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Competition and cooperation in the exploitation of the groundwater resource

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Abstract
We study the exploitation of a common groundwater resource, first as a static and then as a differential game, in order to take into account the strategic and dynamic interactions among the users of the resource. We suppose that firms can form coalitions or can decide not to cooperate. The non-cooperation regime is characterized by pumping that lead to depletion of the aquifer; the cooperation preserves the resource. Open-loop and feedback equilibria have been computed and compared in order to characterize the existence of cooperators and defectors in water resources management.

Keywords Groundwater extraction · Competition and cooperation · Differential game

JEL Classification D62 · D99 · Q15

1 Introduction

During the second half of the twentieth century, groundwater withdrawals have increased up to the point that they now supply water to half of the worlds population. It is said that groundwater is the world's most extracted raw materials, see Jaroslav and Annukka (2007). This extra use has caused water table drawdowns and depletion of groundwater resources in many parts of the world and this highlights the importance of groundwater management. Intensive use of groundwater leads to a wide array of social, economic and environmental consequences such as land subsidence, increases in the vulnerability of agriculture and other uses of the water to climate change, increases in pumping costs (Burke 2003). The open-access nature of natural resources, such

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as groundwater and the accompanying externalities, in combination with the failure to treat natural resources as capital, has made this an attractive research area (Brown 2000), for the development of rules for efficient water allocation among competing uses over time and space (Koundouri and Xepapadeas 2004). The problems created by the growing pressure of water extractions are twofold: one is water scarcity in local watersheds or whole basins created by excessive surface and groundwater withdrawals and the other is water degradation from pollution loads leading to many tracts of rivers and whole aquifers being spoiled, and losing their capacity to sustain ecosystem functioning and human activities. Interest in water resources conflict resolution has increased over the last decades and various quantitative and qualitative methods have been proposed for conflict resolution in water resources management. Gisser and Sanchez (1980) analyze aquifer management regime and find that welfare gains from policy intervention are insignificant compared with competitive outcomes. They conclude that this dependence is negligible if the capacity of the aquifer is large. Gisser and Sanchez's theoretical prediction is that if the storage capacity of the aquifer is relatively large, then the two behaviors would be very close. For an overview of these results, it is possible to see Koundouri (2004). Several studies have used game theory to provide frameworks for studying the strategic actions of individual players to develop more broadly acceptable solutions. In particular, authors assume that farmers behave myopically in the calculation of the private solution, that is, farmers take decisions over a short period of time, without considering the impact of the other users on the available stock. Other authors propose differential games in order to explore the behavior of farmers in the long run. Negri (1989) characterizes analytical solutions of the water table level at the steady state for two types of Nash equilibria, open-loop and feedback solutions, and for the socially optimal case, also referred as Pareto optimal case. He shows that the dependence between the socially optimal solution and the open-loop solution is positive and captures the pumping cost externality. This dependence between the two kinds of solutions is positive and represents the inefficiency of private exploitation. Provencher and Burt (1993) prove, using a discrete time dynamic programming, that if the objective function of the problem is concave, the feedback solution is inefficient, in comparison with the socially optimal solution. Rubio and Casino (2001, 2003) adapt the Gisser and Sanchez model as a differential game and derive analytical solutions of the socially optimal, open-loop and feedback cases over an infinite planning horizon. They also confirm Negri's result, strategic behavior exacerbates the inefficiency of private solutions. Moreover, they confirm the Gisser and Sanchez rule when the strategic externality is considered, for large aquifers, the dependent solutions get closer at the steady state. Esteban and Albiac (2011) take ecosystem damages into consideration in modeling aquifer management regimes and they show that by including these environmental externalities into the analytical framework, results can change substantially.

In our paper, we adapt the model proposed by Rubio and Casino (2001) and we introduce the environmental damages due to over-exploitation of the groundwater resource as in Esteban and Albiac (2011). Rubio and Casino compare socially optimal and private extraction of a common property aquifer, as well as all papers quoted, where only these two kinds of equilibria are considered. Our extension to existing literature is that we consider heterogeneous farmers in terms of behavior of the exploitation of the water resource. In particular, we consider N identical firms that differ in terms of their choice to cooperate or defect. In a water conflict, in fact, different groups or individuals can be modeled as players. Each player can make choices unilaterally and the combined choices of all players together determine the possible outcomes of the conflict. Instead of unilaterally moving, counties also may decide to cooperate and form coalitions leading to Pareto optimal outcomes. Game theory techniques provide an effective and precise language for discussing specific water conflicts. A systematic study of a strategic water dispute provides insights about how the conflict can be better resolved and may suggest solutions of the problem of the exploitation of the groundwater. We study the exploitation of a common groundwater resource first as a static and then as a differential game in order to take into account the strategic and dynamic interactions between the users of the resource by several farmers. In particular, we analyze open-loop and feedback equilibria and we illustrate the implications of the different strategies on extraction rates and groundwater table levels. Results show that both equilibria depend on the number of cooperators and defectors and that the difference between them is very small. So results establish that potential benefits coming from the regulation of the resource will be relatively small.

The paper is organized in the following way. Section 2 presents the model, while Sect. 3 describes the analytical resolution of the static game. Section 4 proposes the differential game and computes open-loop Nash equilibria. Moreover, it studies also the Pareto optimal solution and a comparative statics about the parameters of the model. Section 5 computes feedback Nash equilibria and makes a numerical analysis that allows us to compare the two different solutions. Section 6 concludes.

2 The basic model

We assume that the global demand for irrigation is a negatively sloped linear function, defined as follows

$$W = g + kP \tag{1}$$

where W is the amount of the groundwater pumped, P is the price of water, $k < 0$, is the price coefficient and $g > 0$, is the intercept of the water demand function.

