GPS Solut (2012) 16:147–155
DOI 10.1007/s10291-011-0213-0

ORIGINAL ARTICLE

Temporal correlation for network RTK positioning

Robert Odolinski

Received: 13 May 2010 / Accepted: 3 February 2011 / Published online: 1 March 2011

Abstract Temporal correlation in network real-time kinematic (RTK) data exists due to unmodeled multipath and atmospheric errors, in combination with slowly changing satellite constellation. If this correlation is neglected, the estimated uncertainty of the coordinates might be too optimistic. In this study, we compute temporal correlation lengths for network RTK positioning, i.e., the appropriate time separation between the measurements. This leads to more realistic coordinate uncertainty estimates, and an appropriate surveying strategy to control the measurements can be designed. Two methods to estimate temporal correlation lengths are suggested. Several monitor stations that utilize correction data from two SWEPOS^{TM} Network RTK services, a standard service and a project-adapted service with the mean distance between the reference stations of approximately 70 and 10–20 km, are evaluated. The correlation lengths for the standard service are estimated as 17 min for the horizontal component and 36–37 min for the vertical component. The corresponding estimates for the project-adapted service are 13–17 and 13–16 min, respectively. According to the *F* test, the proposed composite first-order Gauss–Markov autocovariance function shows a significantly better least-squared fit to data compared to the commonly used one-component first-order Gauss–Markov model. A second suggested method is proposed that has the potential of providing robust correlation lengths without the need to fit a model to the computed autocovariance function.

R. Odolinski (✉)
GNSS Research Centre, Curtin University of Technology,
Perth, WA, Australia
e-mail: Robert.odolinski@curtin.edu.au

Keywords GNSS · Network RTK · Temporal correlation · Correlation length

Introduction

Temporal correlation is important when estimating the uncertainty (GUM 1994) in positioning with network RTK (real-time kinematic). Temporal correlations appear due to multipath and antenna effects, atmospheric errors including ionosphere and troposphere, and slowly changing satellite constellation. Ignoring temporal correlations can lead to overly optimistic uncertainty estimation of coordinates, in particular for a short time of observation.

Howind et al. (1999) present a correlation study for double differencing and baselines of 46–458 km length and investigate the reliability of station coordinates considering temporal correlation. The temporal correlation is described by a first-order autoregressive process, using correlation lengths of 5–20 min to fill up the nondiagonal elements of the variance covariance matrix. They concluded that temporal correlation must be taken into account to obtain reliable station coordinates, especially for deformation analysis where coordinate changes of up to 2 cm might appear otherwise. Tiberius (2001) uses this process to populate the elements in the variance–covariance matrix to compute the variance of the mean value. This value is subsequently compared to the variance of the mean value based on a diagonal variance–covariance matrix, containing no temporal correlation. One of the main conclusions is that the estimated standard deviation of the mean value becomes too optimistic if the temporal correlation is neglected. Teunissen and Amiri-Simkooei (2008) present the least-square variance component estimation method for estimating the parameters in the GPS stochastic model for raw range observables and position coordinates. They exemplify the estimation of the autocovariance function and the significance of results by the standard deviation of the estimated variance components.

Kjørsvik (2002) demonstrates that the autocorrelation function for network RTK positioning, based on the virtual reference station (VRS) concept and short baselines of 7 and 12 km, decreases significantly slower than that for single base RTK positioning. Vollath et al. (2002) and Emardson et al. (2009) present correlation length estimates for different error sources, including multipath, ionosphere, and troposphere, all based on network RTK and the VRS concept. These estimates are modeled assuming a first-order Gauss–Markov process. The correlation lengths are obtained at the normalized autocovariance value of 1/e, which is a well-known approach for single GNSS error sources. The ionospheric correlation length is estimated as 7–17 min and the geometric correlation length as 8–73 min, which includes local effects such as troposphere. These estimates are based on the time series of 24 h of VRS data and baselines that are 19–31 km to the closest reference station, for different parts of the world, including Germany, USA, Japan, and Australia. Emardson et al. (2009) estimate the correlation lengths as 4 min for multipath effects, 17 min for ionospheric effects, and 112 min for tropospheric effects, based on a 40-km baseline in the Swedish reference station Network SWEPOS™.

The main purpose of this study is to quantify the temporal correlation for network RTK positioning by estimating correlation lengths. An important difference with respect to previous studies is the emphasis on estimating correlation lengths for position coordinates and not for single-error sources as in Vollath et al. (2002) and Emardson et al. (2009). Furthermore, many of the other studies mentioned above did not directly intend to quantify and estimate these kinds of correlation lengths. The correlation length estimates in this study are specifically for network RTK rovers and especially for the users of SWEPOS.

