatures. For better understanding, we use the term "MW function" in this paper. The MW function is

$$A_{\mathrm{MW}} = L_{\mathrm{WL}} - P_{\mathrm{NL}} = -\lambda_{\mathrm{WL}} N_{\mathrm{WL}} + (b^{\mathrm{r}}_{A_{\mathrm{MW}}} - b^{\mathrm{s}}_{A_{\mathrm{MW}}}) + \varepsilon_{A_{\mathrm{MW}}} \tag{17}$$

$$b^{\mathrm{r}}_{A_{\mathrm{MW}}} = (\alpha_{\mathrm{WL}} b^{\mathrm{r}}_{L_1} + \beta_{\mathrm{WL}} b^{\mathrm{r}}_{L_2}) - (\alpha_{\mathrm{NL}} b^{\mathrm{r}}_{P_1} + \beta_{\mathrm{NL}} b^{\mathrm{r}}_{P_2}) \tag{18}$$

$$b^{\mathrm{s}}_{A_{\mathrm{MW}}} = (\alpha_{\mathrm{WL}} b^{\mathrm{s}}_{L_1} + \beta_{\mathrm{WL}} b^{\mathrm{s}}_{L_2}) + (\alpha_{\mathrm{NL}} b^{\mathrm{s}}_{P_1} + \beta_{\mathrm{NL}} b^{\mathrm{s}}_{P_2}) \tag{19}$$

The second function is the difference of L_{IF} and P_{IF},

$$A_{\mathrm{IF}} = -\lambda_{\mathrm{IF}} \Delta N_{\mathrm{IF}} + (b^{\mathrm{r}}_{L_{\mathrm{IF}}} - b^{\mathrm{r}}_{P_{\mathrm{IF}}}) - (b^{\mathrm{s}}_{L_{\mathrm{IF}}} - b^{\mathrm{s}}_{P_{\mathrm{IF}}}) + \varepsilon_{A_{\mathrm{IF}}} \tag{20}$$

The wide-lane receiver bias (WRB) $b^{\mathrm{r}}_{A_{\mathrm{MW}}}$ of (18) is a function of the receiver biases, whereas the wide-lane satellite bias $b^{\mathrm{s}}_{A_{\mathrm{MW}}}$ of (19) refers to the satellite biases.

The above expressions represent well-known functions of the basic code and phase equations. No assumptions about the receiver and satellite code and phase biases have been made in these expressions.

Traditional PPP model

The traditional PPP model of Zumberge et al. (1997) uses (3) and (4). For this discussion, we use (5) and (6) in (3) and (4), resulting in the model

$$P_{\mathrm{IF}} = \rho + cdt^{\mathrm{r}}_{P_{\mathrm{IF}}} - cdt^{\mathrm{s}}_{P_{\mathrm{IF}}} + T + \varepsilon_{P_{\mathrm{IF}}} \tag{21}$$

$$\begin{aligned} L_{\mathrm{IF}} = \rho + cdt^{\mathrm{r}}_{P_{\mathrm{IF}}} - cdt^{\mathrm{s}}_{P_{\mathrm{IF}}} + T - \lambda_{\mathrm{IF}} N_{\mathrm{IF}} + (b^{\mathrm{r}}_{L_{\mathrm{IF}}} - b^{\mathrm{r}}_{P_{\mathrm{IF}}}) \\ - (b^{\mathrm{s}}_{L_{\mathrm{IF}}} - b^{\mathrm{s}}_{P_{\mathrm{IF}}}) + \varepsilon_{L_{\mathrm{IF}}} \end{aligned} \tag{22}$$

It can be seen that the ionosphere-free code and phase functions contain the actual clock errors and the code and phase biases. The satellite code clock $cdt^{\mathrm{s}}_{P_{\mathrm{IF}}}$ is available from the International GNSS Service (IGS) by means of the precise clock products (Kouba 2009; Kouba and Héroux 2001; Dow et al. 2009), and the receiver code clock $cdt^{\mathrm{r}}_{P_{\mathrm{IF}}}$ is to be estimated.

If the satellite code clock $cdt^{\mathrm{s}}_{P_{\mathrm{IF}}}$ is applied to the phase observation (22) and the receiver code clock $cdt^{\mathrm{r}}_{P_{\mathrm{IF}}}$ is estimated together with the troposphere and ambiguity parameters, then the estimated ambiguity parameter $-\lambda_{\mathrm{IF}} N_{\mathrm{IF}} + (b^{\mathrm{r}}_{L_{\mathrm{IF}}} - b^{\mathrm{r}}_{P_{\mathrm{IF}}}) - (b^{\mathrm{s}}_{L_{\mathrm{IF}}} - b^{\mathrm{s}}_{P_{\mathrm{IF}}})$ is a linear function of the integer ambiguity and the code and phase biases of the receiver and the satellite. Therefore, the estimated ambiguity parameter will be real-valued. As a result, resolving the integer ambiguity using (21) and (22) is not feasible.

From the user point of view, a tracking network is required that provides the satellite code clock corrections $cdt^{\mathrm{s}}_{P_{\mathrm{IF}}}$. Considering one epoch, suppose the user observes n satellites. The number of observations is $2n$ in (21) and (22). The number of unknown parameters is $3 + 1 + 1 + n$, which includes three coordinates, one receiver code clock $cdt^{\mathrm{r}}_{P_{\mathrm{IF}}}$, one troposphere delay, and n ionosphere-free ambiguities $-\lambda_{\mathrm{IF}} N_{\mathrm{IF}} + (b^{\mathrm{r}}_{L_{\mathrm{IF}}} - b^{\mathrm{r}}_{P_{\mathrm{IF}}}) - (b^{\mathrm{s}}_{L_{\mathrm{IF}}} - b^{\mathrm{s}}_{P_{\mathrm{IF}}})$. The degree of freedom is $2n - (3 + 1 + 1 + n) = n - 5$, which means a minimum of five satellites are required using the traditional PPP model.

