Knowl Inf Syst (2013) 34:653–669
DOI 10.1007/s10115-012-0496-7

REGULAR PAPER

A method based on interval-valued fuzzy soft set for multi-attribute group decision-making problems under uncertain environment

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Received: 5 January 2011 / Revised: 7 October 2011 / Accepted: 29 March 2012 /
Published online: 13 April 2012

Abstract In this paper, we develop a new method for multiple attributes group decision-making problems under uncertain environment, in which the information about attribute weights is incompletely known or completely unknown, and each maker's decision information is expressed by an interval-valued fuzzy soft set. Moreover, this paper takes account of the decision makers' attitude toward risk. In order to get the weight vector of the attributes, we construct the score matrix of the final fuzzy soft set. From the score matrix and the given attribute weights information, we establish an optimization model to determine the weights of attributes. For the special situations where the information about attribute weights is completely unknown, we establish another optimization model. By solving this model, we get a simple and exact formula, which can be used to determine the attribute weights. According to these models, a method based on interval-valued fuzzy soft set, which considers the decision makers' risk attitude under uncertain environment, is given to rank the alternatives. Finally, a numerical example is used to illustrate the applicability of the proposed approach.

Keywords Multi-attribute group decision making (MAGDM) · Fuzzy soft set · Interval-valued fuzzy soft set · Incomplete weight information

1 Introduction

The objective of MADM is to find the most desirable alternatives from a set of available alternatives versus the selected criteria [25]. The key information required in a multi-attribute decision model includes attribute values, attribute weights and a mechanism to synthesize this information into an aggregated value or assessment for each alternative [17]. However, an engineering or management decision information is often vague, imprecise and uncertain, by nature. Moreover, due to the increasing complexity of the socioeconomic environment

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and the lack of knowledge or data about the problem domain, a single expert or decision maker often cannot comprehensively consider the whole aspect of decision problem. Therefore, a general trend in the literature is to investigate group decision models with incomplete information.

On the one hand, the decision makers cannot provide deterministic alternative values but fuzzy numbers instead. For example, literature [3,5,8,15,23,27], respectively, does the research about the decision problems with the attribute values as triangular fuzzy number, interval number, fuzzy set, rough set, intuitionistic fuzzy set and vague set etc. However, these theories are associated with an inherent limitation, which is inadequacy of the parameterization tool associated with these theories. For example, the methods of interval mathematics are not sufficiently adaptable for problems with different uncertainties, and they cannot appropriately describe a smooth changing of information, unreliable, not adequate and defective information, partially contradicting aims, and so on. Fuzzy set is progressing rapidly, but there exists a difficulty: how to set the membership function in each particular case [10]. Yet, the soft set that was initiated by Molodtsov [10], as a new mathematical tool, can deal with uncertainties, which is free from the above limitations. In our study, we will first apply the interval-valued fuzzy soft set theory to multi-attribute group decision-making problems.

In recent years, research on soft set theory has become active and great progress has been achieved in the theoretical aspect. At the same time, there has been some progress concerning practical applications of soft set theory, especially the use of soft sets in decision making. Maji et al. [9] introduced the definition of reduct-soft-set and described the application of soft set theory to a decision-making problem. Mushrif et al. [11] proposed a new classification algorithm of the natural textures, which was based on the notions of soft set theory. Zou and Xiao [28] presented data analysis approaches of soft set under incomplete information. Roy and Maji [14] proposed a novel method of object recognition from an imprecise multi-observer data and a decision-making application of fuzzy soft sets. Although the algorithm was proved incorrect by Kong and Gao et al. [7], the fuzzy soft sets and multi-observer concepts are valuable to successive researchers. Feng et al. [2] presented an adjustable approach to fuzzy soft set-based decision making and gave some illustration.

On the other hand, the estimation of the attribute weights plays an important role in multiple attributes decision making. Due to the complexity and uncertainty of the real-world decision-making problems and the inherent subjective nature of human thinking, the information about attribute weights is usually incomplete. Therefore, the decision makers only can give incomplete information about attribute weights. The incomplete attribute weight information has also been extensively investigated from different perspectives. Kim et al. [6] presented an interactive procedure for multiple attribute group decision making with incomplete information and described some theoretical models to establish group's pairwise dominance relations with group's utility ranges by using a separable linear programming technique. Park et al. [12] fused all individual interval-valued intuitionistic fuzzy decision matrices into the collective interval-valued intuitionistic fuzzy decision matrix by using the IIFHG operator and constructed the score matrix of the collective interval-valued intuitionistic fuzzy decision matrix and established an optimization model to determine the weights of attributes. Park [13] had provided characterization of dominance and PO for decision alternatives and discovered the fact that the set of NDA implied the set of potentially optimal acts when weights were incomplete. This was done via developing a zero-one mixed integer program for establishing dominance. Wei [19] established an optimization model based on the basic ideal of traditional technique for order performance by similarity to ideal solution, by which the attribute weights could be determined. Wei et al. [18] established an optimization model based on the negative ideal solution and max-min operator, by which the attribute weights could be determined. Xu and Chen [21] had developed an interactive method to solve fuzzy multiple group decision-making problems. The method transformed the fuzzy decision matrices into their expected decision matrices and constructed the normalized expected decision matrices. In our article, we will first study interval-valued fuzzy soft set decision-making problem with incomplete attribute weight information. To determine the attribute weights, we have constructed the score matrix of the collective interval-valued fuzzy soft matrix. From the score matrix and the given attribute weights information, we establish an optimization model to determine the weights of attributes. Especially, for the situation where the information about the attribute weights is completely unknown, we have provided a simple and exact formula for obtaining the attribute weights.

