0.99999999 confidence ambiguity resolution with GPS and Galileo

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Abstract In this short contribution it is demonstrated how integer carrier phase cycle ambiguity resolution will perform in near future, when the US GPS gets modernized and the European Galileo becomes operational. The capability of ambiguity resolution is analyzed in the context of precise differential positioning over short, medium and long distances. Starting from dual-frequency operation with GPS at present, particularly augmenting the number of satellites turns out to have beneficial consequences on the capability of correctly resolving the ambiguities. With a 'double' constellation, on short baselines, the confidence of the integer ambiguity solution increases to a level of 0.99999999 or beyond.

Introduction

High-precision GNSS positioning results are obtained with carrier phase measurements, once the integer cycle ambiguities have been successfully resolved. During the last decade much experience has been gained on fast and precise positioning with GPS as a dual-frequency system. The modernization of the GPS and the advent of Galileo will together lead to a truly *multi*-frequency civil Global Navigation Satellite System, enhancing the capability of resolving the carrier phase ambiguities. In this paper both the aspect of a third frequency and the number of satellites are addressed. In addition, the expected (combined) system's performance on ambiguity resolution is presented over time and for locations in Europe.

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Ambiguity resolution

The results in the present analysis rely on the use of the LAMBDA method. The LAMBDA method can handle, in an integral way, all carrier phase cycle ambiguities of a combined GPS-Galileo system, in exactly the same way, and without any modification, as it handles ambiguities in single- and dual-frequency GPS positioning at present. The integer least-squares principle embodied in the LAMBDA method has been proven to be optimal (Teunissen 1999). The *integer least-squares* estimator is best in the sense of maximizing the probability of correct integer estimation, i.e., in maximizing the *ambiguity success-rate*. This measure expresses, given a certain scenario, how successful one can expect to be in resolving the integer carrier phase ambiguities correctly. As an exact evaluation of this success-rate is complicated in the context of integer least-squares estimation, an approximation is used in this analysis, based on so-called bootstrapping. The bootstrap success-rate is a guaranteed (and hence safe) lower bound for the success-rate; the actual integer least-squares success-rate will be larger than, or at least equal to the value given. An introduction to the LAMBDA method and the success-rate can be found in, for example, Joosten and Tiberius (2000) and Joosten and Tiberius (2002).
The ambiguity success-rate – a statistical probability – is a number between 0 and 1 (to be interpreted as 0% and 100%, respectively). In this analysis usually the counterpart is presented, namely, the ambiguity *fail*-rate which simply equals one minus the success-rate. The fail-rate should be as small as possible, and ideally zero. In that case there would be no uncertainty at all about the obtained integer ambiguity values. The fail-rates given here pertain to resolving *all* ambiguities together, as we aim at an integral, all-in-once solution.

Results and analysis

Details of the satellite constellation and the frequency and signal plan for both modernized GPS and future Galileo,

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underlying the present analysis, are described in Tiberius et al. (2002). This reference also gives some considerations on signal tracking of a combined GPS-Galileo receiver.
Table 1 summarizes the GPS and Galileo signals used. The Galileo E2-L1-E1 signal, as an overlay on the GPS L1 signal, is denoted here as E1 for convenience. The Galileo E5a signal is an overlay on the GPS L5 signal.
Ambiguity resolution performance is assessed for three different baseline lengths. On a *short* baseline differential atmospheric delays are assumed to be completely absent (zero). These delays are to be accounted for on medium and long baselines. A (differential) tropospheric zenith delay and (double difference) ionospheric slant delays are included as unknown parameters, but the uncertainty in the values for these parameters has been restricted, dependent on the baseline length. Variations are tolerated to a reasonably small extent on a *medium* baseline (in the order of a few centimeters), and to a much larger extent on a *long* baseline (in the order of several decimeters; see, for example, Odijk 2000). The short baseline is typically only of a few kilometers length, the medium baseline some tens of kilometers (20–30 km, or longer when, for instance, corrective information from a network of active GNSS reference stations is used), and the long baseline can be hundreds of kilometers.
A full day period of 24 h was 'sampled' at a 2-min interval. The ambiguity fail-rates obtained are by default summarized into the 95th (sample) percentile value (and, for reasons of numerical accuracy, the fail-rate has been bounded at 10^{-8}; smaller values are anyhow presented as 10^{-8}). Single epoch solutions are considered, i.e., instantaneous positioning. With only one epoch of data no distinction needs to be made between kinematic and static positioning, and cycle slips are not an issue. Pseudorange code and carrier phase data are used together (with decimeter and millimeter noise, respectively). As a default location, the city of Copenhagen in Denmark is used, at 55°40′N, 12°35′E and 50-m height. A 10° satellite elevation cutoff angle is maintained.