Decoupled clock model

Unlike the traditional PPP model which applies the satellite code clock $cdt^{\mathrm{s}}_{P_{\mathrm{IF}}}$ for both the code and phase observations, one method featuring separate satellite code clock and satellite phase clock has been proposed by Collins (2008). As the satellite clocks are decoupled for code and phase observations, this model is called the decoupled clock model.

By substituting (5) and (6) into (3), and (7), (8), and (14) into (4), the ionosphere-free functions are transformed to

$$P_{\mathrm{IF}} = \rho + (cdt^{\mathrm{r}}_{P_{\mathrm{IF}}} - cdt^{\mathrm{s}}_{P_{\mathrm{IF}}}) + T + \varepsilon_{P_{\mathrm{IF}}} \tag{23}$$

$$L_{\mathrm{IF}} = \rho + (cdt^{\mathrm{r}}_{L_{\mathrm{IF}}} - cdt^{\mathrm{s}}_{L_{\mathrm{IF}}}) + T - \lambda_{\mathrm{IF}}(17N_1 + 60N_{\mathrm{WL}}) + \varepsilon_{L_{\mathrm{IF}}} \tag{24}$$

The decoupled clock model consists of three expressions (23), (24), and

$$A_{\mathrm{MW}} = (b^{\mathrm{r}}_{A_{\mathrm{MW}}} - b^{\mathrm{s}}_{A_{\mathrm{MW}}}) - \lambda_{\mathrm{WL}}N_{\mathrm{WL}} + \varepsilon_{A_{\mathrm{MW}}} \tag{25}$$

In this model the terms $(cdt^{\mathrm{s}}_{P_{\mathrm{IF}}}, cdt^{\mathrm{s}}_{L_{\mathrm{IF}}}, b^{\mathrm{s}}_{A_{\mathrm{MW}}})$ and $(cdt^{\mathrm{r}}_{P_{\mathrm{IF}}}, cdt^{\mathrm{r}}_{L_{\mathrm{IF}}}, b^{\mathrm{r}}_{A_{\mathrm{MW}}})$ are called the satellite and receiver decoupled clock parameters, respectively.

Network solution

If all parameters, i.e., the coordinates, the decoupled clocks, the troposphere delay, and the integer ambiguities, were to be estimated using the three model equations, the number of unknown parameters would be greater than the number of observations, resulting in a singular solution. The solution to the singularity problem is to fix a minimum number of parameters. This technique is called in adjustments imposing minimal constraints or defining the datum.

First, we choose a reference receiver and set its $cdt^{\mathrm{r}}_{P_{\mathrm{IF}}}$ and $cdt^{\mathrm{r}}_{L_{\mathrm{IF}}}$, and $b^{\mathrm{r}}_{A_{\mathrm{MW}}}$ parameters to zero. This defines the clock datum for the network. Second, we set all N_1 and N_{WL} ambiguities of the observed satellites at the reference receiver in (24) and (25) to arbitrary integer values. This defines the ambiguity datum for the reference receiver. Third, we add a non-reference receiver in the network and choose a reference satellite for this receiver. Two ambiguities, N_1 and N_{WL} in (24) and (25), of the reference satellite are set to arbitrary integer values. In this case, the other ambiguities for this non-reference receiver are estimated with respect to the N_1 and N_{WL} ambiguities of the chosen reference satellite. This defines the ambiguity datum for the chosen non-reference receiver. Forth, we repeat the third step for all other non-reference receivers in the network. It should be noted that the reference satellite chosen for the non-reference receiver in the third and forth steps could be different. In other words, each receiver has its own ambiguity datum, and there is no relationship between the ambiguity datum for each receiver in the network.

By implementing the above procedure for defining the clock datum and the ambiguity datum in the network, we can resolve the datum defect implied in (23) to (25). Suppose there are m receivers observing n common satellites. There are *3n* observations per receiver and *3mn* observations for the network. For the reference receiver, the number of unknown parameters is $3 + 1 + 0 + 3n$, which includes three coordinates, one troposphere delay, 0 receiver decoupled clocks, and $3n$ satellite decoupled clocks. All of the $2n$ ambiguities of n observed satellites are fixed to define the ambiguity datum at the reference receiver. For the remaining $m - 1$ receivers that are not used to define the clock datum, the number of unknown parameters is $3(m - 1) + (m - 1) + 3(m - 1) + 3n + 2(n - 1)(m - 1)$, which includes $3(m - 1)$ coordinates, $(m - 1)$ troposphere delays, $3(m - 1)$ receiver decoupled clocks, $3n$ satellite decoupled clocks, and $2(n - 1)(m - 1)$ ambiguities. Note that we estimate $2(n - 1)(m - 1)$ ambiguities for n satellites because $2(m - 1)$ ambiguities are fixed to define the ambiguity datum at the $m - 1$ receivers. Since the n satellites are observed by all receivers, only $3n$ satellite decoupled clocks should be estimated in the network solution. Therefore, for a network consisting of one reference receiver and $m - 1$ non-reference receivers, the number of unknown parameters is $3m + m + 3(m - 1) + 3n + 2(n - 1)(m - 1)$. For example, if $m = 4$, the number of observations is $3 \times 4 \times n = 12n$ and the number of unknown parameters is $12 + 4 + 9 + 3n + 6(n - 1) = 9n + 19$. The corresponding degree of freedom is $3n - 19$. This means that in order to resolve the datum defect issue, at least seven common satellites are required in the network solution. When the number of receivers increases, the minimal number of common satellites decreases. More receivers and more common satellites will further increase the redundancy in the network solution.