As in Rubio and Casino (2001), we adapt the model proposed by Gisser and Sanchez (1980) to study the strategic behavior effects in the exploitation of the groundwater resource. In particular, we suppose that access to the aquifer is restricted by land ownership and consequently we have that the number of farmers is fixed and finite over time. We assume that N is the number of farmers and w_h is the pumping rate of the farmer h. The individual demand function is

$$w_h = \delta_h(g + kP) \quad h = 1, \ldots, N \tag{2}$$

where $0 < \delta_h < 1$ and $\sum_{h=1}^{N} \delta_h = 1$.

Let

$$p(w_h) = \frac{w_h}{k\delta_h} - \frac{g}{k} \quad h = 1, \ldots, N \tag{3}$$

the revenue of the farmer h is

$$\int p(w_h)\mathrm{d}w_h = \frac{1}{2k\delta_h} w_h^2 - \frac{g}{k} w_h \quad h = 1, \ldots, N \tag{4}$$

The total cost of extraction depends directly with the pumping rate and inversely on the level of the water table

$$C(H, W) = (c_0 + c_1 H)W \tag{5}$$

where H is the height of the aquifer, i.e., the water table elevation above some arbitrary level that is considered as being the bottom of the aquifer. The fixed cost linked to the hydrologic cone is $c_0 > 0$ and the marginal pumping cost per acre foot of water pumped per foot of lift is $c_1 < 0$.

The individual farmer's extraction cost is

$$C_h(H, w_h) = (c_0 + c_1 H)w_h \quad h = 1, \ldots, N \tag{6}$$

The differential equation which describes the dynamics of the water table is obtained as the difference between natural recharge and net extractions

$$\dot{H} = \frac{1}{AS}[R + (\alpha - 1)W] \quad H(0) = H_0 \tag{7}$$

where R denotes the deterministic and natural recharge, AS is the area of the aquifer, $0 < \alpha < 1$ is the constant return flow coefficient of irrigation water.

Moreover, we introduce the cost of environmental damages

$$\bar{C}(W) = \beta[-(\alpha - 1)W - R] \tag{8}$$

This cost is defined as the volume depleted from the aquifer in each period $[-(\alpha - 1)W - R]$ multiplied by β. Aquifer depletion is the difference between net extractions $(1 - \alpha)W$ and recharge R. Parameter β is the cost of damages to ecosystems from each cubic meter of aquifer depletion.

The individual farmer's damage cost is

$$\bar{C}(w_h) = \beta[-(\alpha - 1)w_h - R\delta_h] \quad h = 1, \ldots, N \tag{9}$$

Finally, the farmer's h net revenues are equal to the willingness-to-pay for groundwater minus the extraction costs of the resources and minus the damage costs

$$\frac{1}{2k\delta_h} w_h^2 - \frac{g}{k} w_h - (c_0 + c_1 H)w_h - \beta[-(\alpha - 1)w_h - R\delta_h] \quad h = 1, \ldots, N \tag{10}$$

2.1 Game rules and payoffs

First of all, we suppose that all farmers are identical, so using symmetry we can write that $\delta_h = \frac{1}{N}$. The objective of each country is to maximize its profits choosing if it is better to cooperate or not. Let us assume that a non-trivial coalition is composed by m signatory countries ($i = 1, \ldots, m$) and consequently the remaining $N - m$ are considered nonsignatory countries ($j = m+1, \ldots, N$). So we have a simple structure in which only one coalition composed by m countries exists, while the other $N - m$ countries are defectors.

3 The static game

We suppose that all agents are myopic and do not make serious considerations about the future effects that their water withdrawal produces. So they maximize their current profit, solving the following static game.

Each cooperator determines w_i by solving the optimization problem

$$\max_{w_i} \sum_{i=1}^{m} \pi_i = \max_{w_i} \left\{ \frac{N}{2k} \sum_{i=1}^{m} w_i^2 - \frac{g}{k} \sum_{i=1}^{m} w_i - (c_0 + c_1 H) \sum_{i=1}^{m} w_i \right. \\ \left. - \beta \left[-(\alpha - 1) \sum_{i=1}^{m} w_i - \frac{R}{N} \right] \right\}$$

that gives us the total profit of the cooperative venture.

Each defector determines w_j by solving the optimization problem

$$\max_{w_j} \pi_j = \max_{w_j} \left\{ \frac{N}{2k} w_j^2 - \frac{g}{k} w_j - (c_0 + c_1 H) w_j - \beta \left[-(\alpha - 1) w_j - \frac{R}{N} \right] \right\}$$

Assuming interior optimum, the first-order conditions give a system of linear equations in the unknowns w_i

$$\max_{w_i} \sum_{i=1}^{m} \pi_i = 0 \Longleftrightarrow \frac{N}{k} w_i - \frac{g}{k} - c_0 - c_1 H + \beta(\alpha - 1) = 0 \tag{11}$$

Assuming again interior optimum, the first-order conditions give a system of linear equations in the unknowns w_j

$$\max_{w_j} \pi_j = 0 \Longleftrightarrow \frac{N}{k} w_j - \frac{g}{k} - c_0 - c_1 H + \beta(\alpha - 1) = 0 \tag{12}$$

Equations (11) and (12) give a linear system of N equations with N unknowns. However, it is straightforward to see that any cooperator faces the same optimization problem, and analogously for defectors. So, adding the first m equations and the remaining $N - m$ equations we get a system of two equations in two unknowns w_i and w_j. In this manner, we obtain two reaction functions, that are

$$w_i = \frac{k}{N}\left[c_0 + c_1 H + \frac{g}{k} - \beta(\alpha - 1)\right] \tag{13}$$

$$w_j = \frac{k}{N}\left[c_0 + c_1 H + \frac{g}{k} - \beta(\alpha - 1)\right] \tag{14}$$

