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Teachers' reflection on PISA items and why they are so hard for students in Serbia

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Abstract The study explores how teachers perceive and go about students' thinking in connection to particular mathematical content and how they frame the notion of applied mathematics in their own classrooms. Teachers' narratives are built around two released PISA 2012 mathematics items, the 'Drip rate' and 'Climbing Mount Fuji' (will be referred to as the *Fuji* item). Teachers show concordance as to the reasons that could make either of the items difficult for students and are able to provide more examples justifying their reasoning for the 'Fuji' item. Suggestions linked to making the items more familiar to the students mostly relate to de-contextualization of the items' content towards a more formal mathematical record. The teachers agree that students need only basic mathematical knowledge, at a level learned during elementary school, in order to solve these problems. Yet, at the same time, many teachers have difficulty clearly verbalising which procedures students are expected to follow to be able to solve the tasks. Disagreement among the teachers is noticeable when labelling the most difficult part(s) of each of the selected items. Mathematics teachers show openness for learning on how to create math problems we examined in this study, but question the purpose and meaning in incorporating more such problems in their own teaching.

Keywords Teachers' narratives · Mathematics · Mathematics instruction · PISA

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Introduction

Mathematics competence is steadily observed as the basis for further student progress in a number of domains (Ellis and Berry 2005; Baumert et al. 2010). At the same time, the importance of students' problem solving competencies (Hattikudur et al. 2016) and the way mathematical competence in curriculum is operationalized worldwide (Clements and Sarama 2004) remain important topics for math educators across the globe. Conversely, examining students' competences in mathematics cannot be done without examining the teacher competence, as the latter is considered to be an important component of student outcomes (Hattie 2008; Santagata and Yeh 2016). In particular, teachers' ability to explain the reasoning behind a particular solving procedure (Schoenfeld 2014), their perception as to how students relate to mathematical concepts (Crespo 2000) and support students' learning (Krstić 2015), are frequently seen as some of the most prominent features of teacher competence in mathematics. Yet how particular teacher competences are enacted in the classroom very much relies on the teachers' notions as to how the teaching and the learning should take place (Fives and Buehl 2012; König et al. 2014; Radišić and Baucal, 2015). On the understanding that teachers' competences are among the key factors relevant to student outcomes, we examine how teachers reflect on students' thinking in connection to two PISA items—the '*Drip rate*' and the '*Fuji*', and how they frame the notion of applied mathematics in their own classrooms in an attempt to understand the possible factors related to teachers and teaching that are shaping student competences in mathematics.

Theoretical framework

Over the years, researchers in the domain of mathematics education have convincingly argued that understanding of mathematics entails of more than just knowledge of mathematical concepts, principles, and their structures (Schoenfeld 1992; Stein et al. 1996; Pepin et al. 2016; Cai and Ding 2017; Rittle-Johnson 2017). It includes the capacity of the learner to engage in the processes of mathematical thinking (Romberg 1992) allowing that same learner to frame and solve problems, look for patterns or make inferences from data, to abstract, explain or justify their own thinking when confronted with mathematical content and ideas. Thus, mathematical competence is viewed both as a process of achieving that understanding and as the result of the process itself, while the relations between concepts, principles and structures are viewed as bi-directional and iterative (Schneider et al. 2011).

There is a recognition that students are expected to learn to observe mathematics not as a static and strict system of concepts, procedures and structures to be absorbed, but rather, also, as a dynamic field of accumulating knowledge, meaning making, and (re)creating knowledge when engaging with the mathematical content (Romberg 1992; Pepin et al. 2016; Cai and Ding 2017). However, abundant empirical evidence points to large disparities in the level of mathematical competences of students around the world, and large scale assessments, like Trends in Mathematics and Science Study—TIMSS (Mullis et al. 2016) and Programme for International Student Assessment—PISA (OECD 2016), reinforce this empirical evidence.

Confronted with the idea of the need to develop such instructional environments which constitute students' learning of mathematics as a 'dynamic playground', many researchers have been prompted to examine the constituents of effective teaching. Successful and effective teaching has been regarded as a complex task, associated with many factors, but at the same time "no well-defined algorithm is available to guarantee [its] successful solution" (Li and Kaiser 2011, p. 4). Nonetheless, its quality has been seen as one of the key milestones contributing to students' learning and successful outcomes (Li and Kaiser 2011; Dyer & Sherin, 2016).

In the context of mathematics teaching, teacher competence has been viewed as pivotal in relation to the improvement of classroom instruction together with students' own competences in the domain (Fennema and Franke 1992; Li and Kaiser 2011; Richland et al. 2012). At the same time, research on teacher competence has been somewhat dual in its nature (Radišić and Baucal, 2015). While a number of researchers have largely focused on teacher cognition, others follow the notion of teacher beliefs[1] (Santagata and Yeh 2016). The latter suggests that teachers' beliefs about the nature of mathematics and about the teaching and learning of mathematics affect their everyday instruction (Fives and Buehl 2012; Fives and Gregoire-Gill 2014). Departing from the work of Shulman (1987) and his conceptualization of the pedagogical content knowledge (e.g., work of the Michigan group—Ball et al. 2005; Ball et al. 2008) and the COOACTIV study—Baumert et al. 2010), the former focuses on understanding the nature and types of knowledge that make teachers effective (Dyer & Sherin, 2016; Santagata and Yeh 2016),

Recently, however, attempts have been made to put these two perspectives into a dialogue (Santagata and Yeh 2016). For example, Sherin and colleagues put emphasis on the concept of 'noticing', defining it as a professional vision, lenses, through which teachers enact and envision their own teaching. In their work (Sherin and van Es 2009; Sherin et al. 2011), focus is given to teachers' abilities to attend to and interpret student thinking through the course of the instruction, while Blömeke and colleagues address the issue by incorporating perception, interpretation, and decision-making processes in their conceptualization of teacher competence (Blömeke et al. 2015).

These recent attempts are important given the different frameworks as to what constitutes 'teaching' and the complex list of tasks in which teachers are expected to be proficient in while supporting the instruction of their students (Kilpatrick et al. 2001; Schoenfeld and Kilpatrick 2008). Among them, understanding students' mathematical thinking has been observed as an important component of mathematics teaching expertise (Ball 1997; Didis et al. 2016) implying that learning takes place in interaction with the teacher and that teachers' skills in reflecting on students thinking are important part of that process. Numerous studies on how teachers understand student thinking (Ball 2001; Furtak et al. 2016) have highlighted different facets of students' thinking upon which teachers need to and often focus (e.g., Leinhardt 1986; Schoenfeld 2014). These include correcting errors in a procedure (Empson and Jacobs 2008) and deeply rooted misconceptions (Pierson 2008), or identifying students' strategies during particular problem solving situations (Empson and Jacobs 2008; Hattikudur et al. 2016), thus creating a basis for later comparisons and inferences across classes while reflecting on how to respond to different students' ideas and how to adjust their own instructional choices (Jacobs et al. 2010; Santagata and Yeh 2016; Dyer and Sherin 2016).

While an agreement exists among the mathematics educators that students should observe mathematics as a dynamic process of discovering and (re)creating knowledge (Romberg 1992,), there are continuing debates about the different roles both students and teachers play

[1] We distinguish here between the concepts of teacher cognition linked to particular teachers' self-reflections and knowledge about their teaching, subject matter and/or students; and an understanding of beliefs as propositions, which may be consciously or unconsciously held, are evaluative in nature, as such held true by the individual, and are therefore saturated with emotive commitment serving as a guide to thought and behaviour. (Borg 2001, p. 186).

in the process of joint learning (Säljö 2009; Krstić 2015; Cai and Ding 2017), especially if we bear in mind that mathematical procedures which may be instinctive for adults are often very challenging and far from obvious to students irrespective of their age.

