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Real options analysis of investment in carbon capture and sequestration technology

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Abstract Among a comprehensive scope of mitigation measures for climate change, CO_2 capture and sequestration (CCS) plays a potentially significant role in industrialised countries. In this paper, we develop an analytical real options model that values the choice between two emissions-reduction technologies available to a coal-fired power plant. Specifically, the plant owner may decide to invest in either full CCS (FCCS) or partial CCS (PCCS) retrofits given uncertain electricity, CO_2, and coal prices. We first assess the opportunity to upgrade to each technology independently by determining the option value of installing a CCS unit as a function of CO_2 and fuel prices. Next, we value the option of investing in either FCCS or PCCS technology. If the volatilities of the prices are low enough, then the investment region is dichotomous, which implies

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that for a given fuel price, retrofitting to the FCCS (PCCS) technology is optimal if the CO_2 price increases (decreases) sufficiently. The numerical examples provided in this paper using current market data suggest that neither retrofit is optimal immediately. Finally, we observe that the optimal stopping boundaries are highly sensitive to CO_2 price volatility.

Keywords Real options analysis · CCS · Geometric Brownian motion · Mutually exclusive options

1 Introduction

Since the 1970s, as global greenhouse gas (GHG) emissions have increased significantly due to human activities, so have temperatures. Global average sea levels have been rising, global average air and ocean temperatures have been increasing, and wind patterns as well as snow, ice, and frozen ground have been changing (IPCC 2005). Carbon dioxide (CO_2) is referred to as the most critical anthropogenic GHG, annual emissions of which grew by about 80% between 1970 and 2004 (IPCC 2007) mainly due to fossil-fuel combustion and deforestation. Continuing CO_2 emissions at or above current rates would result in further warming and more changes to the global climate during the 21st century.

Serious consideration is currently being given by industrialised countries to reducing their CO_2 emissions. These countries, known as Annex 1 (forty countries and separately the European Union), joined the 1997 Kyoto Protocol and have agreed to reduce their CO_2 emissions to an average of 5% below 1990 levels during the period 2008–2012. In order to implement its commitments, the European Union introduced a CO_2 Emission Trading Scheme (EU ETS) that allocates CO_2 emission permits to its facilities in the power sector, iron and steel manufacturing, and other heavy industries. Such facilities may emit CO_2 annually up to their allowance limits, and any additional emission requires purchase of surplus permits from counterparties. Thus, the negative externality of CO_2 emissions may be reflected in the cost of purchasing additional permits.

A wide range of mitigation options is now available or proposed to be available by 2030. These options include better end-use efficiency improvements, conversion to less carbon-intensive fuels (e.g., switching from coal to gas), nuclear power, renewable energy sources (such as hydropower, wind, and solar), and CO_2 capture and sequestration (CCS) technology. However, since primary energy use will continue to rely on fossil fuels in the near term, CCS technology could play a key intermediate role in alleviating climate change. Moreover, CCS is more likely to reduce overall mitigation costs and allow additional flexibility in attaining GHG emission reduction (IPCC 2005). Nevertheless, according to Hildebrand and Herzog (2008), capturing almost all emissions, or full capture, is a policy that is less likely to progress either new coal-fired plants or CCS technology in the near term. The implementation of full capture at a coal-fired power plant has a critical effect on plant technology, operation, and economics. On the other hand, partial capture of the emissions could be a very good replacement at the first step. In effect, it could provide plant owners with additional flexibility in offsetting emissions costs without the burdensome capital investment or efficiency loss associated with full CCS.

This paper considers the perspective of a coal-fired power plant owner that must decide how to mitigate its CO_2 emissions by investing in either partial (PCCS) or full (FCCS) CCS technology. The former may correspond to either retrofitting only some of the generators in a power plant or capturing some of the CO_2 emissions. We assume that the power plant is operating at its rated capacity in a CO_2-constrained environment that requires the purchase of permits for any CO_2 emissions. Given uncertainty in electricity, coal, and CO_2 prices, following the standard smooth-fit techniques developed by Dixit and Pindyck (1994), we value each mutually exclusive mitigation option via the real options approach and determine when to adopt it assuming discretion over timing and technology choice. We use an approach that is similar to the one described in Décamps et al. (2006), which extends the analysis of Dixit (1993) by providing some conditions under which the optimal investment region is dichotomous under price uncertainty.

Herbelot (1992) also applies option valuation techniques in a similar study, but it analyses the investment situation of a coal-fired power plant that has to reduce its sulfur emissions by either switching to lower-sulfur coal or investing in an emission control system. The two stochastic variables in this study (allowance price and coal price premium)[1] follow correlated geometric Brownian motion (GBM) processes. It develops a discrete-time binomial model to evaluate numerically the investment opportunity. Pindyck (2002) proposes a continuous-time model of environmental policy adoption that takes into account uncertainty over both environmental change and the social costs of environmental damage. The analytical solution to this problem is formalised in Adkins and Paxson (2008), which examines an asset depending on both uncertain revenues and operation costs that has a renewal opportunity. It provides a stochastic two-factor real options model that is solved analytically. While Wickart and Madlener (2007) also uses the real options approach to consider a two-factor model, i.e., the mutually exclusive investment choice between combined heat-and-power production and a conventional heat-only generation system, it accounts for uncertainty in one variable at a time. Abadie and Chamarro (2008a), on the other hand, assumes two sources of risk, viz., the price of emissions allowance and the price of electricity, and evaluates the option to install a CCS unit in a coal-fired power plant via a lattice-based approach. It models the electricity and CO_2 emissions permit prices as evolving according to correlated geometric mean-reverting (GMR) and GBM processes, respectively, and obtains the allowance price thresholds above which it is optimal to invest in CCS immediately. The results indicate that current permit prices do not lead to an immediate adoption of this technology. Abadie and Chamarro (2008b), applying binomial lattices, studies the choice between investing either in a natural gas combined cycle (NGCC) power plant or in an integrated gasification combined cycle (IGCC) power plant.

The contribution of our paper is twofold: (i) we analyse the incentives for CCS retrofits, and (ii) we expand the real options theory for mutually exclusive investment under uncertainty to the case with two risk factors. We examine two situations:

[1] The difference between low-sulfur and high-sulfur coal prices.

- Individual investment options, when investing in FCCS and PCCS technologies are analysed independently.
- Mutually exclusive options, when the decision to invest in either FCCS or PCCS technology is explored.

Having more than one stochastic variable and following the same procedure as in Adkins and Paxson (2008), we evaluate the individual investment options analytically. Moreover, we calculate an optimal stopping boundary for the CO_2 permit price, depending on the fuel price, above which it is optimal to invest in FCCS/PCCS technology immediately. The results of our study suggest that at current CO_2 and coal prices, adopting the emission-reduction policy is not optimal, although both technologies (FCCS and PCCS) are in the money. This general conclusion is thoroughly consistent with previous studies, such as Abadie and Chamarro (2008a). However, as a result of applying different approaches and using different stochastic models for prices, the CO_2 thresholds may, unsurprisingly, differ in comparable studies.

Evaluating the mutually exclusive options, we generalise the theory proposed by Décamps et al. (2006) into a two-dimensional space. We introduce an indifference region around the intersection of the NPVs of the projects, over which it is optimal to wait before investing in either technology. As the FCCS technology produces higher cash flows than the PCCS one along with a significantly larger sunk capital cost, the optimal investment region may become dichotomous. After evaluating each project separately, we have two different option values and, correspondingly, two optimal stopping boundaries: $C^{*(pccs)}(F)$ and $C^{*(fccs)}(F)$. If the CO_2 price is less than $C^{*(pccs)}(F)$, then the plant owner waits until the CO_2 price reaches this value via either an increase in the CO_2 price or a decrease in the fuel price. However, for high values of CO_2, around the indifference curve, the solution to the separate valuation is no longer optimal. Over this region, there are two critical thresholds, $C_L^*(F)$ and $C_U^*(F)$ ($C_L^*(F) < C_U^*(F)$). When the current CO_2 price is included in $[C^{*(pccs)}(F), C_L^*(F)]$, it is optimal to invest immediately in PCCS technology, while for those values greater than $C_U^*(F)$, it is optimal to invest immediately in FCCS. For values in $[C_L^*(F), C_U^*(F)]$, however, it is optimal to wait. Since there is no analytical solution to valuing the mutually exclusive option to retrofit, we propose an algorithm in order to solve this two-factor real options problem numerically. After valuing the mutually exclusive options, we show that without considering the waiting opportunity over the indifference region, the plant owner may lose a modest amount of money by investing immediately. We then explore how these variables, viz., the CO_2 emission allowance and coal prices, may interact in affecting the time of adoption. Finally, we focus on the effects of price volatility on such mutually exclusive mitigation options.

The remainder of this paper is organised as follows. In Sect. 2, we briefly describe the CCS technology. In Sect. 3, we introduce the models of stochastic prices, calculate the analytical solution of the option value of investing in FCCS and PCCS technologies independently, and derive the value of the opportunity to choose between the two technologies. The optimal stopping boundaries are also calculated in both individual and mutually exclusive options. Using the data provided in Sect. 4, we then, in Sect. 5, discuss the implication of our study addressing two numerical examples along with a sensitivity analysis. Finally, Sect. 6 concludes and provides directions for future research in this area.

2 Carbon capture and sequestration technology

CCS is a process by which CO_2 is separated from industrial and energy-related sources. It is then transported for geological storage, ocean storage, or mineral carbonation in order to be isolated permanently from the atmosphere or for use in industrial processes (IPCC 2005). A power plant equipped with CCS technology requires additional energy for capture, transport, and storage, which causes a reduction in overall efficiency of the plant.

According to IPCC (2005), there are three types of CO_2 capture systems:

- Post-combustion, which captures CO_2 from the flue gas and is applied in existing power plants;
- Pre-combustion, in which CO_2 in the fuel is separated before combustion, which is more costly and applicable only to new fossil fuel plants;
- Oxyfuel combustion, which uses high purity of oxygen that causes CO_2 with high concentrations in flue gas to be easily separated. However, it is more expensive because of a higher energy requirement to produce pure oxygen.

After CO_2 is captured, it can be transported from the source to the storage site either through pipelines or using ships. However, for a large amount of CO_2 over short distances, pipelines are preferred, although smaller volumes of CO_2, specifically for larger distances overseas, may be transported with ships (IPCC 2005).

Installing FCCS technology with access to geological or ocean storage, a coal-fired power plant can capture up to 85–95% of its CO_2 emissions (IPCC 2005), while using approximately 10–40% more energy than before. However, achieving this CO_2 capture is likely to be too expensive and almost impossible in near term. With regard to this difficulty, Hildebrand and Herzog (2008) considers a lower rate of capturing, PCCS, as a reasonable first step in putting CCS into action. A coal-fired power plant equipped with PCCS could lower its CO_2 emissions down to a gas-fired power plant's, i.e., a capture of nearly 45–65%. FCCS technology could cause up to 60% increase in the capital cost of a pulverised coal power plant, while this increase for PCCS is extremely less. Moreover, a power plant with PCCS requires less energy than a power plant with FCCS, thereby limiting the efficiency loss.