These reaction functions allow us to compute the optimal groundwater stock. In fact, from differential equation (17) we have:

$$\dot{H} = 0 \Longleftrightarrow R + m(\alpha - 1)w_i + (N - m)(\alpha - 1)w_j = 0$$

and substituting (13) and (14) we obtain the following:

Proposition 1 *There exists a unique myopic stationary equilibrium where the elevation of the water table is given by:*

$$H^*_{MY} = -\frac{R}{c_1 k(\alpha - 1)} - \frac{g}{c_1 k} - \frac{c_0}{c_1} + \frac{\beta(\alpha - 1)}{c_1} \tag{15}$$

and the individual demand functions, respectively, of a cooperator and of a defector, are:

$$w^*_{i(MY)} = w^*_{j(MY)} = -\frac{R}{N(\alpha - 1)} + \frac{\beta(1 - k)(\alpha - 1)}{N} \tag{16}$$

We observe that the myopic equilibrium H^*_{MY} does not depend on the number of cooperators and defectors. It is also independent of the number N of countries that exploit the resource.

The individual demand functions of cooperators and defectors are identical and do not depend on the number of countries which join the agreement but only on the number of countries N which exploit the resource.

4 The differential game

In the optimal control problem, the objective of each country is to maximize its discounted profit choosing if it is better to cooperate or defect. We propose a differential game, in which we calculate open-loop Nash equilibria in order to determine both the optimal paths of the extraction levels and the dynamic of the water table. Time t is continuous, with $t \in [0, +\infty[$ and countries discount future costs using the constant rate $r > 0$. We use a non-cooperative game framework solved in a backward order. As a result of the first stage, m cooperators and $N - m$ defectors exist and, in the second stage, cooperators and defectors choose their abatement levels.

Fixed the extraction levels of defectors, cooperators bind to a level of extraction that maximizes the discounted value of the aggregate payoff of m countries:

$$\pi_i = \max_{w_i} \sum_{i=1}^{m} \int_0^{+\infty} e^{-rt} \left\{ \frac{N}{2k} {w_i}^2 - \frac{g}{k} w_i - (c_0 + c_1 H) w_i - \beta \left[-(\alpha - 1) w_i - \frac{R}{N} \right] \right\} \mathrm{d}t$$

Given the extraction levels of cooperators, defectors bind to a level of extraction that maximizes the discounted value of their payoff:

$$\pi_j = \max_{w_j} \int_0^{+\infty} e^{-rt} \left\{ \frac{N}{2k} {w_j}^2 - \frac{g}{k} w_j - (c_0 + c_1 H) w_j - \beta \left[-(\alpha - 1) w_j - \frac{R}{N} \right] \right\} \mathrm{d}t$$

In both cases, the dynamic of water table is the same:

$$\dot{H} = \frac{1}{AS} \left[R + (\alpha - 1) W \right] \quad H(0) = H_0 \tag{17}$$

The results obtained are proposed in the following:

Proposition 2 *The unique stationary open-loop Nash equilibrium for the water table and the rate of extraction for cooperators and defectors is*

$$H^* = -\frac{R}{r c_1 k (\alpha - 1)[m\mu + (N - m)\xi]} - \frac{1}{c_1} \left[\frac{g}{k} + c_0 - \beta(\alpha - 1) \right]$$

where

$$\mu = \left[\frac{AS}{rASN - (\alpha - 1) c_1 m k} \right] \quad \textit{and} \quad \xi = \left[\frac{AS}{rASN - (\alpha - 1) c_1 k} \right]$$

and

$$w_i^* = -\frac{\mu R}{(\alpha - 1)[m\mu + (N - m)\xi]}$$
$$w_j^* = -\frac{\xi R}{(\alpha - 1)[m\mu + (N - m)\xi]}$$

Proof See “Appendix”. □

Now, we consider the system composed by the adjoint equations and the dynamic of the water table

$$\begin{cases} \dot{\lambda_i} = r\lambda_i + c_1 m w_i \\ \dot{\lambda_j} = r\lambda_j + c_1 w_j \\ \dot{H} = \frac{1}{AS}\left[R + (\alpha-1)\sum_{i=1}^{m} w_1 + (\alpha-1)\sum_{j=m+1}^{N} w_j\right] \end{cases}$$

Substituting (28) and (29), we obtain the following system

$$\begin{cases} \dot{\lambda_i} = q\lambda_i + lH + L \\ \dot{\lambda_j} = p\lambda_j + tH + Q \\ \dot{H} = -s\lambda_i - v\lambda_j + Hz + M \end{cases}$$

where

$$q = r - \frac{c_1 mk(\alpha-1)}{NAS} > 0, \quad p = r - \frac{c_1 k(\alpha-1)}{NAS} > 0, \quad l = \frac{{c_1}^2 mk}{N} < 0$$
$$L = \frac{c_1 mk}{N}\left[\frac{g}{k} + c_0 - \beta(\alpha-1)\right], \quad s = \frac{mk(\alpha-1)^2}{N\{AS\}^2} < 0, \quad t = \frac{{c_1}^2 k}{N} < 0$$
$$Q = \frac{c_1 k}{N}\left[\frac{g}{k} + c_0 - \beta(\alpha-1)\right], \quad v = \frac{(N-m)k(\alpha-1)^2}{N\{AS\}^2} < 0,$$
$$z = \frac{c_1 k(\alpha-1)}{AS} < 0$$
$$M = \frac{1}{AS}\left[R + k(\alpha-1)\left(\frac{g}{k} + c_0 - \beta(\alpha-1)\right)\right]$$

The equilibrium point of the system is

$$\bar{H} = -\frac{Mpq + sLp + vQq}{sl + vt + zpq}$$
$$\bar{\lambda_i} = \frac{l(Mpq + sLp + vQq)}{q(sl + vt + zpq)} - \frac{L}{q}, \quad \bar{\lambda_j} = \frac{t(Mpq + sLp + vQq)}{p(sl + vt + zpq)} - \frac{Q}{p}$$

which is the same obtained in Proposition 2.