The correlation lengths can be used to form appropriate surveying strategies to control and evaluate the measurements. This will result in more realistic coordinate uncertainty estimates compared to estimates obtained from a short observation span. Suitable uncertainty levels for a revisit of a point are presented in Odolinski (2010), computed by the law of error propagation with a confidence level of at least 95%. As an example, the uncertainty level for the horizontal component was estimated to ±60 mm and for the vertical component ±80 mm (ellipsoidal height). If the deviation for the revisit from the previous measurement(s) exceeds these uncertainty levels, a further investigation into gross errors and falsely estimated integer ambiguities should be conducted. Since the uncertainty levels are computed based on a diagonal variance–covariance matrix in the law of error propagation, with no temporal correlation, the revisit should be performed with an appropriate time separation between the measurements. Consequently, correlation length estimates for position coordinates are needed.

Two methods to calculate correlation lengths are suggested. The first method is a composite first-order Gauss–Markov process modeling the autocovariance function. The second method utilizes data from the computed autocovariance function, and by comparing the variance of the mean value, with and without temporal correlation, the correlation length is obtained. The first section introduces the concepts and methods. The section on results provides estimates and analysis of the correlation lengths for different baselines and identifies the composite first-order Gauss–Markov process as the choice to model the autocovariance.

Method

The autocovariance function is introduced, and two different methods to estimate correlation lengths are suggested. Whereas the equations are known, some novelty can be found in the composite first-order Gauss–Markov autocovariance function. This model contains additional parameters and has its origin in the well-known one-component model. The second suggested method estimates correlation lengths without the need to fit a model to the autocovariance function by least squares.

Autocovariance function

A stationary time series has a constant mean value and variance. The probability density function is in this case only dependent on the time lag $\tau = t_1 - t_2$ and not on the absolute times t_1 and t_2. The autocovariance function for a stationary process is defined as follows:

$$C_{yy}(\tau) \equiv E[\{y(t) - \mu\}\{y(t+\tau) - \mu\}] \quad (1)$$

where y contains the measured values, t is time, μ is the true mean value approximated by the mean value $\bar{y}$ of all measured values, and τ is the lag time. Eq. (1) is estimated as follows:

$$\hat{C}_{yy}(k) = \frac{1}{N-k}\sum_{i=1}^{N-k}[y_i - \bar{y}][y_{i+k} - \bar{y}] \quad (k = 0, 1, \ldots, M) \text{ and } (M \ll N) \quad (2)$$

where $k = 0, 1, \ldots, M$ is the number of sampling time increments with $M \ll N$ much smaller than the total length of the time series N, and $\tau = k\Delta t$ with Δt being the time interval between the contiguous observations.

Initially, it is important to correct for low-frequency trends in the time series. In simple cases, only the mean value needs to be removed, and in other cases a more complicated trend removal is necessary. Trends should be removed with care to avoid introduction of erroneous data into the time series during the subtraction. The variance of the least squares estimated autocovariance function, using equal weights and for the case with a true mean value, is given by (Teunissen and Amiri-Simkooei 2008):

$$\sigma^2_{\hat{C}_{yy}(k)} = \frac{\hat{C}_{yy}(0)^2}{a_k(N-k)}, \left(a_0 = \frac{1}{2} \text{ and } a_k = 1 \text{ for } k \neq 0\right) \quad (3)$$

This expression shows that increasing the time lag yields larger standard deviations for the estimates of the autocovariance function (2), which makes sense since less data are used.

Composite first-order Gauss–Markov process

The correlation length in the following approach is determined when the autocovariance function (2) reaches $\hat{C}_{yy}(0)/e$. If the measurements are separated by this correlation length, some correlations still remain but are reduced to $\hat{C}_{yy}(0)/e \approx 0.37 \cdot \hat{C}_{yy}(0)$, which are conventionally considered independent measurements. One way to estimate this correlation length is to assume a first-order Gauss–Markov process with autocovariance function:

$$C_{yy}(\tau) = ae^{-\tau/\tau_c} \quad (4)$$

where a is the amplitude and τ_c is the correlation length, and the parameters are estimated by least squares. Eq. (4) is a well-known model of the autocovariance for several GNSS error sources such as ionosphere, troposphere, and multipath (Vollath et al. 2002). However, Eq. (4) is appropriate for the analysis of one error source at a time, and GNSS observations contain all error sources combined. Problems appear particularly for small time lags τ when white noise, multipath errors, and antenna effects are assumed to dominate. Instead of (4), a composite first-order Gauss–Markov autocovariance function is proposed:

$$C_{yy}(\tau) = a_0 + a_1 e^{-\tau/\tau_1} + a_2 e^{-\tau/\tau_2} \quad (5)$$

where a_1 and the correlation length τ_1 are assumed to be associated with multipath errors and antenna effects and a_2 and the correlation length τ_2 are assumed to pertain to the atmospheric effects, which include ionosphere and troposphere. Finally, a_0 is assumed to model the remaining part which the other parameters are unable to handle. These assumptions are based on the varying correlation length estimated for different error sources (Vollath et al. 2002; Emardson et al. 2009).