Ball and colleagues note that mathematical knowledge “held and expressed by students is often incomplete and difficult to understand” (Ball et al. 2005, p. 17). And while other ‘adults’ can avoid dealing with these, by virtue of their position, teachers are in a unique situation to professionally interpret, correct, and extend on students’ knowledge on an everyday basis. At the same time, engaging with student ideas may not only benefit the students themselves, but may also facilitate teachers’ instructional change as it informs their teaching (Fennema and Franke 1992; Cobb et al. 1993; Fennema et al. 1996). Evidence currently exists to support both claims (Black et al. 2003; Ball et al. 2008; Clarke 2008; Teuscher et al. 2016).

Within the school context, the understanding of mathematics takes place mostly through direct instruction[2] and by working on mathematical tasks (individually or in a collaborative manner). The latter, we conceive as a problem or a set of problems that orient students’ attention towards a particular mathematical idea, providing them with an opportunity to develop or use previously gained concepts and routines in the domain. According to Doyle (1983), mathematical problems “influence learners by directing their attention to particular aspects of content and by specifying ways of processing information” (p. 161). Thus, mathematical problems with which students grapple shape not only the content of their learning, but also how they come to feel and think about that content and how they make sense of mathematics in general. However, not every task is the same, nor is the type of demand it makes on the students (i.e., the need for interpretation, flexibility, construction of meaning) does vary to a great extent (Stein et al. 1996). At the same time, the context of the mathematical task can hinder the solving process, especially if it is set in a way students are not accustomed to (Wyndhamn and Säljö 1997; Verschaffel et al. 2000; Verschaffel et al. 2015), making them prone to give solutions not relevant to the situation described in the task itself (Palm 2008).

For teachers, mathematical tasks allow them to directly observe, interpret and build understanding about students’ thinking while they engage with a given problem (Ball 1997; Didis et al. 2016). The situation enables them to build their own knowledge corpus related to students’ difficulties and misconceptions (Hill et al. 2008), to better understand what students already know or how they progress through a given topic and how their mathematical thinking evolves as they use both informal and formal strategies in grappling with the mathematical content (Crespo 2000). However, observing mathematics from the students’ perspective does not come naturally to all teachers (Smith 2001).

Student mathematical competence through the PISA lenses

PISA is one of the major large-scale assessments that incorporate mathematics in its survey of student competences. The idea to include mathematics in the assessment stems from the notion that a growing number of problems students and young people encounter within both every day and professional situations demand some knowledge of mathematics (OECD 2013; Wijaya et al. 2014). As such, PISA focuses on individual capacities to reason mathematically and/or use mathematical concepts and procedures, across different education systems mirrored through three aspects included in the assessment itself. These are the mathematical processes

[2] Here we refer to interaction between a student and a teacher, irrespective of the actual mode of teaching (e.g. inquiry based, whole class).

(e.g., formulating situation mathematically or interpreting particular outcome), the mathematical content (i.e., actual content in the assessment items) and the contexts in which the assessment items are located (e.g., personal, scientific) (p. 27, OECD 2013) (PISA framework, see the introductory section of this volume).

Again, besides the league tables that show how much above or below of the set average a country operates at PISA in different domains, patterns of student competences can also be observed through the type of problems students are more or less successful in, for example in mathematics, a comparison similar to the one teachers can observe in their classrooms.

From cycle to cycle in PISA, Serbia is performing below the international PISA average, with almost 40% of the students below the PISA level 2[3] in the 2012 cycle,[4] implying that these students can only use their mathematical knowledge and skills in a familiar context with information given explicitly, applying routine procedures. Additional analyses support this claim when observed for students who reach this level and beyond (the majority of students are concentrated around levels 2 and 3), with only a relative lower share of students at the highest levels (5 and 6),[5] that is those who are able to formulate assumptions, model solutions of the problem using prior knowledge and multiple resources of information, and who are able to explain why they use a particular strategy to others (Baucal and Pavlović Babić, 2010; Pavlović Babić and Baucal 2013).

Teaching mathematics and teacher education within the Serbian context

Examining students' mathematical competence would be incomplete without briefly discussing mathematics classrooms in Serbia, which are still, to a large extent, knowledge-oriented,[6] with the practices being largely teacher centred (Radišić and Baucal, 2016). For a long period, mathematics is the domain in which students in Serbia seek help the most through private tutoring (Pešić and Stepanović 2004). Teaching and assessment are seen as separate processes (Radišić et al., 2014), with the focus on the appropriation of academic knowledge and skills (Pavlović Babić and Baucal, 2013) as an opposite to competences and understanding. Students' role is, for the most part, receptive, as lecturing is the dominant form of teaching (Mincu 2009; Radišić and Baucal, 2016). Students' active role in the process of mathematical understanding is incidental rather than part of widespread practices.

At the same time, the way the mathematics curriculum, in both elementary and secondary education programmes is organised, supports the notion that knowledge appropriation is paramount. The curriculum is organised[7] through a large discrete set of themes connected

[3] Considered as the minimal level of functional literacy.
[4] Serbia did not partake in the PISA 2015 cycle.
[5] According to our own analysis the share of Serbian student at highest levels (5th and 6th) should be about 7% and this is less than 5%
[6] In the sense of appropriating factual knowledge, not building on students' competences and problem solving strategies.
[7] e.g. Rule book on teaching plan and programme for grammar school - Official Gazette: 5/1990–1, 3/1991–1, 3/1992–1, 17/1993–106, 2/1994–59, 2/1995–1, 8/1995–1, 23/1997–1, 2/2002–4, 5/2003–1, 10/2003–1, 11/2004–1, 18/2004–1, 24/2004–2, 3/2005–1, 11/2005–163, 2/2006–1, 6/2006–129, 12/2006–2, 17/2006–6, 1/2008–1, 8/2008–3, 1/2009–1, 3/2009–24, 10/2009–64, 5/2010–1, 7/2011–12, 4/2013–175, 14/2013–2, 17/2013–1, 18/2013–8, 5/2014–6, 4/2015–5, 18/2015–1, 11/2016–563, 13/2016–10 (correction); Rule book on teaching plan and programme for educational profile of electrical engineer in telecommunications - Official Gazette: 9/2007–1, 5/2011–133, 10/2014–1, 10/2014–140, 8/2015–46. The rule books for array of educational profiles in electrical engineering, economy and law were inspected for the purpose of this text. Above listed are given as examples.

linearly and, if any kind of hierarchy does exists, it is not grounded in the type of underlying intellectual activity, but rather the complexity of the given topic. At Upper Secondary level,[8] the curriculum content is not contextualised to the particular vocation of the vocational schools.

Mathematics teachers (i.e., mathematics subject teachers)[9] are educated at Faculties of Mathematics where mathematics is observed as a formal discipline. Even in tracks that prepare future subject teachers, mathematics as the formal discipline is much more present opposed to skills teachers need in working with children at different levels (e.g., methods of teaching).[10] At the same time, mathematicians' professional associations emphasise the academic nature of mathematics in all in-service training programmes which they also largely provide.

Focus of the analyses

Departing from the idea that mathematical understanding is equally a process and an outcome, that as a process it is both dynamic and continuous, shared between teachers and the students (Schoenfeld 1992; Stein et al. 1996; Pepin et al. 2016; Cai and Ding 2017; Rittle-Johnson 2017), and relying on the concept of noticing (Sherin and van Es 2009), we are set to examine how teachers observe students' thinking in connection to two PISA items—the '*Drip rate*' and '*Fuji*'. Observing teachers' capacity to 'put themselves into students' shoes' as one of important facets of their competence for teaching mathematics (Black et al. 2003; Hill et al. 2008; Santagata and Yeh 2016), we examine (a) how teachers perceive typical math problems students at upper secondary level in Serbia encounter in relation to the two chosen PISA items; (b) how they elaborate on the two items in relation to any difficulties students may encounter while they are trying to solve the items (i.e., related misconceptions) or the necessary competences that are at play when students are seeking strategies in order to successfully solve the problems; and, finally, (c) how do teachers themselves frame the notion of applied mathematics in their own classrooms?