The stability properties of the stationary state $(\bar{H}, \bar{\lambda}, \bar{\lambda})$ can be analyzed evaluating the following associated Jacobian matrix:

$$J(H, \lambda_i, \lambda_j) = \begin{bmatrix} q & 0 & l \\ 0 & p & t \\ -s & -v & z \end{bmatrix}$$

We have that:

$$Det\ J(\bar{H}, \bar{\lambda_i}, \bar{\lambda_j}) = c_1 k(\alpha - 1)[c_1 km(m - N - 1) + N^2 rAS)] < 0$$

instead

$$\begin{aligned} Tr\ J(\bar{H}, \bar{\lambda_i}, \bar{\lambda_j}) &= 2NrAS + c_1 k(\alpha - 1)(n - m - 1) < 0 \\ &\iff m > \frac{2NrAS + c_1 k(\alpha - 1)(N - 1)}{c_1 k(\alpha - 1)} \end{aligned}$$

These results imply that the steady state of the system is a saddle point and that there exists an optimal path which leads to it.

Now, we want to compare the stationary equilibrium of the open-loop case with the value obtained in the myopic case. We value the difference of the aquifer heights

$$H^* = -\frac{R}{rc_1 k(\alpha - 1)[m\mu + (N - m)\xi]} - \frac{1}{c_1}\left[\frac{g}{k} + c_0 - \beta(\alpha - 1)\right]$$

and

$$H^*_{MY} = -\frac{R}{c_1 k(\alpha - 1)} - \frac{g}{c_1 k} - \frac{c_0}{c_1} + \frac{\beta(\alpha - 1)}{c_1}$$

So, we have

$$H^* - H^*_{MY} = \frac{-R[c_1 mk(\alpha - 1) - rAS(m^2 - m + N)]}{(rASN - (\alpha - 1)c_1 mk)(rASN - (\alpha - 1)c_1 k)}$$

We observe that this difference depends on the number of agents that exploit the resource and on the number of cooperators. This means that the open-loop Nash equilibrium is more efficient with respect to the equilibrium obtained in the myopic case.

4.1 The Pareto optimum

The goal of this section is to estimate the inefficiency of equilibria when cooperators and defectors coexist, compared with the efficient solution of the problem represented by the Pareto optimum. We analyze the difference between the Pareto optimal solution and any other solution computed, which is defined inefficiency, in terms of stock. For the Pareto optimal solution, we consider that only the grand coalition exists and so $m = N$. This problem captures the socially optimal exploitation of groundwater and in this case the level of the stock at the steady state is

$$H^*_{PO} = -\frac{R}{rc_1 k(\alpha - 1)N\mu} - \frac{1}{c_1}\left[\frac{g}{k} + c_0 - \beta(\alpha - 1)\right]$$

For the private exploitation of groundwater, we suppose that there are no coalitions, so $m = 0$. In this case, the level of the stock at the steady state is

$$H^*_{NC} = -\frac{R}{rc_1k(\alpha-1)N\xi} - \frac{1}{c_1}\left[\frac{g}{k} + c_0 - \beta(\alpha-1)\right]$$

This result coincides with that proposed by Rubio and Casino (2001). If we consider how the level of stock varies with m, we have the following:

Proposition 3 *When the number of cooperators m increases (respectively, decreases) the level of the height of the aquifer, at the steady state, increases (decreases) both in the open-loop case and in the Pareto optimum one.*

Proof See "Appendix". □

Now, we compare analytically the efficiency of the different level of stock solutions, at the steady state. In order to define the inefficiency in terms of stock, we compute the difference between steady-state stock levels obtained from open-loop Nash equilibrium and the Pareto optimal solution. The difference is

$$H^*_{PO} - H^* = \frac{R}{rc_1k(\alpha-1)}\left\{\frac{(N-m)(\mu-\xi)}{N\mu[m\mu+(N-m)\xi]}\right\} \tag{18}$$

where

$$\frac{R}{rc_1k(\alpha-1)} < 0,\ N\mu = \left[\frac{AS}{rASN-(\alpha-1)c_1mk}\right] > 0,$$
$$[m\mu+(N-m)\xi] = \frac{(NAS)^2r-(\alpha-1)c_1km(N-m-1)}{[rASN-(\alpha-1)c_1mk][rASN-(\alpha-1)c_1k]} > 0$$

and

$$\mu - \xi = \frac{AS(\alpha-1)c_1k(m-1)}{[rASN-(\alpha-1)c_1mk][rASN-(\alpha-1)c_1k]} < 0$$

and so the difference (18) is positive. This means that non-cooperative solution is inefficient, for all values of m.

4.2 Comparative statics

In this section, we propose an analysis of the steady state $(H, w_i, w_j) = (H^*, w_i^*, w_j^*)$ when the parameters of the model change. First of all, we are interested in the variation of the equilibrium, in relation to the number of players and in particular to those who cooperate and those that do not participate in the coalition. It is possible to see that, if the number of countries N that exploit the groundwater resource increases, then always the steady state of water table and the rate of extraction, both for cooperators and for defectors, decreases. In fact, we have

$$\frac{\partial H^*}{\partial N} = \frac{-R[c_1kmrAS(1-\alpha)(m^2+2N-1)+c_1^2k^2m^2(1-\alpha)^2+Nr^2(AS)^2(2m^2-2m+N)]}{rAS[N^2rAS+c_1km(N-m+1)(1-\alpha)]^2} < 0$$

$$\frac{\partial w_i^*}{\partial N} = \frac{R[c_1^2k^2m(1-\alpha)^2+rASkc_1m(1-\alpha)(m-1)+2rASNc_1k(1-\alpha)+r^2(AS)^2N^2]}{(\alpha-1)[rASN^2+c_1mk(1-\alpha)+c_1mkN(1-\alpha)-c_1m^2k(1-\alpha)]^2} < 0$$

$$\frac{\partial w_j^*}{\partial N} = \frac{R[c_1^2k^2m^2(1-\alpha)^2+rASkc_1m^2(1-\alpha)+rASNc_1mk(1-\alpha)+r^2(AS)^2N^2]}{(\alpha-1)[rASN^2+c_1mk(1-\alpha)+c_1mkN(1-\alpha)-c_1m^2k(1-\alpha)]^2} < 0$$

