# Additional Empirical Evidence on Real Convergence: A Fractionally Integrated Approach

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**Abstract:** We examine the real convergence hypothesis for 14 OECD countries looking at the fractional order of integration of the differences of real GDP per capita in these countries with respect to the United States. Using parametric procedures, the results vary depending on how we specify the I(0) disturbances. If they are white noise, convergence is achieved for Canada and Australia, and with autocorrelated disturbances, this hypothesis is satisfied for France and the Netherlands. However, allowing for a break at World War II, evidence of convergence is obtained for all countries. JEL no. C32, O41
*Keywords:* Real convergence; fractional integration; long memory

## 1 Introduction

We have witnessed in recent years an emerging body of empirical literature on convergence in per capita output across different economies. The interest on this subject may be explained, at least in part, as a prediction test of the neoclassical growth model (Solow 1956) as opposed to the "new" endogenous growth models (Romer 1986; Lucas 1988). As it is well known, the neoclassical model predicts (under some assumptions) that per capita output in an economy will converge to each country's steady state (conditional convergence) or to a common steady state (unconditional convergence), regardless of its initial per capita output level. On the contrary, in endogenous growth models there is no tendency for income levels to converge, since divergence can be generated by relaxing some of the neoclassical assumptions (e.g., by incorporating nonconvexities in the production function).

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*Remark:* The authors gratefully acknowledge financial support from the Ministerio de Ciencia y Tecnologia (SEC2002-01839, Spain). We especially thank the managing editor, H. Lehment, and an anonymous referee for their helpful comments and suggestions. Please address correspondence to Fernando Pérez de Gracia, Universidad de Navarra, Facultad de Ciencias Economicas, Edificio Biblioteca, Entrada Este, E-31080 Pamplona, Spain; e-mail: fgracia@unav.es

 DOI: 10.1007/s10290-006-0057-9

In a time series context, stochastic convergence asks whether permanent movements in one country's per capita output are associated with permanent movements in other countries' output, that is, it examines whether common stochastic elements matter, and how much persistent the differences among countries are. Thus, stochastic convergence implies that output differences between economies cannot contain unit roots or time trends. Using this methodology, Bernard and Durlauf (1991) find that they can only reject the presence of a unit root for the difference France–Italy, among the G7 countries. Bernard and Durlauf (1995) find little evidence of income convergence when analyzing convergence among 15 OECD countries. The same result is obtained by Cellini and Scorcu (2000), who can only reject the nonconvergence hypothesis for the pairs United States–Germany, United States–Japan, and France–Italy. On the other hand, Carlino and Mills (1993) and Loewy and Papell (1996) find support for convergence among the U.S. regions, a result that might be explained due to the more homogenous nature of the economies studied by these authors.

When the convergence tests take into account the possibility of structural breaks, the evidence of convergence is reinforced. Greasley and Oxley (1997) find evidence of convergence between Belgium and the Netherlands, France and Italy, Australia and the United Kingdom, and Sweden and Denmark. St. Aubyn (1999) finds similar results between the United States and the United Kingdom, and between Australia and Japan. Cellini and Scorcu (2000) detect stochastic convergence only for the United States and Canada, and the United States and the United Kingdom when they allow for structural breaks. Strazicich et al. (2004) analyze the differences in per capita income of fifteen OECD countries with respect to the United States allowing for two structural breaks, and reject the unit root hypothesis in eleven of the fifteen countries examined, supporting thus the stochastic convergence hypothesis.

All the above approaches examine real convergence by means of testing if the series of interest is I(0) stationary or if it contains unit roots. In the latter case, the series is not mean reverting and convergence is not satisfied. In this paper, we define real convergence as mean reversion in the differences in per capita output among countries, and we test this hypothesis using a methodology based on fractional integration. In doing so, we avoid the strong dichotomy produced by the I(0)/I(1) specifications, and consider a wider variety of I($d$) models, with $d$ not necessarily constrained to be 0 or 1, but being possibly a real number. Then, if $d < 1$, the series is mean reverting and convergence is satisfied. This approach has already been applied in Michelacci and Zaffaroni (2000), Silverberg and Verspagen (1999), and Dolado et al. (2002a). Michelacci and Zaffaroni (2000) use a log-periodogram regression estimate, which is highly biased in small samples. To avoid this small-sample bias problem, Silverberg and Verspagen (1999) employ a nonparametric FGN estimator due to Beran (1994) and Sowell's (1992) maximum likelihood method, while Dolado et al. (2002a) use the fractional Dickey–Fuller test proposed by the authors in Dolado et al. (2002b). When the convergence hypothesis is analyzed by means of these methodologies, the results are mixed. Michelacci and Zaffaroni could not reject the hypothesis that all the OECD countries are nonstationary and mean reverting ($0.5 < d < 1$). Therefore, according to these authors, convergence takes place, although at a hyperbolic very slow rate. However, Silverberg and Verspagen (1999) find significant long memory (with $d > 1$) in the GDP per capita relative to the United States and, thus, no evidence of convergence, although their overall conclusions depend on the application of the FGN model. Dolado et al. (2002a) show that, after dealing with small-sample bias and a deterministic trend, there is strong evidence in favor of an order of integration between 0 and 1. Therefore, and similarly to Michelacci and Zaffaroni (2000), their results support convergence according to a long-memory process. In this paper, we employ both parametric and semiparametric techniques of fractional integration, which have several distinguishing features compared with the previous procedures. In particular, the use of the parametric approach of Robinson (1994a) permits us to include dummy variables to take into account for the potential presence of breaks in the series. The outline of the paper is as follows. In Section 2, we describe the methods that are employed in the article. Section 3 covers the empirical analysis. Section 4 deals with the possibility of structural breaks, while Section 5 offers some conclusions.

## 2 Long-Memory Processes and Convergence

An I(0) process $\{u_t, t = 0, \pm 1, ...\}$ can be defined as a covariance stationary process with a spectral density function that is positive and finite at the zero frequency. In this context, we say that a time series $\{x_t, t = 0, \pm 1, ...\}$ is I($d$) if:

$$\begin{aligned} (1-L)^d x_t &= u_t, \qquad t = 1, 2, ..., \\ x_t &= 0, \qquad t \leq 0,^{1} \end{aligned} \tag{1}$$

where $u_t$ is I(0) and $L$ means the lag operator ($Lx_t = x_{t-1}$). Note that the polynomial above can be expressed in terms of its binomial expansion, such that for all real $d$,

$$(1-L)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j = 1 - dL + \tfrac{d(d-1)}{2} L^2 - \ldots .$$

The macroeconomic literature has stressed the cases of $d = 0$ and 1; however, $d$ can be any real number. Clearly, if $d = 0$ in (1), $x_t = u_t$, and a "weakly autocorrelated" $x_t$ is allowed for. However, if $d > 0$, $x_t$ is said to be a long-memory process, also called "strongly autocorrelated" because of the strong association between observations widely separated in time. And as $d$ increases beyond 0.5 and through 1, $x_t$ can be viewed as becoming "more nonstationary" in the sense, for example, that the variance of partial sums increases in magnitude.

To determine the degree of integration is important from both economic and statistical viewpoints. If $d = 0$, the series is covariance stationary and possesses "short memory," with the autocorrelations decaying fairly rapid. If $d$ belongs to the interval (0, 0.5), $x_t$ is still covariance stationary; however, the autocorrelations take much longer time to disappear than in the previous case. If $d \in [0.5, 1)$, the series is no longer covariance stationary, but it is still mean reverting, with the effect of the shocks dying away in the long run. Finally, if $d \geq 1$, $x_t$ is nonstationary and non–mean reverting. Thus, the fractional differencing parameter $d$ plays a crucial role in describing the persistence behavior of the series: the higher $d$ is, the higher the association between the observations will be.