3 Problem formulation

3.1 Assumptions

We take the perspective of the owner of a baseload coal-fired power plant with infinite lifetime[2] intending to reduce its CO_2 emissions by investing in either PCCS or FCCS

[2] Although a coal-fired power plant has a typical lifetime of forty years, for simplicity, in this paper, we assume that it has an infinite lifetime. This is justified by the impact of discounting the cashflows that are

technology. Since the timing of the retrofit is at the discretion of the owner, the option is perpetual. Additionally, we assume that the investment is entirely irreversible and cannot be scrapped once installed, nor is it possible to suspend the CCS unit to allow venting. The option of switching from one technology to another is also assumed to be impracticable in this study. Three sources of uncertainty are taken into consideration: fuel input price, F_t (in \$/MWh), electricity output price, E_t (in \$/MWh$_e$), and CO_2 permit price, C_t (in \$/tCO_2). Future revenues and costs of the investment are discounted at a subjective constant annual rate, μ. After investing in either technology, the electricity production of the plant, Q (in MWh$_e$/year would remain the same as before; however, the overall efficiency of the plant will decline due to further energy requirements. Finally, once the retrofit decision is made, the CCS technology is installed immediately, i.e., there is no time-to-build problem as in Majd and Pindyck (1987).

3.2 NPV of mitigation projects

We assume that E_t, F_t, and C_t evolve stochastically according to the following GBM processes:[3]

$$dE_t = \alpha_E E_t\, dt + \sigma_E E_t\, dz_t^E \tag{1}$$

$$dF_t = \alpha_F F_t\, dt + \sigma_F F_t\, dz_t^F \tag{2}$$

$$dC_t = \alpha_C C_t\, dt + \sigma_C C_t\, dz_t^C \tag{3}$$

where $\{\alpha_i < \mu; i = E, F, C\}$ and $\{\sigma_i; i = E, F, C\}$ are, respectively, the drift and the volatility parameters, and dz_t^i stands for the increment of standard Brownian motion process. Moreover, we suppose that the prices are correlated, i.e., $\mathcal{E}(dz_t^i\, dz_t^j) = \rho_{ij}\, dt$ for $\{(i, j) = (E, F), (E, C), (F, C)\}$. Therefore, the net expected discounted profit of an existing power plant without any CCS, conditional on current prices E, F, and C, is given by:

$$\begin{aligned} V(E, F, C) &= Q\mathcal{E}\left[\int_0^{\infty} (E_t e^{-\mu t} - \epsilon_F F_t e^{-\mu t} - \epsilon_C C_t e^{-\mu t})\, dt | E_0 = E, F_0 = F, C_0 = C\right] \\ &= Q\left[\frac{E}{\mu - \alpha_E} - \frac{\epsilon_F F}{\mu - \alpha_F} - \frac{\epsilon_C C}{\mu - \alpha_C}\right] \end{aligned} \tag{4}$$

where ϵ_F and ϵ_C represent the heat rate (in MWh/MWh$_e$) and the emission rate (in tCO_2/MWh$_e$), respectively, of a power plant without CCS. Thus, the expected net

Footnote 2 continued
several decades in the future. Plus, assuming that all equipment lasts forever removes any complication from having to compare technologies with different lifetimes.

[3] As suggested in Pindyck (1999), although long-run energy prices are mean-reverting, since their rate of mean reversion is low, the GBM assumption may be acceptable in many applications.

present value (NPV) of investing in retrofit project $j = \{pccs, fccs\}$ can be calculated as follows:

$$V^{(j)}(E,F,C) = Q\left[\frac{E}{\mu-\alpha_E} - \frac{\epsilon_F^{(j)}F}{\mu-\alpha_F} - \frac{\epsilon_C^{(j)}C}{\mu-\alpha_C}\right] - I^{(j)} - V(E,F,C)$$
$$\Rightarrow V^{(j)}(F,C) = Q\left[\frac{(\epsilon_F-\epsilon_F^{(j)})F}{\mu-\alpha_F} + \frac{(\epsilon_C-\epsilon_C^{(j)})C}{\mu-\alpha_C}\right] - I^{(j)} \tag{5}$$

where $I^{(j)}$ includes the initial sunk capital cost of the retrofit to technology j together with all other costs, such as additional operating and maintenance costs, which are discounted at the constant rate μ. Here, $\epsilon_C^{(j)}$ and $\epsilon_F^{(j)}$ are the CO_2 emissions and heat rate, respectively, with retrofit j. From Eq. (5), it is revealed that the expected NPV of mitigation no longer depends on the electricity price since the plant's electricity output is unaffected. As we could expect, the expected NPV is decreasing in F and increasing in C because of the negative coefficient $(\epsilon_F - \epsilon_F^{(j)})$ and the positive coefficient $(\epsilon_C - \epsilon_C^{(j)})$, respectively. Intuitively, CCS technology reduces the plant's efficiency, which increases its post-retrofit heat rate while decreasing its CO_2 emissions rate. Accordingly, the value of the opportunity to mitigate, $W^{(j)}(F,C)$, depends only on the fuel price and CO_2 permit price.

3.3 Valuation of the mitigation options

3.3.1 Individual investment options

Using dynamic programming, we first derive the value of the option to invest in PCCS and FCCS, independently. The Bellman equation, as the primary equation of optimisation theory, states that the rate of return on the option, $\mu W^{(j)}(F,C)$, must equal the expected rate of capital gain on it, $\mathcal{E}[dW^{(j)}(F,C)]/dt$ (see Dixit and Pindyck 1994 for more details):

$$\mu W^{(j)}(F,C) = \mathcal{E}[dW^{(j)}(F,C)]/dt \tag{6}$$

Thus, by applying Itô's lemma to the right-hand side of Eq. (6), the option to invest in j must satisfy the following partial differential equation (PDE):

$$\mu W^{(j)}(F,C) = \alpha_F F W_F^{(j)}(F,C) + \frac{1}{2}\sigma_F^2 F^2 W_{FF}^{(j)}(F,C) + \alpha_C C W_C^{(j)}(F,C)$$
$$+\frac{1}{2}\sigma_C^2 C^2 W_{CC}^{(j)}(F,C) + \rho\sigma_F\sigma_C FC W_{FC}^{(j)}(F,C) \tag{7}$$

where the subscripts denote the partial derivatives, e.g., $W_F^{(j)}(F,C) = \frac{\partial W^{(j)}(F,C)}{\partial F}$, and $\rho = \frac{\mathcal{E}(dz_t^F dz_t^C)}{dt}$.[4]

[4] Since the electricity price is not relevant to retrofits, from now on, we define $\rho = \rho_{FC}$.

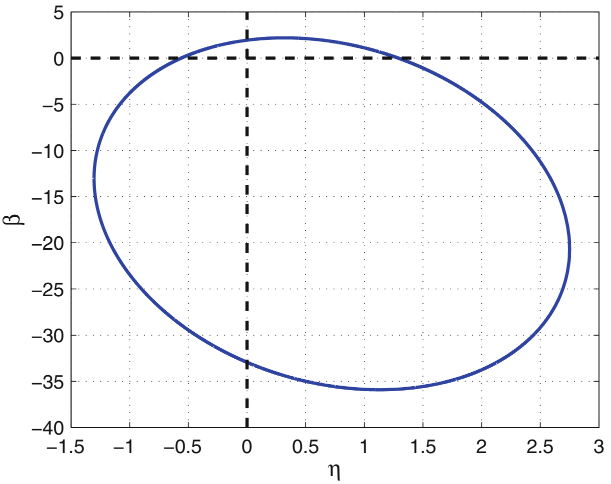

Fig. 1 Function $H(\beta, \eta) = 0$

A general solution to the PDE, Eq. (7), is of the power form as follows:

$$W^{(j)}(F, C) = A^{(j)} F^{\beta^{(j)}} C^{\eta^{(j)}}; \quad 0 < F < \infty, \; 0 < C < C^{*(j)}(F) \tag{8}$$

where $A^{(j)}$, $\beta^{(j)}$, and $\eta^{(j)}$ are endogenous coefficients, depending on F, which are to be determined together with the free boundary, $C^{*(j)}(F)$. Substituting Eq. (8) into Eq. (7) yields:

$$\begin{aligned} H\left(\beta^{(j)}, \eta^{(j)}\right) = \alpha_F \beta^{(j)} + \frac{1}{2}\sigma_F^2 \beta^{(j)}(\beta^{(j)} - 1) + \alpha_C \eta^{(j)} + \frac{1}{2}\sigma_C^2 \eta^{(j)}(\eta^{(j)} - 1) \\ + \rho \sigma_F \sigma_C \beta^{(j)} \eta^{(j)} - \mu = 0 \end{aligned} \tag{9}$$

Equation (9) is that of an ellipse in η and β that passes through all four axes (Adkins and Paxson 2008) and is graphed in Fig. 1 using the data provided in Table 1. This implies that Eq. (8) can have the form:

$$W^{(j)}(F, C) = A_1^{(j)} F^{\beta_1^{(j)}} C^{\eta_1^{(j)}} + A_2^{(j)} F^{\beta_2^{(j)}} C^{\eta_2^{(j)}} + A_3^{(j)} F^{\beta_3^{(j)}} C^{\eta_3^{(j)}} + A_4^{(j)} F^{\beta_4^{(j)}} C^{\eta_4^{(j)}} \tag{10}$$

where,

$$\begin{aligned} \eta_1^{(j)} > 0 \quad &\text{and} \quad \beta_1^{(j)} < 0 \\ \eta_2^{(j)} < 0 \quad &\text{and} \quad \beta_2^{(j)} > 0 \\ \eta_3^{(j)} > 0 \quad &\text{and} \quad \beta_3^{(j)} > 0 \\ \eta_4^{(j)} < 0 \quad &\text{and} \quad \beta_4^{(j)} < 0 \end{aligned} \tag{11}$$

Table 1 Price and plant parameter values

Parameter	Description	Value
α_F	Growth rate of coal price	0.04
α_C	Growth rate of CO_2 price	0.03[a]
σ_F	Volatility of coal price	0.05[b]
σ_C	Volatility of CO_2 price	0.47[a]
ρ	Correlation between coal and CO_2 prices	0.20[c]
μ	Discount rate	0.08
Φ	Capacity of the plant (MW_e)	500
Q	Annual energy production of the plant (MWh_e)	4,380,000
F_0	Current price of coal (\$/MWh)	15.5[d]
C_0	Current price of CO_2 (\$/tCO_2)	25.59[a]

[a] Abadie and Chamarro (2008a)'s data (using daily futures price data from ICE)
[b] Abadie and Chamarro (2008b)'s data (using yearly average prices gathered by the US Energy Information Administration)
[c] Since there is little information on CO_2 permit prices, we first assume a reasonable positive correlation coefficient between CO_2 and fuel prices. We then show how any changes in this coefficient may affect the results
[d] The current price of coal is \$95/tCO_2. According to ORNL (2009), a ton of coal on average produces 22 GJ (6.11 MWh) of energy. Thus, \$95/tCO_2 divided by 6.11 MWh/tCO_2 yields approximately \$15.5/MWh

However, by imposing limiting boundary conditions on F and C, we can eliminate the last three terms in Eq. (10). When the fuel price, F, tends to infinity, the option value becomes worthless, therefore, the coefficients $A_2^{(j)}$ and $A_3^{(j)}$ in Eq. (10) must be zero to prevent from diverging. Similarly, for low values of C (close to zero) it is not justifiable to invest in any CCS technology, i.e., the option value is worthless and the coefficient $A_4^{(j)}$ in Eq. (10) must be zero, too. We then end up with the following option value function:

$$W^{(j)}(F,C) = A_1^{(j)} F^{\beta_1^{(j)}} C^{\eta_1^{(j)}}, \quad 0 < F < \infty,\ 0 < C < C^{*(j)}(F) \tag{12}$$

which can be rewritten as:

$$W^{(j)}(F,C) = A^{(j)} F^{\beta^{(j)}} C^{\eta^{(j)}}, \quad 0 < F < \infty,\ 0 < C < C^{*(j)}(F) \tag{13}$$

where $\eta^{(j)} > 0$ and $\beta^{(j)} < 0$. To prove uniqueness of the solution, standard techniques for such elliptic PDEs usually rely on proof by contradiction, which are outlined in Appendix A.