It is also possible to prove that the steady state of the water table and the rate of extraction of defectors decreases with the number of signatories

$$\frac{\partial H^*}{\partial m} = \frac{R[c_1k(\alpha-1)-NrAS][m^2c_1k(\alpha-1)-NrAS(2m-1)]}{rAS[c_1km(m-N-1)(\alpha-1)+N^2rAS]^2} > 0$$
$$\frac{\partial w_j^*}{\partial m} = \frac{-Rkc_1(2m-1-N)[rASN(1-2m)+c_1m^2k(\alpha-1)]}{[rASN^2+c_1mk(1-\alpha)+c_1mkN(1-\alpha)-c_1m^2k(1-\alpha)]^2} > 0$$

However, the steady state of the rate of extraction of cooperators can increase or decrease with respect to the number of countries belonging to the agreement. The partial derivative of steady state of extraction of cooperators is

$$\frac{\partial w_i^*}{\partial m} = \frac{-Rkc_1(2m-1-N)[-rASN+c_1k(\alpha-1)]}{[rASN^2+c_1mk(1-\alpha)+c_1mkN(1-\alpha)-c_1m^2k(1-\alpha)]^2}$$

which can be positive or negative.

Table 1 shows the effects on the equilibrium $(H, w_i, w_j) = (H^*, w_i^*, w_j^*)$ of changing the parameters of the model. These effects have been estimated from the partial derivatives of the equilibrium coordinates H^*, w_i^* and w_j^*. About the natural recharge, the equilibrium coordinates increase, instead with respect to the area of aquifer AS, the share of H^* decreases while the fitness of w_i^* decreases and w_j^* increases. The equilibrium share of the demand function of cooperators and defectors does not depend on the cost of damages to ecosystems from aquifer depletion β and on the fixed cost linked with the hydrologic cone c_0, instead with respect to these parameters the equilibrium share of H^* goes down or up.

Table 1 The effects on the equilibrium (H^*, w_i^*, w_j^*) of the parameters model

	H^*	w_i^*	w_j^*
$\nearrow R$	$\nearrow$	$\nearrow$	$\nearrow$
$\nearrow \alpha$	$\searrow\nearrow$	$\searrow\nearrow$	$\searrow\nearrow$
$\nearrow k$	$\searrow\nearrow$	$\nearrow$	$\nearrow$
$\nearrow c_1$	$\searrow\nearrow$	$\nearrow$	$\searrow$
$\nearrow c_0$	$\searrow\nearrow$	$\longleftrightarrow$	$\longleftrightarrow$
$\nearrow AS$	$\searrow$	$\nearrow$	$\searrow$
$\nearrow \beta$	$\nearrow$	$\longleftrightarrow$	$\longleftrightarrow$
$\nearrow r$	$\searrow$	$\nearrow$	$\searrow$

5 Feedback Nash equilibrium

In this section, we focus on Nash equilibria in feedback strategies. In the feedback information structure, farmers observe the level of the resource during the planning period, i.e., they consider informations about the state of the water table over time. It is more credible for the farmers to make decisions about their behavior assuming that strategies of the other farmers depend not only on time but on the state of the groundwater resource.

The Hamilton–Jacobi–Bellman equation for cooperators is:

$$rV_i = \max_{w_i} \left[\sum_{i=1}^{m} \left\{ \frac{N}{2k} {w_i}^2 - \frac{g}{k} w_i - (c_0 + c_1 H) w_i - \beta \left[-(\alpha - 1) w_i - \frac{R}{N} \right] \right\} \right. \\ \left. + V_i' \frac{1}{AS} \left(R + (\alpha - 1) \sum_{i=1}^{m} w_i + (\alpha - 1) \sum_{j=m+1}^{N} w_j \right) \right] \tag{19}$$

The Hamilton–Jacobi–Bellman equation for defectors is:

$$rV_j = \max_{w_j} \left[\left\{ \frac{N}{2k} {w_j}^2 - \frac{g}{k} w_j - (c_0 + c_1 H) w_j - \beta \left[-(\alpha - 1) w_j - \frac{R}{N} \right] \right\} \right. \\ \left. + V_j' \frac{1}{AS} \left(R + (\alpha - 1) \sum_{i=1}^{m} w_i + (\alpha - 1) \sum_{j=m+1}^{N} w_j \right) \right] \tag{20}$$

$V_i(H)$ and $V_j(H)$ denote the optimal control value functions associated with the optimization problem assigned, V_i' and V_j' are the first derivatives with respect to the state variable H. The optimal value of the control variables must satisfy the first-order conditions that are:

$$\frac{N}{k}w_i - \frac{g}{k} - c_0 - c_1 H + \beta(\alpha - 1) + V_i' \frac{(\alpha - 1)}{AS} = 0$$
$$\frac{N}{k}w_j - \frac{g}{k} - c_0 - c_1 H + \beta(\alpha - 1) + V_j' \frac{(\alpha - 1)}{AS} = 0 \quad (21)$$