User solution

From the user point of view, a tracking network is required that provides the satellite decoupled clocks $(cdt^{\mathrm{s}}_{P_{\mathrm{IF}}}, cdt^{\mathrm{s}}_{L_{\mathrm{IF}}}, b^{\mathrm{s}}_{A_{\mathrm{MW}}})$ resulting from the network solution. The clock datum defined by the reference receiver can be retained, which means no additional clock datum is required in the user solution. However, the ambiguity datum must be defined by choosing one reference satellite and setting the N_1 and N_{WL} ambiguities of the reference satellite to arbitrary integer values. It should be noted that the reference satellite in the user solution can differ from those chosen in the network solution. In fact, each receiver should define its own ambiguity datum. The integer cycle ambiguity datum difference will be absorbed by the receiver decoupled clock parameters as pointed out in Shi and Gao (2010) and as can also be seen from (24) and (25). From this perspective, the estimated receiver decoupled clocks become relative clocks with respect to the ambiguity datum.

By applying the satellite decoupled clocks $(cdt^{\mathrm{s}}_{P_{\mathrm{IF}}}, cdt^{\mathrm{s}}_{L_{\mathrm{IF}}}, b^{\mathrm{s}}_{A_{\mathrm{MW}}})$ and setting the ambiguity datum in

(23) to (25), the unknown parameters become estimable. More specifically, the integer wide-lane and N_1 ambiguities can be directly estimated in the function model.

Method summary

In summary, the satellite decoupled clocks $(cdt^{s}_{P_{IF}}, cdt^{s}_{L_{IF}}, b^{s}_{A_{MW}})$ are required to remove the satellite clock and bias errors in (23) to (25). By defining the ambiguity datum and estimating the receiver decoupled clocks $(cdt^{r}_{P_{IF}}, cdt^{r}_{L_{IF}}, b^{r}_{A_{MW}})$ containing the receiver clock and bias errors, the integer wide-lane and N_1 ambiguities can be directly estimated in the user solution.

Suppose n satellites are observed. The number of observations is $3n$ in (23) to (25). The ambiguity datum is defined by fixing the wide-lane and N_1 ambiguities of the reference satellite to arbitrary integer values. The number of unknown parameters is $3 + 3 + 1 + 2(n - 1)$, which includes three coordinates, three receiver decoupled clocks, one troposphere delay, and $2(n - 1)$ wide-lane and N_1 ambiguities. The degree of freedom is $3n - (3 + 3 + 1 + 2(n - 1)) = n - 5$, which means a minimum of five satellites are required to apply the decoupled clock model.

Single-difference between-satellites method

The function model consists of the ionosphere-free functions (21) and (22), plus the MW function (17). A sequential solution is adopted. The wide-lane ambiguity is fixed first because its long wavelength of 86.9 cm makes resolution feasible for a very short period of observations. The fixed wide-lane ambiguities are then treated as known integer values in the subsequent N_1 ambiguity resolution.

Network solution

Both the integer wide-lane and N_1 ambiguities can be obtained by rounding the real-valued wide-lane and N_1 ambiguities in the network solution. The corresponding products are the satellite wide-lane and N_1 FCB corrections. Both corrections are computed as the differences of the real-valued and the integer ambiguities.

Wide-lane ambiguity fixing The development starts with the MW function (17) and applies it to the between-satellite single-difference,

$$\Delta A^{j,i}_{\mathrm{MW}} = A^{j}_{\mathrm{MW}} - A^{i}_{\mathrm{MW}} = -\lambda_{\mathrm{WL}} \Delta N^{j,i}_{\mathrm{WL}} - \Delta b^{s\,j,i}_{A_{\mathrm{MW}}} + \varepsilon_{\Delta A^{j,i}_{\mathrm{MW}}} \tag{26}$$

where the double superscripts indicate the differencing operation. The differencing has canceled the receiver wide-lane biases $b^{r}_{A_{\mathrm{MW}}}$ seen in (18). The satellite wide-lane biases have been found to be quite stable over several consecutive days (Wang and Gao 2007; Ge et al. 2008). As a result, epoch-averaging can be applied to determine the satellite wide-lane FCB correction. A certain period of time is required to allow the single-differenced MW function (26) to reach convergence. Then, the integer wide-lane ambiguity can be obtained by rounding the real-valued wide-lane ambiguity to its nearest integer value as

$$\overline{\Delta N}^{j,i}_{\mathrm{WL}} = -\left\langle \frac{\Delta A^{j,i}_{\mathrm{MW}}}{\lambda_{\mathrm{WL}}} \right\rangle \tag{27}$$

where $\langle * \rangle$ denotes rounding of the real value to the nearest integer value. The satellite wide-lane FCB correction in unit of meters is calculated as

$$\Delta b^{s\,j,i}_{A_{\mathrm{MW}}} = -\Delta A^{j,i}_{\mathrm{MW}} - \lambda_{\mathrm{WL}} \overline{\Delta N}^{j,i}_{\mathrm{WL}} \tag{28}$$

This equation is used to determine the satellite wide-lane FCB corrections at a receiver. By averaging the satellite wide-lane FCB corrections over the receivers in the network, a correction with high precision can be obtained. This process can be repeated for singe-difference observations of other satellite phase pairs, resulting in a unique set of satellite wide-lane FCB corrections to be broadcast to the users.

N_1 ambiguity fixing The first step in resolving the N_1 ambiguity is to apply (14) to the between-satellite single-difference,

$$\lambda_{\mathrm{IF}} \Delta N^{j,i}_{\mathrm{IF}} = 17 \lambda_{\mathrm{IF}} \Delta N^{j,i}_{1} + 60 \lambda_{\mathrm{IF}} \Delta N^{j,i}_{\mathrm{WL}} \tag{29}$$

where $\Delta N^{j,i}_{\mathrm{IF}} = N^{j}_{\mathrm{IF}} - N^{i}_{\mathrm{IF}}$, N^{i}_{IF} and N^{j}_{IF} are the estimated ionosphere-free integer ambiguities in (22). Next, we apply (20) to single-differences, giving

$$\Delta A^{j,i}_{\mathrm{IF}} = -\lambda_{\mathrm{IF}} \Delta N^{j,i}_{\mathrm{IF}} - (\Delta b^{s\,j,i}_{L_{\mathrm{IF}}} - \Delta b^{s\,j,i}_{P_{\mathrm{IF}}}) + \varepsilon_{\Delta L_{\mathrm{IF}}} \tag{30}$$

and note, again, that the receiver biases have been canceled. Substituting (29) into (30) and rearranging the equation leads to

$$\Delta A^{j,i}_{\mathrm{IF}} + 60 \lambda_{\mathrm{IF}} \Delta N^{j,i}_{\mathrm{WL}} = -17 \lambda_{\mathrm{IF}} \Delta N^{j,i}_{1} - (\Delta b^{s\,j,i}_{L_{\mathrm{IF}}} - \Delta b^{s\,j,i}_{P_{\mathrm{IF}}}) + \varepsilon_{\Delta L_{\mathrm{IF}}} \tag{31}$$