There exist many approaches of estimating and testing the fractional differencing parameter $d$. Earlier studies test the long-memory hypothesis using the rescaled-range (R/S) method, which is suggested by Hurst (1951), and is defined as

$$R/S = \frac{\max_{1 \leq j < T} \sum_{t=1}^{j} (x_t - \overline{x}) - \min_{1 \leq j < T} \sum_{t=1}^{j} (x_t - \overline{x})}{\left( \frac{1}{T} \sum_{t=1}^{T} (x_t - \overline{x})^2 \right)^{\frac{1}{2}}},$$

[1] For an alternative definition of fractional integration (the type I class), see Marinucci and Robinson (1999).

where $\overline{x}$ is the sample mean of the process $x_t$. The specific estimate of $d$ (Mandelbrot and Wallis 1968) is given by:

$$\hat{d} = \frac{\log(R/S)}{\log T} - \frac{1}{2}.$$

Mandelbrot and Wallis (1969), Mandelbrot (1972, 1975), and Mandelbrot and Taqqu (1979) analyze the properties of this procedure.[2] A problem with this statistic is that the distribution of its test statistic is not well defined and is sensitive to short-term dependence and heterogeneities of the underlying data generating process. Lo (1991) develops a modified R/S method that addresses these drawbacks of the classical R/S method.

Another method widely used in the empirical work is the one proposed by Geweke and Porter-Hudak (GPH 1983), which is a semiparametric procedure. They use it to obtain an estimate of the fractional differencing parameter $d$ based on the slope of the spectrum around the zero frequency. However, this method has some potential problems. First, it is too sensitive to the choice of the bandwidth parameter numbers, and in the presence of short-range dependence such as autoregressive or moving average terms, the GPH estimator is known to be biased in small samples (see, e.g., Agiakloglou et al. 1992).

In the context of parametric approaches, Sowell (1992) analyzes the exact maximum likelihood estimates of the parameters of the fractionally ARIMA($p$, $d$, $q$) model

$$\phi(L)(1-L)^d x_t = \theta(L)\varepsilon_t, \qquad t = 1, 2, \ldots, \tag{2}$$

where $\phi(L)$ and $\theta(L)$ are polynomials of orders $p$ and $q$, respectively, with all zeroes of $\phi(L)$ and $\theta(L)$ outside the unit circle, and $\varepsilon_t$ is white noise. He uses a recursive procedure that allows quick evaluation of the likelihood function in the time domain, which is given by

$$(2\pi)^{-T/2}\left|\Sigma\right|^{-1/2} \exp\left(-\frac{1}{2}X'_T \Sigma^{-1} X_T\right),$$

where $X_T = (x_1, x_2, \ldots, x_T)'$ and $X_T \sim \mathrm{N}(0, \Sigma)$.[3]

[2] Beran (1994) shows how to implement the R/S procedure.

[3] Other parametric methods of estimating $d$ based on the frequency domain were proposed by Fox and Taqqu (1986) and Dahlhaus (1989), among others. Small-sample properties of these and other estimates were examined in Smith et al. (1997) and Hauser (1999), and more recently in Silverberg and Verspagen (2003).

In this article we use both parametric and semiparametric methods. First, we present a parametric testing procedure developed by Robinson (1994a), which permits us to test I($d$) statistical models in raw time series. Then a semiparametric method (Robinson 1995b) will be described. These two methods have several distinguishing features that make them particularly relevant in comparison with other procedures. Thus, they have a standard null limit behavior, unlike other methods in which the limit distribution has to be calculated numerically on a case-by-case simulation study. Moreover, the limit distribution in Robinson (1994a) is unaffected by the inclusion of deterministic trends or the type of I(0) disturbances used to specify the short-run components of the series. Another useful property of these procedures is that they do not require Gaussianity, and a moment condition only of order two is necessary.

### *2.1 A Parametric Testing Procedure*

Robinson (1994a) proposed a Lagrange Multiplier (LM) test of the null hypothesis:

$$H_0 : d = d_0 \,, \tag{3}$$

in the model

$$y_t = \beta' z_t + x_t, \qquad t = 1, 2, \ldots, \tag{4}$$

and (1), for any given real value $d_0$, where $y_t$ is the time series we observe; $\beta = (\beta_1, \ldots, \beta_k)'$ is a $(k \times 1)$ vector of unknown parameters; and $z_t$ is a $(k \times 1)$ vector of deterministic regressors that may include, for example, an intercept, (e.g., $z_t \equiv 1$), or an intercept and a linear time trend (in case of $z_t = (1, t)'$). The functional form of the test statistic (denoted by $\hat{r}$), is given in Appendix A. Based on the null hypothesis $H_0$ in (3), Robinson (1994a) established that under certain regularity conditions:[4]

$$\hat{r} \rightarrow_d \mathrm{N}(0,\ 1) \quad \text{as} \quad T \rightarrow \infty \,, \tag{5}$$

and also the Pitman efficiency of the tests against local departures from the null. Thus, we are in a classical large-sample testing situation: an approximate one-sided $100\,\alpha$ percent level test of $H_0$ (3) against the alternative: $H_1 : d > d_0$ ($d < d_0$) will be given by the rule: "Reject $H_0$ if $\hat{r} > z_\alpha$

[4] These conditions are very mild regarding technical assumptions which are satisfied by (1) and (4).

($\hat{r} < -z_\alpha$)," where the probability that a standard normal variate exceeds $z_\alpha$ is $\alpha$. As in other standard large-sample testing situations, Wald and LR test statistics against fractional alternatives will have the same null and local limit theory as the LM tests presented here. Sowell (1992) employed essentially such a Wald testing procedure but it requires an efficient estimate of $d$ (see, equation (2)), and while such estimates can be obtained, the LM procedure seems computationally more attractive. A problem with the parametric procedures is that the model must be correctly specified. Otherwise, the estimates are liable to be inconsistent. In fact, misspecification of the short-run components of the process may invalidate the estimation of the long-run parameter.

### *2.2 A Semiparametric Estimation Procedure*

There exist several methods for estimating $d$ in a semiparametric way. Examples are the log-periodogram regression estimate (LPE), initially proposed by Geweke and Porter-Hudak (1983) and modified later by Künsch (1986) and Robinson (1995a), the average periodogram estimate, (APE, Robinson 1994b) and a Gaussian (Whittle) estimator (Robinson 1995b).

The Gaussian semiparametric method of Robinson (1995b) is basically a local "Whittle estimate" in the frequency domain, using a band of frequencies that degenerates to zero. The estimate ($\hat{d}$) is described in Appendix B. Under finiteness of the fourth moment and other conditions, Robinson (1995b) proves the asymptotic normality of this estimate, while Lobato (1999) extended it to the multivariate case. We have decided to use in this article the Whittle estimator primarily because of its computational simplicity. Note that we do not need to employ any additional user-chosen numbers in the estimation (as is the case with the LPE and the APE, where a trimming number is also requried). Also, we do not have to assume Gaussianity in order to obtain an asymptotic normal distribution, Robinson's (1995b) estimate being more efficient than the LPE.

## 3 Data and Test Results

The data used in this section are annual log real GDP per capita in 1990 Geary-Khamis PPP-adjusted dollars. The series runs from 1870 to 2001 for 14 OECD countries (Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, the Netherlands, Norway, Sweden, the

United Kingdom, and the United States) and from 1885 to 2001 for Japan. The data for the period 1870–1994 have been obtained from Maddison (1995) and these series have been updated using the GGDC (Groningen Growth and Development Center) database 2002. As indicator of real convergence, we use the differences of per capita GDP of each of the 14 countries with respect to the U.S. economy. This indicator has been widely used in other empirical works (e.g., St. Aubyn 1999; Silverberg and Verspagen 1999; etc.).