They define the optimal strategies of pumping as functions of the height of aquifer. From (21) we have that:

$$w_i = \frac{k}{N}\left[-V_i' \frac{(\alpha - 1)}{AS} + \frac{g}{k} + c_0 + c_1 H - \beta(\alpha - 1)\right]$$
$$w_j = \frac{k}{N}\left[-V_j' \frac{(\alpha - 1)}{AS} + \frac{g}{k} + c_0 + c_1 H - \beta(\alpha - 1)\right] \quad (22)$$

Substituting these pumping levels in Eqs. (19) and (20), and setting $D = \frac{g}{k} + c_0 - \beta(\alpha - 1)$, we obtain the following nonlinear differential equations:

$$rV_i = (V_i')^2 \left[\frac{-mk(\alpha - 1)^2}{2N(AS)^2}\right] + V_i' \left[\frac{R}{AS} - V_j' \frac{k(N - m)(\alpha - 1)^2}{N(AS)^2} + \frac{k(\alpha - 1)(D + c_1 H)}{AS}\right] - \frac{mk(D + c_1 H)^2}{2N} + \frac{\beta R}{N} \quad (23)$$

$$rV_j = (V_j')^2 \left[\frac{(1 - 2N - 2m)k(\alpha - 1)^2}{2N(AS)^2}\right] + V_j' \left[\frac{R}{AS} - V_i' \frac{km(\alpha - 1)^2}{N(AS)^2} + \frac{k(\alpha - 1)(D + c_1 H)}{AS}\right] - \frac{k(D + c_1 H)^2}{2N} + \frac{\beta R}{N} \quad (24)$$

In order to compute the solutions of the above equations, given the linear quadratic structure of the model, we guess that the optimal value functions are quadratic and consequently the equilibrium strategies are linear with respect to the state variable. Precisely, we postulate quadratic functions of the form

$$V_i = \frac{1}{2}A_i H^2 + B_i H + C_i \quad (25)$$
$$V_j = \frac{1}{2}A_j H^2 + B_j H + C_j \quad (26)$$

where A, B and C are constant parameters of the unknown value functions which are to be determined. Substituting Eqs. (25) and (26) and their derivatives in Eqs. (23) and (24), we obtain a system of six algebraic Riccati equations for the coefficients of the value functions.

$$rA_i = \frac{-mk(\alpha - 1)^2}{N(AS)^2} A_i^2 + \frac{2k(\alpha - 1)c_1}{AS} A_i - \frac{2k(N - m)(\alpha - 1)^2}{N(AS)^2} A_i A_j - \frac{kmc_1^2}{N}$$
$$rB_i = \frac{-mk(\alpha - 1)^2}{N(AS)^2} A_i B_i + \frac{k(\alpha - 1)c_1}{AS} B_i + \left[\frac{R}{AS} + \frac{k(\alpha - 1)D}{AS}\right] A_i$$

$$
\begin{aligned}
&-\frac{k(N-m)(\alpha-1)^2}{N(AS)^2}(A_iB_j+A_jB_i)-\frac{kmDc_1}{N}\\
rC_i &= \frac{-mk(\alpha-1)^2}{2N(AS)^2}B_i^2+\left[\frac{R}{AS}+\frac{k(\alpha-1)D}{AS}\right]B_i-\frac{k(N-m)(\alpha-1)^2}{N(AS)^2}B_iB_j\\
&-\frac{kmD^2}{2N}+\frac{\beta R}{N}\\
rA_j &= \frac{(1-2N-2m)k(\alpha-1)^2}{N(AS)^2}A_i^2+\frac{2k(\alpha-1)c_1}{AS}A_j-\frac{2km(\alpha-1)^2}{N(AS)^2}A_iA_j-\frac{kc_1^2}{N}\\
rB_j &= \frac{(1-2N-2m)k(\alpha-1)^2}{N(AS)^2}A_jB_j+\frac{k(\alpha-1)c_1}{AS}B_j+\left[\frac{R}{AS}+\frac{k(\alpha-1)D}{AS}\right]A_j\\
&-\frac{km(\alpha-1)^2}{N(AS)^2}(A_iB_j+A_jB_i)-\frac{kDc_1}{N}\\
rC_j &= \frac{(1-2N-2m)k(\alpha-1)^2}{2N(AS)^2}B_j^2+\left[\frac{R}{AS}+\frac{k(\alpha-1)D}{AS}\right]B_j-\frac{km(\alpha-1)^2}{N(AS)^2}B_iB_j\\
&-\frac{kD^2}{2N}+\frac{\beta R}{N}
\end{aligned}
\tag{27}
$$

This system does not have an analytical solution. Equation (27) have four pairs of solutions, but only one satisfies the stability condition $\frac{\mathrm{d}\dot{H}}{\mathrm{d}H} < 0$. To obtain this condition, we substitute the linear strategies in the dynamic constraint of the water table which yields the following differential equation

$$
\begin{aligned}
\dot{H} &= \frac{R}{AS}+\frac{km(\alpha-1)}{NAS}\left[-V_i'\frac{(\alpha-1)}{AS}+D+c_1H\right]\\
&+\frac{k(N-m)(\alpha-1)}{NAS}\left[-V_j'\frac{(\alpha-1)}{AS}+D+c_1H\right]
\end{aligned}
$$

and the stability condition

$$
\frac{(\alpha-1)k}{NAS}\left\{\frac{-m(\alpha-1)}{AS}A_i-\frac{(N-m)(\alpha-1)}{AS}A_j+Nc_1\right\}<0
$$

Using this inequality, it is possible, in the following numerical application, to be able to select the value functions and to analyze the stability of coalitions for the feedback Nash equilibrium.