The left side is computable since $\Delta N^{j,i}_{\mathrm{WL}}$ is known from (27). Denoting the left side by $\Delta A^{j,i}_{1}$ and labeling the difference of the single-difference phase and code biases by $\Delta b^{s\,j,i}_{A_1}$, we obtain

$$\Delta A^{j,i}_{1} = -17 \lambda_{\mathrm{IF}} \Delta N^{j,i}_{1} - \Delta b^{s\,j,i}_{A_1} + \varepsilon_{\Delta L_{\mathrm{IF}}} \tag{32}$$

$$\Delta b^{s\,j,i}_{A_1} = \Delta b^{s\,j,i}_{L_{\mathrm{IF}}} - \Delta b^{s\,j,i}_{P_{\mathrm{IF}}} \tag{33}$$

The integer single-differenced ambiguity $\Delta N_1^{j,i}$ is contaminated by $\Delta b_{A_1}^{s\,j,i}$. Therefore, the determination of this single-differenced bias term is the key to resolving the integer single-differenced ambiguity $\Delta N_1^{j,i}$.

Similar to the procedure used to determine the single-differenced satellite wide-lane FCB corrections, a certain period of time is needed to allow the real-valued single-differenced ambiguity (32) to reach convergence. Then, the integer ambiguity $\Delta N_1^{j,i}$ can be obtained by rounding as follows:

$$\overline{\Delta N}_1^{j,i} = -\left\langle \frac{\Delta A_1^{j,i}}{17\lambda_3} \right\rangle, \tag{34}$$

and the single-differenced satellite N_1 FCB corrections in unit of meters can be calculated, using the just computed $\overline{\Delta N}_1^{j,i}$, as

$$\Delta b_{A_1}^{s\,j,i} = -\Delta A_1^{j,i} - 17\lambda_{IF}\overline{\Delta N}_1^{j,i} \tag{35}$$

Equation (35) can be used to determine the satellite N_1 FCB correction $\Delta b_{A_1}^{s\,j,i}$ at a receiver. By averaging the satellite N_1 FCB corrections from multiple receivers in the network, a precise value can be obtained. Applying this process to other between-satellite differences, a set of the satellite N_1 FCB corrections with high precision can be obtained and then broadcast to the users.

User solution

By applying the satellite wide-lane and N_1 FCB corrections determined in the network solution to remove the satellite biases and a single-difference between-satellites operator to remove the receiver biases, the integer property of the wide-lane and N_1 ambiguities in the user solution can be recovered.

Wide-lane ambiguity fixing At the user site, a single-difference between the same satellite pair (i,j) as in (26) is also performed,

$$\begin{aligned}\Delta A_{\mathrm{MW\_}u}^{j,i} &= A_{\mathrm{MW\_}u}^{j} - A_{\mathrm{MW\_}u}^{i} \\ &= -\lambda_{\mathrm{WL}}\Delta N_{\mathrm{WL\_}u}^{j,i} - \Delta b_{A_{\mathrm{MW}}}^{s\,j,i} + \varepsilon_{A_{\mathrm{MW\_}u}^{j,i}}\end{aligned} \tag{36}$$

where all terms have the same definition as used in (26), but now for the user solution. The satellite wide-lane FCB corrections $\Delta b_{A_{\mathrm{MW}}}^{s\,j,i}$ of (28) determined in the network solution are applied to the single-differenced MW combination, giving

$$\Delta A_{\mathrm{MW\_}u}^{j,i} + \Delta b_{A_{\mathrm{MW}}}^{s\,j,i} = -\lambda_{\mathrm{WL}}\Delta N_{\mathrm{WL\_}u}^{j,i} \tag{37}$$

It is thus clear that by applying the satellite wide-lane FCB correction $\Delta b_{A_{\mathrm{MW}}}^{s\,j,i}$ determined in the network solution, the integer property of the single-differenced wide-lane ambiguity $\Delta N_{\mathrm{WL\_}u}^{j,i}$ in the user solution can be recovered.

N_1 ambiguity fixing At the user site, the single-differencing between the same satellite pair (i,j) as in (32) is applied as

$$\Delta A_{1\_u}^{j,i} = -17\lambda_{IF}\Delta N_{1\_u}^{j,i} - \Delta b_{A_1}^{s\,j,i} + \varepsilon_{L_{\mathrm{IF}}\_u} \tag{38}$$

where all terms have the same meaning as used in (32), but now for the user solution. The satellite N_1 FCB corrections $\Delta b_{A_1}^{s\,j,i}$ of (35) determined in the network solution are applied to (38), giving

$$\Delta A_{1\_u}^{j,i} + \Delta b_{A_1}^{s\,j,i} = -17\lambda_{\mathrm{IF}}\Delta N_{1\_u}^{j,i} \tag{39}$$

It is clear that by applying the satellite N_1 FCB corrections $\Delta b_{A_1}^{s\,j,i}$ determined in the network solution, the integer property of the single-differenced N_1 ambiguity $\Delta N_{1\_u}^{j,i}$ can be recovered from (39).