Though not reported in the paper, we initially performed the procedures described in Section 2 to the individual series, obtaining in all cases strong evidence of unit roots. Moreover, we also performed classical methods (Dickey and Fuller 1979; Phillips and Perron 1988; Kwiatkowski et al. 1992; etc.) and the results were all consistent with stochastic trends or unit roots. In the light of this, we look at the order of integration of the differenced series with respect to the United States. Initially, we perform the parametric procedure described in Section 2.1. Denoting each of the time series by $y_t$, we employ through the model given by (1) and (4), with $z_t = (1, t)'$, $t \geq 1$, $z_t = (0, 0)'$ otherwise. Thus, under the null hypothesis $H_0$ in (3):

$$y_t = \beta_0 + \beta_1 t + x_t, \qquad t = 1, 2, \ldots, \tag{6}$$

$$(1 - L)^{d_0} x_t = u_t, \qquad t = 1, 2, \ldots, \tag{7}$$

and we treat separately the cases $\beta_0 = \beta_1 = 0$ a priori; $\beta_0$ unknown and $\beta_1 = 0$ a priori; and both $\beta_0$ and $\beta_1$ unknown, i.e., we consider the cases of no regressors in the undifferenced regression (6), an intercept, and an intercept and a linear time trend, respectively.[5] We will model the I(0) process $u_t$ to be both white noise and to have parametric autocorrelation.

The test statistic reported across Tables 1 and 2 is the one-sided one corresponding to $\hat{r}$ in Appendix A, so that significant positive values of this are consistent with orders of integration higher than $d_0$, whereas significant negative ones are consistent with alternatives of form: $d < d_0$. A notable feature observed in Table 1(i), in which $u_t$ is taken to be white noise (when the form of $\hat{r}$ significantly simplifies) and $\beta_0 = \beta_1 = 0$ a priori, is the fact that the value of the test statistic monotonically decreases with $d_0$. Such monotonicity is a characteristic of any statistic, given correct specification

[5] According to some authors, this can be seen as testing for long-run unconditional convergence ($\beta_0 = \beta_1 = 0$); conditional convergence ($\beta_0$ unknown and $\beta_1 = 0$), and convergence as a catch-up ($\beta_0$ and $\beta_1$ unknown).

Table 1: *Testing the Order of Integration with Respect to the United States with White Noise Disturbances*

| | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|
| *(i): With no regressors* | | | | | | | | | |
| Austria | 23.20 | 16.28 | 9.26 | 3.24 | **−0.60** | −2.78 | −4.04 | −4.80 | −5.28 |
| Australia | 25.06 | 19.11 | 9.94 | 2.65 | **−1.32** | −3.34 | −4.43 | −5.06 | −5.47 |
| Belgium | 23.25 | 15.97 | 9.85 | 5.43 | 2.27 | **−0.01** | −1.68 | −2.92 | −3.83 |
| Canada | 22.35 | 17.93 | 10.36 | 3.04 | **−1.25** | −3.33 | −4.37 | −4.97 | −5.36 |
| Denmark | 15.40 | 12.56 | 8.67 | 4.74 | **1.54** | **−0.82** | −2.48 | −3.61 | −4.37 |
| Finland | 25.45 | 20.42 | 13.14 | 5.56 | **0.76** | −1.72 | −3.13 | −4.03 | −4.64 |
| France | 18.47 | 13.52 | 8.89 | 4.81 | 1.72 | **−0.49** | −2.07 | −3.18 | −3.95 |
| Germany | 20.25 | 15.44 | 10.36 | 5.25 | **1.24** | **−1.37** | −2.96 | −3.90 | −4.48 |
| Italy | 22.56 | 18.23 | 12.28 | 5.98 | **1.52** | **−1.10** | −2.69 | −3.71 | −4.39 |
| Japan | 22.71 | 18.75 | 12.51 | 5.12 | **0.09** | −2.45 | −3.76 | −4.49 | −4.93 |
| Netherlands | 18.97 | 12.02 | 7.20 | 3.68 | **1.07** | **−0.82** | −2.21 | −3.21 | −3.95 |
| Norway | 23.96 | 19.01 | 12.25 | 5.63 | **1.36** | **−1.11** | −2.67 | −3.71 | −4.42 |
| Sweden | 23.47 | 16.27 | 9.05 | 4.27 | **1.20** | **−0.91** | −2.43 | −3.52 | −4.29 |
| United Kingdom | 24.30 | 17.60 | 9.38 | 3.89 | **0.50** | −1.70 | −3.14 | −4.08 | −4.70 |
| *(ii): With an intercept* | | | | | | | | | |
| | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 |
| Austria | 23.20 | 16.98 | 9.32 | 3.20 | **−0.57** | −2.73 | −3.99 | −4.76 | −5.25 |
| Australia | 25.06 | 19.15 | 8.50 | **1.27** | **−1.61** | −3.24 | −4.24 | −4.89 | −5.33 |
| Belgium | 23.25 | 16.88 | 10.04 | 5.43 | 2.29 | **0.02** | −1.65 | −2.87 | −3.79 |
| Canada | 22.35 | 15.29 | 6.11 | **0.64** | −2.08 | −3.60 | −4.49 | −5.05 | −5.42 |
| Denmark | 15.40 | 11.44 | 7.75 | 4.47 | **1.62** | **−0.66** | −2.35 | −3.52 | −4.35 |
| Finland | 25.45 | 20.20 | 11.73 | 5.10 | **1.41** | **−0.87** | −2.44 | −3.53 | −4.29 |
| France | 18.47 | 13.37 | 8.60 | 4.79 | 1.87 | **−0.30** | −1.91 | −3.06 | −3.88 |
| Germany | 20.25 | 15.43 | 10.09 | 5.17 | **1.30** | **−1.31** | −2.92 | −3.89 | −4.48 |
| Italy | 22.56 | 17.50 | 11.04 | 5.56 | 1.80 | **−0.61** | −2.22 | −3.30 | −4.06 |
| Japan | 22.71 | 20.09 | 14.05 | 6.03 | **0.73** | −2.06 | −3.55 | −4.39 | −4.89 |
| Netherlands | 18.97 | 12.77 | 7.31 | 3.67 | **1.08** | **−0.81** | −2.20 | −3.21 | −3.94 |
| Norway | 23.96 | 18.45 | 11.02 | 5.54 | 2.28 | **0.04** | −1.65 | −2.92 | −3.83 |
| Sweden | 23.47 | 16.27 | 9.05 | 4.27 | **1.20** | **−0.91** | −2.43 | −3.52 | −4.29 |
| United Kingdom | 24.30 | 17.92 | 8.73 | 3.28 | **0.35** | −1.73 | −3.19 | −4.13 | −4.75 |
| *(iii): With an intercept and a linear time trend* | | | | | | | | | |
| | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 |
| Austria | 23.12 | 17.02 | 9.40 | 3.21 | **−0.57** | −2.73 | −3.99 | −4.76 | −5.25 |
| Australia | 22.90 | 15.71 | 7.23 | **1.41** | **−1.60** | −3.25 | −4.24 | −4.86 | −5.28 |
| Belgium | 21.53 | 16.10 | 10.20 | 5.52 | 2.29 | **0.01** | −1.65 | −2.87 | −3.78 |
| Canada | 14.13 | 9.24 | 4.47 | **0.53** | −2.08 | −3.60 | −4.50 | −5.05 | −5.41 |
| Denmark | 14.06 | 10.74 | 7.55 | 4.44 | **1.62** | **−0.66** | −2.35 | −3.52 | −4.31 |
| Finland | 20.17 | 14.58 | 9.34 | 4.77 | **1.41** | **−0.86** | −2.44 | −3.53 | −4.29 |
| France | 18.28 | 13.23 | 8.58 | 4.79 | 1.87 | **−0.30** | −1.91 | −3.06 | −3.88 |
| Germany | 20.12 | 15.27 | 10.05 | 5.17 | **1.30** | **−1.31** | −2.92 | −3.88 | −4.47 |
| Italy | 21.18 | 15.97 | 10.43 | 5.48 | 1.80 | **−0.62** | −2.22 | −3.30 | −4.06 |
| Japan | 22.08 | 18.09 | 12.11 | 5.57 | **0.73** | −2.05 | −3.54 | −4.38 | −4.89 |
| Netherlands | 17.42 | 12.18 | 7.39 | 3.72 | **1.08** | **−0.81** | −2.20 | −3.21 | −3.94 |
| Norway | 19.65 | 14.24 | 9.33 | 5.28 | 2.28 | **0.04** | −1.65 | −2.90 | −3.82 |
| Sweden | 17.16 | 12.59 | 8.20 | 4.20 | **1.20** | **−0.91** | −2.43 | −3.52 | −4.29 |
| United Kingdom | 17.78 | 12.51 | 7.45 | 3.36 | **0.35** | −1.76 | −3.19 | −4.13 | −4.75 |

*Note:* In bold: Nonrejection values of the null hypothesis at the 5 percent significance level. The data correspond to the differences of the log real GDP per capita of each country with respect to the United States.