We now use a value-matching and two smooth-pasting conditions along with Eq. (9) to solve for the four unknowns:

$$A^{(j)} F^{\beta^{(j)}} C^{\eta^{(j)}} = Q \left[\frac{\left(\epsilon_F - \epsilon_F^{(j)}\right)}{\mu - \alpha_F} F + \frac{\left(\epsilon_C - \epsilon_C^{(j)}\right)}{\mu - \alpha_C} C \right] - I^{(j)} \quad \text{on } C = C^{*(j)}(F) \tag{14}$$

$$A^{(j)} \beta^{(j)} F^{\beta^{(j)}-1} C^{\eta^{(j)}} = Q \frac{\left(\epsilon_F - \epsilon_F^{(j)}\right)}{\mu - \alpha_F} \quad \text{on } C = C^{*(j)}(F) \tag{15}$$

$$A^{(j)} \eta^{(j)} F^{\beta^{(j)}} C^{\eta^{(j)}-1} = \frac{\left(\epsilon_C - \epsilon_C^{(j)}\right)}{\mu - \alpha_C} \quad \text{on } C = C^{*(j)}(F) \tag{16}$$

Rearranging Eq. (16), we obtain the coefficient $A^{(j)}$ as follows:

$$A^{(j)} = \frac{Q\left(\epsilon_C - \epsilon_C^{(j)}\right)}{\eta^{(j)}(\mu - \alpha_C)} F^{-\beta^{(j)}} [C^{*(j)}(F)]^{1-\eta^{(j)}} \tag{17}$$

Substituting this into Eq. (15) gives the following equation for the optimal stopping boundary:

$$C^{*(j)}(F) = \frac{\eta^{(j)} \left(\epsilon_F - \epsilon_F^{(j)}\right)(\mu - \alpha_C)}{\beta^{(j)} \left(\epsilon_C - \epsilon_C^{(j)}\right)(\mu - \alpha_F)} F \tag{18}$$

Finally, a linear relationship between $\beta^{(j)}$ and $\eta^{(j)}$ using Eq. (14) is given by:

$$\beta^{(j)} = \frac{Q\left(\epsilon_F - \epsilon_F^{(j)}\right)\left(\eta^{(j)} - 1\right) F}{(\mu - \alpha_F)\, I^{(j)} - Q\left(\epsilon_F - \epsilon_F^{(j)}\right) F}, \tag{19}$$

which is decreasing in $\eta^{(j)}$, because of the negative coefficient $\left(\epsilon_F - \epsilon_F^{(j)}\right)$ and the positive denominator.[5] If we impose this line on $H(\beta^{(j)}, \eta^{(j)}) = 0$, then it intersects the function at two points, which we now try to obtain. In Fig. 2, using the data for PCCS technology, provided in Table 2, we show the intersections of the two lines, for the lowest and the highest value of F in our range of data, and the ellipse $H(\beta^{(j)}, \eta^{(j)}) = 0$.

After substituting the exponent $\beta^{(j)}$ from Eq. (19) into Eq. (9), we end up with the following quadratic polynomial:

$$a(\eta^{(j)})^2 - b\eta^{(j)} - c = 0 \tag{20}$$

[5] The denominator, $[(\mu - \alpha_F)I^{(j)} - Q(\epsilon_F - \epsilon_F^{(j)})F]$, is positive because $(\mu - \alpha_F)$ is positive and $(\epsilon_F - \epsilon_F^{(j)})$ is negative.

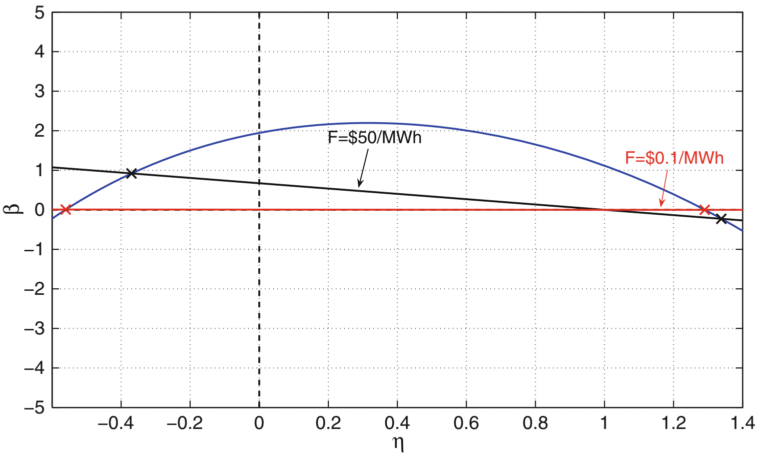

Fig. 2 The intersection of function $H(\beta, \eta) = 0$ (data from Table 1) and Eq. (19) for PCCS technology (data from Table 2), e.g., when $F = \$50/\mathrm{MWh}$, $\eta_1^{(pccs)} = 1.33$ and $\beta_1^{(pccs)} = -0.21$

Table 2 CCS parameter values

Parameter	Description	PC	PC with PCCS	PC with FCCS
ϵ_C	Emission rate (tCO_2/MWh_e)	0.80	0.32[a]	0.08[b]
ϵ_F	Heat rate (MWh/MWh_e)	2.42	2.55	2.8
O & M	Additional operation and maintenance ($\$/MWh_e$)	–	1.4	1.5
T & S	Transport and storage ($\$/tCO_2$)	–	9	9
K	Initial capital cost of retrofit (m$)	–	130	331.57
$I^{(j)}$[c]	Total retrofit investment cost (m$)	–	443.17	768.475

[a] Capture of nearly 60% of the CO_2 emissions
[b] Capture of nearly 90% of the CO_2 emissions
[c] $I^{(j)} = K^{(j)} + \frac{Q}{\mu}(\mathrm{O\ \&\ M}) + \frac{Q}{\mu}(\mathrm{T\ \&\ S})(\epsilon_C - \epsilon_C^{(j)})$

where,

$$a = \left(\frac{1}{2}\sigma_F^2 + \frac{1}{2}\sigma_C^2 - \rho\sigma_F\sigma_C\right)\left(\left(\epsilon_F - \epsilon_F^{(j)}\right) QF\right)^2 - \left(\sigma_C^2 - \rho\sigma_F\sigma_C\right)(\mu - \alpha_F)\left(\epsilon_F - \epsilon_F^{(j)}\right) QI^{(j)}F + \frac{1}{2}\sigma_C^2(\mu - \alpha_F)^2\left(I^{(j)}\right)^2 \tag{21}$$

$$b = \left(\frac{1}{2}\sigma_F^2 + \frac{1}{2}\sigma_C^2 - \rho\sigma_F\sigma_C + \alpha_F - \alpha_C\right)\left(\left(\epsilon_F - \epsilon_F^{(j)}\right) QF\right)^2 - \left(\sigma_C^2 - \rho\sigma_F\sigma_C - \frac{1}{2}\sigma_F^2 + \alpha_F - 2\alpha_C\right)(\mu - \alpha_F)\left(\epsilon_F - \epsilon_F^{(j)}\right) QI^{(j)}F + \left(\frac{1}{2}\sigma_C^2 - \alpha_C\right)(\mu - \alpha_F)^2\left(I^{(j)}\right)^2 \tag{22}$$

$$
\begin{aligned}
c = {} & (\mu - \alpha_F)\left(\left(\epsilon_F - \epsilon_F^{(j)}\right) QF\right)^2 \\
& + \left(\alpha_F - \frac{1}{2}\sigma_F^2 - 2\mu\right)(\mu - \alpha_F)\left(\epsilon_F - \epsilon_F^{(j)}\right) QI^{(j)}F \\
& + \mu(\mu - \alpha_F)^2 \left(I^{(j)}\right)^2
\end{aligned} \tag{23}
$$

Since $(\epsilon_F - \epsilon_F^{(j)}) < 0$, $(\mu - \alpha_F) > 0$, and the volatility of coal price is always less than that of the CO_2 price ($\sigma_F < \sigma_C$), coefficients a and c are positive. The discriminant $\Delta = b^2 + 4ac$ is, therefore, positive, which ensures the existence of two real and distinct roots:

$$
\eta_1^{(j)} = \frac{b + \sqrt{b^2 + 4ac}}{2a} \tag{24}
$$

$$
\eta_2^{(j)} = \frac{b - \sqrt{b^2 + 4ac}}{2a} \tag{25}
$$

In Appendix B, we prove that $\eta_1^{(j)}$ is always greater than 1; as a result, the corresponding $\beta_1^{(j)}$ calculated from Eq. (19) is negative. On the other hand, $\eta_2^{(j)}$ is negative, thus the corresponding $\beta_2^{(j)}$ is positive. It is observed that the boundary condition $W^{(j)}(F, C) \to 0$ as $F \to \infty$ appears superfluous and seems entirely guaranteed by value-matching and smooth-pasting conditions. Therefore, the unknowns $\eta^{(j)}$, $\beta^{(j)}$, and $A^{(j)}$ in Eq. (13) are calculated, respectively, via Eqs. (24), (19), and (17). Figure 2 shows that, for this choice of data, $\eta_1^{(j)}$ is increasing in F while $\beta_1^{(j)}$ is decreasing. Equation (19) also substantiates the inverse relationship between $\beta_1^{(j)}$ and $\eta_1^{(j)}$. A list of the calculated unknowns for some values of F are reported in Appendix C.