Method summary

In summary, the IGS satellite code clock $cdt_{P_{\mathrm{IF}}}^{s}$ is required to remove the satellite clock and code bias errors in (21) and (22). The estimated ambiguity parameter in (22) is the real-valued ionosphere-free ambiguity. By using a single-differencing between-satellites operator of (36) and applying the single-differenced wide-lane FCB corrections of (28), the integer property of the wide-lane ambiguities can be recovered in (37). After the integer wide-lane ambiguities are obtained, the real-valued N_1 ambiguities can be computed from the real-valued ionosphere-free ambiguities using (31). By using the single-differencing between-satellites operator of (38) and applying the single-differenced N_1 FCB corrections of (35), the integer property of the N_1 ambiguities can be recovered in (39). Once the single-differenced integer wide-lane and N_1 ambiguities are obtained in (37) and (39), the single-differenced integer ionosphere-free ambiguities can be reconstructed in (29). By setting the single-differenced integer ionosphere-free ambiguity of the reference satellite to arbitrary integer value, the undifferenced integer ionosphere-free ambiguity can be obtained.

Suppose n satellites are observed. The number of observations is $2n$ in (21) and (22). One reference satellite is required for single-differencing of the wide-lane and N_1 ambiguities. The number of unknown parameters is $3 + 1 + 1 + n$, which includes three coordinates, one receiver code clock, one troposphere delay, and n ionosphere-free ambiguities. The degree of freedom is $2n - (3 + 1 + 1 + n) = n - 5$, which means a minimum of five satellites are required to apply the single-difference between-satellites method.

Integer phase clock model

The integer phase clock model proposed by Laurichesse et al. (2008) consists of the ionosphere-free functions (21), (22), and the MW function (17). In addition to the IGS satellite code clock for the code observation in (21), another satellite phase clock is required for the phase observation in (22). As the usage of the satellite phase clock is for the integer N_1 ambiguity resolution, this method is called the integer phase clock model.

Network solution

The wide-lane ambiguity resolution in the network solution is achieved by rounding the MW function (17). The by-product is the wide-lane satellite bias as the fractional part of MW function. On the other hand, the integer N_1 ambiguities are estimated using (22) with another by-product of the satellite phase clock.

Wide-lane ambiguity fixing The determination of the integer wide-lane ambiguity begins with the MW function (17), and the integer wide-lane ambiguity is computed by means of rounding. Similar to the clock datum definition in the decoupled clock model, a reference receiver is chosen and its wide-lane receiver bias $b^{\mathrm{r}}_{A_{\mathrm{MW}}}$ is set to zero. Then, the integer part of the MW function (17) is attributed to the integer wide-lane ambiguity as

$$\overline{N}_{\mathrm{WL}} = -\left\langle \frac{A_{\mathrm{MW}}}{\lambda_{\mathrm{WL}}} \right\rangle, \tag{40}$$

and the fractional part is attributed to the WSB as

$$b^{\mathrm{s}}_{A_{\mathrm{MW}}} = -A_{\mathrm{MW}} - \lambda_{\mathrm{WL}}\overline{N}_{\mathrm{WL}} \tag{41}$$

The computed WSB is used to calculate the other WRBs in the network solution. Eventually, a set of WSB $b^{\mathrm{s}}_{A_{\mathrm{MW}}}$ are determined in the network solution and broadcast to the users.

N_1 ambiguity fixing The ionosphere-free code and phase observations (21) and (22) are involved in resolving the integer N_1 ambiguity. We substitute (14) into (22) in order to replace the ionosphere-free ambiguity with N_1 and N_{WL}. Since the wide-lane ambiguities N_{WL} have already been fixed, it can be moved to the left side. Moving the geocentric satellite distance and the tropospheric term also to the left results in the rearrangement of (22) as

$$L_{\mathrm{IF}} - 60\lambda_{\mathrm{IF}}N_{\mathrm{WL}} - \rho + T = cdt^{\mathrm{r}}_{L_{\mathrm{IF}}} - cdt^{\mathrm{s}}_{L_{\mathrm{IF}}} - 17\lambda_{\mathrm{IF}}N_1 + \varepsilon_{L_{\mathrm{IF}}} \tag{42}$$

Calling the left side Q_{IF}, we obtain

$$Q_{\mathrm{IF}} = (cdt^{\mathrm{r}}_{L_{\mathrm{IF}}} - cdt^{\mathrm{s}}_{L_{\mathrm{IF}}}) - 17\lambda_{\mathrm{IF}}N_1 \tag{43}$$

where Q_{IF} can be calculated since all terms on the left side of (42) are known.

The determination of satellite phase clocks $cdt^{\mathrm{s}}_{L_{\mathrm{IF}}}$ in (43) can be explained as follows. First, we use the reference receiver chosen in "Wide-lane ambiguity fixing" section and set its $cdt^{\mathrm{r}}_{L_{\mathrm{IF}}}$ to zero. Second, the ambiguity datum at the reference receiver is defined by setting all N_1 ambiguities of the observed satellites to arbitrary integer values. Note that since the wide-lane ambiguities have already been fixed, it is not necessary to fix the wide-lane ambiguities as the ambiguity datum. Third, initial estimates of the satellite integer phase clocks $cdt^{\mathrm{s}}_{L_{\mathrm{IF}}}$ are derived as the fractional parts of $-Q_{\mathrm{IF}} - 17\lambda_{\mathrm{IF}}N_1$ in (43). Only the satellite phase clocks for observed satellites are determined in this step. Forth, a new receiver is added. With the initial estimates of satellite phase clocks $cdt^{\mathrm{s}}_{L_{\mathrm{IF}}}$ obtained in the third step, the difference $Q_{\mathrm{IF}} + cdt^{\mathrm{s}}_{L_{\mathrm{IF}}}$ can be calculated. Applying rounding as done for other methods, the integer part is attributed to the N_1 ambiguity and the fractional part is the receiver phase clock $cdt^{\mathrm{r}}_{L_{\mathrm{IF}}}$. Moreover, the satellite phase clocks $cdt^{\mathrm{s}}_{L_{\mathrm{IF}}}$ for those satellites which are not observed in the third step can also be obtained once the integer N_1 ambiguity and the receiver phase clock $cdt^{\mathrm{r}}_{L_{\mathrm{IF}}}$ are known. The newly calculated satellite phase clocks $cdt^{\mathrm{s}}_{L_{\mathrm{IF}}}$ are then added to those obtained in the third step. Fifth, another receiver is added, and the fourth step is repeated until a complete set of satellite phase clocks $cdt^{\mathrm{s}}_{L_{\mathrm{IF}}}$ are obtained.