Table 2: *Testing the Order of Integration with Respect to the United States with Bloomfield (1) Disturbances*

*(i): With no regressors*

| | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|
| Austria | 11.27 | 6.92 | 3.82 | **1.14** | **−0.55** | −1.67 | −2.32 | −2.87 | −3.19 |
| Australia | 13.52 | 9.51 | 4.98 | 1.70 | **−0.47** | −1.64 | −2.52 | −3.09 | −3.52 |
| Belgium | 11.20 | 5.51 | 2.16 | **0.09** | **−1.21** | −2.07 | −2.62 | −3.04 | −3.29 |
| Canada | 10.92 | 8.65 | 5.17 | 2.09 | **−0.38** | −2.00 | −2.89 | −3.43 | −3.76 |
| Denmark | 4.76 | 3.14 | **1.48** | **−0.14** | **−1.13** | −1.88 | −2.53 | −2.93 | −3.28 |
| Finland | 12.44 | 9.03 | 5.13 | **1.34** | **−0.93** | −2.16 | −2.92 | −3.30 | −3.59 |
| France | 6.16 | 3.05 | **0.91** | **−0.64** | −1.74 | −2.39 | −2.92 | −3.33 | −3.58 |
| Germany | 7.85 | 4.53 | 2.26 | **0.39** | **−1.04** | −2.06 | −2.85 | −3.40 | −3.88 |
| Italy | 9.89 | 7.18 | 3.92 | **1.02** | **−0.93** | −2.10 | −2.83 | −3.27 | −3.59 |
| Japan | 11.71 | 9.01 | 5.85 | 2.61 | **0.31** | **−1.29** | −2.37 | −3.03 | −3.45 |
| Netherlands | 6.75 | 2.57 | **0.23** | **−1.27** | −2.12 | −2.76 | −3.21 | −3.47 | −3.79 |
| Norway | 11.18 | 7.76 | 3.99 | **0.83** | **−1.00** | −2.11 | −2.72 | −3.13 | −3.44 |
| Sweden | 11.08 | 7.81 | 3.71 | **0.69** | **−1.25** | −2.35 | −2.92 | −3.29 | −3.58 |
| United Kingdom | 11.55 | 6.86 | 2.41 | **0.07** | **−1.16** | −2.10 | −2.62 | −3.13 | −3.54 |

*(ii): With an intercept*

| | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|
| Austria | 11.27 | 7.64 | 3.71 | **1.13** | **−0.67** | −1.71 | −2.32 | −2.85 | −3.18 |
| Australia | 13.52 | 9.60 | 4.30 | **0.14** | **−1.35** | −2.23 | −2.88 | −3.26 | −3.58 |
| Belgium | 11.20 | 6.33 | 2.31 | **−0.04** | **−1.29** | −2.13 | −2.67 | −3.08 | −3.33 |
| Canada | 10.92 | 7.07 | 2.62 | **0.02** | **−1.39** | −2.28 | −2.93 | −3.28 | −3.57 |
| Denmark | 4.76 | 2.68 | **0.77** | **−0.28** | **−1.28** | −1.88 | −2.35 | −2.87 | −3.22 |
| Finland | 12.44 | 8.19 | 3.60 | **0.24** | **−1.24** | −2.08 | −2.71 | −3.09 | −3.40 |
| France | 6.16 | 3.29 | **0.88** | **−0.64** | −1.76 | −2.33 | −2.85 | −3.26 | −3.52 |
| Germany | 7.85 | 4.72 | 2.26 | **0.35** | **−0.96** | −1.94 | −2.75 | −3.34 | −7.78 |
| Italy | 9.89 | 6.01 | 2.50 | **0.12** | **−1.39** | −2.25 | −2.97 | −3.37 | −3.68 |
| Japan | 11.71 | 10.00 | 6.68 | 2.85 | **0.38** | **−1.11** | −2.04 | −2.72 | −3.17 |
| Netherlands | 6.75 | 2.97 | **0.27** | **−1.26** | −2.10 | −2.74 | −3.19 | −3.45 | −3.78 |
| Norway | 11.18 | 7.05 | 2.63 | **−0.06** | **−1.31** | −2.12 | −2.55 | −3.01 | −3.30 |
| Sweden | 11.08 | 6.15 | 2.12 | **−0.16** | **−1.37** | −2.16 | −2.69 | −3.12 | −3.39 |
| United Kingdom | 11.55 | 7.09 | 2.03 | **−0.30** | **−1.27** | −2.04 | −2.51 | −2.98 | −3.38 |

*(iii): With an intercept and a linear time trend*

| | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|
| Austria | 11.14 | 7.69 | 3.80 | **1.15** | **−0.67** | −1.72 | −2.32 | −2.84 | −3.16 |
| Australia | 11.55 | 7.30 | 3.33 | **0.18** | **−1.35** | −2.26 | −2.87 | −3.15 | −3.27 |
| Belgium | 9.50 | 5.60 | 2.63 | **0.07** | **−1.29** | −2.14 | −2.68 | −3.07 | −3.49 |
| Canada | 4.98 | 2.83 | **1.33** | **−0.16** | **−1.39** | −2.28 | −2.93 | −3.28 | −3.57 |
| Denmark | 3.91 | 2.23 | **0.75** | **−0.31** | **−1.28** | −1.88 | −2.35 | −2.86 | −3.19 |
| Finland | 8.29 | 4.67 | 2.02 | **0.05** | **−1.25** | −2.08 | −2.72 | −3.09 | −3.40 |
| France | 6.35 | 3.07 | **0.86** | **−0.64** | −1.76 | −2.33 | −2.85 | −3.26 | −3.53 |
| Germany | 7.67 | 4.61 | 2.21 | **0.34** | **−0.96** | −1.94 | −2.75 | −3.32 | −3.80 |
| Italy | 8.69 | 5.25 | 2.28 | **0.03** | **−1.39** | −2.25 | −2.97 | −3.37 | −3.68 |
| Japan | 11.41 | 8.32 | 5.36 | 2.55 | **0.38** | **−1.07** | −2.01 | −2.67 | −3.22 |
| Netherlands | 5.72 | 2.52 | **0.23** | **−1.21** | −3.10 | −2.74 | −3.19 | −3.45 | −3.78 |
| Norway | 7.38 | 4.41 | 1.72 | **−0.13** | **−1.31** | −2.11 | −2.55 | −3.01 | −3.29 |
| Sweden | 6.35 | 3.67 | **1.51** | **−0.23** | **−1.37** | −2.16 | −2.69 | −3.12 | −3.38 |
| United Kingdom | 6.79 | 3.55 | **1.39** | **−0.25** | **−1.28** | −1.92 | −2.51 | −2.98 | −3.38 |

*Note:* In bold: Nonrejection values of the null hypothesis at the 5 percent significance level. The data correspond to the differences of the log real GDP per capita of each country with respect to the United States.

and adequate sample size. We also see in this table that we cannot reject the unit-root null hypothesis in practically any of the countries, the only two exceptions being France and Belgium, where the unit root is rejected but $H_0$ (3) cannot be rejected with $d = 1.25$. Thus, using this simple model (with no regressors and white noise disturbances) we do not find any evidence of convergence among the OECD countries.

Table 1 gives results with (ii) $\beta_1 = 0$ a priori (no time trend in the undifferenced regression) and (iii) both $\beta_0$ and $\beta_1$ unrestricted, still with white noise $u_t$. In every case, $\hat{r}$ is monotonic and, moreover, while there are sometimes large differences in the value of $\hat{r}$ across Table 1(ii) and (iii) for the same series/$d_0$ combination, the conclusions suggested by both seem very similar, that on the whole the extreme nonstochastic trends ($d = 0$), are inappropriate while the unit root ($d = 1$) is seldom rejected.[6] In fact, the only country where we find evidence of mean reversion is Canada, where the unit root is rejected and $d = 0.75$ is not. This hypothesis cannot be rejected for Australia but here the unit root cannot either be rejected.