We may, finally, be interested in simplifying the option value function by substituting $A^{(j)}$ into Eq. (13) and combining Eqs. (19) and (18). We then have:

$$
\begin{aligned}
W^{(j)}(F, C) = \frac{Q\left(\epsilon_C - \epsilon_C^{(j)}\right)}{\eta^{(j)}(\mu - \alpha_C)} \left[C^{*(j)}(F)\right]^{1-\eta^{(j)}} C^{\eta^{(j)}}, \\
0 < F < \infty,\ 0 < C < C^{*(j)}(F)
\end{aligned} \tag{26}
$$

where $\eta^{(j)}$ is calculated from Eq. (24) and

$$
C^{*(j)}(F) = \frac{\eta^{(j)}(\mu - \alpha_C)}{(\eta^{(j)} - 1)(\mu - \alpha_F)} \frac{(\mu - \alpha_F)I^{(j)} - Q\left(\epsilon_F - \epsilon_F^{(j)}\right)F}{Q\left(\epsilon_C - \epsilon_C^{(j)}\right)} \tag{27}
$$

3.3.2 Mutually exclusive options

Now, we would like to consider the mutually exclusive option to retrofit with either PCCS or FCCS technology. By plotting the expected NPV of each technology, we note that there will be an indifference curve, $C_I(F)$, where they intersect, and if the volatilities are low enough, then it may be the case that the option value for investment in PCCS technology is greater than that for the FCCS technology. In this event, an indifference region will open up around the indifference curve, in which case it is optimal to wait before investing in either technology.

This dichotomous option, which includes the option value functions of both technologies, must satisfy the Bellman equation [Eq. (9)]. Following the same methodology as in Sect. 3.3.1, over the indifference region, $\{(F, C) \mid 0 < F < \infty,\ C_L^*(F) < C < C_U^*(F)\}$, it must have the form:

$$\Psi(F, C) = D_1 F^{\delta_1} C^{\gamma_1} + D_2 F^{\delta_2} C^{\gamma_2} + D_3 F^{\delta_3} C^{\gamma_3} + D_4 F^{\delta_4} C^{\gamma_2} \tag{28}$$

where,

$$\begin{aligned} &D_1,\ D_2,\ D_3,\ D_4 \ > 0 \\ &\delta_1 < 0 \quad \text{and} \quad \gamma_1 > 0 \\ &\delta_2 > 0 \quad \text{and} \quad \gamma_2 < 0 \\ &\delta_3 > 0 \quad \text{and} \quad \gamma_3 > 0 \\ &\delta_4 < 0 \quad \text{and} \quad \gamma_4 < 0 \end{aligned} \tag{29}$$

However, the limiting boundary conditions of F help us to get rid of the last two terms in Eq. (28). For low values of F (close to zero), the option value of investing in PCCS becomes worthless, and the mutually exclusive option value equals the option value of investing in FCCS. This occurs if $D_4 = 0$ and the coefficients D_1, δ_1, and γ_1 tend to, respectively, $A_1^{(fccs)}$, $\beta_1^{(fccs)}$, and $\eta_1^{(fccs)}$. On the other hand, for large values of $F(F \rightarrow \infty)$, the option value of investing in FCCS becomes worthless and the mutually exclusive option value approaches the option value of investing in PCCS. This condition holds if $D_3 = 0$ and the coefficients D_2, δ_2, and γ_2 tend to $A_2^{(pccs)}$, $\beta_2^{(pccs)}$, and $\eta_2^{(pccs)}$, respectively. We, finally, end up with the following option value:

$$\Psi(F, C) = D_1 F^{\delta_1} C^{\gamma_1} + D_2 F^{\delta_2} C^{\gamma_2} \tag{30}$$

where,

$$\begin{aligned} &D_1,\ D_2 \ > 0 \\ &\delta_1 < 0 \quad \text{and} \quad \gamma_1 > 0 \\ &\delta_2 > 0 \quad \text{and} \quad \gamma_2 < 0 \end{aligned} \tag{31}$$

Intuitively, in the indifference region, when the fuel price decreases and the CO_2 permit price increases, investment in FCCS becomes more likely. Therefore, for any value of (F, C) in this region, the first term on the right-hand side of Eq. (30) can be interpreted as the value of the option to upgrade to FCCS. On the other hand, since the PCCS technology requires less energy than the FCCS one and captures less CO_2, it is more profitable when the fuel price increases and CO_2 permit price decreases. Thus, we interpret the second term on the right-hand side of Eq. (30) as the value of the option to upgrade to PCCS for any value of (F, C) in the indifference region. Now, the power coefficients, which are the two roots of Eq. (9), are to be determined along with the endogenous coefficients, D_1 and D_2, as well as the upper, $C_U^*(F)$, and lower, $C_L^*(F)$, free boundaries that indicate where the intermediate option value curve value-matches and smooth-pastes with the expected NPV curves of the FCCS and PCCS technologies, respectively.

Substituting Eq. (30) into Eq. (7) yields:

$$\left(\alpha_F\delta_1 + \frac{1}{2}\sigma_F^2\delta_1(\delta_1 - 1) + \alpha_C\gamma_1 + \frac{1}{2}\sigma_C^2\gamma_1(\gamma_1 - 1) + \rho\sigma_F\sigma_C\delta_1\gamma_1 - \mu\right) D_1F^{\delta_1}C^{\gamma_1}$$
$$+ \left(\alpha_F\delta_2 + \frac{1}{2}\sigma_F^2\delta_2(\delta_2 - 1) + \alpha_C\gamma_2 + \frac{1}{2}\sigma_C^2\gamma_2(\gamma_2 - 1) + \rho\sigma_F\sigma_C\delta_2\gamma_2 - \mu\right) D_2F^{\delta_2}C^{\gamma_2} = 0 \tag{32}$$

which holds if and only if,

$$\alpha_F\delta_1 + \frac{1}{2}\sigma_F^2\delta_1(\delta_1 - 1) + \alpha_C\gamma_1 + \frac{1}{2}\sigma_C^2\gamma_1(\gamma_1 - 1) + \rho\sigma_F\sigma_C\delta_1\gamma_1 - \mu = 0 \tag{33}$$
$$\alpha_F\delta_2 + \frac{1}{2}\sigma_F^2\delta_2(\delta_2 - 1) + \alpha_C\gamma_2 + \frac{1}{2}\sigma_C^2\gamma_2(\gamma_2 - 1) + \rho\sigma_F\sigma_C\delta_2\gamma_2 - \mu = 0 \tag{34}$$

These two equations together with the following six value-matching and smooth-pasting conditions are used to solve for the eight unknowns ($D_1, D_2, \delta_1,\ \gamma_1,\ \delta_2,\ \gamma_2$, $C_L^*(F)$, and $C_U^*(F)$):

$$\Psi(F, C) = Q\left[\frac{\left(\epsilon_F - \epsilon_F^{(pccs)}\right)}{\mu - \alpha_F}F + \frac{\left(\epsilon_C - \epsilon_C^{(pccs)}\right)}{\mu - \alpha_C}C\right] - I^{(pccs)} \quad \text{on } C = C_L^*(F) \tag{35}$$

$$\Psi_F(F, C) = Q\frac{\left(\epsilon_F - \epsilon_F^{(pccs)}\right)}{\mu - \alpha_F} \quad \text{on } C = C_L^*(F) \tag{36}$$

$$\Psi_C(F, C) = Q\frac{\left(\epsilon_C - \epsilon_C^{(pccs)}\right)}{\mu - \alpha_C} \quad \text{on } C = C_L^*(F) \tag{37}$$

$$\Psi(F, C) = Q\left[\frac{\left(\epsilon_F - \epsilon_F^{(fccs)}\right)}{\mu - \alpha_F}F + \frac{\left(\epsilon_C - \epsilon_C^{(fccs)}\right)}{\mu - \alpha_C}C\right] - I^{(fccs)} \quad \text{on } C = C_U^*(F) \tag{38}$$

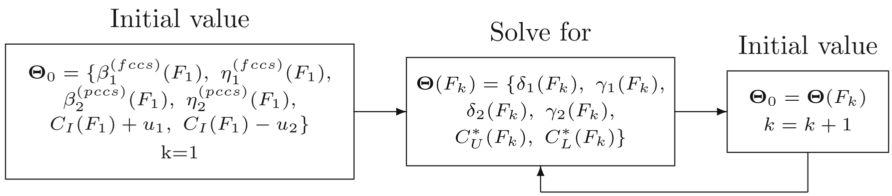

Fig. 3 Numerical solution heuristic

$$\Psi_F(F, C) = Q\frac{\left(\epsilon_F - \epsilon_F^{(fccs)}\right)}{\mu - \alpha_F} \quad \text{on } C = C_U^*(F) \tag{39}$$

$$\Psi_C(F, C) = Q\frac{\left(\epsilon_C - \epsilon_C^{(fccs)}\right)}{\mu - \alpha_C} \quad \text{on } C = C_U^*(F) \tag{40}$$

From Eqs. (39) and (40), which are linear functions of D_1 and D_2, we can calculate D_1 and D_2 in terms of the other unknowns:

$$D_1 = Q\frac{\left(\epsilon_C - \epsilon_C^{(fccs)}\right)(\mu - \alpha_F)\delta_2 C_U^*(F) - \left(\epsilon_F - \epsilon_F^{(fccs)}\right)(\mu - \alpha_C)\gamma_2 F}{(\mu - \alpha_F)(\mu - \alpha_C)\left(\gamma_1\delta_2 - \gamma_2\delta_1\right) F^{\delta_1} C_U^*(F)^{\gamma_1}} \tag{41}$$

$$D_2 = Q\frac{\left(\epsilon_F - \epsilon_F^{(fccs)}\right)(\mu - \alpha_C)\gamma_1 F - \left(\epsilon_C - \epsilon_C^{(fccs)}\right)(\mu - \alpha_F)\delta_1 C_U^*(F)}{(\mu - \alpha_F)(\mu - \alpha_C)(\gamma_1\delta_2 - \gamma_2\delta_1) F^{\delta_2} C_U^*(F)^{\gamma_2}} \tag{42}$$

By substituting these coefficients into Eqs. (35–38), we reduce the system of eight equations to a new system of six non-linear equations with six unknowns, $\Theta(F) = \{\delta_1(F), \gamma_1(F), \delta_2(F), \gamma_2(F), C_U^*(F), C_L^*(F)\}$, which must be solved numerically.

With an appropriate guess of the starting values using the `fsolve` command in Matlab, we can solve this system numerically. First, we discretise the values of the fuel price, e.g., in the ascending set $\{0, F_1, F_2, F_3, \ldots\}$. Starting from F_1, the most reasonable guess for the initial values of $\delta_1(F_1)$ and $\gamma_1(F_1)$ might be $\beta_1^{(fccs)}(F_1)$ and $\eta_1^{(fccs)}(F_1)$, respectively, calculated from Eqs. (24) and (19). Similarly, we can use $\beta_2^{(pccs)}(F_1)$ and $\eta_2^{(pccs)}(F_1)$, Eqs. (25) and (19), as an appropriate choice for the initials of $\delta_2(F_1)$ and $\gamma_2(F_1)$, respectively. However, the only information we have on the initials of $C_U^*(F_1)$ and $C_L^*(F_1)$ is that they surround the indifference point, $C_I(F_1)$. Therefore, we consider $C_I(F_1) + u_1$ and $C_I(F_1) - u_2$ as the initials of $C_U^*(F_1)$ and $C_L^*(F_1)$, respectively. Here, u_1 and u_2 may be chosen randomly, e.g., from the interval $(0, 1)\$/tCO_2$. Using these initial values, we solve the problem for $\Theta(F_1)$. Next, we use the calculated $\Theta(F_1)$ as the initial values for the unknown parameters $\Theta(F_2)$ and solve for them similarly. Successively, in each step k, the previous calculated $\Theta(F_{k-1})$ can be used as the initial value of the current step and solve the system for $\Theta(F_k)$ (see Fig. 3).