The parameter τ_2 is henceforth denoted "correlation length", since atmospheric errors have been estimated to have the longest correlation length, and all other errors are at this point assumed to be uncorrelated or close to uncorrelated. The nonlinear function (5) is fitted to the estimated autocovariance function (2). The vector of the function f is defined as

$$f(\tau) = a_0 + a_1 e^{-\tau/\tau_1} + a_2 e^{-\tau/\tau_2} - \hat{C}_{yy}(\tau) \quad (6)$$

where $\tau = k\Delta t$, $k = 0, 1, \ldots, M$, and $\Delta t = 1$ s. The first-order Taylor series expansion of the function f for row k is given by

$$\begin{aligned} J_k &= \begin{bmatrix} \frac{\partial f(k)}{\partial a_0} & \frac{\partial f(k)}{\partial a_1} & \frac{\partial f(k)}{\partial \tau_1} & \frac{\partial f(k)}{\partial a_2} & \frac{\partial f(k)}{\partial \tau_2} \end{bmatrix} \\ &= \begin{bmatrix} 1 & e^{-k/\tau_1} & \frac{k}{\tau_1^2} a_1 e^{-k/\tau_1} & e^{-k/\tau_2} & \frac{k}{\tau_2^2} a_2 e^{-k/\tau_2} \end{bmatrix} \end{aligned} \quad (7)$$

The initial values for the unknown parameters $c = [a_0, a_1, \tau_1, a_2, \tau_2]$ are obtained by trial and error. The least-squares solution of the linearized equation system with unit observational weights is

$$\delta c = -(J^T J)^{-1} J^T f \quad (8)$$

where $c + \delta c$ represents the updated parameters. The system (8) is iterated until $\delta c \approx 0$. The a posteriori standard deviation of unit weight is then calculated as

$$\hat{\sigma}_0 = \sqrt{\frac{\varepsilon^T \varepsilon}{M - m}} = \sqrt{\frac{\varepsilon^T \varepsilon}{M - 5}} \quad (9)$$

In this expression, ε represents the residuals, M is the number of observations or rows in the J matrix, and m is the number of unknowns, which in this case are the five unknown parameters a_0, a_1, τ_1, a_2, and τ_2. The variance–covariance matrix of the estimated parameters is

$$\hat{C}_{cc} = \hat{\sigma}_0^2 (J^T J)^{-1} \quad (10)$$

Equal weights are pragmatically chosen in (8) since 1 month and 1 Hz frequency data are used for the computation of (2). Consequently, the autocovariance function (2) at time lag $k = M$ has a high redundancy. However, the estimates in the autocovariance function (2) are correlated for some time lags since the same observations are used for different lags. Nevertheless, equal weighting should give realistic a_0, a_1, τ_1, a_2, and τ_2 estimates, although probably with somewhat overoptimistic standard deviations for the estimated parameters.

Combining ionospheric and tropospheric effects into one atmospheric effect modeled by a_2 and τ_2 might not be fully correct. It has been shown that these effects have significantly different separate correlation lengths.

According to empirical data, the correlation coefficients between the parameters obtained from (10) might in some cases become considerably large. This correlation can be explained by algebraic correlation in the formulation of the least-squares solution due to the J matrix (7). On the other hand, the condition number of the design matrix J is in all cases sufficiently low to assure a large number of significant digits of the estimated parameters. If another first-order Gauss–Markov process term is added into (5) to separately model the ionospheric and tropospheric effects, which in reality would be desirable, it would result in even higher correlation between the parameters. The consequence could be an overparameterization leading to interpretation and numerical problems when estimating the parameters.

The correlation between the parameters in (5) leads to the realization that it is not necessarily true that all atmospheric effects are being absorbed by a_2 and τ_2 but can partly be absorbed by a_0, a_1, and τ_1 as well and vice versa. Nevertheless, the correlation lengths can still be regarded as an appropriate time separation to obtain uncorrelated measurements for revisits with network RTK. The composite first-order Gauss–Markov autocovariance function (5) will henceforth be denoted "composite model", and the one-component first-order Gauss–Markov autocovariance function (4) will be denoted "one-component model".

The variance of the mean value and the number of effective observations

We introduce another robust technique for estimating correlation lengths and compare it with the previous approach. A similar exposition can be found in Tiberius (2001), but with the purpose of showing the differences between the variance of the mean value obtained by neglecting or considering temporal correlation and not to specifically estimate correlation lengths.

The number of "effective observations" equals the number of uncorrelated measurements. This number can be used to estimate the correlation lengths between measurements. Consider a large number of measurements, for example $n = 3600$ measurements with 1-s interval. With a correlation length of 10 min, only $n_* = 6$ measurements out of 3,600 are effective observations. This means that only 6 out of 3,600 measurements are uncorrelated measurements.