5.1 Numerical analysis

We consider numerical simulations in order to compare myopic, open-loop and feedback extraction strategies and to show, in particular, how cooperation and defection among farmers influence these results. Simulations are carried out in Maple 17 and the parameter values are the same presented by Gisser and Sanchez (1980). We have that the number of countries is $N = 500$, while the number of cooperators is not fixed,

$0 \leq m \leq 500$. The initial water table elevation is $H_0 = 3400$ ft above sea level, the natural recharge is $R = 173{,}000$ ac ft/yr, the aquifer area is $AS = 135{,}000$ ac/yr, the return flow coefficient is $\alpha = 0.27$, the intercept pumping cost function is $c_0 = 125$ dollars/ac ft, while the slope pumping cost function is $c_1 = -0.035$ dollars/ac ft/ft of lift, the intercept water demand function is equal to $g = 470{,}375$ ac ft/yr, the slope water demand function is $k = -3259$ ac ft/yr, the cost of damages is $\beta = 0, 05$ dollars/ac ft and the constant rate is $r = 0.05$.

We obtain the following results

m	H^*_{MY}	H^*_{OL}	H^*_{FB}
0	1526.362	1526.420	1526.419
50	1526.362	1526.670	1526.667
100	1526.362	1527.432	1527.422
150	1526.362	1528.705	1527.685
200	1526.362	1530.488	1530.457
250	1526.362	1532.782	1531.911
300	1526.362	1535.588	1534.543
350	1526.362	1538.910	1537.863
400	1526.362	1542.749	1542.709
450	1526.362	1547.110	1546.085
500	1526.362	1551.998	1551.998

The numerical simulation shows that both the stationary open-loop Nash equilibrium and the stationary feedback Nash equilibrium are influenced by the numbers of cooperators and they increase if m increases. Instead, the myopic Nash equilibrium is independent both of the number of countries that join a coalition and also of N. We observe that the stationary open-loop Nash equilibrium is higher than the stationary feedback Nash equilibrium also if the difference is very small. Moreover, the value of the myopic Nash equilibrium corresponds to the case in which all countries are defectors, so we have the following relationship among the stationary values of the water table

$$H^*_{\mathrm{MY}} < H^*_{\mathrm{FB}} < H^*$$

This inequality confirms the effect of strategic externality, but also the inefficiency of the private solution which is represented by the myopic equilibrium, if compared with the socially optimal exploitation that corresponds to the open-loop and feedback Nash equilibria when $N = m$.

6 Concluding remarks

In this paper, we have extended the literature quoted, examining the formation of agreements in a static and dynamic context of a common groundwater resource exploitation. We have developed the model proposed by Rubio and Casino (2001) taking also ecosystem damages in consideration and we have supposed that countries can cooperate or defect. In the literature, only the socially optimal exploitation or the private exploitation of the aquifer are proposed. These strategies correspond to cases of full cooperation or total defection. The contribution we have added is to analyze the possibility that countries decide to form coalitions to preserve water resources or to defect, causing greater exploitation of aquifer. So, our aim is to consider N countries that differ in terms of choice of their behavior. We have shown as in the static context, the myopic stationary equilibrium does not affect by the number of cooperators and defectors. Instead, in the dynamic game, the number of cooperators and non-cooperators affect the steady states of the water table and the rate of extraction for cooperators and defectors. In particular, we have illustrated the implications of the different strategies, open-loop and feedback solutions, on extraction rates and groundwater table levels showing that in the stationary open-loop Nash equilibrium the level of the water is higher than in the stationary feedback Nash equilibrium, also if this difference is very small.

Moreover, we have confirmed how private solutions are inefficient compared to the Pareto optimum, in terms of stock and we have proposed a comparative statics analysis with respect to all parameters which characterize the model.

Appendix

Proof of Proposition 2 In order to solve the problem proposed, we use the maximum principle. Let us define the current value of the Hamiltonian $\mathcal{H}$ in the standard way.

$$\mathcal{H}_i = \sum_{i=1}^{m} \left\{ \frac{N}{2k} {w_i}^2 - \frac{g}{k} w_i - (c_0 + c_1 H) w_i - \beta \left[-(\alpha - 1) w_i - \frac{R}{N} \right] \right\}$$
$$+ \frac{\lambda_i}{AS} \left[R + (\alpha - 1) \sum_{i=1}^{m} w_i + (\alpha - 1) \sum_{j=m+1}^{N} w_j \right] \quad i = 1, \ldots, m$$
$$\mathcal{H}_j = \frac{N}{2k} {w_j}^2 - \frac{g}{k} w_j - (c_0 + c_1 H) w_j - \beta \left[-(\alpha - 1) w_j - \frac{R}{N} \right]$$
$$+ \frac{\lambda_j}{AS} \left[R + (\alpha - 1) \sum_{i=1}^{m} w_i + (\alpha - 1) \sum_{j=m+1}^{N} w_j \right] \quad j = m+1, \ldots, N$$

where λ_i and λ_j are the adjoint variables. We obtain the following set of necessary conditions for an interior open-loop equilibrium:

$$\frac{\partial \mathcal{H}_i}{\partial w_i} = 0 \iff \frac{N}{k} w_i - \frac{g}{k} - c_0 - c_1 H + \beta(\alpha - 1) + \frac{\lambda_i}{AS}(\alpha - 1) = 0 \quad i = 1, \ldots, m$$
$$\frac{\partial \mathcal{H}_j}{\partial w_j} = 0 \iff \frac{N}{k} w_j - \frac{g}{k} - c_0 - c_1 H + \beta(\alpha - 1) + \frac{\lambda_j}{AS}(\alpha - 1) = 0$$
$$j = m+1, \ldots, N$$

which give us:

$$w_i = \frac{k}{N}\left[\frac{g}{k} + c_0 + c_1 H - \beta(\alpha - 1) - \frac{\lambda_i}{AS}(\alpha - 1)\right] \quad i = 1, \ldots, m \tag{28}$$

$$w_j = \frac{k}{N}\left[\frac{g}{k} + c_0 + c_1 H - \beta(\alpha - 1) - \frac{\lambda_j}{AS}(\alpha - 1)\right] \quad j = m+1, \ldots, N \tag{29}$$