Table 3 NPV and option values

Mitigation technology (j)	NPV (m\$) $V^{(j)}(F_0, C_0) - I^{(j)}$	Option value (m\$) $W^{(j)}(F_0, C_0)$
PCCS	412.20	608.92
FCCS	200.59	809.12

4 Data

Data are reported in Tables 1 and 2. Parameters of CO_2 and coal price models and the data for FCCS technology are roughly adopted with Abadie and Chamarro (2008a,b)'s choice of parameters. The coal price in our study evolves according to a GBM process, while the electricity price, which represents the efficiency loss from the CCS retrofit, follows a GMR process with a low rate of mean reversion (0.125) in Abadie and Chamarro (2008b). The PCCS technology is proposed considering emissions reduction and initial capital cost provided by Hildebrand and Herzog (2008).

5 Numerical examples

5.1 Individual investment options

We first consider a super critical pulverised coal (SCPC) power plant that has the option to invest in PCCS/FCCS technology in order to reduce its CO_2 emissions. Given current prices, we find the optimal stopping boundaries for independently investing in PCCS and FCCS as follows:

$$C^{*(pccs)}(F_0) = \$66.33/\mathrm{tCO_2}$$
$$C^{*(fccs)}(F_0) = \$92.12/\mathrm{tCO_2}$$

As we would expect, the critical CO_2 price for investing in PCCS technology is noticeably less than that for investing in FCCS technology. This difference between the free boundaries can be attributed to the high option value of waiting (the difference between the option value and the NPV, which are reported in Table 3) for FCCS (\$608.53m) in comparison to that for PCCS (\$196.72m). Both technologies are in the money, i.e., if the plant owner has to invest now or never, then she would invest immediately. On the other hand, she would lose a large amount of money by killing the waiting opportunity, specifically by investing in FCCS technology. Clearly, the NPVs of investing in FCCS and PCCS are more sensitive to C than to F, because the coefficient of C, in Eq. (5), is larger than the coefficient of F for both technologies.

The optimal stopping boundaries for each technology are graphed in Figs. 4 and 5. As expected, these boundaries are strictly increasing with respect to F, i.e., the higher the fuel price is, the less likely the plant owner is to adopt the emission-reducing policy. It is also revealed that the boundaries are approximately linear with respect to F. This results from small changes in $\eta^{(j)}$ for different values of F, e.g., in Table 5, it is observed that $\eta^{(pccs)}$ ranges from 1.2919 to 1.3339, which causes an approximate

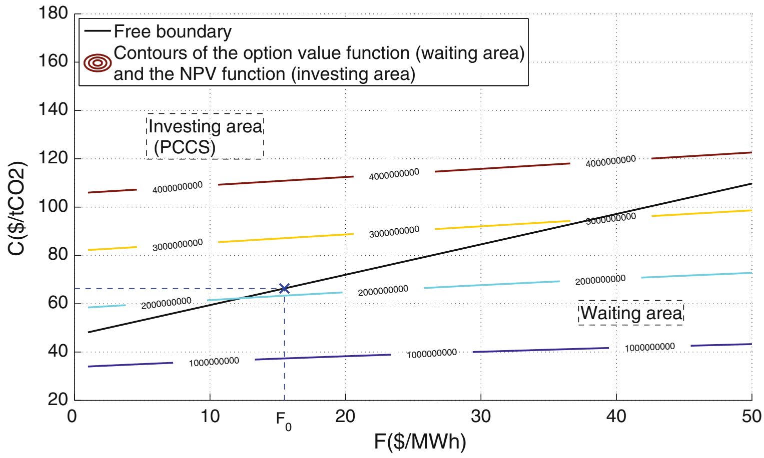

Fig. 4 Free boundary $C^{*(pccs)}(F)$ as a function of F for PCCS retrofit

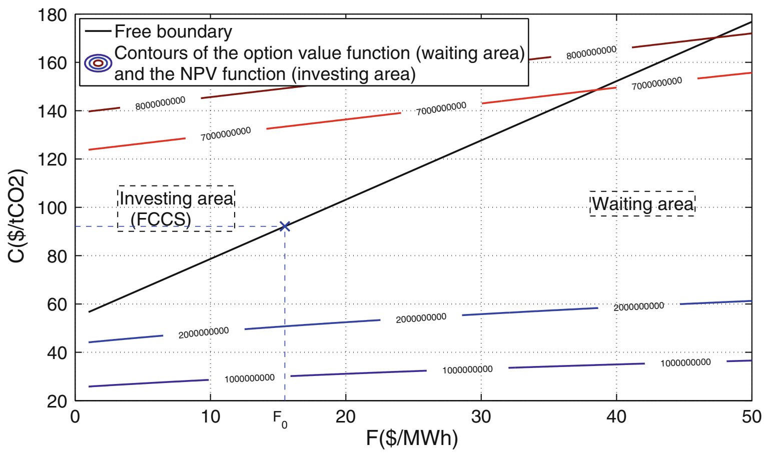

Fig. 5 Free boundary $C^{*(fccs)}(F)$ as a function of F for FCCS retrofit

linear relationship between $C^{*(j)}(F)$ and F in Eq. (27). These lines can be estimated as follows:

$$C^{*(pccs)}(F) = 46.8520 + 1.2570F \tag{43}$$

$$C^{*(fccs)}(F) = 54.1245 + 2.4523F \tag{44}$$

The NPV and the option value of investing in PCCS and FCCS are, respectively, graphed in Figs. 6 and 7. From these graphs, the distinction between the NPV and the option value of investing in FCCS compared with PCCS is clearly visible. Furthermore, the FCCS expected NPV is more sensitive to both F and C.

Our results for investing in the FCCS technology are similar to those of Abadie and Chamarro (2008a). Although the option value of investing in such CCS technology in

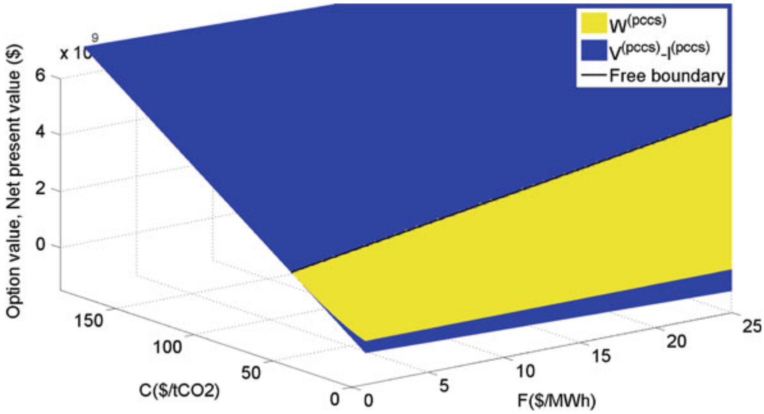

Fig. 6 NPV and option value for PCCS

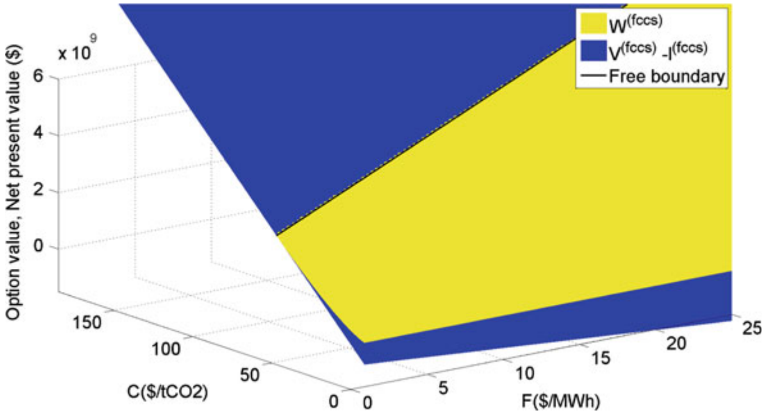

Fig. 7 NPV and option value for FCCS

both studies are nearly equal, the NPV calculated in Abadie and Chamarro (2008a) is almost twice as much as the value calculated in this paper, which may be due to our different choice of model for the fuel price as the source of cost in our model. The use of a GMR process with high volatility (50%) and high mean-reversion rate (0.96) for the electricity price in Abadie and Chamarro (2008a) precipitates adoption in comparison with our study with the assumption of a GBM process for the fuel price. This results in a higher NPV and, thus, a lower critical threshold ($\$73.54/tCO_2$) calculated in Abadie and Chamarro (2008a) in comparison with the value calculated in our study ($\$92.12/tCO_2$).

5.1.1 Sensitivity analysis

Figure 8 shows the optimal stopping boundary, $C^{*(fccs)}(F)$, for different values of σ_F and σ_C. The solid line shows the boundary for the base case values of the volatilities, $\sigma_F = 0.05$ and $\sigma_C = 0.47$. It is revealed that $C^{*(fccs)}(F)$ is more sensitive to changes

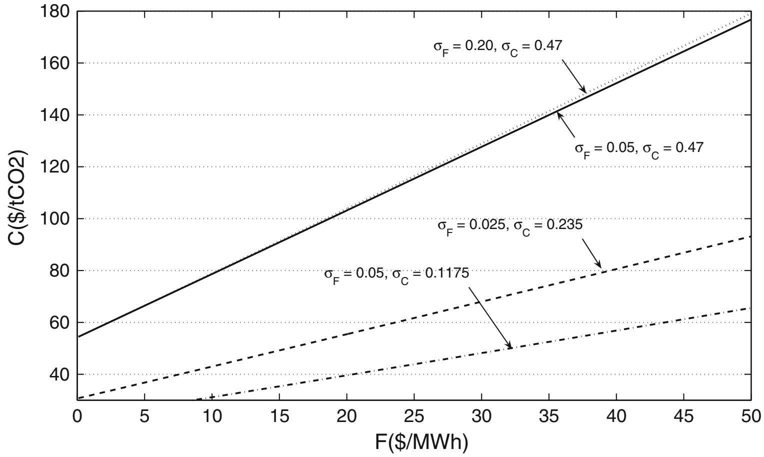

Fig. 8 FCCS Free boundary sensitivity analysis with respect to volatilities

in the CO_2 price volatility than in the fuel price volatility. By letting σ_C to be fixed at its base value, if we increase the value of σ_F to 0.2 (a 300% increase), then a negligible increase in $C^{*(fccs)}(F)$ is observed. On the other hand, a 75% decrease in σ_C (to 0.1175) can make a significant downward change in $C^{*(fccs)}(F)$. This is intuitively because the CCS technology is more exposed to the CO_2 price than to the fuel price. In general, increasing uncertainty over the prices raises the value of waiting and, thus, shifts the optimal stopping boundary upward.

