

Combinatorial and robust optimisation models and algorithms for railway applications

Laura Galli

Received: 18 May 2010 / Revised: 10 August 2010 / Published online: 18 August 2010
© Springer-Verlag 2010

Abstract The following is a summary of the author's Ph.D. thesis supervised by Alberto Caprara and Paolo Toth and defended on April 16, 2009 at the Università di Bologna. The thesis is written in English and is available from the author upon request. Railway systems represent a challenging area for operations research, especially when highly-complex and data-intensive applications, such as large-scale transportation networks, are at stake. One of the main issues concerns imperfect information. The classic notion of Robust Optimisation, as a way to represent and handle mathematically systems with not precisely known data, did not prove to be successfully applicable in the railway setting. For this reason a new paradigm has been defined recently in [Liebchen et al. \(2007\)](#): Recoverable robustness. Here we present our research on recoverable robust optimisation models for two important railway problems: Train platforming and Rolling stock planning.

Keywords Mixed integer programming · Robust optimisation · Train platforming · Rolling stock planning

MSC classification (2000) 90B06 · 90B25 · 90C11 · 90C20 · 90C57 · 90C59 · 90C90

1 Solution to the train platforming problem

Train platforming is concerned with finding an assignment of trains to platforms in a railway station and defining the corresponding routes. Routing a train in a railway station means finding two paths that link the arrival and departure directions of the given train to a stopping platform. The train platforming problem we consider in this

L. Galli (✉)
DEIS, Università di Bologna, Viale Risorgimento 2, 40136 Bologna, Italy
e-mail: l.galli@unibo.it

work is based on a rather general formulation proposed by the main Italian Infrastructure Manager (RFI), where, for each train, we aim at defining the stopping platform, together with the arrival and departure paths. This version also allows modifications (shifts) on the train schedule (within some range). This last feature is particularly important, since it represents a feedback among two consecutive planning phases: timetabling and platforming. Another important feature is the presence of a quadratic objective function to penalize train conflicts in the network.

We illustrate an ILP formulation for the problem based on the concept of pattern, a tuple containing all the information about the resources to be assigned to a train (i.e., paths, platform and shifts). We propose an exact algorithm based on a branch-and-cut-and-price framework, since the number of variables and constraints is too large to be dealt with simultaneously. We also present an effective way to linearize the objective function, which leads to strong bounds for the continuous relaxation of the model.

The algorithm is tested using real-world data of the Italian railways. The results represent an important step forward in railway planning. For the instances in our case study, we show that we can produce solutions that are much better than those produced by a simple heuristic currently in use, and that often turn out to be (nearly-)optimal. This work is presented in [Caprara et al. \(2007\)](#).

2 Recoverable robustness

Robust optimisation seeks for good solutions although the input data is not known exactly. More precisely, a robust solution is a solution that is guaranteed to be feasible for every realization of the input data (i.e., for all scenarios). This feature guarantees the given solution against any input occurrence, but it also represents the main drawback of robust optimisation methods. In fact it makes the approach extremely over conservative. In real-world applications, the scenario set can be extremely broad, thus the cost of a robust solution can get easily very large and unacceptable for the planner. In many cases, such a solution does not even exist. This rules out robust optimisation for many applications. On the other hand, in most real-world problems, the level of robustness required is far less strict, because solutions may be recovered after the data has changed. Recovery can almost always be applied in real-world situations and ignoring this aspect sets a huge gap between mathematical approaches and their practical utilization.

To overcome this principal weakness of classical robustness, the concept of Recoverable Robustness has been introduced by [Liebchen et al. \(2007\)](#). Formally speaking, a solution is recovery-robust against certain scenarios and for a certain limited type of recovery, if for each of the considered scenarios the solution can be recovered to a feasible one within the limits of the type of recovery.

3 Recovery-robust train platforming

In many cases, as in our real-world study on two main stations of the Italian railway network, the capacity of a station is a bottleneck of the whole system, so tight

planning is required. As a consequence, even small disturbances to the schedule can widely affect the smooth operation of the system. In this study we construct a platforming plan which can be operated with only a limited amount of total delay and without re-planning of platforms or paths in all likely scenarios, i.e., in those scenarios with a limited amount of small disturbances. In other words, we seek a recovery-robust platforming.