The variance–covariance matrix is given by the autocovariance function (1) as

$$Q_{ij} = C_{yy}(\tau_{ij}) = C_{yy}(|i-j|) \quad (i,j = 0,1,\ldots,n) \tag{11}$$

where τ_{ij} is the time lag between measurement i and j. The variance of the mean value is then given by the law of error propagation as

$$\sigma_{\bar{x}}^2 = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n} Q_{ij} \tag{12}$$

If the measurements are uncorrelated, then $Q_{ij} = 0$ for $i \neq j$ and $Q_{ii} = \sigma^2$. The variance of the mean value for the case without correlations is then defined as

$$\sigma_{\bar{x}_0}^2 = \frac{1}{n^2}\sum_{i=1}^{n} Q_{ii} = \frac{1}{n^2} n\sigma^2 = \frac{\sigma^2}{n} \tag{13}$$

If the variance σ^2 of the measurements and the mean value are known or realistically estimated, the number of effective observations can be calculated. Eq. (12) is set equal to (13), giving

$$\begin{aligned} \frac{\sigma^2}{n_*} &= \sigma_{\bar{x}}^2 = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n} Q_{ij} \\ n_* &= \frac{\sigma^2}{\sigma_{\bar{x}}^2} = \frac{\sigma^2}{\frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n} Q_{ij}} = \frac{n^2\sigma^2}{\sum_{i=1}^{n}\sum_{j=1}^{n} Q_{ij}} \end{aligned} \tag{14}$$

The ratio of n to n_*, multiplied by Δt, gives

$$\Omega = \frac{n\Delta t}{n_*} \tag{15}$$

the correlation length Ω. The correlation length can finally be compared to τ_2 obtained from the composite model (5).

Rover data from the SWEPOS Network

SWEPOS is a permanent GNSS reference station network in Sweden. We used data from three monitor stations (Vetlanda, Nol, and Marieholm) reconfigured as permanent rovers. These monitor stations are all located in the southern part of Sweden and receive correction data from two SWEPOS Network RTK (VRS) services, the standard service (NRTK) and a project-adapted service (PRTK), with the mean distance between the reference stations of approximately 70 and 10–20 km, respectively (Fig. 1). By continuously logging RTK positions as latitude, longitude, and ellipsoidal height with 1 Hz frequency, the stations are used to evaluate the expected quality of the network RTK positioning. Vetlanda is a monitor station for the standard service (NRTK) located 26 km from the closest reference station. It receives the VRS corrections as a regular user, while Nol and Marieholm (PRTK) are located 7 and 3 km from the closest reference station and receive corrections for a fixed and pre-determined VRS position. The latitudes and longitudes were transformed to northing, easting in the SWEREF 99 reference frame, and finally compared with 4 weeks of data used as true benchmarks and computed with the Bernese software v 5.0.

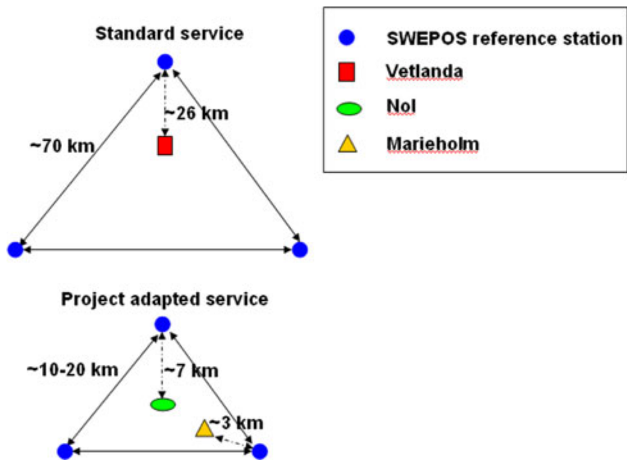

Fig. 1 Location of the monitor stations Vetlanda (NRTK), Nol, and Marieholm (PRTK)

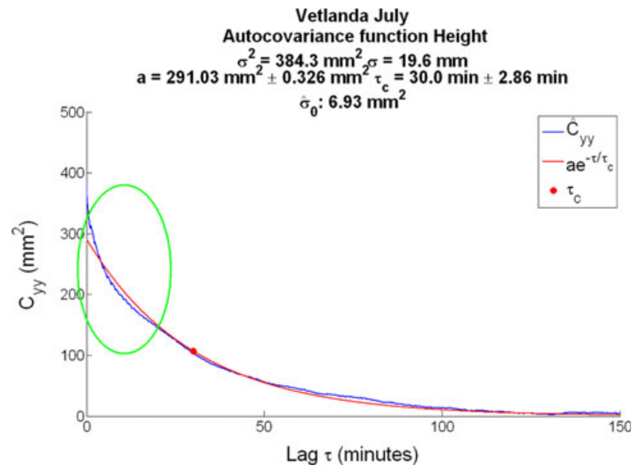

Fig. 2 Autocovariance function (2) and a least-squares one-component model (4) for the height component. Data refer to station Vetlanda and July 2009