Following the above procedure, the integer N_1 ambiguities can be obtained. In addition, a set of satellite phase clocks $cdt^{\mathrm{s}}_{L_{\mathrm{IF}}}$ are determined in the network solution and broadcast to the users.

User solution

The required corrections in the user solution are the wide-lane satellite bias correction, the IGS satellite code clock, and the satellite phase clock. First, by applying the wide-lane satellite bias correction and the satellite-averaged wide-lane receiver bias correction, the wide-lane ambiguity resolution in the user solution can be obtained. Second, the integer N_1 ambiguities can be directly estimated with the satellite phase clock determined in the network solution.

Wide-lane ambiguity resolution Regarding the user solution, the MW observation at the user site can be derived as

$$A_{\mathrm{MW}\_u} = -\lambda_{\mathrm{WL}}N_{\mathrm{WL}\_u} + (b^{\mathrm{r}}_{A_{\mathrm{MW}\_u}} - b^{\mathrm{s}}_{A_{\mathrm{MW}}}) \tag{44}$$

By applying the wide-lane satellite bias corrections (41) to (44), we can obtain

$$A_{\mathrm{MW}\_u} + b^{\mathrm{s}}_{A_{\mathrm{MW}}} = -\lambda_{\mathrm{WL}} N_{\mathrm{WL}\_u} + (b^{\mathrm{r}}_{A_{\mathrm{MW}\_u}} - b^{\mathrm{s}}_{A_{\mathrm{MW}}}) + b^{\mathrm{s}}_{A_{\mathrm{MW}}} = -\lambda_{\mathrm{WL}} N_{\mathrm{WL}\_u} + b^{\mathrm{r}}_{A_{\mathrm{MW}\_u}} \tag{45}$$

It can be seen that the wide-lane ambiguity, after correcting for the wide-lane satellite bias, is still contaminated by the wide-lane receiver bias $b^{\mathrm{r}}_{A_{\mathrm{MW}\_u}}$. Since this wide-lane receiver bias is the same for all observed satellites, it can be obtained by averaging the fractional parts of the real-valued ambiguities from all observed satellites as

$$\bar{b}^{\mathrm{r}}_{A_{\mathrm{MW}\_u}} = \frac{1}{n}\sum_{i=1}^{n}\left[\left(A_{\mathrm{MW}\_u} + b^{\mathrm{s}}_{A_{\mathrm{MW}}}\right) - \lambda_{\mathrm{WL}}\left\langle \frac{A_{\mathrm{MW}\_u} + b^{\mathrm{s}}_{A_{\mathrm{MW}}}}{\lambda_{\mathrm{WL}}}\right\rangle\right]_i \tag{46}$$

where n represents the number of observed satellites. By substituting the wide-lane receiver bias (46) into (45), we can obtain the integer wide-lane ambiguity as

$$A_{\mathrm{MW}\_u} + b^{\mathrm{s}}_{A_{\mathrm{MW}}} - \bar{b}^{\mathrm{r}}_{A_{\mathrm{MW}\_u}} = -\lambda_{\mathrm{WL}} N_{\mathrm{WL}\_u} \tag{47}$$

through which the wide-lane ambiguity resolution in the user solution can be achieved.

N_1 ambiguity resolution Regarding the user positioning determination, we should first substitute (14) into (22) and then move the wide-lane ambiguities fixed in "Wide-lane ambiguity resolution" section to the left side as

$$L_{\mathrm{IF}} - 60\lambda_{\mathrm{IF}} N_{\mathrm{WL}} = \rho + cdt^{\mathrm{r}}_{L_3} - cdt^{\mathrm{s}}_{L_3} + T - 17\lambda_{\mathrm{IF}} N_1 + \varepsilon_{L_{\mathrm{IF}}} \tag{48}$$

Equations (21) and (48) are used to compute the user positioning. The IGS satellite code clock $cdt^{\mathrm{s}}_{P_{\mathrm{IF}}}$ and the phase clock $cdt^{\mathrm{s}}_{L_{\mathrm{IF}}}$ determined in "N_1 ambiguity resolution" section are used in (21) and (48), respectively. Along with the user coordinates, the receiver code clock $cdt^{\mathrm{r}}_{P_{\mathrm{IF}}}$, the receiver phase clock $cdt^{\mathrm{r}}_{L_{\mathrm{IF}}}$, the troposphere delay, and the integer N_1 ambiguities can be directly estimated.

Method summary

In summary, the integer wide-lane ambiguities N_{WL_u} in the user solution are first estimated through (47), following the WSB correction $b^{\mathrm{s}}_{A_{\mathrm{MW}}}$ in (45) and the WRB correction $\bar{b}^{\mathrm{r}}_{A_{\mathrm{MW}\_u}}$ in (46). Thus, the ionosphere-free N_{IF} ambiguity estimation in the phase observation L_{IF} in (22) becomes the N_1 ambiguity estimation in (48). The satellite phase clocks $cdt^{\mathrm{s}}_{L_{\mathrm{IF}}}$ are applied to L_{IF} in (48), and the IGS code clocks $cdt^{\mathrm{s}}_{P_{\mathrm{IF}}}$ are applied to P_{IF} in (21). Therefore, the receiver code clock $cdt^{\mathrm{r}}_{P_{\mathrm{IF}}}$ and receiver phase clocks $cdt^{\mathrm{r}}_{L_{\mathrm{IF}}}$ are the remaining clock parameters to be estimated, which is to say, by applying separate satellite code/phase clocks and estimating separate receiver code/phase clocks, the integer N_1 ambiguity can be directly estimated in (48).