Method

Sample

The sample involved 20 mathematics teachers from six Upper Secondary schools in Belgrade; given the fact PISA surveys 15-year-old students who are in Serbia attending the Upper Secondary programmes. Ten teachers were employed as mathematics teachers in three grammar schools partaking in the project, whereas the remaining ten were sampled from three vocational schools (VET). The three VET schools included various tracks in economy or

[8] Compulsory primary education starts at the age of 7, lasting 8 years. Upper secondary education takes part through different vocational education profiles or in grammar schools, for age groups 15 to 19.
[9] Taught by subject area teachers in grades 5 to 8 of elementary school and all grades at upper secondary level.
[10] Conclusion is drawn based on syllabus analyses for mathematics teacher programme at the undergraduate and master level, as well as other mathematics programmes offered by Faculties of Mathematics at different universities in Serbia. Current analyses included syllabuses from Faculty of Mathematics (University of Belgrade), Faculty of Sciences (University of Novi Sad) and Faculty of Sciences and Mathematics (University of Nis). According to the Serbian bylaws and rulebooks, a person is allowed to apply for a position of a mathematics subject teacher after graduating from any of the accredited programmes in mathematics.

electrical engineering.[11] All grammar schools included social science and arts, and a science track. Teachers' work-load requires that teachers have either student groups from different tracks (or educational profiles in VET schools) or just one. The former was the case for all the teachers partaking in the study. At the time when the study was conducted, all teachers taught students in Grade One of Upper Secondary school.

In all schools, teachers were contacted through school principals and school professional bodies, who gave an approval for conducting the study, leaving the final decision to the teachers' voluntary participation. All teachers in the sample obtained their degrees in mathematics or mathematics and computer science at the Faculty of Mathematics at the University of Belgrade, while their work experience varied from 2 years in service to over 25 years. The majority of sampled teachers is women, which is common for the education system in Serbia at all levels of pre-university education.

Data collection and materials

The data were collected following a semi-structured interview guide comprised of questions on how teachers assess the difficulty of the two chosen mathematics items, clarity of instruction for each, how they frame desired procedural steps for solving the tasks, observe necessary mathematical competence in doing so and where in their opinions possible difficulties for the students lay (Appendix 1). All topics were discussed for each of the items, firstly the item '*Drip rate*', followed by the '*Fuji*' item. The interview was concluded by teachers giving accounts on own pre-service and in-service training related to generating and teaching similar mathematical problems and desirability in providing students with similar examples in their own teaching. Each interview lasted between 30 and 40 min. Before the interview, teachers were once again familiarised with the purpose of the study.

The two items, around which teachers' narratives were organised—the '*Drip rate*' and '*Fuji*' were selected among the PISA 2012 released mathematics items list.[12] Previous analyses of students' achievement in PISA 2012 showed that these items are among the relatively more difficult ones for students in Serbia, with only 5.7% of students in total able to solve items at levels 5 and 6 (Pavlović Babić and Baucal, 2013). As both '*Fuji*' and the '*Drip rate*' belong to these groups, they were chosen as the material for this study.

The '*Drip rate*' item implies that students have to solve tasks which require them to vary different parameters like double the time an infusion runs or to calculate its volume (see Fig. 1 for details). '*Fuji*' involves students calculating average number of visitors in a given period, route time or step length for a specific trail (see Fig. 2 for details). While both parts of the '*Drip rate*' fall into mathematical content area of change and relationships[13] with occupational context, '*Fuji*' is a combination of two content areas, quantity and change and relationships. The context remains societal throughout the task. No interventions were made on the released items, while in the study an official translation for Serbia was used (provided to the research team by the representatives of the national PISA team).

[11] VET schools with such educational programs were contacted as they represent the majority of VET schools in Serbia, when observed across the span of possible education fields.

[12] Full overview is available at: https://www.oecd.org/pisa/pisaproducts/pisa2012-2006-rel-items-maths-ENG.pdf

[13] PISA items in mathematics include the following content area: change and relationships, space and shape, quantity, and uncertainty and data. The Contexts include: personal occupational, societal and scientific.

DRIP RATE

Infusions (or intravenous drips) are used to deliver fluids and drugs to patients.

Nurses need to calculate the drip rate, *D*, in drops per minute for infusions.

They use the formula $D = \dfrac{dv}{60n}$ where

d is the drop factor measured in drops per millilitre (mL)
v is the volume in mL of the infusion
n is the number of hours the infusion is required to run.

Question 1

A nurse wants to double the time an infusion runs for.
Describe precisely how *D* changes if *n* is **doubled** but *d* and *v* do not change.

....

....

....

Question 3:

Nurses also need to calculate the volume of the infusion, *v*, from the drip rate, *D*.
An infusion with a drip rate of 50 drops per minute has to be given to a patient for 3 hours. For this infusion the drop factor is 25 drops per millilitre.
What is the volume in mL of the infusion?

Volume of the infusion .... ml.

Fig. 1 Overview of the 'drip rate' item

Data analysis

The data gathered from the semi-structured interviews were captured with an audio recorder and transcribed verbatim. The first round of the analyses was done on five transcripts following the principles of thematic analysis (Braun and Clarke 2006) on each of the major sections within the interview guide. Within each of the sections, transcripts were read through thoroughly and initial codes were organised into meaningful groups. This process was continued on the remaining 15 transcripts following the constant comparative analytic process

CLIMBING MOUNT FUJI

Mount Fuji is a famous dormant volcano in Japan.

Question 1: CLIMBING MOUNT FUJI

Mount Fuji is only open to the public for climbing from 1 July to 27 August each year. About 200 000 people climb Mount Fuji during this time.

On average, about how many people climb Mount Fuji each day?

A 340
B 710
C 3400
D 7100
E 7400

Question 2: CLIMBING MOUNT FUJI

The Gotemba walking trail up Mount Fuji is about 9 kilometres (km) long.

Walkers need to return from the 18 km walk by 8 pm.

Toshi estimates that he can walk up the mountain at 1.5 kilometres per hour on average, and down at twice that speed. These speeds take into account meal breaks and rest times.

Using Toshi's estimated speeds, what is the latest time he can begin his walk so that he can return by 8 pm?

....

Question 3: CLIMBING MOUNT FUJI

Toshi wore a pedometer to count his steps on his walk along the Gotemba trail.

His pedometer showed that he walked 22 500 steps on the way up.

Estimate Toshi's average step length for his walk up the 9 km Gotemba trail. Give your answer in centimetres (cm).

Answer: .... cm

Fig. 2 Overview of the 'climbing Mountain Fuji' item

(Merriam 1998) contrasting the 2 items and determining whether teachers teach in grammar schools or VET establishments. In this way, we were able to account for both the different themes within a particular section and possible similarities and differences in the narratives concerning the two items. Themes were repeatedly reviewed and refined to gain the most precise illustration of the data. Supporting quotes were selected. As the patterns were established across the transcripts, discussion between the first and the second author supported the refinement of themes. An overview of the themes is depicted in Fig. 3.