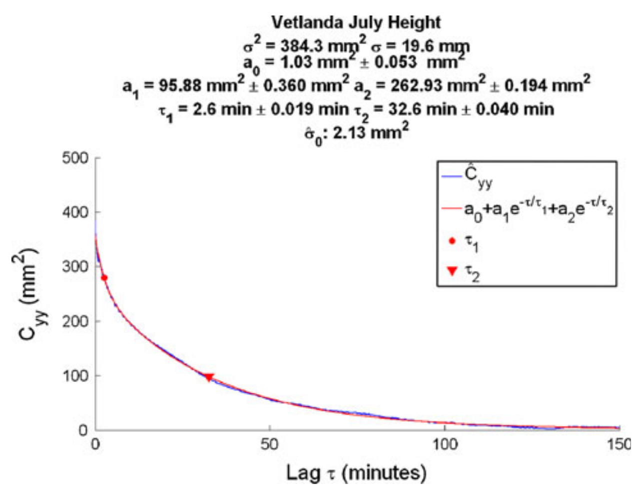

Fig. 3 Autocovariance function (2) and least-squares composite model (5) for the height component. Data refer to Vetlanda and July 2009

Results

Estimates and analyses of the correlation lengths for different baselines are presented, and justification is provided in support of modeling the autocovariance with the composite model. Correlation lengths are then estimated by the second method and compared.

One-component model versus composite model

We first verify that the composite model (5) provides a significantly better least-squared fit to data compared to a one-component model (4). We begin with removing the mean value of each coordinate component prior to the calculation of the autocovariance function (2). The one-component model (4) is then assumed for Vetlanda. Using data for July 2009, the results for the height component are seen in Fig. 2. The blue line is the estimated autocovariance function (2), whereas the red line denotes the estimated one-component model (4). The parameter τ_c is marked as a red dot. The standard deviation of unit weight $\hat{\sigma}_0$ is calculated by (9), using $m = 2$ for the two unknowns a and τ_c. As stated earlier, problems appear particularly for small time lags in the area of the green circle where white noise, multipath errors, and antenna effects are dominating.

Figures 3, 4, and 5 represent the height, northing, and easting components of station Vetlanda and the composite model (5). The autocovariance function (2) is denoted in blue, whereas the red line denotes the composite model (5) estimated according to (6–8). The parameter τ_1 is marked as a red dot and is assumed to show the correlation length for multipath errors and antenna effects. The red triangle indicates the assumed correlation length τ_2 for atmospheric effects. Consequently, τ_2 is the time lag for which the measurements are assumed to be uncorrelated or close to uncorrelated, because when both the ionospheric and tropospheric effects are uncorrelated, no other temporal correlation is assumed to remain. The standard deviations for a_0, a_1, τ_1, a_2, and τ_2 are the square root of the diagonal elements of the variance–covariance matrix (10). The standard deviation of unit weight is calculated by (9) with five unknowns.

Inspection of Figs. 2 and 3 and the respective standard deviation of unit weight reveal that the composite model (5) has a smaller sum of squared residuals than the one-component model (4). This is expected since additional parameters in a model usually tend to decrease the sum of the squared residuals. In order to investigate the significance of this improvement, the standard deviation of unit weight for (4) and (5) is provided for all monitor stations and the height component in Table 1. The data are Vetlanda (July through September), Marieholm (April and May), and Nol (April and May).

The second and fourth columns represent the number of degrees of freedom for the two models, which is seen to be 2,995 or greater for all cases; hence, the noticeable differences in the standard deviation of unit weight are indeed significant. This can easily be verified by the F test (Bjerhammar 1973):

$$F = \frac{\left(\frac{\varepsilon_2^T\varepsilon_2-\varepsilon_5^T\varepsilon_5}{p_2-p_1}\right)}{\frac{\varepsilon_5^T\varepsilon_5}{f_5}} = \frac{\left(\frac{\varepsilon_2^T\varepsilon_2-\varepsilon_5^T\varepsilon_5}{5-2}\right)}{\frac{\varepsilon_5^T\varepsilon_5}{M-5}} \tag{16}$$

where ε_2 and p_1 are the residuals and number of parameters for the one-component model (4), and ε_5 and p_2 are the respective values for the composite model. The degree of freedom for the composite model is f_5. The null hypothesis is that the composite model does not provide a significantly better least-squared fit to data, with distribution $F_{p2-p1,\ f5}$. The F ratio is compared to an upper critical value in a one-side tailed table of the F-distribution, and even for a significance level at 1%, the composite model (5) exceeds this tabular value by far. Consequently, the null hypothesis is rejected, and it can be concluded that (5) shows a significantly better least-squared fit to data than (4).