In connection with the power properties of Robinson's (1994a) tests, it must be stressed that it is only in a local sense that they are optimal, and doubtless they could be bettered against nonlocal departures of interest by some optimal point procedure. In view of this, there is some satisfaction in the fact that $d < 0.75$ and $d > 1.25$ are always decisively rejected in Table 1. On the other hand, this significant result might be due in large part to unaccounted for I(0) autocorrelation in $u_t$, even bearing in mind the monotonicity of $\hat{r}$ in $d_0$. Thus, we also fitted AR models to $u_t$. The results are not reported though it is important to stress that we observed a lack of monotonicity in $\hat{r}$ with respect to $d_0$ in practically all series. This could be explained in terms of model misspecification as it is argued, for example, in Gil-Alana and Robinson (1997). However, it may also be due to the fact that the AR coefficients are Yule–Walker estimates and thus, though they are smaller than one in absolute value, they can be arbitrarily close to 1. A problem then may occur in that they may be capturing the order of integration of the series by means, for example, of a coefficient of 0.99 in case of using AR(1) disturbances.[7]

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[6] In most of the cases, the coefficients associated with the time trend were found to be insignificantly different from zero, while those related to the intercept are statistically significant in practically all cases.

[7] Note that a similar problem faces all other standard unit (or fractional) root tests if the roots are close to the unit root circle.

In order to solve this problem, we use other less conventional forms of I(0) processes. One that seems especially relevant and convenient in the context of the present tests is that proposed by Bloomfield (1973), in which the spectral density function is given by:

$$f(\lambda; \sigma^2; \tau) = \frac{\sigma^2}{2\pi} \exp\left(2 \sum_{r=1}^{m} \tau_r \cos(\lambda r)\right), \tag{8}$$

where $m$ is a parameter describing the short-run dynamics of the series. Like the stationary AR(p) model, the Bloomfield (1973) model has exponentially decaying autocorrelations and thus we can use a model like this for $u_t$ in (7).

The results based on the Bloomfield (1973) exponential model (with $m = 1$) are displayed in Table 2. Other values of $m$ were also employed and the results were very similar to those reported in the table. We see that monotonicity is achieved for all series and all values of $d_0$. Starting with no regressors (Table 2(i)), we observe that the unit root null hypothesis cannot be rejected for many series; $d = 1.25$ cannot be rejected for Japan, while $d = 0.75$ (and even $d = 0.50$) cannot be rejected for many other countries. We see that France and the Netherlands present the strongest evidence of mean reversion, since $d = 1$ is rejected in favor of alternatives with $d < 1$. In fact, $H_0$ (3) cannot be rejected for these two countries when $d = 0.50$ and 0.75. These two hypotheses cannot be rejected in the case of Denmark along with $d = 1$. For Austria, Belgium, Finland, Germany, Italy, Norway, Sweden, and the United Kingdom, the nonrejection values take place at $d = 0.75$ and 1. Finally, Australia, Canada, and Japan present the highest degrees of nonstationarity and the nonrejection values occur here at $d = 1$ (for Australia and Canada) and at $d = 1$ and 1.25 (for Japan). Including an intercept or an intercept and a linear trend, the conclusions remain the same, and the highest evidence of convergence is achieved for France and the Netherlands, followed by Denmark, Sweden, and the United Kingdom, though in these three countries the unit root hypothesis cannot be rejected.

To be more precise on the appropriate order of integration of each of the series, we recompute the tests of Robinson (1994a), but this time for a range of values of $d_0 = 0$, (0.01), 2. Table 3 reports for each series and each type of regressors the 95 percent confidence intervals of those values of $d_0$ where $H_0$ (3) cannot be rejected.[8] The left-hand side of the table

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[8] The confidence intervals were built up according to the following strategy: first, a value of $d$ was chosen from a grid; then, the test statistic was formed testing the null for this

Table 3: *95 Percent Confidence Intervals for the Nonrejection Values of d*

| | White noise disturbances | | | Bloomfield (1) disturbances | | |
|---|---|---|---|---|---|---|
| | No regressors | Intercept | Linear trend | No regressors | Intercept | Linear trend |
| Austria | [0.84 – 1.10] | [0.84 – 1.10] | [0.84 – 1.10] | [0.69 – 1.14] | [0.69 – 1.24] | [0.68 – 1.24] |
| Australia | [0.80 – 1.02] | [0.73 – 1.00] | [0.74 – 1.00] | [0.76 – 1.24] | [0.63 – 1.05] | [0.61 – 1.05] |
| Belgium | [1.07 – 1.49] | [1.08 – 1.49] | [1.08 – 1.49] | [0.55 – 1.10] | [0.56 – 1.08] | [0.57 – 1.11] |
| Canada | [0.82 – 1.03] | **[0.69 – 0.94]** | **[0.68 – 0.94]** | [0.79 – 1.16] | [0.57 – 1.06] | [0.46 – 1.06] |
| Denmark | [1.00 – 1.36] | [1.00 – 1.38] | [1.00 – 1.38] | [0.48 – 1.13] | [0.37 – 1.15] | [0.35 – 1.15] |
| Finland | [0.95 – 1.23] | [0.98 – 1.33] | [0.98 – 1.34] | [0.73 – 1.10] | [0.62 – 1.08] | [0.56 – 1.08] |
| France | [1.01 – 1.41] | [1.03 – 1.44] | [1.03 – 1.44] | **[0.43 – 0.98]** | **[0.40 – 0.99]** | **[0.41 – 0.99]** |
| Germany | [0.98 – 1.28] | [0.98 – 1.29] | [0.98 – 1.29] | [0.58 – 1.13] | [0.56 – 1.17] | [0.56 – 1.17] |
| Italy | [1.00 – 1.32] | [1.02 – 1.39] | [1.02 – 1.39] | [0.70 – 1.10] | [0.58 – 1.04] | [0.56 – 1.04] |
| Japan | [0.91 – 1.14] | [0.95 – 1.20] | [0.95 – 1.20] | [0.84 – 1.29] | [0.87 – 1.37] | [0.85 – 1.38] |
| Netherlands | [0.94 – 1.37] | [0.94 – 1.38] | [0.94 – 1.38] | **[0.32 – 0.87]** | **[0.36 – 0.86]** | **[0.34 – 0.86]** |
| Norway | [0.98 – 1.32] | [1.07 – 1.50] | [1.07 – 1.50] | [0.69 – 1.12] | [0.58 – 1.09] | [0.51 – 1.10] |
| Sweden | [0.95 – 1.27] | [0.97 – 1.36] | [0.97 – 1.36] | [0.67 – 1.06] | [0.54 – 1.05] | [0.46 – 1.05] |
| United Kingdom | [0.92 – 1.24] | [0.90 – 1.23] | [0.90 – 1.23] | [0.57 – 1.13] | [0.53 – 1.15] | [0.47 – 1.14] |

*Note:* We mark in bold those intervals where the highest value is smaller than 1. The data correspond to the differences of the log real GDP per capita of each country with respect to the United States.

corresponds to the case of white noise $u_t$, while the right-hand side reports the results based on Bloomfield (1973) disturbances. We have marked in bold those intervals where the highest value of the interval is smaller than 1, implying mean reversion and thus, real convergence. Starting with white noise $u_t$, we see that convergence is only achieved for Canada in the case of an intercept and/or a linear time trend. However, if we allow for autocorrelated disturbances, the unit root cannot be rejected for this country, and convergence is then achieved only for the cases of France and the Netherlands. For the remaining countries, even though there are many cases where $d < 1$ cannot be rejected, the unit root is included in all the intervals, making impossible to draw clear conclusions about the existence or not of convergence.