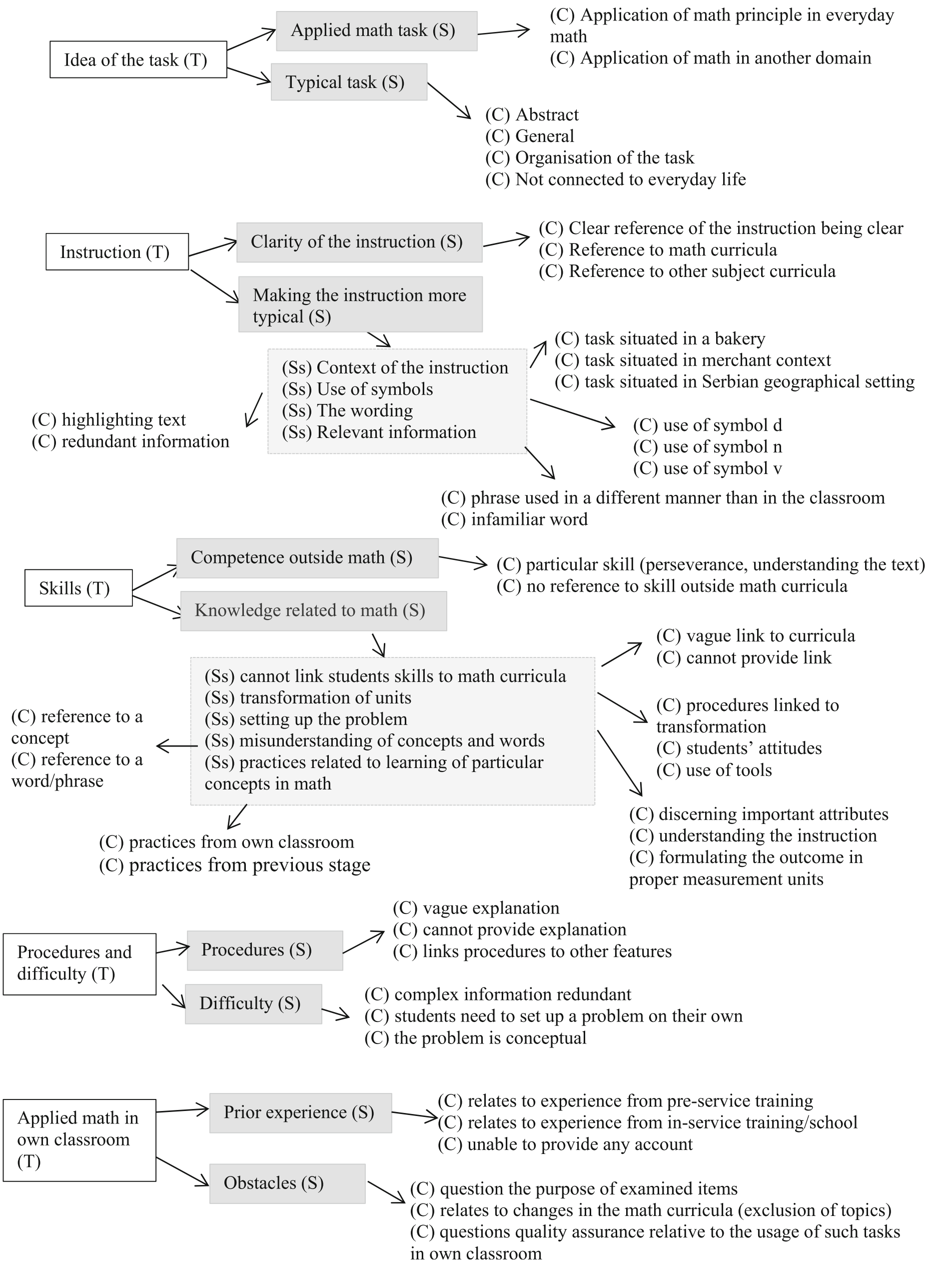

Fig. 3 Overview of the themes (T), subthemes (S&Ss) and example codes (C) across the data set (both items)

Results

Understanding a mathematical task as a set of problems that orient students' attention towards a particular mathematical concept (idea) and a 'field' in which students can utilise and further develop routines they have previously mastered, one of our major concerns was whether the teachers partaking in the study will observe the two chosen PISA items as mathematical tasks.

For the teachers both the '*Drip rat*e' and '*Fuji*' were considered as such irrespective whether they work in VET or a grammar school. Again for all, both are examples of applied mathematical problems, and in their opinion either of them is not conceived as a mathematical problem students in Serbia typically encounter in class. In all the teachers' narratives, we found an idea that typical mathematical problems in Serbian classrooms are abstract and more general, not related to the everyday life situations (example 1).

> The task is [*the Drip rate*] … let me say is like this …it is an application of mathematics, this is a task within the context of biology and physics, it is an applied math task, well now we officially don't do such tasks, our tasks are entirely of different kind, entirely different set up, it is not something related to the everyday context, they are more general and abstract, but this is a mathematical task.
> [....]
> Something like this [*Fuji*] is more likely they will encounter with than the previous one [*the Drip rate*], because the previous had a bit of physics, something is dripping, this is arithmetical mean [*refers to the mount Fuji task*] the first part, then the second is just an expansion of that, they have a lot of physics here as well, you transform the units of measurement, ask for an answer in centimetres, calculate time and so on, but again this is mathematics, an application of mathematics.
> Example 1—On the idea of typical mathematical problems (Teacher, grammar school)

Depending on the teachers' stance that both the '*Drip rate*' and the '*Fuji*' are related to the mathematical knowledge, their narratives will be organised as to depict (1) how they perceive the instruction for the students in both items (section—Instruction given in the items—"is it clear enough?"), (2) how they conceptualise skills and procedures students need in the process (section—The skills students need and their typical mistakes), and (3) possible difficulties students may encounter while solving the items (section—What are the difficult parts?). Finally, we will present (4) how teachers frame the notion of applied mathematics in their own classrooms (section—Applied mathematics in their own classrooms).

Instruction given in the items—"is it clear enough?"

In discussing the clarity of the item instruction for both items, the teachers were unified that "as they are", both items are clear in terms of the instructions given. However, despite this agreement, their comments differed relative to the each item on what could make the item instruction more 'typical' for the Serbian students. In the case of the '*Drip rate*' item for which the teachers claim that "there is no problem with the instruction, we simply don't do this in class", and that in this item "dependence of size is used" whereas in "mathematics this is not done", four subthemes emerged as to how to make the item instruction more typical. For each, we found no differences between the teachers working in VET or grammar schools.

The first suggestion related to the item's context. What was described as the 'hospital' context in most cases is seen as something students typically do not encounter, and that if a similar task would be created using a 'merchant' context (e.g., a bakery), it would appeal more to them. Secondly, what all teachers saw as the major drawback of the '*Drip rate*' item was coded as "an inappropriate use of the symbols" (i.e., letters d, v, n), meaning that the way symbols are laid out in the item does not correspond to their usual use in the classroom. For example the letter 'd' is commonly used for length and 'v' for speed. Irrespective of the fact that the meaning of each symbol is explained in the item instruction, teachers very strongly argued that relabelling to known symbols would enable students to observe the context of the task as something more similar to their school experience. Many of the arguments for this were also provided from the teachers own practice.

Reflecting the wording used, a number of teachers noticed that a sentence within the question 1 of the item 'A nurse wants to double the time an infusion runs for.' would have been more in line with the regular classroom discourse if the word "double" would have been replaced by "prolong" or a phrase such as 'lasts twice as long' would have been used. The fourth and final subtopic that emerged in the teacher narratives related to the way the text of the '*Drip rate*' item is highlighted or rather the lack of it. The teachers were unified that in a textual task of this length, they would expect all the important information to be in '**bold text'** across the task. In the current version, this is done only for the question one (Fig. 1). This finding implies teachers' perceiving that it is on the task developer to mark all the relevant information, and not the responsibility of the students. Again the finding is also in line with the teachers' perception that typical mathematical problems in Serbia are more general and abstract, stripped of information irrelevant for the task itself.

Clarity of the item instruction was also a prominent feature in the teachers' narratives for the '*Fuji*' item, but at the same time this item, in particular, was seen as a task with "redundant information" for which "Serbian geographical locations and names could have been used" to make the item more familiar.[14] A sample of teachers' suggestions relative to the redundancy of the information is given in example 2. Text in bold is seen as the redundant.

Example 2—the 'Fuji' item (question 2)
The Gotemba walking trail **up Mount Fuji** is about 9 km (km) long.
Walkers need to return from the 18 km walk by 8 pm.
Toshi **estimates that he can walk up the mountain** at 1.5 km per hour on average, and down at twice that speed. **These speeds take into account meal breaks and rest times**.
Using Toshi's estimated speeds, what is the latest time he can begin his walk so that he can return by 8 pm?