Correlation length estimates

Inspecting Figs. 3, 4, and 5 again we see that the correlation length τ_2 is estimated as 16–18 min for the horizontal component and 33 min for the vertical component. The autocovariance function (2) in these figures might be expected to be noisy. However, when comparing to 24 h of data of July 1, 2009, for Vetlanda and height component

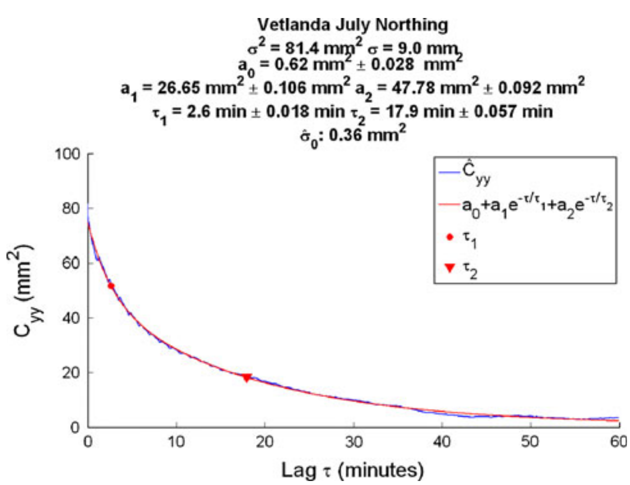

Fig. 4 Autocovariance function (2) and a least-squares composite model (5) for the northing component. Data refer to Vetlanda and July 2009

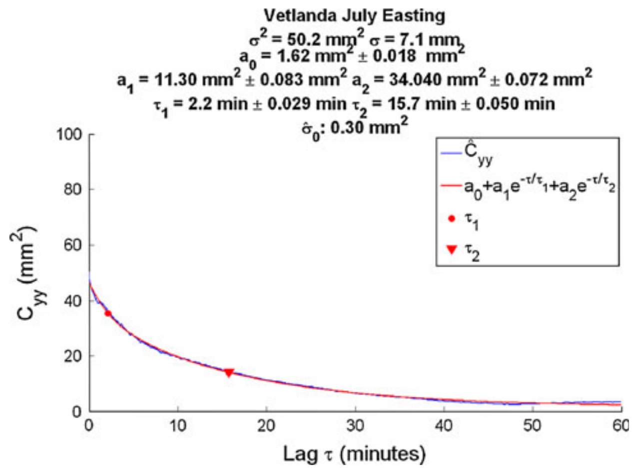

Fig. 5 Autocovariance function (2) and a least-squares composite model (5) for the easting component. Data refer to Vetlanda and July 2009

Table 1 The standard deviation of unit weight and degree of freedom for height and the one-component (4) and composite model (5)

Station/time	One-component		Composite	
	$\hat{\sigma}_0(\mathrm{mm}^2)$	M-2	$\hat{\sigma}_0(\mathrm{mm}^2)$	M-5
Vetlanda/July	6.93	8,998	2.13	8,995
Vetlanda/Aug	8.95	7,798	2.46	7,795
Vetlanda/Sep	6.10	7,198	1.63	7,195
Nol/April	3.47	4,198	0.76	4,195
Nol/May	4.71	4,198	0.98	4,195
Marieholm/April	2.20	3,598	0.56	3,595
Marieholm/May	2.66	2,998	0.37	2,995

Correlation length estimates

(Fig. 6), it can be concluded that the high redundancy resulting from 1 month of 1 Hz data is responsible for the less noisy appearance.

This conclusion is further supported by the estimated standard deviation (3) of the autocovariance function in Tables 2 and 3 for $k = 1$ and M s, where the small values are explained by the high redundancy of data N. For a smaller data set, say 24 h, the standard deviations would be larger. Note that Eq. (3) is for the case with a true mean value used in the calculation of (2); nevertheless, the high redundancy of data probably mitigates this issue.

The horizontal component is in Table 3 given by the mean value of northing and easting. Also, note that the varying sizes of N are explained by time gaps in the data due to re-initialization and removal of gross errors. However, these time gaps have been accounted for in the computation of the autocovariance function (2) and the related standard deviations (3).

Table 4 presents a summary of the correlation length τ_2 estimates for Vetlanda based on 3 months of data from July

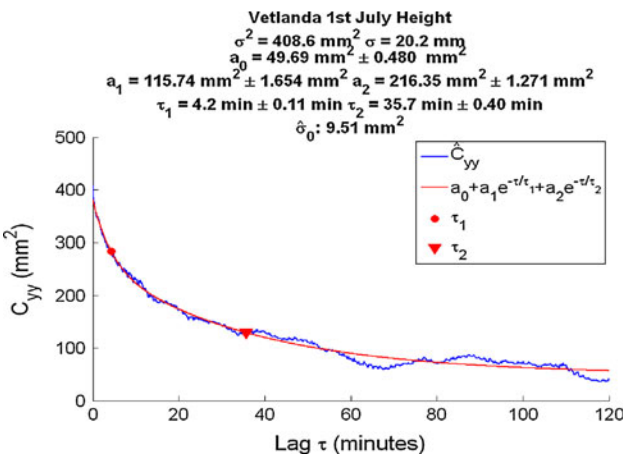

Fig. 6 Autocovariance function (2) and a least-squares composite model (5) for the height component. Data for Vetlanda and July 1, 2009

through September 2009, assuming a composite model (5). The horizontal correlation length τ_2 is given as the mean value of τ_2 for northing and easting. The standard deviations from (10) for τ_2 and the horizontal component are estimated as a combined value between northing and easting. Table 5 shows the respective results for stations Nol and Marieholm, based on monthly data from April and May 2009.