Suppose the user observes n satellites. The number of observations is $2n$ in (3) and (48). The ambiguity datum is required which can be defined by fixing the N_1 ambiguity of the reference satellite to arbitrary integer value. The number of unknown parameters is $3 + 2 + 1 + (n - 1)$, which includes three coordinates, two clocks (receiver code and phase), one troposphere delay, and $n - 1$ N_1 ambiguities. The degree of freedom is $2n - (3 + 2 + 1 + (n - 1)) = n - 5$, which means a minimum of five satellites are required to apply the integer phase clock model.

Method comparison

The user implementation details of the traditional method and the three PPP integer ambiguity resolution methods are listed in Table 1. Our method comparison focuses on the strategy of the integer property recovery, system redundancy, and the necessary corrections.

The methods are classified into two categories according to the characteristics of the satellite clock terms. The first category uses only the satellite code clock. This category includes the traditional PPP model and the single-difference between-satellites method. When the satellite code clock is applied to the phase observation in such models, the estimated receiver clock parameter is the receiver code clock and the corresponding ambiguity parameter is real-valued. For the traditional PPP model, integer phase ambiguity resolution is not feasible since no additional ambiguity corrections are provided to correct the real-valued ambiguities. For the single-difference between-satellites method, the integer property can be recovered and integer phase ambiguity resolution becomes feasible because two additional wide-lane and N_1 FCB corrections are provided to correct the real-valued ambiguities.

The second category is for methods that model satellite code clock and satellite phase clock, which includes the decoupled clock model and the integer phase clock model. The only difference between the two models is the approach for fixing the wide-lane ambiguity. The integer phase clock model utilizes the WSB corrections and the satellite-averaging process to fix the integer wide-lane ambiguity, whereas the decoupled clock model directly estimates the integer wide-lane ambiguity along with other unknowns through the function model. As to the N_1 ambiguity resolution, both models require the satellite code clock $cdt^{\mathrm{s}}_{P_{\mathrm{IF}}}$ and phase clock $cdt^{\mathrm{s}}_{L_{\mathrm{IF}}}$ and estimate the receiver code clock $cdt^{\mathrm{r}}_{P_{\mathrm{IF}}}$ and phase clock $cdt^{\mathrm{r}}_{L_{\mathrm{IF}}}$. As a result, the integer N_1 ambiguity can be directly estimated.

Table 1 Method comparison between the traditional PPP model and the three ambiguity fixing models in the user solution

	Traditional PPP	Decoupled clock	Single-difference between-satellites	Integer phase clock
Satellite clocks	IGS code clock: $cdt^{s}_{P_{IF}}$	Code clock: $cdt^{s}_{P_{IF}}$ Phase clock: $cdt^{s}_{L_{IF}}$ MW bias: $b^{s}_{A_{MW}}$	IGS code clock: $cdt^{s}_{P_{IF}}$	IGS code clock: $cdt^{s}_{P_{IF}}$ Phase clock: $cdt^{s}_{L_{IF}}$
Receiver clocks	Code clock: $cdt^{r}_{P_{IF}}$	Code clock: $cdt^{r}_{P_{IF}}$ Phase clock: $cdt^{r}_{L_{IF}}$ MW bias: $b^{r}_{A_{MW}}$	Code clock: $cdt^{r}_{P_{IF}}$	Code clock: $cdt^{r}_{P_{IF}}$ Phase clock: $cdt^{r}_{L_{IF}}$
Datum definition or reference satellite	N/A	One ambiguity datum	One reference satellite	One ambiguity datum
N_{WL} ambiguity correction	N/A	N/A	FCB $\Delta b^{s\,j,i}_{A_{MW}}$	WSB $b^{s}_{A_{MW}}$
N_1 ambiguity correction	N/A	N/A	FCB $\Delta b^{s\,j,i}_{A_1}$	N/A
Integer property	Not recovered	Recovered	Recovered	Recovered
Number of observations	$2n$	$3n$	$2n$	$2n$
Number of parameters	$3 + 1 + 1 + n$	$3 + 3 + 1 + 2(n - 1)$	$3 + 1 + 1 + n$	$3 + 2 + 1 + (n - 1)$
Degree of freedom	$n - 5$	$n - 5$	$n - 5$	$n - 5$
Required corrections	$cdt^{s}_{P_{IF}}$	$cdt^{s}_{P_{IF}}$ $cdt^{s}_{L_{IF}}$ $b^{s}_{A_{MW}}$	$cdt^{s}_{P_{IF}}$ $\Delta b^{s\,j,i}_{A_{MW}}$ $\Delta b^{s\,j,i}_{A_1}$	$cdt^{s}_{P_{IF}}$ $cdt^{s}_{L_{IF}}$ $b^{s}_{A_{MW}}$

If n satellites are observed by the user receiver, although the number of observations and unknown parameters is different in these methods, the degree of freedom $n - 5$ is the same for all methods. All methods require at least five satellites for position determination.

As to the broadcast requirements, the decoupled clock model requires three decoupled clocks $cdt^{s}_{P_{IF}}$, $cdt^{s}_{L_{IF}}$, and $b^{s}_{A_{IF}}$ for each satellite. For the single-difference between-satellites method, the IGS code clock $cdt^{s}_{P_{IF}}$ is required for both code and phase observations. Two satellite wide-lane and N_1 FCB corrections $\Delta b^{s\,j,i}_{A_{MW}}$ and $\Delta b^{s\,j,i}_{A_1}$ are necessary to correct the real-valued wide-lane and N_1 ambiguities. For the integer phase clock model, the satellite WSB correction $b^{s}_{A_{MW}}$ is needed to fix the wide-lane ambiguity. In addition, the IGS code clock $cdt^{s}_{P_{IF}}$ and the satellite phase clock $cdt^{s}_{L_{IF}}$ are required in order to resolve the integer N_1 ambiguity. In summary, all three PPP integer ambiguity resolution methods require three corrections for each satellite. From this point of view, the correction broadcasting burden is the same.