We note that the suggestions given by the teachers are all in the direction of decontextualizing the task, despite the fact these items have been judged as the sort of problem that students may have previously encountered in physics. In teachers' perception, and this is true in relation to teachers both in grammar schools and VET establishments, the familiarly of the item is increased once the item is stripped of the contextual information, which actually makes the item a so called word problem. If suggestions are followed the item becomes general and abstract, which indeed, as the teachers say, is typical of the tasks dominating their classrooms. At the same time, the type of suggestions teachers have given represent a continuation of the theme visible in the '*Drip rate*' item on the necessity to mark all the important information in the task and to match it with the typical classroom practice.

[14] This suggestion is not in accord with the official instructions given for the item as part of the translation procedures followed in the PISA Survey.

The skills students need and their typical mistakes

In setting the scene to foster teacher thinking of how the students would go about the two items, we explored how they perceive skills and competences students need to solve the items and the underlying mathematical knowledge. For the latter, all teachers were able to provide us with a clear list, whereas in the case of the former "patience", "concentration" and "to understand a text one has read" were named by just a few. Nonetheless, a distinctive pattern in teacher answers was visible when naming the specific subject related knowledge. All teachers, in both grammar and vocational schools, referred to both knowledge of mathematics and physics (Table 1), stressing on many occasions that "mathematically speaking, these are not difficult math problems", and that basic mathematical competence is necessary for the students to solve them, such as the four computational operations or knowledge of proportions, which all teachers emphasised is "something students learn in elementary school".

From their answers, it is clear that teachers are not clearly referencing particular aspects of the mathematics curriculum or students' competences when thinking about the possible problems students may encounter as they solve the items. Whereas in the case of the '*Drip rate*' item, the prevalent theme was that this is a type of the task "students simply give up on" and "would have problems in setting it up in the first place", without further naming the concrete problems students may come across; for the '*Fuji*' item these deliberations were much more concrete. We have identified four discrete subthemes relative to this item alone—transformation of units of measurement and calculating the arithmetical mean, using what teachers called the "big numbers". All teachers indicated how this is something students dislike doing, always asking if they can rely on their phone calculators. The second topic related to students possibly misunderstanding some words like the word 'pedometer', while the third evolved around the idea that many students may have problems in setting up the mathematical problem based on the information given in the text. Although in the narratives of all teachers at least two possible problems students may encounter were identified, all identified the fourth theme—will the students be able to calculate the correct number of days in a month? Teachers here referred to the actual number of days July and August have, as well as the fact whether students will know how to count the number of days between first and the last day of the month (i.e., 30 or 31 days). For the former, most teachers wondered whether students would remember a game they all learn when they were 7 years old on how to determine a number of the days in the month.[15]

What are the difficult parts?

When asked about procedures students would be expected to follow in solving the problems, none of the teachers provides a clear narrative that explains the procedural steps a student would need to use in order to solve the task correctly. This pattern is consistently visible in teachers from both VET establishments and the grammar schools. Rather they comment on the general aspects students should look into (e.g., "first to read the task carefully and take notice

[15] The game says that starting from the knuckle on your pinky finger one can calculate the number of days in the month. Each knuckle represents 31 days and an indent in between them 30 days. The game is usually taught already in Grade 1 of elementary school by the class teachers.

Table 1 Subject knowledge perceived by teachers relevant to the solving procedures of the two items

The list of subject knowledge observed in the teacher narratives	The Drip rate	Climbing Mount Fuji
Mathematics		
4 computational operations	✓	✓
Proportions	✓	
Fractions and the dependence of numerators	✓	
Equations	✓	
How many days in the month is there?		✓
Physics		
Units of measurement (and their conversion)	✓	✓
Dependence between speed, time and distance		✓

which units are changing and which stay constant") or focus on the logic of the task and how they would try to make it more familiar to the students (Example 3).

Example 3—example from narratives related to the 'drip rate' item

I would try to address first that they try to realise first what the infusion is, what does this picture explain, to see first if they understand what they should be doing ..so something is dripping.. so let's see if we have a small wheel there and if we turn it down, if it drips slower will it need more time to drip, and the volume is constant or if we turn it up does it mean it will drip faster, that we talk about this a bit first.

(Teacher, vocational school)

I think I would try to instruct them to try to link all the unit of measurement in the task to the language of mathematics and its formulas… for example here when it says double the time, that they introduce a t and that they translate conditions from here [*refers to the task*] into the language of mathematics.

(Teacher, grammar school)

More focused and concrete narratives are visible once teachers orient their attention to the particular part of the task, which they have previously named as the most difficult one relative to the particular item. The reasons why a particular question is labelled as the "most difficult" tend to overlap, thus reducing the number of themes visible in the teacher narratives (e.g., question 2 in the '*Fuji*' item—"there are too many demands and information given"), while teachers' views on what is the most difficult question within the item differ. Teachers are rather divided between questions 2 and 3 in the '*Fuji*' item, and questions 1 and 3 of the '*Drip rate*'. Such reflections, however, unravel the extent of the different understandings that teachers possess when it comes to student thinking. While in some cases the amount of information indicates complexity, for others this is when students "are requested to think" in the form of abstract reasoning, which they consider is needed in question 1 of the '*Drip rate*' item.

Applied mathematics in their own classrooms

Through the narratives, teachers clearly referred to the items as applied mathematical problems, atypical for students in Serbia. To some extent, we argue that the latter is also true for the teachers. None of them reported having had the possibility to create or work with such tasks during their pre-service training, and only a few mentioned some scarce opportunities during the in-service training. Although all teachers were open to the idea of incorporating more of such tasks in their own teaching, two major concerns appeared in their narratives—"what do we expel from the curriculum to find time for this?", and at the end "will this kind of problems become obligatory?". So despite the fact that an initial positive attitude was visible, it was soon replaced by a concern of what the purpose of introducing such problems would be, and, in addition, whether someone in the system will monitor if they have more applied tasks during the class or not. Interestingly, the introduction of applied mathematics is seen as an 'either-or' situation. Teachers do not see the contextualisation of tasks as part of their responsibility as mathematics educators in the twenty-first century. Rather, this very contextualisation is seen by them as an additional topic teachers will need to cover, and they do not see it as a tool that adds to the variety of the tasks and strategies that they usually prepare for the students within the given curricular topics. For them, a full instalment of contextual tasks is only possible if the entire approach to teaching mathematics and the curricula will change. Within the current system, teachers do not find it feasible, but rather see it as an additional thing they would need to add to the "currently overcrowded curriculum" they are "already struggling with" in both grammar and VET schools.

Discussion

In an attempt to further unravel teachers' competences relevant for teaching/learning mathematics (Li and Kaiser 2011; Richland et al. 2012; Radišić et al., 2014; Radišić and Baucal, 2016), we have focused on examining how teachers perceive students' thinking in connection to two PISA items—the '*Drip rate*' and *the* '*Fuji*' *item*. However, the way this study was contextualised has forced teachers not to link their reflections to a particular teaching situation they have just experienced, but rather demanded from them a meta-perspective grounded in the knowledge and experience they have previously built (Hill et al. 2008).

The analyses began with the reflections related to the idea of typical mathematical problems that students encounter within lessons in the classrooms of Serbia. From that perspective, both the '*Drip rate*' and the '*Fuji*' items were judged as atypical, since they relate to applied mathematics which is not the focus of the mathematics curriculum in either the grammar or VET schools. Also, although applied tasks are, to some extent present in Serbian classrooms, the context of the chosen items is more typical for physics, than mathematics; while the way in which the items are formulated deviates from the abstract, formal and more general math problems students usually deal with in class (Anić and Pavlović Babić 2015). Given the general differences between the grammar schools and VET tracks, it did come as a surprise that we found no differences between what is considered a typical task among the teachers working in the two different school settings. For all, mathematics is a formal discipline and learning of mathematics is not seen as a situated learning (Radišić and Baucal, 2015, 2016).