Table 1
Basic GPS (L) and Galileo (E) signal parameters. Binary phase shift keying (BPSK) and binary offset carrier (BOC) signals are considered, with code rate indicated – preceded by sub-carrier frequency for the BOC signal – as multiples of 1.023 MHz

Signal	Frequency f (10.23 MHz)	Modulation
L1	154	BPSK(1)
L2	120	BPSK(1)
L5	115	BPSK(10)
E1	154	BOC(2,2)
E5a, E5b	115, 117.5	BPSK(10)
E6	125	BPSK(5)

Triple frequency

Figure 1 shows the ambiguity fail-rate for Galileo. A (nominal) 27 satellite constellation is used. As can be seen from the graph at left, the effect of adding a third frequency signal is negligible on the medium and long baselines. On the medium baseline the ambiguity fail-rate reduces only from 33% to 31%. On the long baseline, ambiguity resolution is definitely not feasible using just a single epoch of data; the ambiguity fail-rate is 0.99 or larger, anyhow.
From the graph at right, it can be seen that modifying the choice of the third frequency, i.e., using E5a – close to E5b (see Table 1) – instead of E6, brings hardly any changes to the ambiguity fail-rate.
When (carrier phase) ranging is carried out simultaneously on two or more frequencies, traditionally the idea of forming so-called wide-lane carriers is popular. The 'ordinary' wide lane of GPS L1 and L2 has a wavelength of

$$\lambda_{L1-L2} = \frac{c}{f_{L1} - f_{L2}},$$

which is 0.86 m (with c the speed of light). The wide lane which can be formed with the Galileo E6 and E5b carriers has a wavelength of λ_{E6-E5b}=3.91 m. Choosing carrier frequencies more close together yields an even larger wavelength; for instance, combining the Galileo E5b and

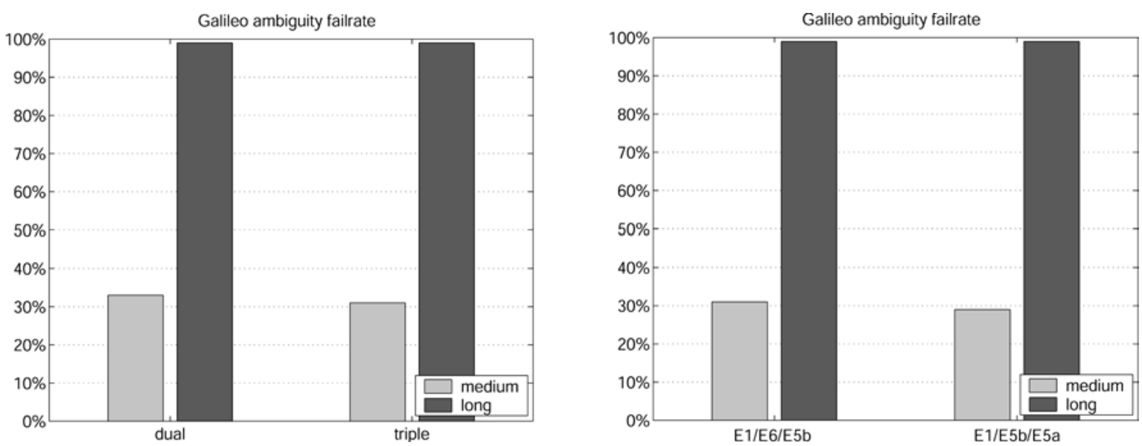

Fig. 1
Ambiguity fail-rate (95th percentile over 24 h) for medium and long baseline, with Galileo 27 satellite constellation. At *left*, dual frequency (E1, E5b) versus triple frequency (E1, E5b, E6); at *right*, two triple-frequency scenarios (E1, E5b, E6 versus E1, E5b, E5a). Undifferenced measurement standard deviation values are σ_{E1}=0.30 m, σ_{E5b}=σ_{E6}=σ_{E5a}=0.10 m

E5a would yield a wavelength of $\lambda_{E5b-E5a}$=11.72 m. One may be inclined to believe that resolution of the ambiguities is enhanced by choosing carrier frequencies more close together, as it results in a longer wavelength. In itself, there is nothing wrong with the reasoning of creating a longer wavelength when two frequencies are taken more close together, but it represents only half of the truth (see the study on long baselines in Teunissen et al. 1999). When ranging is carried out with *two* carriers, *two* ambiguities have to be resolved (or two admissible integer combinations of these instead). Only then does the full, final high precision become available for the coordinates. And, when three carriers are employed, all three ambiguities need to be resolved. One has to consider the *overall* success-rate (ibid.), taking into account measurement precision, observation scenario and modeling aspects. The differences in Fig. 1 at right, although the (long) wide-lane wavelength did increase here by a factor of 3, are marginal.