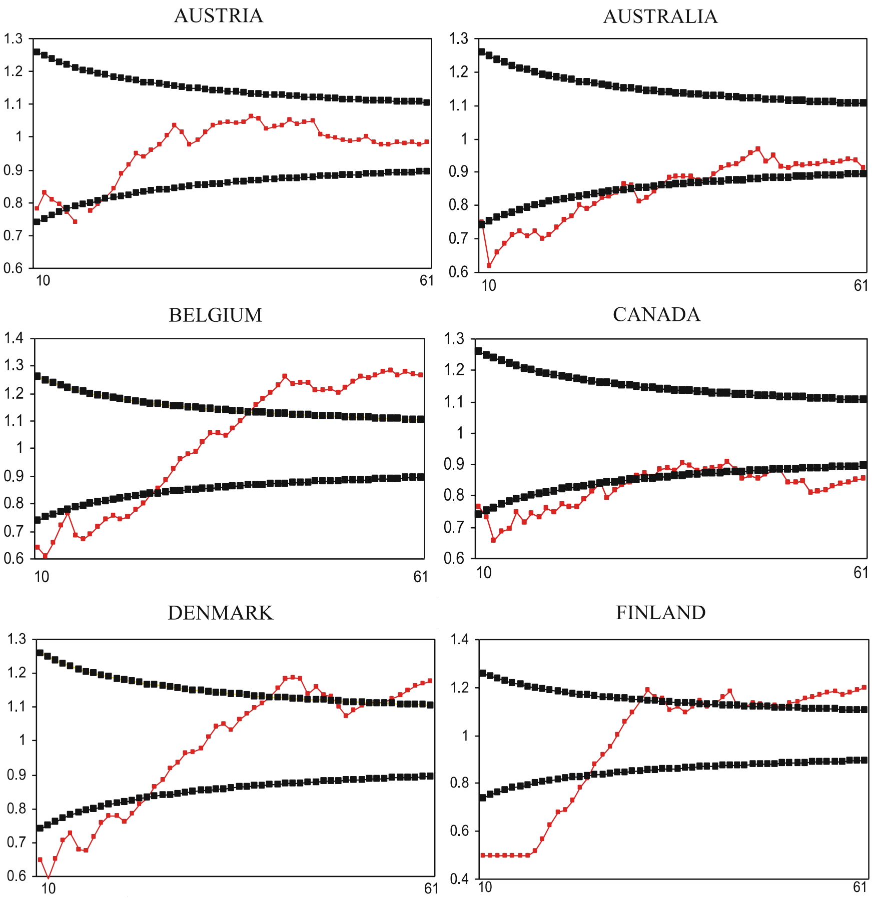
It is important to note that the results presented so far have nothing to do with the estimation of the fractional differencing parameter and the simple computation of diagnostic departures from real values of $d$. Alternatively we could have obtained point estimates of $d$. In fact, we tried with Sowell's (1992) method and the results were completely in line with Robinson's

value; if the null is rejected at the 5 percent level, this value of $d$ was discarded, otherwise, it was kept; an interval is then obtained after considering all the values of $d$ in the grid.

(1994a) parametric approach.[9] However, we have considered it more convenient to report the confidence intervals of the values where the null cannot be rejected in order to obtain better comparisons with the unit root case.

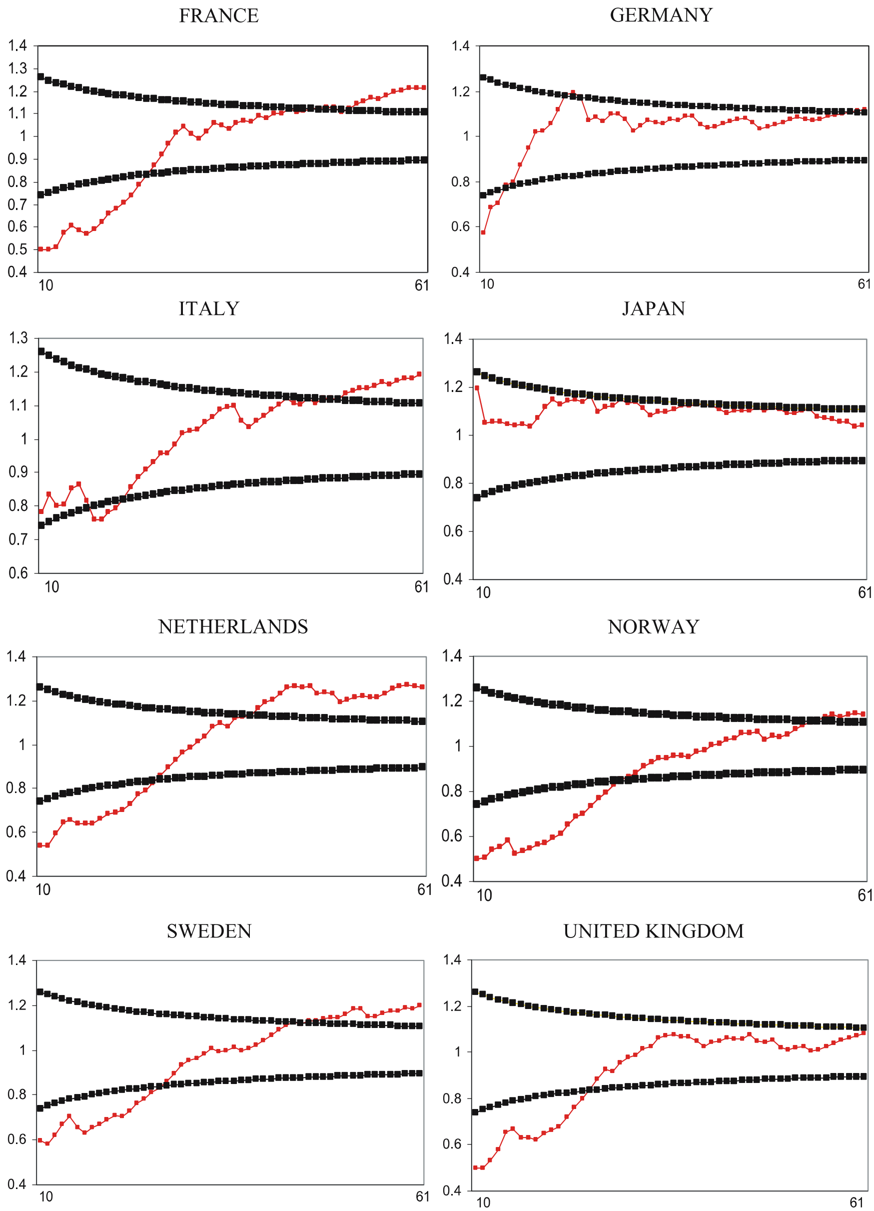
Next we perform the semiparametric procedure described in Section 2.2. Figure 1 reports the results based on the Gaussian semiparametric method

Figure 1: *Gaussian Estimate of Robinson Based on First-Differenced Data for a Range of Values* $m = 10, T/2$




[9] Note that Robinson's (1994a) method is based on the LM principle and uses an approximation to the likelihood function.

Figure 1: *continued*




*Note:* The horizontal axis refers to the bandwidth parameter number $m$, while the vertical one refers to the estimated value of $d$. The estimates were calculated based on the first-differenced data, adding then 1 to obtain the proper orders of integration.

of Robinson (1995b), i.e., $d_1$ in Appendix B for a range of values of $m$ from 10 to $T/2$.[10] Since the time series are clearly nonstationary, the analysis is carried out based on the first-differenced data, adding then 1 to the estimated values of $d$ to obtain the proper orders of integration. We also include in the figure the 95 percent confidence interval corresponding to the unit root case (i.e., $d = 1$). We see that for most of the countries, the results are a bit ambiguous, and they strongly depend on the choice of the bandwidth number $m$. Only for Australia and Canada, the results support the hypothesis of convergence with $d$ smaller than 1 and below the unit root interval. For Austria and Germany, practically all the values are within the interval while for the remaining countries, the results are less conclusive, finding values below the interval for small $m$, within the interval for some other values of $m$, and above the unit root interval for $m$ close to $T/2$. Thus, for most of the countries, the results are very sensitive to the choice of $m$. This might imply that there is a substantial correlation in $u_t$, because using too large $m$ might induce the bias in the estimation of $d$ from the short-run dynamics of the data. Moreover, the presence of structural breaks is another issue that should be taken into account, and this is examined in the following section.