The adjoint equations are:

$$\begin{aligned} \dot{\lambda_i} &= r\lambda_i + c_1 m w_i \quad i = 1, \ldots, m \\ \dot{\lambda_j} &= r\lambda_j + c_1 w_j \quad j = m+1, \ldots, N \end{aligned}$$

and the transversality conditions being:

$$\begin{aligned} &\lim_{t \longrightarrow +\infty} e^{-rt}\lambda_i(t) \geq 0 \quad \lim_{t \longrightarrow +\infty} e^{-rt}\lambda_i(t)H(t) = 0 \quad i = 1, \ldots, m \\ &\lim_{t \longrightarrow +\infty} e^{-rt}\lambda_j(t) \geq 0 \quad \lim_{t \longrightarrow +\infty} e^{-rt}\lambda_j(t)H(t) = 0 \quad j = m+1, \ldots, N \end{aligned}$$

Taking into account that, at the steady state, $\dot{H} = \dot{\lambda} = 0$, the first-order conditions and the adjoint equations are used to determine the stationary equilibrium

$$\begin{aligned} &\dot{\lambda_i} = 0 \Longleftrightarrow \lambda_i = -c_1 m[g + k(c_0 + c_1 H - \beta(\alpha - 1))]\left[\frac{AS}{rASN - (\alpha - 1)c_1 mk}\right] \\ &i = 1, \ldots, m \\ &\dot{\lambda_j} = 0 \Longleftrightarrow \lambda_j = -c_1[g + k(c_0 + c_1 H - \beta(\alpha - 1))]\left[\frac{AS}{rASN - (\alpha - 1)c_1 k}\right] \\ &j = m+1, \ldots, N \end{aligned}$$

and moreover

$$\dot{\lambda_i} = 0 \Longleftrightarrow w_i = -\frac{r\lambda_i}{mc_1} \quad \text{and} \quad \dot{\lambda_j} = 0 \Longleftrightarrow w_j = -\frac{r\lambda_j}{c_1}$$

From differential equation, we have

$$\dot{H} = 0 \Longleftrightarrow R + m(\alpha - 1)w_i + (N - m)(\alpha - 1)w_j = 0$$

By substitution of λ_i and λ_j, we obtain the steady-state value of water table

$$H^* = -\frac{R}{rc_1 k(\alpha - 1)[m\mu + (N - m)\xi]} - \frac{1}{c_1}\left[\frac{g}{k} + c_0 - \beta(\alpha - 1)\right]$$

where

$$\mu = \left[\frac{AS}{rASN - (\alpha - 1)c_1 mk} \right] \quad \text{and} \quad \xi = \left[\frac{AS}{rASN - (\alpha - 1)c_1 k} \right]$$

and so

$$\lambda_i^* = \frac{\mu c_1 m R}{r(\alpha - 1)[m\mu + (N - m)\xi]}$$
$$\lambda_j^* = \frac{\xi c_1 R}{r(\alpha - 1)[m\mu + (N - m)\xi]}$$

and

$$w_i^* = -\frac{\mu R}{(\alpha - 1)[m\mu + (N - m)\xi]}$$
$$w_j^* = -\frac{\xi R}{(\alpha - 1)[m\mu + (N - m)\xi]}$$

□

Proof of Proposition 3 We have

$$\frac{\partial H^*}{\partial m} = \frac{R[c_1 k(\alpha - 1) - NrAS][m^2 c_1 k(\alpha - 1) - NrAS(2m - 1)]}{rAS[c_1 km(m - N - 1)(\alpha - 1) + N^2 rAS]^2} > 0$$
$$\frac{\partial H_{PO}^*}{\partial m} = \frac{R}{rNAS} > 0$$

□

References

Brown, G.: Renewable natural resource management and use without markets. J. Econ. Lit. **38**(4), 875–914 (2000)

Burke, J.: Groundwater for irrigation: productivity gains and the need to manage hydro-environmental risk. In: Llamas, M.R., Custodio, E. (eds.) Intensive Use of Groundwater: Challenges and Opportunities, Chapter 3, pp. 59–80. Taylor and Francis, London (2003)

Esteban, E., Albiac, J.: Groundwater and ecosystems damages: questioning the Gisser–Sanchez effect. Ecol. Econ. **70**, 2062–2069 (2011)

Gisser, M., Sanchez, D.: Competition versus optimal control in groundwater pumping. Water Resour. Res. **16**, 638–642 (1980)

Jaroslav, V., Annukka, L.: Groundwater resources sustainability indicators. IHP-VI Series on Groundwater 14 (2007)

Koundouri, P.: Current issues in the economics of groundwater resource management. J. Econ. Surv. **18**(5), 703–740 (2004)

Koundouri, P., Xepapadeas, A.: Estimating accounting price for common pool natural resources: a distance function approach. Water Resour. Res. **40**(6), S617 (2004)

Negri, D.: The common property aquifer as a differential game. Water Resour. Res. **25**, 9–15 (1989)

Provencher, B., Burt, O.: The externalities associated with the common property exploitation of groundwater. J. Environ. Econ. Manag. **24**, 139–158 (1993)

Rubio, S.J., Casino, B.: Competitive versus efficient extraction of a common property resource. The groundwater case. J. Econ. Dyn. Control **25**, 1117–11137 (2001)

Rubio, S.J., Casino, B.: Strategic behavior and efficiency in the common property extraction of groundwater. Environ. Resour. Econ. **26**, 73–87 (2003)