More satellites

Figure 2 compares, for GPS, the baseline 24 satellite constellation with an augmented, 30 satellite constellation (realized basically by occupying also the (spare) fifth slot in each orbit) for dual-frequency ranging. The dual-frequency fail-rate for the medium baseline is almost halved from 48% with 24 satellites to 28% with 30 satellites. The effect of just six extra satellites (globally) can be clearly seen. The GPS constellation at present is, by the way, actually closer to the latter than the former situation.

Combined GPS and Galileo

A high-end geodetic receiver may eventually track all three civil GPS signals, on L1, L2 and L5, and all three Galileo signals, on E1 and E5 (open service) and E6 (commercial service). Navigation receivers may rely on GPS L1 and L5, and on Galileo E1 and E5. The latter (double) dual-frequency scenario is analyzed in the following. We consider, in Fig. 3 at right, a combined GNSS constellation, with 24 GPS and 27 Galileo satellites.

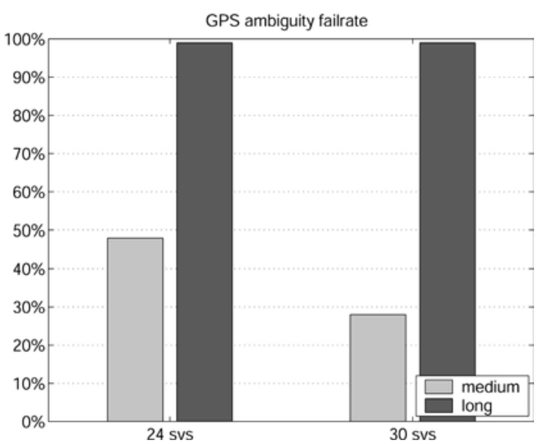

Fig. 2
Ambiguity fail-rate (95th percentile over 24 h) for medium and long baseline with GPS 24 satellite and 30 satellite constellation; dual frequency (L1, L2). Undifferenced standard deviation values are $\sigma_{L1}=\sigma_{L2}$=0.30 m

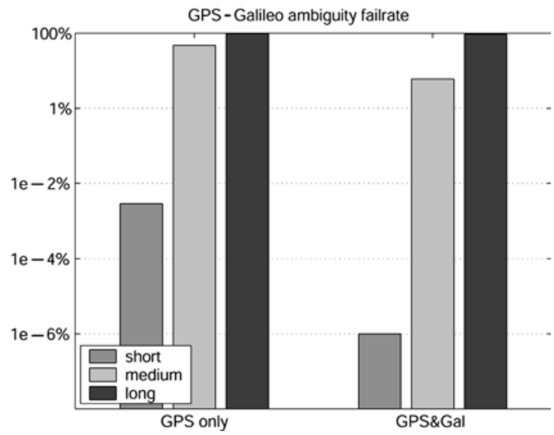

Fig. 3
Dual-frequency ambiguity fail-rate (95th percentile over 24 h) for short, medium and long baselines, with GPS-only (L1, L2) at *left* and GPS-Galileo (L1, L5, E1, E5b) satellite constellation at *right*. Undifferenced standard deviation values are $\sigma_{L1}=\sigma_{L2}$=0.30 m, $\sigma_{L5}=\sigma_{E5b}$=0.10 m, σ_{E1}=0.15 m

