4OR-Q J Oper Res (2009) 7:109–112
DOI 10.1007/s10288-008-0075-7

PHD THESIS

Models and algorithms for combinatorial optimization problems arising in railway applications

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Received: 28 February 2008 / Published online: 27 May 2008

Abstract This is a summary of the author's PhD thesis supervised by Alberto Caprara and Paolo Toth and defended on 29 May 2007 at the Università di Bologna. The thesis is written in English and is available from the author upon request. This work deals with Railway Optimization, and in particular it focuses on the Train Timetabling Problem (in the basic version on a corridor and in the extension to a railway network), and on the Train Unit Assignment Problem. Integer Linear Programming (ILP) formulations are proposed for both problems, and their continuous and Lagrangian relaxations are used to obtain optimal and heuristic solutions to real-world instances.

Keywords Train Timetabling · Train Unit Assignment · Integer Linear Programming · Relaxation · Branch-and-Cut-and-Price · Heuristic algorithm

MSC classification (2000) 90B06 · 90C10 · 90C57 · 90C59 · 90C90

1 Railway Optimization

Among railway optimization problems, the most relevant ones concern the main phases that are needed in the planning and operational processes related to railway systems. The most studied areas consider Line Planning, Train Timetabling, Train Platforming, Train Unit Assignment, Train Unit Shunting and Crew Planning. For surveys on railway optimization problems see, e.g. Caprara et al. (2007) and Cordeau et al. (1998).

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In the following, we present our research on the Train Timetabling Problem, both on a corridor and on a network, and on the Train Unit Assignment Problem.

2 A column generation approach to Train Timetabling on a corridor

We consider a single one-way line (corridor), together with a set of trains. Each train is assigned by the train operator an *ideal* timetable (specifying the departure time from its first station, the arrival time at its destination station, and the arrival and departure times for the intermediate stations), representing the most desirable timetable for the train. Train Timetabling aims at determining an optimal timetable, satisfying a set of track capacity constraints. Namely, overtaking between trains occur only within a station, and for each station there are lower bounds on the time interval between two consecutive arrivals and two consecutive departures. In order to satisfy these constraints, the ideal timetable can be modified: one is allowed to anticipate or delay the departure time of each train from its first station (*shift*), and to increase the stopping time interval at the intermediate stations (*stretch*). Moreover, one can cancel the train. Each train is assigned an ideal profit, which is achieved if the train is scheduled according to its ideal timetable. The profit is decreased if we apply shift and/or stretch to obtain a feasible timetable. The objective is to maximize the sum of the profits of the scheduled trains.

The Train Timetabling Problem is NP-hard. We propose a space–time graph representation for the problem, in which nodes represent arrivals or departures at a station in given time instants, and paths represent feasible timetables for trains. We present an ILP formulation, based on path variables, and propose an exact Branch-and-Cut-and-Price algorithm. The upper bounds are computed by solving the LP-relaxation of the model. Since there is an exponentially large number of possible paths for a train, our approach is based on column generation techniques. Moreover, we have a very large number of constraints, which are handled by separation algorithms. Furthermore, we develop constructive heuristic algorithms, based on the optimal solution of the LP-relaxation, and combined with local search procedures to improve the best solution found. The algorithms are tested on a set of real-world instances from Rete Ferroviaria Italiana (the main Italian Railway Infrastructure Company), and are compared to the results obtained by a Lagrangian approach described in Caprara et al. (2002). The results show that the LP upper bound is often significantly stronger than the Lagrangian upper bound, and the heuristic algorithms improve the solutions found by the Lagrangian heuristic. The exact algorithm finds optimal solutions for small-size instances.

Moreover, we propose an alternative version of the overtaking constraints, involving an arbitrary number of trains, that are stronger than those in the ILP above, involving train pairs. The associated separation concerns the problem of computing the Stable Set of Maximum Weight in a particular graph given by the edge intersection of a comparability graph and a complete k-partite graph. We develop a heuristic approach (based on flow techniques) and an exact method (based on dynamic programming) for handling the separation problem.

This problem and the methods used for solving it are presented in Cacchiani et al. (2008).

3 Freight transportation in railway networks

We study a freight transportation problem in railway networks, where a certain number of passenger trains have already a prescribed timetable which cannot be changed, and train operators would like to schedule freight trains, according to an *ideal* timetable. The aim is to schedule as many freight trains as possible, while changing the ideal timetable as little as possible, and satisfying a set of track capacity constraints.

This is an extension of the basic version of the Train Timetabling Problem on a corridor to a railway network. We propose a graph representation of the problem, and present an ILP model. The model is based on arc and node binary variables, and it is a generalization of the model proposed in Caprara et al. (2002). The track capacity constraints are relaxed in a Lagrangian way and near-optimal Lagrangian multipliers are determined through subgradient optimization. At each iteration of the subgradient procedure, a constructive heuristic algorithm based on the Lagrangian profits is applied. Finally, when the subgradient procedure has converged, some paths are fixed in the solution and subgradient optimization is re-applied to the resulting problem. We present computational results on real-world instances of Rete Ferroviaria Italiana. The case we consider is a very complex railway network. It is composed of:

- many different alternative routes for going from a station to another one,
- the so-called *railway nodes*, i.e., big stations where we can have many different lines and different routes also inside the station,
- some single tracks that can be used in both directions.

The tests performed on the case study show the effectiveness of our approach. Results of this research are reported in Cacchiani et al. (2006).

4 Solving a real-world Train Unit Assignment Problem

We face a real-world Train Unit Assignment Problem for an operator running trains in a regional area. A train unit is a self-contained train with an engine and passenger seats. Given a set of timetabled train trips, each with a required number of passenger seats, and a set of train units, each with a given number of available seats, the problem calls for a minimum-cost assignment of the train units to trips, possibly combining more than one train unit for a given trip, that fulfills the seat requests. Furthermore, the problem presents constraints for the feasible sequencing of trips and for the maintenance of train units. The problem is solvable efficiently if there is a unique train unit type and NP-hard for the general version considered in our case study. We propose a graph representation for the problem, and an ILP formulation in which the seat requirement constraints are stated in a "strong" form, derived from the description of the convex hull of the variant of the knapsack polytope arising when the sum of the variables is restricted not to exceed two. We consider the LP-relaxation of the model, which is solved through column generation. We develop an LP-based heuristic method, which presents three main components: (1) a diving rule to fix the value of some of the variables following the current LP optimal solution, re-optimizing the LP after the addition of these fixing constraints, (2) a simple constructive heuristic procedure based on the current dual LP solution that is applied at each iteration of the column-generation based procedure, and (3) a refinement procedure that is applied to improve each solution produced by the constructive heuristic procedure in (2).

In addition, we illustrate a relaxation of the problem that can be used to derive lower bounds, and is generally faster to solve. The aim is to find a set of trips that are pairwise "incompatible", i.e., they cannot be assigned to the same train unit (of whatever type), and then optimally solve the sub-instance restricted to these trips. This set of trips is found by computing a Maximum Weighted Stable Set on a comparability graph, that can be computed efficiently by flow techniques. We present computational results on a real-world case study, obtaining considerable improvements over the practitioners' solutions in relatively small computing times. The described methods are presented in Cacchiani et al. (2007).

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