As seen in Tables 4 and 5, the magnitude of the correlation length τ_2 differs a few minutes from month to month. This is more obvious in Table 4 for the height component of Vetlanda where the value for August differs from that of July and September by 9 min. Imprecise tropospheric modeling caused by different environments at the reference stations and the receiver might cause these effects. Hence, an analysis of the humidity at the weather station Målilla located close to Vetlanda was performed. No significant difference between July and September was found for the humidity.

This correlation length difference could also be explained by geomagnetic storms affecting the ionosphere. Geomagnetic storms are measured by a kp index provided from NOAA observatories for stations mostly located in North America and one at Hartland, UK. Three days were found to have quite active geomagnetic storms in August, http://www.swpc.noaa.gov/ftpmenu/warehouse/2009.html where geomagnetic field activity ranged from quiet to major storm levels for August 30. A mean value of τ_2 for the height component and for these 3 days was estimated as 56.3 min, with $\sigma_{\tau 2} = 0.22$ min and $\hat{\sigma}_0 = 3.0\ \text{mm}^2$. It was thus concluded that the geomagnetic storms indeed seem to affect the correlation length for August. In July and September, only minor indications of sun activity were found for 2 days, and consequently, these days were not further investigated.

Number of effective observations

This approach, based on (11–15), provides correlation lengths without the need to fit a model to the computed autocovariance function as in (5). Additionally, the approach can provide information as to whether or not the estimated value τ_2 is reasonable.

In order to compute the number of effective observations, the autocovariance function (2) is used, which subsequently yields the correlation lengths Ω via (11–15). For these computations, we use k up to M listed in Tables 2 and 3. Also, the same monitor stations as in the previous approach are used. The results are given in Tables 6 and 7. Note again that all values for the horizontal component are given by the mean value of northing and easting. The monthly data were also divided into daily data, yielding one Ω_i for each day i. Thus, a standard deviation estimate of Ω could be computed by computing the RMS around the "true" monthly based value of Ω divided by the square root of the number of days according to

$$\sigma_\Omega = \sqrt{\frac{\sum_{i=1}^{n} \frac{(\Omega_i - \Omega)^2}{n}}{n}} \tag{17}$$

where Ω_i is the correlation length based on M and 24 h of data for each day i, n is the total number of days for 1 month, and Ω is the correlation length based on 1 month of data.

Table 2 Standard deviation of the autocovariance function (3) for all monitor stations, height component, and $k = 1$ and M s

Station/time	N	M	$\hat{C}_{yy}(0)$ (mm²)	$\sigma_{\hat{C}_{yy}(1)}$(mm²)	$\sigma_{\hat{C}_{yy}(M)}$(mm²)
Vetlanda/July	2,025,868	9,000	384.3	0.27230	0.30747
Vetlanda/Aug	2,178,566	7,800	458.9	0.31250	0.33450
Vetlanda/Sep	2,016,945	7,200	275.5	0.19560	0.21575
Nol/April	2,188,851	4,200	83.8	0.05716	0.06083
Nol/May	2,204,497	4,200	107.5	0.07299	0.07742
Marieholm/April	2,119,917	3,600	64.9	0.04507	0.04825
Marieholm/May	2,237,374	3,000	73.6	0.04961	0.05323

Table 3 Standard deviation of the autocovariance function (3) for all monitor stations, horizontal component, and $k = 1$ and M s

Station/time	N	M	$\hat{C}_{yy}(0)$ (mm^2)	$\sigma_{\hat{C}_{yy}(1)}$(mm^2)	$\sigma_{\hat{C}_{yy}(M)}$(mm^2)
Vetlanda/July	2,025,868	3,600	65.8	0.04665	0.05233
Vetlanda/Aug	2,178,566	3,600	61.8	0.04204	0.04471
Vetlanda/Sep	2,016,945	3,600	46.7	0.03317	0.03636
Nol/April	2,188,851	4,200	19.4	0.01325	0.01410
Nol/May	2,204,497	3,600	24.6	0.01670	0.01789
Marieholm/April	2,119,917	3,600	14.3	0.00989	0.01058
Marieholm/May	2,237,374	3,600	15.7	0.01054	0.01133

Table 4 Correlation length τ_2 estimates and standard deviations for Vetlanda based on data from July through September 2009

Station/time	Horizontal		Vertical	
	τ_2 (min)	$\sigma_{\tau 2}$ (min)	τ_2 (min)	$\sigma_{\tau 2}$ (min)
Vetlanda/July	16.8	0.054	32.6	0.040
Vetlanda/Aug	18.8	0.083	41.4	0.065
Vetlanda/Sep	16.7	0.139	32.6	0.065
Mean:	17.4		35.5	

Table 5 Correlation length τ_2 estimates and standard deviations for Nol and Marieholm based on data from April and May 2009