The correlation between the two stochastic variables may also affect the value and the time of adopting the emission-reduction policy. Figure 9 shows that a high positive correlation between the two GBM processes makes the adoption more accessible by reducing the critical threshold. Intuitively, high positive correlation reduces the risk of large differences between the two variables because any increase (decrease) in one variable may be accompanied by an increase (decrease) in the other. Hence, due to decrease in overall uncertainty, investment is optimal sooner.

As we would expect, the larger the sunk capital cost of the investment is, the less likely the plant owner is to invest. This is illustrated in Fig. 10 that compares the optimal stopping boundary for the base value of the capital cost with the boundaries for an increase of 100% as well as a decrease of 50%. Finally, we can generalise the results from the sensitivity analysis of the FCCS technology to that of the PCCS one.

5.2 Mutually exclusive options

Now, suppose that the PC power plant has to choose between two alternative technologies: PCCS or FCCS. As discussed earlier in Sect. 3.3.2, using the data provided in Sect. 4, we first plot the expected PV of each technology to realise whether or not their intersection can lead to an indifference region. Figure 11 illustrates that the PCCS technology, which has a lower sunk capital cost, is uniformly dominated by the FCCS one. In this case, for CO_2 prices greater than the optimal boundary of FCCS

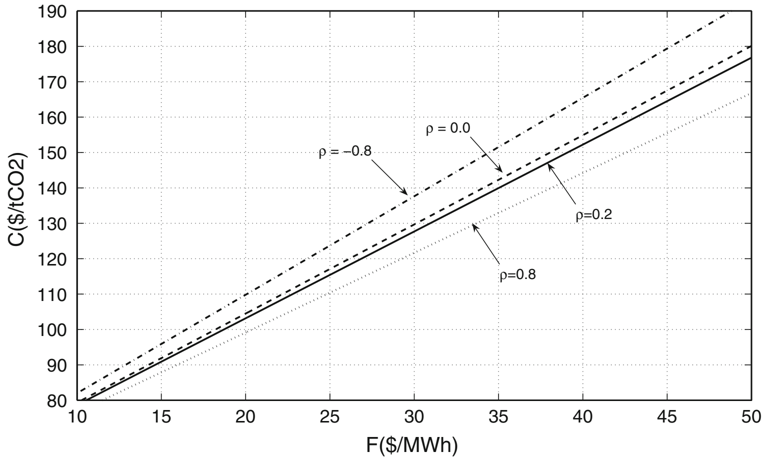

Fig. 9 FCCS free boundary sensitivity analysis with respect to the correlation coefficient

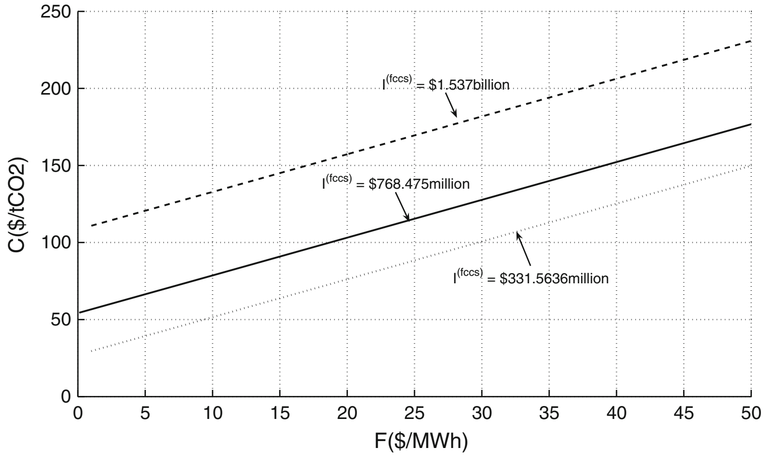

Fig. 10 FCCS free boundary sensitivity analysis with respect to the capital cost

($C^{*(fccs)}(F)$), we invest immediately in FCCS, while for those prices less than this critical boundary, we wait.

Although the data here suggest that the PCCS technology would be skipped, it may be plausible that future innovations favour it. In order to determine how the methodology of Sect. 3.3.2 may cope with such an outcome, we modify the data such that the optimal investment region becomes dichotomous. As discussed in Décamps et al. (2006), a sufficient condition in order to have a dichotomous optimal investment region is that the PCCS retrofit generate slightly lower output flow than the FCCS retrofit, but at a considerably lower sunk capital cost. We would also require the volatilities of the prices to be relatively low, otherwise the optimal investment region would never be dichotomous. Concerning this, we propose a superior PCCS technology in which

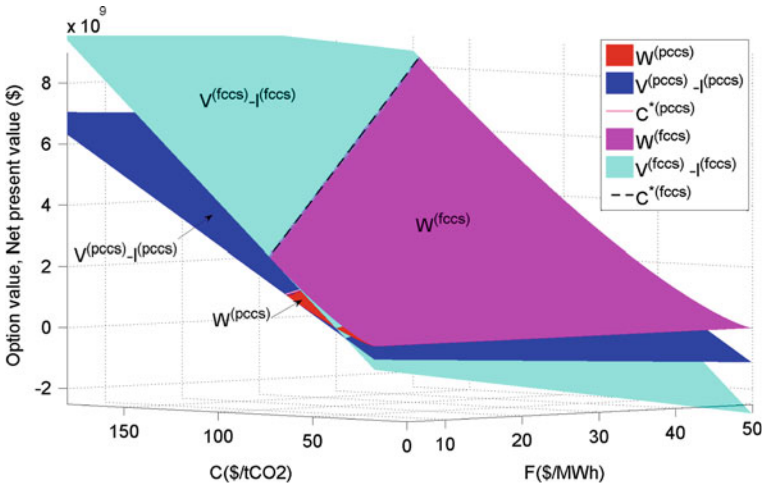

Fig. 11 NPV and option value (separate valuation) indicate that the PCCS technology is uniformly dominated by the FCCS one ($\sigma_F = 0.05$ and $\sigma_C = 0.47$)

the CO_2 emissions rate drops to $0.14tCO_2/MWh_e$ (capture of nearly 82%) while the initial capital cost is reduced to \$75m. All other parameters are kept unchanged. We now plot the expected NPV of each technology in Fig. 12. It is observed that the option value of investing in PCCS is greater than that of investing in FCCS. This fact results in an indifference region opening up around the indifference line in which it is optimal for the investor to wait before investing in either technology. It should be mentioned that our solution to the individual investment options holds over the range $[0, C^{*(pccs)}(F)]$. We now need to evaluate the intermediate option and to find the two thresholds: C_L^* and C_U^*.

The intermediate option value as well as the thresholds are calculated using the algorithm in Fig. 3 and graphed in Figs. 13 and 14, respectively. It is revealed that for low values of CO_2: (i) for a constant CO_2 price, when the fuel price increases, it is more attractive to wait for PCCS, and when it decreases, it is more attractive to invest immediately; (ii) for a constant fuel price, increasing the CO_2 price results in investing in PCCS technology in order to reduce plant's CO_2 emissions. Over the indifference region: (i) for any constant CO_2 price, as the fuel price increases, investing in PCCS becomes more economical, and as it decreases, investing in FCCS is preferred; (ii) for a constant fuel price, when the CO_2 price increases, it is more attractive to invest in FCCS, and when it decreases, it is more attractive to invest in PCCS because FCCS technology captures more CO_2 emissions than the PCCS technology does. Given the current price $F_0 = 15.5(\$/MWh)$, we find the free boundary $C^{*(pccs)}(F_0) = \$50.83/tCO_2$. As the CO_2 price ($\$25.59/tCO_2$) is currently below this free boundary, no retrofit is immediately adopted. However, suppose that the current CO_2 price given $F_0 = \$15.5/MWh$ is located exactly on the indifference line, i.e., $C_I(F_0) = \$136.21/tCO_2$. The expected NPVs of investing in FCCS and PCCS, which are identical, and the mutually exclusive intermediate option value of investing in either technology are

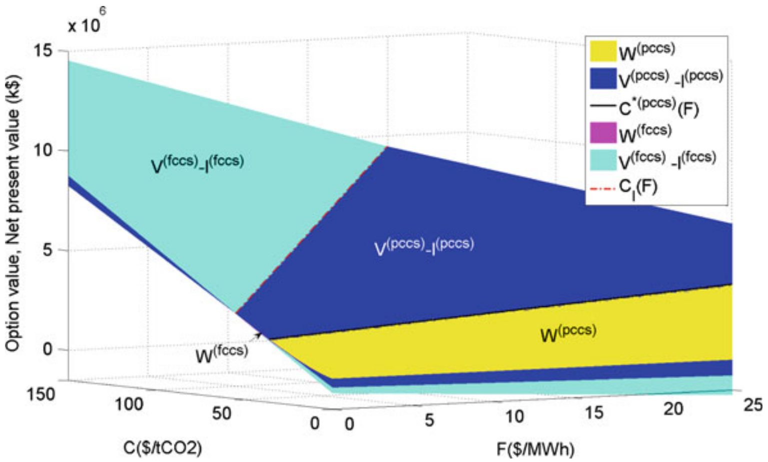

Fig. 12 NPV and option value with enhanced PCCS technology (separate valuation) indicate that the option value of investing in PCCS ($W^{(pccs)}$) is greater than that of investing in FCCS ($W^{(fccs)}$), thereby resulting in an indifference region around the indifference line ($C_I(F)$)($\sigma_F = 0.05$ and $\sigma_C = 0.47$)

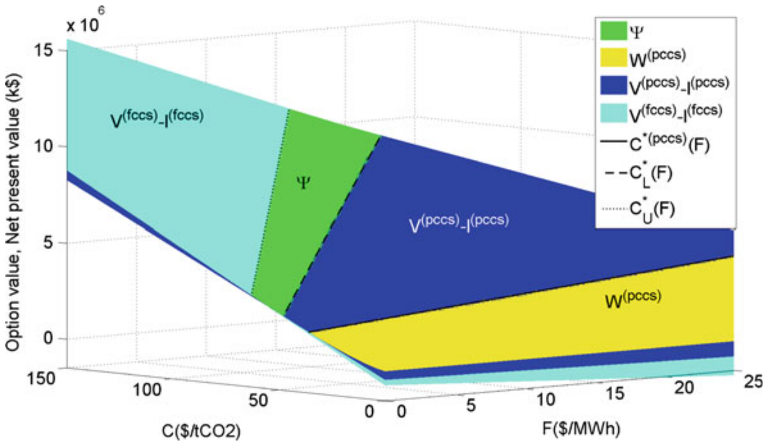

Fig. 13 NPV and option value with enhanced PCCS technology (mutually exclusive options) show that for CO_2 prices less than $C^{*(pccs)}(F)$, we wait for PCCS, while for those prices between $C^{*(pccs)}(F)$ and $C^*_L(F)$, we invest immediately in PCCS; over the indifferent region (Ψ), we wait to invest either in PCCS or FCCS, and for CO_2 prices greater than $C^*_U(F)$, we invest immediately in FCCS ($\sigma_F = 0.05$ and $\sigma_C = 0.47$)

given in Table 4. The option value of waiting before investing in either technology is then \$20.042m which shows that by investing in any technology without considering this waiting opportunity we may lose an amount equal to 0.28% of the NPV of investing. Although such a high CO_2 price is not currently plausible, future international agreements on emissions may make result in such prices. For example, in Sweden, the CO_2 tax is \$145/tCO_2 (Swedish Government Budget Bill 2008).