If we remember that the mathematical problems that students encounter will shape not only the content of their learning, but also how they come to think about that content and visualise mathematics in general (Doyle 1983), and that the students are expected to learn to observe mathematics as a dynamic field in which they engage in the processes of mathematical thinking, (Romberg 1992; Pepin et al. 2016), we may question such outcomes to be credible in a typical classroom context in Serbia. Rather, we see that the idea of mathematics as an abstract discipline is strongly present in teachers' reflections. In this sense, mathematics is seen as disconnected from everyday life, and learning is through practicing decontextualized, formal math tasks that are reinforced through the current practices.

Neither of the items is seen by the teachers as ‘mathematically difficult’. They merely require basic knowledge that students will have acquired during elementary school. At the same time, the ‘*Drip rate*’ item is seen as the type of the problem that students would not even grapple with, but rather give up at the very beginning, since it involves ‘thinking’ not covered in class. To some extent, this contradicts previous findings on types of the problems that are more difficult for the students in Serbia. Both items fall into this category (Pavlović Babić and Baucal, 2013), but not according to the teachers’ understanding of what constitutes ‘mathematically difficult’ tasks. Rather, a lack of familiar milieu is what is hindering the process in their accounts, which again is in line with the findings of the general importance of context for successful problem solving (Wyndhamn and Säljö 1997; Verschaffel et al. 2000). The finding is also in line with the findings of Anić and Pavlović Babić (2015). The authors have modified particular PISA items to examine the influence of irrelevant information in mathematical problems. The more the task resembled to the typical abstract form, the higher was students’ success rate in solving the tasks. The idea of ‘mathematically not difficult’ task also implies teachers’ notion that if students would be shown just a formula the item is grounded in, it would be easy for them to solve it, demonstrating that contextualising mathematics (mathematization of the experience, i.e., creating a mathematical model of a situation) for them has no meaning in their teaching/learning practice.

At the same time, teachers’ reasoning on the topic of difficult and less difficult parts within each item also uncovers disparities in their vision of what constitutes a difficult task—is it a task where a student is confronted with more information or a task in which (s)he needs to reason? Such narratives point to the conclusion these two processes are mutually exclusive in teachers’ mind-set, providing us with a particular insight into how they actually envision the process of thinking evolves. Another implication may be related to the particular need of incorporating more systemic knowledge on particular developmental processes relevant to learning in both pre-service and in-service training of mathematics teachers.

Correcting errors in a procedure (Empson and Jacobs 2008) or well established misconceptions (Pierson 2008) are prominent features of reasoning about how students think in relation to particular mathematical content, together with identifying students’ strategies during particular problem solving situations (Empson and Jacobs 2008; Hattikudur et al. 2016). When comparing the two examined items, more consistently, teachers were able to point to possible difficulties in the procedure or misconceptions relative to the ‘*Fuji*’ task, whereas ambiguity in depicting anticipated procedures for each of the items was equally present. To some extent, we may attribute this to the fact that some procedures may be so obvious to adults (Ball et al. 2005); in this case, adults with abundant professional knowledge, that they themselves may find it irrelevant to exactly explain each step. However, if we bear in mind that mathematical verbalization allows both students and teachers to interact about critical mathematical content (Doabler et al. 2015), verbalisation becomes one of critical aspects of teacher competence as it fosters teachers’ interaction with the students in class and it should be evident when teachers are trying to portray their own concepts of students thinking in connection to particular math content. Evidence of their inability to clearly do so, speaks of the necessity to support this skill through both pre-service and in-service training more systematically.

Lastly, although mathematical task are seen as typically formal and not tied to everyday context, in teacher reflections about the particular PISA items, we also recognise strict reference to specific informal practices relevant to solving mathematical problems (e.g., counting the number of days in the month). The extent to which teachers rely on such practices in their teaching may be an entirely new research topic, but at the same time it does provide actual space for introducing everyday context and tools in the realm of mathematics making it less formal and disconnected from the students' reality.

Given the fact that mathematics is perceived by the teachers as a largely formal discipline, while its content is practised mostly through formal and abstract tasks, we may ask how, in such conditions, students learn that mathematics truly is a dynamic playground where they enact their own skills and create meaningful new knowledge (Romberg 1992; Pepin et al. 2016), together with the fact that the school mathematics is fundamentally no different than the one they encounter in everyday life (OECD 2013; Wijaya et al., 2014). The grounds for the current reasoning of teachers are strongly evident in the in-service training they have received, the opinion of the mathematicians' professional associations and even the curriculum. Meanwhile, the extent to which mathematics is experienced as a 'difficult' and a formal subject in school to the students (Pešić and Stepanović 2004) relates to the very way teacher project such an image. Nevertheless, even in that context, teachers in our study did show a readiness to adopt more contextual problems in own teaching, like the ones in the PISA survey. Despite the fact that this initial readiness was mixed with concerns of the purpose introducing such tasks in their everyday practice, the notion that they may be 'monitored' in any way as to the amount of contextual math problems in their teaching, speaks in favour of how much teachers rely on the system in projecting the image of what mathematics is and how it should be approached by educators and taught in the classroom.

So despite the fact that research discourse focuses upon teacher competencies and how these facilitate an active role of the student (Schoenfeld and Kilpatrick 2008) in the process of achieving mathematical understanding (Cai and Ding 2017; Rittle-Johnson 2017), there remains an image that is still disconnected from the discourse of teachers, mathematicians' associations and the discourses within which official curriculum developers operate (Clements and Sarama 2004). The scarce opportunities to be acquainted with mathematical problems similar to ones in PISA during the process of professional development mean that teachers may have only limited resources to create new sets of examples for their students. Moreover, the enacting changes in their classrooms, as part of attempts to make their teaching more effective, are unlikely to happen unless teachers are systematically supported in doing so by the classroom, school and college environment in which they operate, and not just by the array of research results produced by the academics.

In this study, teachers' narratives show clearly how the dominant teaching/learning practices can contribute individual student outcomes. This indicates that, even in the case of international studies such as PISA, particular league tables need to be viewed in the context of the actual practice taking place in schools. On the other hand, if we think about the PISA items as a tool for further deepening our understanding of the mathematics enacted in the classroom, through subtle modifications in the items, proposed by the teachers, these may help us to create a gradient of how both teachers and the students depict mathematical tasks relative to their formal characteristics and enable us to sketch when we actually leave the 'formal mathematics zone' and enter into the realm of 'applied mathematics'. Such a road map may be observed as a tool in future used by both teachers and teacher educators.

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Appendix 1. The interview guide

Introductory section provided the participants with the study aims and some basic information relative to the country participation in the PISA programme.

Now we will show you two cards with math items and we will discuss each and how they can be solved, as well as how you perceive these problems. These items were chosen by examining students' achievement in the cycle 2012 and comparing it with the achievement of students in other participating countries.

Here is the first item. Please take a look at it.

1. What is your general impression about this item? Is this a math item? Is the picture accompanying the item appropriate and explains the math item?
2. Is the instruction given in the problem clear enough? In your experience what kind of students might have difficulty to understand instructions for this math item (e.g. language difficulties)?
3. Would you suggest some modifications that would make instruction for this problem more understandable to the students (language, presentation, information...)
4. Do you find all parts of the item easy or difficult for our students? Which part of the items do you find more difficult?
5. What kind of competencies do the students need to solve this item?
6. What students need to know previously and to be able to do in order to solve this math item?
7. How would you explain to the students to think about this math problem while solving it? What is it you expect students to do while solving this math item?
8. To what extent you think our students deal with such items at their math lessons? (if answered they do not deal enough what might facilitate the change towards dealing with them more often in class)

   [Questions 1 to 8 were asked for each item. The 'Drip rate' was the first item and 'Fuji' the second]
9. If you look at your in-service training and seminars organised by the mathematical association are these kind of problems something you tackle in these?
10. Do you feel you as a teacher might need additional support to introduce more problems like these in you mathematics lessons or do you believe their amount in your teaching is enough? (If the answer is yes what kind of support you think would enable you to introduce more of this kind of math problems in your teaching? Should the support come from actors at the school level, mathematics teachers' associations or the policy makers?)