## 4 A Potential Break at World War II

The implication of structural changes in unit-root tests has attracted the attention of many authors. Thus, Perron (1989) found that the 1929 crash and the 1973 oil crisis were a cause of nonrejection of unit roots in many macroeconomic time series and that, when these were taken into account, deterministic models were preferable. This question has also been specifically studied by Christiano (1992), Krol (1992), Zivot and Andrews (1992), Mill (1994), Bai and Perron (1998), etc. In the context of fractional processes, Granger and Hyung (1999), Bos et al. (1999, 2002), and Diebold and Inoue (2001) come to the conclusion that I($d$) models and structural change are easily confused.

[10] Some attempts to calculate the optimal bandwidth numbers have been examined in Delgado and Robinson (1996) and Robinson and Henry (1996). However, in the case of the Whittle estimator, the use of optimal values has not been theoretically justified. Other authors, such as Lobato and Savin (1998), use an interval of values of $m$ but we have preferred to report the results for the whole range of values of $m$.

In this section, we consider the possibility of a structural break at World War II. Greasley and Oxley (1997), Li and Papell (1999), St. Aubyn (1999), Cellini and Scorcu (2000), Strazicich et al. (2004), and recently Attfield (2003) analyze real income convergence taking into account structural breaks. Table 4 is similar to Table 3 above, but referring now to the pre- and post-war data, i.e., we report the confidence intervals of the nonrejection values of *d* for the differenced series with respect to the United States. In both cases, we assume that $u_t$ is white noise. Starting with the data ending in 1945, the first thing we observe is that convergence is only obtained for Australia with no regressors and for Canada with an intercept and with a linear trend. However, a very different picture is obtained if we look at the results for the post-war data. Here, if we do not include regressors, mean reversion is obtained for all countries except Austria, Germany, and Japan. Including an intercept, the unit root is practically never rejected, the only exception being Belgium and Italy, while in case of a linear time trend, the unit root is never rejected. Economically speaking, this result might appear surprising. Thus, while the unit root is rejected in favor of convergence for most countries if no regressors are included, the unit root null is rarely rejected if an intercept is included, implying

Table 4: *95 Percent Confidence Intervals for the Nonrejection Values of d with Data before and after 1945*

| | Before World War II | | | After World War II | | |
|---|---|---|---|---|---|---|
| | No regressors | Intercept | Linear trend | No regressors | Intercept | Linear trend |
| Austria | [0.91 – 1.30] | [0.82 – 1.32] | [0.84 – 1.32] | [0.61 – 1.01] | [1.32 – 1.80] | [1.28 – 1.74] |
| Australia | **[0.65 – 0.91]** | [0.62 – 1.01] | [0.61 – 1.02] | **[0.43 – 0.84]** | [0.34 – 1.17] | [0.40 – 1.16] |
| Belgium | [1.02 – 1.43] | [1.04 – 1.44] | [1.05 – 1.44] | **[0.58 – 0.91]** | **[0.67 – 0.99]** | [0.81 – 1.23] |
| Canada | [0.77 – 1.07] | **[0.65 – 0.97]** | **[0.65 – 0.97]** | **[0.59 – 0.87]** | [0.54 – 1.34] | [0.68 – 1.31] |
| Denmark | [1.08 – 1.42] | [1.08 – 1.49] | [1.08 – 1.49] | **[0.45 – 0.78]** | [0.33 – 1.30] | [0.52 – 1.16] |
| Finland | [1.00 – 1.34] | [1.01 – 1.44] | [1.01 – 1.44] | **[0.62 – 0.96]** | [0.50 – 1.56] | [0.81 – 1.53] |
| France | [1.17 – 1.49] | [1.12 – 1.48] | [1.12 – 1.48] | **[0.46 – 0.78]** | [0.36 – 1.73] | [0.90 – 1.58] |
| Germany | [0.78 – 1.17] | [0.65 – 1.10] | [0.68 – 1.10] | [0.93 – 1.53] | [0.97 – 1.50] | [0.96 – 1.63] |
| Italy | [1.21 – 1.53] | [1.16 – 1.54] | [1.16 – 1.53] | **[0.58 – 0.91]** | **[0.46 – 0.87]** | [1.07 – 1.93] |
| Japan | [0.89 – 1.26] | [0.81 – 1.33] | [0.82 – 1.33] | [0.81 – 1.16] | [0.34 – 1.76] | [1.28 – 1.62] |
| Netherlands | [1.10 – 1.48] | [1.16 – 1.52] | [1.16 – 1.51] | **[0.32 – 0.67]** | [0.22 – 1.43] | [0.86 – 1.34] |
| Norway | [1.10 – 1.40] | [1.11 – 1.45] | [1.11 – 1.45] | **[0.59 – 0.83]** | [0.44 – 1.37] | [0.54 – 1.49] |
| Sweden | [1.03 – 1.34] | [1.01 – 1.39] | [1.01 – 1.39] | **[0.47 – 0.80]** | [0.40 – 1.38] | [0.73 – 1.53] |
| United Kingdom | [0.71 – 1.13] | [0.88 – 1.31] | [0.90 – 1.30] | **[0.46 – 0.83]** | [0.23 – 1.47] | [0.24 – 1.52] |

*Note:* We mark in bold those intervals where the highest value is smaller than 1.

that unconditional convergence is accepted but conditional convergence is rejected, which makes not much economic sense, since unconditional convergence is a subset of conditional convergence. However, we observe that when an intercept is included the intervals are generally wider, and while the unit root cannot be rejected, other values of $d$ smaller than 1 are also plausible, suggesting that conditional convergence might also be satisfied. Therefore, we observe a different behavior in the time series before and after World War II, implying that a structural break might have occurred at that time.

Table 5 reports the 95 percent confidence intervals for the nonrejection values of $d$ in Robinson's (1994a) setup in (1) and (4) with a dummy variable for the break at World War II. We try both a shift and a slope dummy and model $u_t$ in terms of white noise and Bloomfield (with $m = 1$) disturbances. Given the significance of the intercept in most of the previous models, we also include it in the model. Thus, we test the null model

$$y_t = \beta_0 + \beta_1 D_t + x_t, \qquad t = 1, 2, \ldots,$$
$$(1 - L)^{d_0} x_t = u_t, \qquad t = 1, 2, \ldots,$$

Table 5: *95 Percent Confidence Intervals with a Dummy Break*

| | White noise disturbances: With a shift dummy | White noise disturbances: With a slope dummy | Bloomfield (1) disturbances: With a shift dummy | Bloomfield (1) disturbances: With a slope dummy |
|---|---|---|---|---|
| Austria | [0.91 1.06 1.39] | [0.79 **0.94** 1.17] | [0.69 **0.84** 1.02] | [0.51 **0.68** 1.01] |
| Australia | [0.96 1.13 1.38] | [0.78 **0.90** 1.06] | [0.70 **0.86** 1.14] | [0.58 **0.79** 1.19] |
| Belgium | [1.01 1.11 1.28] | [0.90 1.03 1.23] | [0.83 **0.93** 1.08] | [0.59 **0.71** 0.92] |
| Canada | [1.00 1.14 1.33] | [1.03 1.14 1.34] | [0.71 **0.87** 1.23] | [0.83 **0.96** 1.23] |
| Denmark | [0.90 **0.98** 1.11] | [0.93 1.00 1.12] | [0.71 **0.81** 0.94] | [0.85 **0.94** 1.10] |
| Finland | [1.01 1.14 1.37] | [1.02 1.14 1.37] | [0.74 **0.82** 0.95] | [0.76 **0.85** 1.01] |
| France | [1.00 1.11 1.26] | [0.92 1.05 1.26] | [0.83 1.00 1.28] | [0.61 **0.75** 1.03] |
| Germany | [0.99 1.09 1.26] | [0.96 1.14 1.43] | [0.82 **0.96** 1.15] | [0.50 **0.62** 0.82] |
| Italy | [1.14 1.29 1.49] | [0.99 1.14 1.41] | [0.88 1.03 1.30] | [0.76 **0.89** 1.26] |
| Japan | [0.98 1.07 1.21] | [0.91 1.00 1.15] | [0.92 1.07 1.32] | [0.76 **0.96** 1.31] |
| Netherlands | [0.96 1.06 1.18] | [0.81 **0.95** 1.16] | [0.93 1.15 1.49] | [0.53 **0.67** 0.90] |
| Norway | [0.94 1.00 1.10] | [0.95 1.02 1.13] | [0.93 1.02 1.13] | [0.89 **0.99** 1.13] |
| Sweden | [1.02 1.12 1.10] | [1.08 1.17 1.31] | [0.85 **0.95** 1.13] | [0.95 1.07 1.24] |
| United Kingdom | [1.03 1.21 1.40] | [1.05 1.23 1.50] | [0.81 **0.91** 1.08] | [0.72 **0.83** 1.09] |