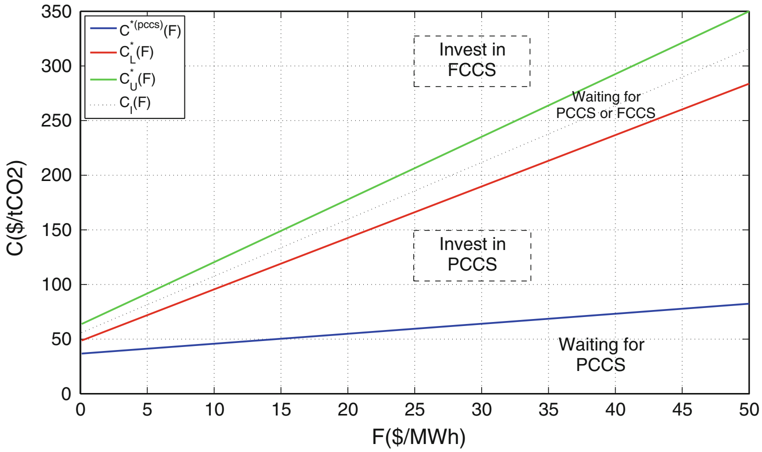

Fig. 14 Free boundaries with enhanced PCCS technology ($\sigma_F = 0.05$ and $\sigma_C = 0.47$)

Table 4 NPVs, option value, and thresholds with enhanced PCCS technology and higher initial CO_2 price

$V^{(pccs)}(F_0, C_I(F_0)) - I^{(pccs)}$	\$7.1776 billion
$V^{(fccs)}(F_0, C_I(F_0)) - I^{(fccs)}$	\$7.1776 billion
$\Psi(F_0, C_I(F_0))$	\$7.1976 billion
$C_L^*(F_0)$	\$121.45/tCO$_2$
$C_U^*(F_0)$	\$151.96/tCO$_2$

5.2.1 Sensitivity analysis

From the previous example with the models of irreversible investments, decreasing the price volatilities reduces the waiting value. This can be seen from Fig. 15, which depicts the optimal stopping boundaries with a 40% decrease in the base values of the price volatilities. Comparing these boundaries to those for the base values, it is observed that both the postponing areas are narrower for the reduced volatilities. On the other hand, the mutually exclusive intermediate option value at the indifference point, $\Psi(F_0, C_I(F_0))$, reduces to \$7.1848 billion which is equivalent to losing 0.10% of the NPV of investing by killing the waiting opportunity. This value, however, rises to \$7.2167 billion with a 40% increase in the base values of the price volatilities, which reveals that we may lose 0.55% of the NPV of investing if we fail to take advantage of waiting. Furthermore, in Fig. 16 we plot the NPVs of FCCS and PCCS technologies and their option values with the price volatilities twice as much as the base values. It is observed that even the enhanced PCCS technology, which has a lower sunk capital cost, is uniformly dominated by the FCCS one. In this case, for CO_2 prices greater than the optimal boundary of FCCS ($C^{*(fccs)}(F)$), we invest immediately in FCCS.

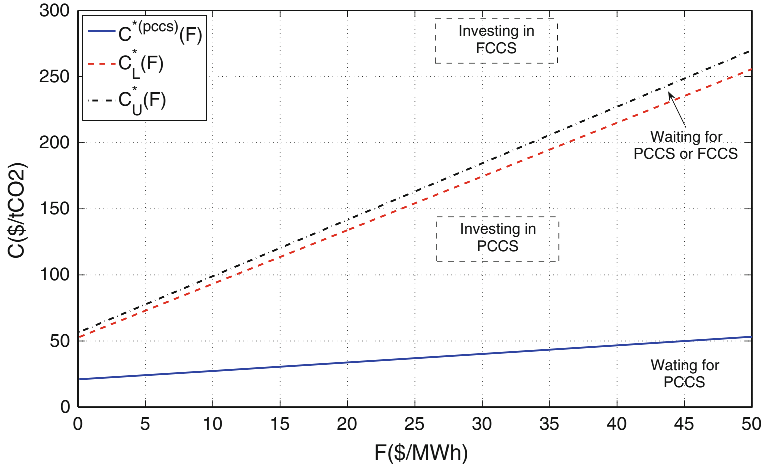

Fig. 15 Free boundaries with enhanced PCCS technology ($\sigma_F = 0.030$ and $\sigma_C = 0.282$)

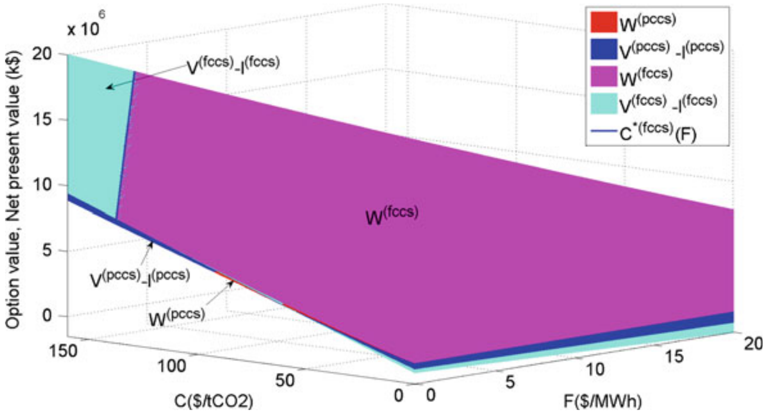

Fig. 16 NPV and option value with enhanced PCCS technology (separate valuation) indicate that the PCCS technology is uniformly dominated by the FCCS one and for CO_2 prices greater (less) than $C^{*(fccs)}(F)$, we invest immediately in (wait for) FCCS ($\sigma_F = 0.10$ and $\sigma_C = 0.94$)

6 Conclusions

As industrialised countries have agreed to reduce their CO_2 emissions, which is assumed to be the most critical anthropogenic GHG, a wide range of mitigation options have been proposed. Among these, the CCS technology is of high importance because fossil fuels continue to be the dominant energy resources in the near term. Capturing almost all emissions is the main objective of policymakers; however, it may critically alter the technology, operation, and economics of a power plant. As a result, in this paper we have analysed both full and partial capture technologies under uncertainty over CO_2 permit and coal prices.

We first have taken the perspective of a coal-fired power plant that has to decide whether to invest, now or anytime in the future, in an emission–reduction technology. Thus, we have examined the opportunity to invest in FCCS and PCCS technologies separately. The options to invest in such technologies have been valued as well as the optimal stopping boundaries. Using current market data, we find that investing in any CCS technology is not optimal. The critical threshold for investing in FCCS given current coal price is $92.12/t$CO_2$, while the current CO_2 price is $25.59/t$CO_2$. By proposing a more achievable PCCS technology, although we could reduce the critical threshold to $56.70/t$CO_2$, it is still not optimal to invest immediately.

We then assume that the plant owner has to decide between investing in either FCCS or PCCS technology simultaneously and introduce the required conditions under which the investment region becomes dichotomous. Regarding these conditions, we propose an enhanced PCCS technology such that its calculated option value from the separate valuation is greater than that of the FCCS technology. Therefore, their NPVs intersect each other at an indifference curve that leads us to value a postponing area where we wait before investing in either technology. Unlike our analytical solution to the separate valuation, this mutually exclusive option value, depending on more than one stochastic variable, must be solved numerically. As such, our solution method is a quasi-analytical one.

The sensitivity of the investment opportunities to changes in the volatilities and the correlation of the stochastic prices as well as in the sunk capital cost has been analysed in this paper. Our numerical examples show that the investment option is highly sensitive to alterations in the volatility of CO_2 price. Generally, increases in volatilities cause increases in optimal boundaries as well as in option values. However, the correlation between the two prices has an opposite impact on the optimal boundaries, such that high positive correlation between prices makes the waiting area narrower.

On the whole, the outcome of this paper is twofold. Firstly, we demonstrate that investing in any CCS technology is not economically advisable in the near term. It would be, however, more attractive should more rigorous climate policies be imposed, e.g., which either increases the CO_2 price level or reduces the uncertainty in the CO_2 price. Secondly, from a theoretical point of view, we develop a two-factor real options model for mutually exclusive investment under uncertainty over two correlated variables.

Although GBM processes are commonly assumed to be good models for energy prices, as examined, e.g., in Pindyck (1999), they may not be suitable for CO_2 permit prices. Moreover, using alternative stochastic processes for energy prices, such as mean-reverting models, as in Abadie and Chamarro (2008a), may result in different outcomes. Considering other possible options, such as the option to suspend the CCS unit to allow venting or the option of switching from one technology to another, may also affect the option value. Finally, a complete model that accounts for the limited lifetime of the equipment or the time-to-build problem would be better able to capture the sequential decision-making challenges faced by a power plant. The methods in this study can be extended to any similar utilities faced with investing in alternative opportunities under uncertainty.

Appendix A: Argument on the Uniqueness of the Solutions Obtained

The method of Adkins and Paxson (2008) used to obtain a solution for the real option value appears successful but does not itself prove that the obtained solution is unique. To prove uniqueness, standard techniques for such elliptic PDEs usually rely on proof by contradiction (see Mattheij et al. 2005 for more details). Taking the individual investment option problem solved in Sect. 3.3.1, if we assume that the solution found W for the real option value is *not* unique and that a second solution $\widetilde{W}$ exists, then the difference $\phi = W - \widetilde{W}$ also satisfies the Bellman Eq. (7). For illustration, we take a simpler form of the governing equation

$$\left(F^2\phi_F\right)_F + \left(C^2\phi_C\right)_C - \mu\phi = 0 \tag{A-1}$$

and assume that the free boundary for W is at $F = F^*(C)$ and the free boundary for $\widetilde{W}$ is at $F = \widetilde{F}(C)$. Then, multiplying the governing equation by ϕ and integrating over the domain $\mathcal{D}$, which is the region $C > 0$ and $F > \max\left(F^*(C), \widetilde{F}(C)\right)$ in which both W and $\widetilde{W}$ are well defined, leads to

$$\begin{aligned} &\int\limits_{\max(F^*,\widetilde{F})} C^2\phi\phi_C\, dF + \int\limits_{\max(F^*,\widetilde{F})} F^2\phi\phi_F\, dC \\ &\quad = \iint\limits_{\mathcal{D}} \mu\phi^2 + C^2(\phi_C)^2 + F^2(\phi_F)^2\, dC\, dF \end{aligned} \tag{A-2}$$

where the right-hand side obviously must be greater than or equal to zero and the left-hand side is dependent only on the values at the free boundary; here, the boundary conditions at $F \to \infty$ and $C = 0$ are already accounted for in the integration by parts by assuming $\phi \to 0$ is a suitable manner. The proof of uniqueness then focuses on showing that this left-hand side cannot be strictly positive leading to $\phi = 0$ and, thus, $W = \widetilde{W}$ everywhere. Adopting this approach, it is trivial to show that two distinct solutions $W \neq \widetilde{W}$ cannot have the same free boundary, $F^*(C) = \widetilde{F}(C)$, as in that case ϕ and its first derivatives are zero on the free boundary, and, hence, the right-hand side of (A-2) is also zero. Indeed, for the case where say $\widetilde{F}(C) \leqslant F^*(C)$ everywhere and for a solution domain of finite extent, a reasonable argument for uniqueness can also be constructed. However, proving uniqueness via this approach for arbitrary $F^*(C) \neq \widetilde{F}(C)$ over a solution domain of infinite extent is more difficult, and an adequate proof remains currently under investigation.