With GPS or Galileo alone, the single epoch ambiguity fail-rate on the medium baseline is generally at the few tens of percent level. In a combined system, with about double the amount of satellites, this fail-rate is only a few percent (Fig. 3 at right). This is still too large for safety-critical applications such as landing aircraft, but it could be fruitfully exploited in land surveying and geodetic applications. The short baseline has not been addressed yet. With GPS dual-frequency operation at present the single epoch fail-rate is usually below the 0.1% level. In Fig. 3 at right it can be seen that a combined system yields a fail-rate of only 0.000001%, even using a single epoch of data (note the logarithmic scale of the vertical axis in Fig. 3). With a combined GPS and Galileo system and integral ambiguity resolution, values with eight or more nines can be achieved as success-rate. This may allow use of precise carrier phase positioning also in safety-critical applications. This level can be maintained also when a differential tropospheric zenith delay needs to be accounted for (for instance, to bridge a height difference between rover and reference). Finally, Fig. 4 gives a snapshot impression of the variation of the ambiguity success-rate with geographic location. The medium baseline was used, although a differential tropospheric delay was not accounted for. For just one epoch in time, the instantaneous *success*-rate (not fail-rate!) was computed for a 0.1×0.1 grid over Europe. It can be clearly seen that combined GPS-Galileo outperforms current dual-frequency GPS (Fig. 4 right versus left). The distinct difference in gray-coding scale between the two graphs should be noticed; with combined GPS-Galileo the success-rate is larger than 0.95 everywhere, for the single epoch considered.

Concluding remarks

It has been shown that the capability of instantaneously resolving the carrier phase ambiguities correctly, with a

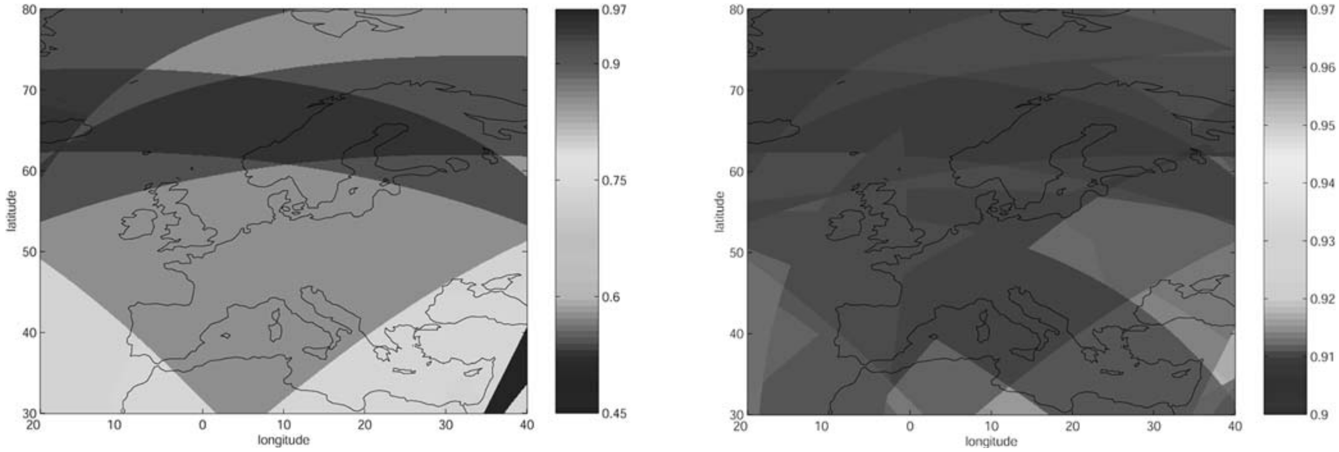

Fig. 4
Ambiguity success-rate over Europe on a medium baseline, at left for dual-frequency GPS (L1, L2), at right for combined GPS-Galileo (L1, L5, E1, E5b). Undifferenced standard deviations values are $\sigma_{L1}=\sigma_{L3}=0.30$ m, $\sigma_{L5}=\sigma_{E5b}=0.10$ m, $\sigma_{E1}=0.20$ m

combined GPS-Galileo system, clearly prevails over present dual-frequency GPS operation. On a short baseline, very high ambiguity success-rate levels can be obtained, using even a single epoch of data. Values of 0.99999999 or larger are not uncommon. This may allow use of precise carrier phase positioning also in safety-critical (navigation) applications and, as only one epoch of data is considered, no distinction needs to be made between a moving and a stationary receiver. On a medium baseline, typically 20–30 km, or longer when corrective information is used from a network of active GNSS reference stations, a success-rate at the level of 95% can be achieved using a combined GPS – Galileo system. On a long baseline, up to 100s km, the instantaneous success-rate is still too low for any practical applicability. It is to be noted that the success- and fail-rates presented for combined GPS and Galileo pertain to resolving the ambiguities of both GPS and Galileo satellites *all together*.

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