Station/time	Horizontal		Vertical	
	τ_2 (min)	$\sigma_{\tau 2}$ (min)	τ_2 (min)	$\sigma_{\tau 2}$ (min)
Nol/April	20.5	0.23	21.9	0.24
Nol/May	17.7	0.28	18.0	0.18
Marieholm/April	14.5	0.29	12.4	0.23
Marieholm/May	13.9	0.22	11.0	0.083
Mean:	16.7		15.8	

Table 6 Correlation length Ω estimates and standard deviations for Vetlanda based on data from July through September 2009

Station/time	Horizontal		Vertical	
	Ω (min)	σ_Ω (min)	Ω (min)	σ_Ω (min)
Vetlanda/July	17.9	0.69	36.7	2.44
Vetlanda/Aug	17.5	0.73	44.0	3.06
Vetlanda/Sep	14.4	0.97	30.1	2.76
Mean:	16.6		36.9	

Table 7 shows that the correlation length Ω for the vertical component is close to the horizontal component for the project-adapted service. The densification of the network to 10–20 km between the reference stations for this service decreases the correlation length Ω for the vertical component, compared with Table 6, due to more effective ionospheric and tropospheric modeling over short baselines.

Table 7 Correlation length Ω estimates and standard deviations for Nol and Marieholm based on data from April and May 2009

Station/time	Horizontal		Vertical	
	Ω (min)	σ_Ω (min)	Ω (min)	σ_Ω (min)
Nol/April	13.5	0.65	14.5	0.87
Nol/May	12.3	0.56	14.2	0.91
Marieholm/April	12.2	0.99	10.0	0.84
Marieholm/May	14.1	0.96	11.7	0.73
Mean:	13.0		12.6	

Conclusions

Two methods indicated by Eqs. (5) and (11–15) have been suggested for estimating correlation lengths for network RTK positioning. The composite first-order Gauss–Markov model (5) provided a significantly better least-squared fit to data compared to the normally used one-component first-order Gauss–Markov model (4) (Figs. 2, 3; Table 1). The F ratio (16) exceeds a one-sided tabular value from the F-distribution by far for a significance level at 1%, and it can thus be concluded that the composite model shows good capability to model the autocovariance function for network RTK positioning. Additionally, the approach in (11–15) has shown the potential of providing robust correlation lengths without the need to fit a model to the computed autocovariance function. This is of great benefit for smaller data sets, e.g., 24-h data sets, since the standard deviation (3) of the autocovariance function worsens for less data and consequently may lead to difficulties in reaching convergence of the least-squares iteration (6–8).

The correlation lengths estimated in this study are expected to be updated and recomputed when more monitoring stations are installed in other parts of Sweden, providing more data with different environmental and spatial characteristics. Future analyses should include different parts of the ionospheric cycle and different weather conditions. Additionally, the correlation lengths differ with latitudes because ionospheric activities differ; the southern part of Sweden is less affected than the northern part. The ionosphere also shows seasonal variations with higher sun activity during the Swedish winter months. Therefore, it is necessary to collect and evaluate data during the winter time and in the northern parts of Sweden.

Acknowledgments The author wants to thank the editor and two anonymous reviewers for the constructive comments and suggestions, significantly improving the quality of this paper. A further acknowledgment is given to Professor Clas-Göran Persson at the Royal Institute of Technology (KTH) and Geodetic Research Department of Lantmäteriet (Swedish mapping, cadastral and land registration authority) for the invaluable suggestions improving the contents of the paper.

References

Bjerhammar A (1973) Theory of errors and generalized matrix inverses. Elsevier, Amsterdam, London, New York

Emardson R, Jarlemark P, Bergstrand S, Nilsson T, Johansson J (2009) Measurement accuracy in network-RTK. SP Sweden Technical Research Institute and Chalmers Technical University. SP report :23

GUM (1994) Guide to the expression of uncertainty In Measurement. international organization of standardization, 1st edn. Geneva Switzerland, Corrected and reprinted 1995

Howind J, Kutterer H, Heck B (1999) Impact of temporal correlations on GPS-derived relative point positions. J Geod 73(5)

Kjørsvik N (2002) Assessing the multi-base station GPS solutions. FIG XXII International Congress, April 19–26, Washington D.C. USA

Odolinski R (2010) Swedish user guidelines for network RTK. In: proceedings of the XXIV FIG International Congress, Sydney, 2010

Teunissen PJG, Amiri-Simkooei AR (2008) Least-squares variance component estimation. J Geod 82

Tiberius C (2001) A univariate analysis of the impact of time correlation. Boll Geod 60(1):33–48

Vollath U, Landau H, Chen X, Doucet K, Pagels C (2002) Network RTK versus single base RTK—understanding the error characteristics. In: ION GPS 2002, Portland, USA, Sept. 24–27, 2002

Author Biography

Robert Odolinski is currently a PhD student at the GNSS Research Centre of Curtin University of Technology in Perth, Australia, where his research theme is PPP-RTK. He was previously a Research Assistant at the Geodetic Research Department of Lantmäteriet (Swedish mapping, cadastral and land registration authority). He holds an MSc in Geodesy and Geoinformatics from the Royal Institute of Technology (KTH) in Stockholm, Sweden.