Appendix B: Characteristics of the roots of Eq. (20)

For simplicity, we rewrite exponents a, b, and c in Eqs. (21–23) as follows:

$$a = \left(\frac{1}{2}\sigma_F^2 + \frac{1}{2}\sigma_C^2 - \rho\sigma_F\sigma_C\right) K_1 + \left(\sigma_C^2 - \rho\sigma_F\sigma_C\right) K_2 + \frac{1}{2}\sigma_C^2 K_3$$

$$b = a + (\alpha_F - \alpha_C)\, K_1 + \left(-\frac{1}{2}\sigma_F^2 + \alpha_F - 2\alpha_C\right) K_2 - \alpha_C K_3$$

$$c = (\mu - \alpha_F) K_1 - \left(\alpha_F - \frac{1}{2}\sigma_F^2 - 2\mu\right) K_2 + \mu K_3$$

where,

$$\begin{aligned} K_1 &= \left(\left(\epsilon_F - \epsilon_F^{(j)}\right) QF\right)^2 \ > 0 \\ K_2 &= -(\mu - \alpha_F)\left(\epsilon_F - \epsilon_F^{(j)}\right) QI^{(j)} F \ > 0 \\ K_3 &= (\mu - \alpha_F)^2 (I^{(j)})^2 \ > 0 \end{aligned}$$

Next, it can be shown that $c > a - b$:

$$\begin{aligned} c - a + b &= (\alpha_F - \alpha_C) K_1 + \left(-\frac{1}{2}\sigma_F^2 + \alpha_F - 2\alpha_C\right) K_2 - \alpha_C K_3 \\ &\quad + (\mu - \alpha_F) K_1 - \left(\alpha_F - \frac{1}{2}\sigma_F^2 - 2\mu\right) K_2 + \mu K_3 \\ &= (\mu - \alpha_C)(K_1 + 2K_2 + K_3) \ > 0 \end{aligned} \tag{B-1}$$

We may now finalise the proof as follows:

$$c > a - b \ \Rightarrow \ b^2 + 4ac > 4a^2 - 4ab + b^2 = (2a - b)^2 \tag{B-2}$$

Thus,

$$-\sqrt{b^2 + 4ac} < (2a - b) < \sqrt{b^2 + 4ac} \tag{B-3}$$

$$\Rightarrow b - \sqrt{b^2 + 4ac} < 2a < b + \sqrt{b^2 + 4ac} \tag{B-4}$$

Therefore, $\eta_1^{(j)} > 1$ and $\eta_2^{(j)} < 1$. On the other hand, $\eta_2^{(j)}$ is not only less than 1, but also less than 0 because $b < \sqrt{b^2 + 4ac}$.

Appendix C: Parameters of Eq. (13)

Table 5 provides the calculated parameters of Eq. 13, $\left\{\eta^{(pccs)}, \beta^{(pccs)}, C^{*(pccs)}(F), A^{(pccs)}\right\}$, for some values of F.

Table 5 Parameters of Eq. (13) for some PCCS

F(\$/MWh)	$\eta^{(pccs)}$	$\beta^{(pccs)}$	$C^{*(pccs)}(F)$	$A^{(pccs)}$	F	$\eta^{(pccs)}$	$\beta^{(pccs)}$	$C^{*(pccs)}(F)$	$A^{(pccs)}$
1	1.2919	−0.0091	48.15	7.4100E+10	26	1.3214	−0.1463	79.52	4.2098E+10
1.5	1.2929	−0.0135	48.77	7.2170E+10	26.5	1.3218	−0.1479	80.15	4.1879E+10
2	1.2938	−0.0177	49.40	7.0435E+10	27	1.3221	−0.1496	80.78	4.1665E+10
2.5	1.2947	−0.0219	50.02	6.8853E+10	27.5	1.3224	−0.1512	81.41	4.1456E+10
3	1.2956	−0.0260	50.65	6.7397E+10	28	1.3228	−0.1528	82.04	4.1253E+10
3.5	1.2964	−0.0300	51.27	6.6048E+10	28.5	1.3231	−0.1544	82.67	4.1055E+10
4	1.2973	−0.0338	51.90	6.4790E+10	29	1.3235	−0.1560	83.29	4.0861E+10
4.5	1.2981	−0.0376	52.53	6.3614E+10	29.5	1.3238	−0.1575	83.92	4.0673E+10
5	1.2989	−0.0414	53.15	6.2510E+10	30	1.3241	−0.1590	84.55	4.0488E+10
5.5	1.2997	−0.0450	53.78	6.1470E+10	30.5	1.3244	−0.1605	85.18	4.0309E+10
6	1.3005	−0.0486	54.41	6.0489E+10	31	1.3247	−0.1620	85.81	4.0133E+10
6.5	1.3012	−0.0520	55.03	5.9561E+10	31.5	1.3250	−0.1635	86.44	3.9961E+10
7	1.3020	−0.0554	55.66	5.8681E+10	32	1.3253	−0.1649	87.07	3.9794E+10
7.5	1.3027	−0.0588	56.29	5.7846E+10	32.5	1.3256	−0.1663	87.70	3.9630E+10
8	1.3034	−0.0620	56.91	5.7052E+10	33	1.3259	−0.1677	88.33	3.9470E+10
8.5	1.3041	−0.0652	57.54	5.6296E+10	33.5	1.3262	−0.1691	88.96	3.9313E+10
9	1.3048	−0.0683	58.17	5.5574E+10	34	1.3265	−0.1704	89.59	3.9160E+10
9.5	1.3054	−0.0714	58.79	5.4885E+10	34.5	1.3268	−0.1718	90.21	3.9010E+10
10	1.3061	−0.0744	59.42	5.4227E+10	35	1.3271	−0.1731	90.84	3.8864E+10
10.5	1.3067	−0.0774	60.05	5.3596E+10	35.5	1.3273	−0.1744	91.47	3.8720E+10
11	1.3073	−0.0802	60.68	5.2992E+10	36	1.3276	−0.1757	92.10	3.8580E+10
11.5	1.3079	−0.0831	61.30	5.2413E+10	36.5	1.3279	−0.1769	92.73	3.8442E+10
12	1.3085	−0.0858	61.93	5.1858E+10	37	1.3281	−0.1782	93.36	3.8308E+10
12.5	1.3091	−0.0886	62.56	5.1324E+10	37.5	1.3284	−0.1794	93.99	3.8176E+10
13	1.3097	−0.0912	63.19	5.0810E+10	38	1.3286	−0.1806	94.62	3.8047E+10
13.5	1.3102	−0.0938	63.81	5.0316E+10	38.5	1.3289	−0.1818	95.25	3.7920E+10
14	1.3108	−0.0964	64.44	4.9841E+10	39	1.3291	−0.1830	95.88	3.7797E+10
14.5	1.3113	−0.0989	65.07	4.9382E+10	39.5	1.3294	−0.1842	96.51	3.7675E+10
15	1.3119	−0.1014	65.70	4.8940E+10	40	1.3296	−0.1854	97.14	3.7556E+10
15.5	1.3124	−0.1038	66.33	4.8514E+10	40.5	1.3299	−0.1865	97.77	3.7439E+10
16	1.3129	−0.1062	66.95	4.8102E+10	41	1.3301	−0.1876	98.39	3.7325E+10
16.5	1.3134	−0.1086	67.58	4.7705E+10	41.5	1.3303	−0.1887	99.02	3.7213E+10
17	1.3139	−0.1109	68.21	4.7320E+10	42	1.3306	−0.1898	99.65	3.7103E+10
17.5	1.3144	−0.1131	68.84	4.6949E+10	42.5	1.3308	−0.1909	100.28	3.6995E+10
18	1.3148	−0.1153	69.47	4.6589E+10	43	1.3310	−0.1920	100.91	3.6889E+10
18.5	1.3153	−0.1175	70.09	4.6241E+10	43.5	1.3312	−0.1931	101.54	3.6785E+10
19	1.3158	−0.1197	70.72	4.5904E+10	44	1.3315	−0.1941	102.17	3.6683E+10
19.5	1.3162	−0.1218	71.35	4.5577E+10	44.5	1.3317	−0.1952	102.80	3.6582E+10

Table 5 continued

F(\$/MWh)	$\eta^{(pccs)}$	$\beta^{(pccs)}$	$C^{*(pccs)}(F)$	$A^{(pccs)}$	F	$\eta^{(pccs)}$	$\beta^{(pccs)}$	$C^{*(pccs)}(F)$	$A^{(pccs)}$
20	1.3166	−0.1239	71.98	4.5260E+10	45	1.3319	−0.1962	103.43	3.6484E+10
20.5	1.3171	−0.1259	72.61	4.4953E+10	45.5	1.3321	−0.1972	104.06	3.6387E+10
21	1.3175	−0.1279	73.24	4.4654E+10	46	1.3323	−0.1982	104.69	3.6293E+10
21.5	1.3179	−0.1299	73.86	4.4365E+10	46.5	1.3325	−0.1992	105.32	3.6199E+10
22	1.3183	−0.1318	74.49	4.4084E+10	47	1.3327	−0.2002	105.95	3.6108E+10
22.5	1.3187	−0.1337	75.12	4.3811E+10	47.5	1.3329	−0.2011	106.58	3.6018E+10
23	1.3191	−0.1356	75.75	4.3545E+10	48	1.3331	−0.2021	107.21	3.5930E+10
23.5	1.3195	−0.1374	76.38	4.3287E+10	48.5	1.3333	−0.2030	107.84	3.5843E+10
24	1.3199	−0.1393	77.01	4.3036E+10	49	1.3335	−0.2039	108.47	3.5758E+10
24.5	1.3203	−0.1411	77.64	4.2792E+10	49.5	1.3337	−0.2049	109.10	3.5674E+10
25	1.3207	−0.1428	78.26	4.2555E+10	50	1.3339	−0.2058	109.73	3.5591E+10
25.5	1.3210	−0.1446	78.89	4.2323E+10					

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