Platforming naturally allows for a limited, simple recovery, i.e., a limited amount of propagated total delay, and it makes sense to assume that only a limited number of trains will reach the station initially delayed, or experience a seminal disturbance on the platform before departure. We define the set of likely scenarios in this spirit. For all scenarios with moderate disturbances, a recovery-robust platforming solution is guaranteed to be adjustable by simple means and at an a priori limited cost, i.e., the total delay will not exceed a certain fixed budget.

An advantageous feature of the platforming problem is that disturbances only occur on the right-hand side of the problem. This enables the use of the robust network-buffering approach, introduced by [Stiller \(2008\)](#), which yields tractable recovery-robust models. Roughly speaking “tractable” means that, out of the infinite (uncountable) set of scenarios, we can restrict attention to a finite and relatively small set in modeling the problem. In addition, network buffering can be easily combined with existing specialized algorithms for the deterministic platforming problem.

In this way we construct recovery-robust platforming plans for the two stations we considered in our computational study on real-world data, namely Genova and Palermo. The results are promising. While nominal optimality is maintained, the propagated delay can be reduced by up to 49%. This work is presented in [Caprara et al. \(2008\)](#).

In a different, but related research we also experiment different robust optimisation models and re-scheduling algorithms in an on-line (i.e., real time) context using a rolling horizon framework. We also design a simulation framework to evaluate and compare the level of robustness of different plans against different recovery strategies, see [Caprara et al. \(2009\)](#).

4 Recovery-robust railway rolling stock planning

In this research we apply the notion of Recoverable Robustness to railway rolling stock planning. In particular, the concept of Recoverable Robustness presented in [Liebchen et al. \(2007\)](#) requires a minimization over a set of recovery algorithms, and it is not obvious how this can be carried out in general. Our purpose is to investigate a way of bringing the theoretical notion closer to practice. In particular, we consider what happens if recovery is done in the best possible way via a unique optimal recovery algorithm. The values computed will clearly represent lower bounds for any set of recovery algorithms. Under mild assumptions, this lower bound can be computed by solving a single mathematical program.

We evaluate the approach on real-life rolling stock planning problems of NS, the main operator of passenger trains in the Netherlands. It arises 2–6 months before the actual train operations, and amounts to assigning the available rolling stock to the trips in a given timetable. The objectives of the problem that is traditionally solved,

called nominal problem, are related to service quality, efficiency, and—to a limited extent—to robustness. Reference [Fioole et al. \(2006\)](#) describes a Mixed Integer Linear Programming (MILP) model for this nominal problem.

The computational results show that, thanks to Benders decomposition, our lower bound can be computed within relatively short time for our case study for a small number of (representative) scenarios. In addition, we present a successful heuristic based on Benders decomposition leading to robust integer solutions whose value is very close to the continuous lower bound. Finally, we show the practical effectiveness of Recoverable Robustness, by proposing an evaluation framework to prove that the robust solution is indeed much easier recoverable than the non-robust optimal solution of the underlying rolling stock scheduling problem on a very large number of scenarios. This work is presented in [Cacchiani et al. \(2008\)](#).

References

- Cacchiani V, Caprara A, Galli L, Kroon L, Maróti G, Toth P (2008) Price of recoverability for railway rolling stock planning. ARRIVAL-TR-0121. Submitted to Transportation Science
- Caprara A, Galli L, Toth P (2007) Solution to the train platforming problem. ARRIVAL-TR-0146. Under revision for Transportation Science
- Caprara A, Galli L, Stiller S, Toth P (2008) Recovery-robust platforming by network buffering. ARRIVAL-TR-0157
- Caprara A, Galli L, Kroon L, Maróti G, Toth P (2009) Robust train routing. ARRIVAL-TR-0216
- Fioole P, Kroon L, Maróti G, Schrijver A (2006) A rolling stock circulation model for combining and splitting of passenger trains. Eur J Operat Res 174:1281–1297
- Liebchen C, Lübbecke M, Möhring R, Stiller S (2007) Recoverable robustness. ARRIVAL-TR-0066
- Stiller S (2008) Extending concepts of reliability. Ph.D. Thesis, Institut für Mathematik, Technische Universität Berlin, Berlin, Germany