where $D_t = 1\mathrm{I}(t > T_b)$ (a shift dummy), or $D_t = (t - T_b)\mathrm{I}(t > T_b)$ (slope dummy), with $T_b = 1945$. In both tables we also display, for each series, the value of $d$ that produces the lowest statistic in absolute value across $d$, this value being thus an approximation to the likelihood estimate. Starting again with white noise $u_t$, we observe that all the intervals include the unit root case, and the lowest statistics correspond to values of $d$ higher than 1 for all countries except Denmark (with a shift dummy) and Austria, Australia, and the Netherlands (with a slope dummy). Including autocorrelated disturbances, which seems to be a more realistic assumption, some of the intervals are strictly below 1, and more importantly, practically all the values of $d$ corresponding to the lowest statistics are smaller than 1 if a slope dummy is considered. The exception is Sweden, with a value higher than 1 with a slope dummy but smaller with a shift dummy. A deeper inspection at the coefficients associated with the dummies showed that those corresponding to the slope dummy were statistically significant at conventional levels (5 percent) in all countries.

## 5 Concluding Remarks

In this article we examine real convergence in some OECD countries by means of fractionally integrated techniques. We look at the orders of integration of the differences in the log real GDP per capita series in Austria, Australia, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, the Netherlands, Norway, Sweden, and the United Kingdom with respect to the United States, which is used as a benchmark country. For this purpose we employ a parametric testing procedure due to Robinson (1994a) and a semiparametric estimation method (Robinson 1995b). We use these procedures, primarily because of the distinguishing features that make them particular relevant in comparison with other methods. Thus, Robinson's (1994a) tests allow us to consider unit- and fractional-root tests with no effect on its null limit distribution, which is also unaffected by the inclusion of deterministic trends and different types of I(0) disturbances. In addition, the tests are the most efficient ones when directed against the appropriate (fractional) alternatives. The reason for using the Gaussian semiparametric method of Robinson (1995b) is based on its computational simplicity, along with the fact that it just requires a single bandwidth parameter, unlike other procedures where a trimming number is also required.

Using the parametric procedure of Robinson (1994a), the results vary substantially depending on how we specify the I(0) disturbances. Thus, if they are white noise, convergence is achieved for the cases of Canada and Australia. However, if we permit autocorrelation, the unit root cannot be rejected for these two countries, and evidence of convergence is only obtained for France and the Netherlands. Similar evidence is obtained when using other parametric approaches like Sowell's (1992) maximum likelihood estimation in the time domain. We also performed the Gaussian semiparametric Whittle procedure of Robinson (1995b). The results here were consistent with the parametric ones for the case of white noise disturbances, finding thus conclusive evidence of real convergence in Australia and Canada. For the remaining countries, the results were very sensitive to the choice of the bandwidth parameter number. The possibility of a structural break, due to World War II, was also taken into account. We performed the same analysis but based on pre- and post-war data. Working with the data ending in 1945, convergence was only achieved for Australia and Canada while using post-war data, the convergence hypothesis was satisfied for all countries except Austria, Germany, and Japan, a result which is partially in line with those in Michelacci and Zaffaroni (2000) and Dolado et al. (2002a), who find reasonably strong evidence of real convergence for most countries in their sample. The different behavior for the pre- and post-war data suggests that dummy variables for the break should be incorporated in the model. In doing so, we obtain stronger evidence of real convergence in all countries. We believe that this finding reinforces the evidence of convergence for OECD countries since the results on convergence, when using time series techniques, are not very optimistic in the literature. Moreover, in terms of the debate between neoclassical and endogenous growth models, the results seem more consistent with the implication on convergence of Solow's neoclassical model.

Several other lines of research are under progress, which should prove relevant to the analysis of these and other macroeconomic data. Multivariate versions of the tests of Robinson (1994a) are being developed and this would lead to an alternative approach to the study of cointegration. The Bloomfield model for the I(0) components is also being developed in a multivariate setup. Other issues such as the endogeneization of the time of the structural breaks in the setup of Robinson (1994a) is being investigated. How this may affect the results presented here will be addressed in future papers.

## Appendix A

The test statistic proposed by Robinson (1994a) is based on the Lagrange Multiplier (LM) principle, and is given by:

$$\hat{r} = \frac{T^{1/2}}{\hat{\sigma}^2}\hat{A}^{-1/2}\,\hat{a}\,,$$

where $T$ is the sample size and

$$\hat{a} = \frac{-2\pi}{T}\sum_{j=1}^{T-1}\psi(\lambda_j)g(\lambda_j;\,\hat{\tau})^{-1}I(\lambda_j);$$

$$\hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \frac{2\pi}{T}\sum_{j=1}^{T-1}g(\lambda_j;\,\hat{\tau})^{-1}I(\lambda_j);$$

$$\hat{A} = \frac{2}{T}\left(\sum_{j=1}^{T-1}\psi(\lambda_j)^2 - \sum_{j=1}^{T-1}\psi(\lambda_j)\hat{\varepsilon}(\lambda_j)' \times \left(\sum_{j=1}^{T-1}\hat{\varepsilon}(\lambda_j)\hat{\varepsilon}(\lambda_j)'\right)^{-1} \times \sum_{j=1}^{T-1}\hat{\varepsilon}(\lambda_j)\psi(\lambda_j)\right)$$

$$\psi(\lambda_j) = \log\left|2\sin\frac{\lambda_j}{2}\right|; \quad \hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial\tau}\log g(\lambda_j;\,\hat{\tau});$$

$$\lambda_j = \frac{2\pi j}{T}; \quad \hat{\tau} = \arg\min\sigma^2(\tau)\,.$$

$I(\lambda_j)$ is the periodogram of $u_t$ evaluated under the null, i.e.,

$$\hat{u}_t = (1-L)^{d_0}y_t - \hat{\beta}'w_t; \quad \hat{\beta} = \left(\sum_{t=1}^{T}w_t w_t'\right)^{-1}\sum_{t=1}^{T}w_t(1-L)^{d_0}y_t;$$

$$w_t = (1-L)^{d_0}z_t\,,$$

and $g$ is a known function related to the spectral density function of $u_t$,

$$f(\lambda;\,\sigma^2;\,\tau) = \frac{\sigma^2}{2\pi}g(\lambda;\,\tau), \quad -\pi < \lambda \le \pi\,.$$

## Appendix B

The estimate of Robinson (1995b) is implicitly defined by:

$$\hat{d} = \arg\min_d\left(\log\overline{C(d)} - 2d\frac{1}{m}\sum_{j=1}^{m}\log\lambda_j\right),$$

$$\text{for } d \in (-1/2,\ 1/2); \quad \overline{C(d)} = \frac{1}{m}\sum_{j=1}^{m}I(\lambda_j)\lambda_j^{2d}, \quad \lambda_j = \frac{2\pi j}{T}, \frac{m}{T} \to 0\,.$$

where $m$ is a bandwidth parameter number.

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