The analysis above has demonstrated that the three methods developed for integer ambiguity resolution in PPP are equivalent since they all based on the same ionosphere-free code and phase combinations [(3), (4), and (17)] and the same phase ambiguities (wide-lane and N_1) are resolved. These methods differ only in their approaches to remove the fractional phase part, such as different parameterizations for clock and bias modeling and corrections as shown in Table 1. Although small numerical difference may exist due to different computational procedures, the three methods will provide equivalent positioning solution and precision once the phase ambiguities are correctly resolved to their integer values.

Conclusions and discussions

We first explained the reason why integer ambiguity resolution is not feasible in the traditional PPP model. The three PPP integer ambiguity resolution methods are then described using the same notation, which helps better understand these methods and is of value for their proper implementation by users. Since no assumption is made during the method derivation, this contribution also supplements the previous method comparison work which ignores the effects of the code biases on the phase ambiguity in PPP.

Some practical differences exist among the three methods, for example, the consideration of the code and phase biases into the separate code and phase clocks (the decoupled clock model), the ambiguity corrections (the single-difference between-satellites method), or both the separate code and phase clocks plus the ambiguity corrections (the integer phase clock model); the procedure for the integer property recovery by estimation (the decoupled clock model), ambiguity correction (the single-difference between-satellites method), or both ambiguity correction and estimation (the integer phase clock model). However, the three methods will provide equivalent results once the phase ambiguities are correctly resolved to the integer values.

The comparison among the three PPP integer ambiguity resolution methods is conducted with respect to integer property recovery method, system redundancy, and required corrections. As all these methods require three corrections to recover the ambiguity integer property and the system redundancy for all methods is equal to n-5 (n denotes the number of observed satellites), the equivalence of these three methods for PPP integer ambiguity resolution in the user solution has been obtained.

Acknowledgments Paul Collins, Jianghui Geng, and Denis Laurichesse are sincerely acknowledged for their valuable discussions and suggestions. We also appreciate the editor and the two anonymous reviewers for their comments on this manuscript. This research was supported by NSERC and the Liaoning Talent Program (Grant No. LR2011007). In addition, the first author is financially supported by Key Laboratory of Precise Engineering and Industry Surveying, National Administration of Surveying, Mapping and Geoinformation (Grant No. PF2012-13), and China Scholarship Council.

References

Collins P (2008) Isolating and estimating undifferenced GPS integer ambiguities. In: Proceedings of ION NTM-2008, Institute of Navigation, San Diego, California, Jan, pp 720–732

Collins P, Bisnath S, Francois L, Héroux P (2010) Undifferenced GPS ambiguity resolution using the decoupled clock model and ambiguity datum fixing. Navigation 57(2):123–135

Dow J, Neilan R, Rizos C (2009) The international GNSS service (IGS) in a changing landscape of global navigation satellite system. J Geod 83(3–4):191–198

Gao Y, Shen X (2002) A new method for carrier phase based precise point positioning. Navigation 49(2):109–116

Ge M, Gendt G, Rothacher M, Shi C, Liu J (2008) Resolution of GPS carrier-phase ambiguities in precise point positioning (PPP) with daily observations. J Geod 82(7):389–399

Geng J, Teferle F, Shi C, Meng X, Dodson A, Liu J (2009) Ambiguity resolution in precise point positioning with hourly data. GPS Solut 13(4):263–270

Geng J, Meng X, Dodson A, Teferle F (2010) Integer ambiguity resolution in precise point positioning: method comparison. J Geod 84(9):569–581

Hatch R (1982) The synergism of GPS code and carrier measurements. In: Proceedings of the third international symposium on satellite Doppler positioning at Physical Sciences Laboratory of New Mexico State University, Feb 8–12, vol 2, pp 1213–1231

Kouba J (2009) A guide to using international GNSS service (IGS) products. http://igscb.jpl.nasa.gov/components/usage.html

Kouba J, Héroux P (2001) Precise point positioning using IGS orbit and clock products. GPS Solut 5(2):12–28

Laurichesse D, Mercier F, Berthias J, Bijac J (2008) Real time zero-difference ambiguities blocking and absolute RTK. In: Proceedings of the ION NTM-2008, Institute of Navigation, San Diego, California, Jan, pp 747–755

Melbourne WG (1985) The case for ranging in GPS-based geodetic systems. In: Proceedings of the first international symposium on precise positioning with the global positioning system, Rockville, MD, USA, 15–19 April

Shi JB, Gao Y (2010) Analysis of the integer property of ambiguity and characteristics of code and phase clocks in PPP using a decoupled clock model. In: Proceedings of ION GNSS-2010, The Institute of Navigation, Portland, Oregon, Sept, pp 2553–2564

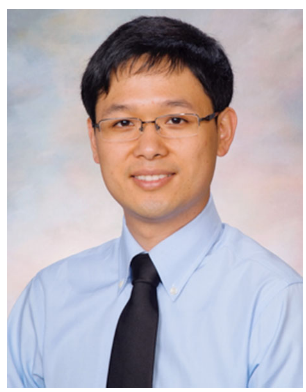
Wang M, Gao Y (2006) GPS un-differenced ambiguity resolution and validation. In: Proceedings of ION GNSS-2006, Institute of Navigation, Fort Worth, TX, Sept, pp 292–300

Wang M, Gao Y. (2007) An investigation on GPS receiver initial phase bias and its determination. In: Proceedings of ION NTM-2007, The Institute of Navigation, San Diego, California, Jan, pp 873–880

Wübbena G (1985) Software developments for geodetic positioning with GPS using TI-4100 code and carrier measurements. In: Proceedings of the first international symposium on precise positioning with the global positioning system, Rockville, MD, April

Zumberge JF, Heflin MB, Jefferson DC, Watkins MM, Webb FH (1997) Precise point positioning for the efficient and robust analysis of GPS data from large networks. J Geophys Res 102(B3):5005–5017

Author Biographies



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