References

Anić, I., & Pavlović Babić, D. (2015). How we can support success in solving mathematical problems? *Teaching Innovations, 28*(3), 36–49.

Ball, D. L. (1997). What do students know? Facing challenges of distance, context, and desire in trying to hear children. In B. Biddle, T. Good, & I. Goodson (Eds.), *International handbook on teachers and teaching* (Vol. II, pp. 679–718). Dordrecht: Kluwer Press.

Ball, D. L. (2001). *Teaching, with respect to mathematics and students. Beyond classical pedagogy: Teaching elementary school mathematics* (pp. 11–22). Mahwah: Lawrence Erlbaum Associates.

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: who knows mathematics well enough to teach third grade, and how can we decide? *American Educator, 30*(3), 14–17 20–22, 43–46.
Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: what makes it special? *Journal of Teacher Education, 59*(5), 389–407.
Baucal, A., & Pavlović Babić, D. (2010). *PISA 2009 u Srbiji: prvi rezultati. Nauči me da mislim, nauči me da učim.* Institut za psihologiju Filozofskog fakulteta u Beogradu i Centar za primenjenu psihologiju.
Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., et al. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal, 47*, 133–180.
Black, P., Harrison, C., Lee, C., Marshall, B., & Wiliam, D. (2003). *Assessment for learning: Putting it into practice*. Maidenhead: McGraw-Hill International.
Blömeke, S., Gustafsson, J. E., & Shavelson, R. (2015). Beyond dichotomies: Viewing competence as a continuum. *Zeitschrift für Psychologie, 223*(1), 3–13.
Borg, M. (2001). Key concepts: teachers' beliefs. *ELT Journal, 55*(2), 186–188.
Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology, 3*, 77–101.
Cai, J., & Ding, M. (2017). On mathematical understanding: perspectives of experienced Chinese mathematics teachers. *Journal of Mathematics Teacher Education, 20*, 5–29.
Clarke, B. (2008). A framework of growth points as a powerful teacher development tool. In D. Tirosh & T. Wood (Eds.), *International handbook of mathematics teacher education: tools and processes in mathematics teacher education* (Vol. 2, pp. 235–256). Rotterdam: Sense Publishers.
Clements, D. H., & Sarama, J. (2004). Learning trajectories in mathematics education. *Mathematical Thinking and Learning, 6*(2), 81–89.
Cobb, P., Wood, T., Yackel, E., & McNeal, E. (1993). Mathematics as procedural instructions and mathematics as meaningful activity: the reality of teaching for understanding. In R. Davis & C. Maher (Eds.), *Schools, mathematics and the world of reality* (pp. 119–134). Needham Heights: Allyn & Bacon.
Crespo, S. (2000). Seeing more than right and wrong answers: prospective teachers' interpretations of students' mathematical work. *Journal of Mathematics Teacher Education, 3*(2), 155–181.
Didis, M. G., Erbas, A. K., Cetinkaya, B., Cakiroglu, E., & Alacaci, C. (2016). Exploring prospective secondary mathematics teachers' interpretation of student thinking through analyzing students' work in modelling. *Mathematics Education Research Journal, 28*, 349–378.
Doabler, C. T., Baker, S. K., Kosty, D. B., Smolkowski, K., Clarke, B., Miller, S. J., & Fien, H. (2015). Examining the association between explicit mathematics instruction and student mathematics achievement. *Elementary School Journal, 115*(3), 303–333.
Doyle, W. (1983). Academic work. *Review of Educational Research, 53*, 159–199.
Dyer, E. B., & Sherin, M. G. (2016). Instructional reasoning about interpretations of student thinking that supports responsive teaching in secondary mathematics. *ZDM Mathematics Education, 48*, 69–82.
Ellis, M. V., & Berry, R. Q. (2005). The paradigm shift in mathematics education: explanations and implications of reforming conceptions of teaching and learning. *Mathematics Educator, 15*(1), 7–17.
Empson, S. B., & Jacobs, V. R. (2008). Learning to listen to children's mathematics. In D. Tirosh & T. Wood (Eds.), *The international handbook of mathematics teacher education: tools and processes in mathematics teacher education* (Vol. 2, pp. 257–281). Rotterdam: Sense Publishers.
Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147–164.) New York.
Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V. R., & Empson, S. B. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education, 27*(4), 403–434.
Fives, H., & Buehl, M. M. (2012). Spring cleaning for the "messy" construct of teachers' beliefs: what are they? Which have been examined? What can they tell us? In K. R. Harris, S. Graham, & T. Urdan (Eds.), *Individual differences and cultural and contextual factors* (pp. 471–499). Washington, DC: APA.
Fives, H., & Gregoire-Gill, M. (2014). *International handbook of research on teachers' beliefs*. New York: Routledge.
Furtak, E. M., Kiemer, K., Kizil Circi, R., Swanson, R., de Leon, V., Morrison, D., & Heredia, S. H. (2016). Teachers' formative assessment abilities and their relationship to student learning: findings from a four-year intervention study. *Instructional Science, 44*, 267–291.
Hattie, J. (2008). *Visible learning: a synthesis of over 800 meta-analyses relating to achievement*. London: Routledge.
Hattikudur, S., Sidney, P. G., & Alibali, M. W. (2016). Does comparing informal and formal procedures promote mathematics learning? The benefits of bridging depend on attitudes toward mathematics. *Journal of Problem Solving, 9*, 13–27.

Hill, H. C., Blunck, M. L., Charalambos, Y. C., Lewis, J. M., Phelps, G. C., Sleep, L., & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: an exploratory study. *Cognition and Instruction, 26*(4), 430–511.

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 41*(2), 169–202.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: helping children learn mathematics*. Washington, D.C.: National Academy Press.

König, J., Blömeke, S., Klein, P., Suhl, U., Busse, A., & Kaiser, G. (2014). Is teachers' general pedagogical knowledge a premise for noticing and interpreting classroom situations? A video-based assessment approach. *Teaching and Teacher Education., 38*, 76–88.

Krstić, K. (2015). Attachment in the student-teacher relationship as a factor of school achievement. *Teaching Innovation, 28*(3), 167–188.

Leinhardt, G. (1986). Expertise in mathematics teaching. *Educational Leadership, 43*, 28–33.

Li, Y., & Kaiser, G. (2011). Expertise in mathematics instruction: advancing research and practice from an international perspective. In Y. Li & G. Kaiser (Eds.), *Expertise in mathematics instruction, an international perspective* (pp. 3–16). New York: Springer.

Merriam, S. R. (1998). *Qualitative research and case study applications in education*. San Francisco: Jossey Bass.

Mincu, M. E. (2009). Myth, rhetoric and ideology in eastern European education. *European Education, 41*(1), 55–78.

Mullis, I. V. S., Martin, M. O., Foy, P., & Hooper, M. (2016). *TIMSS 2015 international results in mathematics.* Retrieved from Boston College, TIMSS & PIRLS International Study Center.

OECD. (2013). *PISA 2012 assessment and analytical framework: mathematics, reading, science, problem solving and financial literacy.* Paris: OECD Publishing.

OECD. (2016). *PISA 2015 results (volume I): excellence and equity in education*. Paris: OECD Publishing.

Palm, T. (2008). Impact of authenticity on sense making in word problem solving. *Educational Studies in Mathematics, 67*(1), 37–58.

Pavlović Babić, D., & Baucal, A. (2013). Inspiriši me, podrži me, PISA 2012 u Srbiji: prvi rezultati. Beograd: Institut za psihologiju.

Pepin, B. E. U., Xu, B., Trouche, L., & Wang, C. (2016). Developing a deeper understanding of mathematics teaching expertise: an examination of three Chinese mathematics teachers' resource systems as windows into their work and expertise. *Educational Studies in Mathematics, 94*(3), 257–274.

Pešić, J., & Stepanović, I. (2004). Škola kao sredina za učenje-učenička percepcija i njihove strategije [School as a learning space—Students' perceptions and strategies]. In D. Plut & Z. Krnjaić (Eds.), *Društvena kriza I obrazovanje – dokument o jednom vremenu [A social crises and education – documenting an era]* (pp. 24–69). Beograd: Institut za psihologiju.

Pierson, J. L. (2008). *The relationship between patterns of classroom discourse and mathematics learning (unpublished dissertation)*. Austin: University of Texas at Austin retrieved at: https://repositories.lib.utexas.edu/bitstream/handle/2152/17898/piersonj.pdf?sequence=2&isAllowed=y

Radišić, J., & Baucal, A. (2015). Portret nastavnika matematike u srednjoj školi: kritička analiza dominantne prakse. Primenjena psihologija. (Portrait of high school math teachers: Critical analysis of dominant practice). *Primenjena psihologija, 8*(1), 25-46.

Radišić, J., & Baucal, A. (2016). "What about when the majority is excluded?": A Critical Eye on Language and Math Classrooms in Serbia. In Surian, A. (Ed.), *Open Spaces for Interactions and Learning Diversities* (pp. 167-178) Sense Publishers.

Radišić, J., Baucal, A., & Videnović, M. (2014). Unfolding the assessment process in a whole class mathematics setting. *Psihološka istraživanja, 17*(2), 137-158.

Richland, L. E., Stigler, J. W., & Holyoak, K. J. (2012). Teaching the conceptual structure of mathematics. *Educational Psychologist, 47*, 189–203.

Rittle-Johnson, B. (2017). Developing mathematics knowledge. *Child Development Perspectives, 0*(0), 1–7.

Romberg, T. A. (1992). Perspectives on scholarship and research methods. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 49–64). New York: Macmillan.

Säljö, R. (2009). Learning, theories of learning, and units of analysis in research. *Educational Psychologist, 44*(3), 202–208.

Santagata, R., & Yeh, C. (2016). The role of perception, interpretation, and decision making in the development of beginning teachers' competence. *ZDM Mathematics Education, 48*, 153–165.

Schneider, M., Rittle-Johnson, B., & Star, J. R. (2011). Relations among conceptual knowledge, procedural knowledge, and procedural flexibility in two samples differing in prior knowledge. *Developmental Psychology, 47*, 1525–1538.

Schoenfeld, A. H. (1992). Learning to think mathematically: problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334–371). New York: Macmillan.

Schoenfeld, A. H. (2014). *Mathematical problem solving*. London: Academic Press.

Schoenfeld, A. H., & Kilpatrick, J. (2008). Toward a theory of proficiency in teaching mathematics. In T. Wood & D. Tirosh (Eds.), *International handbook of mathematics teacher education: Vol. 2. Tools and processes in mathematics teacher education* (pp. 321–354). Rotterdam: Sense Publishers.

Sherin, M. G., & van Es, E. A. (2009). Effects of video club participation on teachers' professional vision. *Journal of Teacher Education, 60*, 20–37.

Sherin, M. G., Jacobs, V. R., & Philipp, R. A. (Eds.). (2011). *Mathematics teacher noticing: seeing through teachers' eyes*. New York: Routledge.

Shulman, L. (1987). Knowledge and teaching: foundations of the new reform. *Harvard Educational Review, 57*, 1–22.

Smith, M. S. (2001). *Practice-based professional development for teachers of mathematics*. Reston: National Council of Teachers of Mathematics.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: an analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*(2), 455–488.

Teuscher, D., Moore, K. C., & Carlson, M. P. (2016). Decentering: a construct to analyze and explain teacher actions as they relate to student thinking. *Journal of Mathematics Teacher Education, 19*, 433–456.

Verschaffel, V., Greer, B., & de Corte, E. (Eds.). (2000). *Making sense of word problems*. Heereweg: Swets & Zeitlinger.

Verschaffel, L., Depaepe, F., & Van Dooren, W. (2015). *Individual differences in word problem solving, The Oxford Handbook of Numerical Cognition* (pp. 1–17). Oxford: Oxford University Press.

Wijaya, A., van den Heuvel-Panhuizen, M., Doorman, M., & Robitzsch, A. (2014). Difficulties in solving context-based PISA mathematics tasks: an analysis of students' errors. *The Mathematics Enthusiast, 11*(3), 555–584.

Wyndhamn, J., & Säljö, R. (1997). Word problems and mathematical reasoning—a study of children's mastery of reference and meaning in textual realities. *Learning and Instruction, 7*(4), 361–382.

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Current Themes of Research:

Jelena Radišić is a postdoc researcher at the Department of Teacher Education and School Research, Faculty of Educational Sciences, University of Oslo. In her research, she examines the following topics: student, teacher and school characteristics affecting academic achievement; assessing the quality and efficacy of the education system (secondary analysis of PISA and TIMSS results); teacher beliefs and practices and their impact on student learning; motivation for learning; mathematics anxiety; emergent literacy.

Most relevant publications in the field of Psychology of Education:

Marković, J., Radišić, J., Jovanović, V. & Ranković, T. (2017). Developing a model for dropout prevention and intervention in primary and secondary schools in Serbia: Assessing the Model Effectiveness. *Psihološka istraživanja, 20*(1), 145–169.

Radišić, J. & Baucal, A. (2016). Using video-stimulated recall to understand teachers' perceptions of teaching and learning in the classroom setting. *Psihološka istraživanja, 19*(2), 165–183.

Kovač Cerović, T., Radišić, J., & Stanković, D. (2015). Bridging the gap between teachers' initial education and induction through student teachers' school practice: case study of Serbia. *Croatian Journal of Education, 17*(2), 43–70.

Radišić, J. & Jošić, S. (2015). Challenges, obstacles and outcomes of applying inquiry method in primary school mathematics: example of an experienced teacher. *Teaching Innovations*, *28*(3), 99–115.

Radišić, J., Videnović, M., & Baucal, A. (2015). Math anxiety – contributing school and individual level factors. European Journal of Psychology of Education, 30(1), 1–20

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Current Themes of Research:

Aleksander Bauca, PhD is a professor at the Department of Psychology at the University of Belgrade. His main field of interest is the socio-cultural developmental psychology and studies of development of new competencies through symmetric (collaborative peer learning) and asymmetric (learning with adults) social interaction. The author's work is also related to improvement of traditional pre-post test research design by integration with the item response theory (IRT) and involvement of qualitative case studies.

Most relevant publications in the field of Psychology of Education:

Budjevac, N., Arcidiacono, F., & Baucal, A. (2017). Reading together: the interplay between social and cognitive aspects in argumentative and non-argumentative dialogues. In: F. Arcidiacono and A. Bova (Eds), Interpersonal argumentation in educational and professional contexts. Springer

Tartas, V., Perret-Clermont, A. N. & Baucal, A. (2016). Experimental micro-histories, private speech and a study of children's learning and cognitive development. Infancia y Aprendizaje, 39(4), pp. 772–811

Radišić, J., Videnović, M., & Baucal, A. (2015). Math anxiety – contributing school and individual level factors. European Journal of Psychology of Education, 30(1), 1–20

Baucal, A. (2013). Two instead of one ZPD: Individual and joint construction in the ZPD. In S. Phillipson, K. Ku, & S. Phillipson (Eds.), Constructing educational achievement: a sociocultural perspective (pp. 161–173). Routledge, London.

Baucal, A. (2012). Ključne kompetencije mladih u Srbiji u PISA 2009 ogledalu [Youth in Serbia: Key competencies in the PISA 2009 mirror]. Belgrade: Institute of Psychology.

Baucal, A., Arcidiacono, F. & Budjevac, N. (2011). Studying interaction in different contexts: a qualitative view. Belgrade: Institute of Psychology.