In this paper, we shall focus on MAGMD problems considering the decision makers' risk attitude under uncertain environment, in which attribute values' information is expressed as interval-valued fuzzy soft set and incomplete attribute weights are identified as a set of linear constrains that may take any form as those in [6,21].

The remainder of this paper is organized as follows: in Sect. 2, we review some basic concepts of soft set, fuzzy soft set, interval-valued fuzzy soft set, similarity between fuzzy sets and point out the difference between fuzzy set and fuzzy soft set. Section 3 develops a novel approach to MAGDM problems considering the decision makers' attitude toward risk under uncertain environment, in which attribute values are expressed as interval-valued fuzzy soft set and incomplete attribute weights are identified as a set of linear constrains. For the special situation where the information about attribute weights is completely unknown, we established another optimization model. By solving this model, we get a simple and exact formula, which can be used to determine the attribute weights. Section 4 gives an illustrative example. Finally, conclusions are discussed in Sect. 5.

2 Preliminaries

2.1 Soft set and fuzzy soft set

Definition 1 (see [10]): A pair (F, E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U.

In other words, the soft set is a parameterized family of subsets of the set U. Every set $F(\varepsilon), \varepsilon \in E$, from this family may be considered as the set of ε-*elements* of the soft set (F, E), or as the set of ε—*approximate* elements of the soft set.

Example 1 A soft set (F, E) describes the attractiveness of the houses which Mr. X is going to buy. $U = \{h_1, h_2, h_3, h_4\}$ is the set of houses under consideration. $E = \{e_1, e_2, e_3, e_4\}$ is the set of parameters. Each parameter is a word or a sentence, which stand for the parameters "expensive," "beautiful," "in the green surroundings" and "in good repair."

Consider the mapping F be a mapping of E into the set of all subsets of the set U. Now consider a soft set (F, E) that describes the "attractiveness of houses for purchase". According to the data collected, the soft set (F, E) is given by

$$(F, E) = \{(e_1, \{h_1, h_3, h_4\}), (e_2, \{h_1, h_2\}), (e_3, \{h_1, h_3\}), (e_4, \{h_2, h_3, h_4\})\},$$

where $F(e_1) = \{h_1, h_3, h_4\}, F(e_2) = \{h_1, h_2\}, F(e_3) = \{h_1, h_3\}$ and $F(e_4) = \{h_2, h_3, h_4\}$. In order to store a soft set in computer, a 2-D table is used to represent the

Table 1 The tabular representation of (F, E)

U	e_1	e_2	e_3	e_4
h_1	1	1	1	0
h_2	0	1	0	1
h_3	1	0	1	1
h_4	1	0	0	1

soft set (F, E). Table 1 is the tabular form of the soft set (F, E). If $h_i \in F\left(e_j\right)$, then $h_{ij} = 1$, otherwise $h_{ij} = 0$, where h_{ij} are the entries (see Table 1).

Definition 2 (see [26]): A fuzzy set $\tilde{A}$ in the universe of discourse U can be represented by a membership function μ_A shown as follows: $\mu_A : U \to [0, 1]$, where μ_A denotes the degree of membership of x belonging to the fuzzy set $\tilde{A}$ and $\mu_A \in [0, 1]$.

Let us consider the family of $\alpha-level$ sets for function μ_A, $F(\alpha) = \{x \in U|\mu_A(x) \geq \alpha\}$, $\alpha \in [0, 1]$.

If we know the family F, we can find the functions $\mu_A(x)$ by means of the following formula:

$$\mu_A(x) = \sup_{\substack{\alpha\in[0,1] \\ x\in F(\alpha)}} \alpha.$$

Thus, every Zadeh's fuzzy set A may be considered as the soft set $(F, [0, 1])$. In soft set theory, membership is decided by adequate parameters, whereas fuzzy set theory depends upon grade of membership.

Because the classical mathematics model usually is too complicated, we cannot find the exact solution. So we introduce the notion of approximate solution and calculate that solution. While in soft set theory, we have the opposite approach to this problem. The initial description of the object has an approximate nature, and we do not need to introduce the notion of exact solution [10].

Definition 3 (see [14]) Let $P(U)$ be the set of all fuzzy subsets in a universe U. Let E be a set of parameters and $A \subseteq E$. A pair (f, A) is called a fuzzy soft set over U, where f is a mapping given by

$$f : A \to P(U). \tag{1}$$

Similar to the viewing a soft set, a fuzzy soft set (f, A) can be viewed $(f, A) = \left\{a = \left\{u_{f_a(U)}|u \in U\right\} |a \in A\right\}$ where the symbol "$a = \left\{u_{f_a(u)}|u \in U\right\}$" indicates that the membership degree of the element $u \in U$ is $f_a(u)$ where $f_a : U \to [0, 1]$ is the membership function of the fuzzy set $f(a)$ [1]. We can see that fuzzy soft set studies the fuzzy power set of universe.

Example 2 Suppose that there are five cars in the universe $U = \{h_1, h_2, h_3, h_4, h_5\}$ and the set of parameters is given by $E = \{e_1, e_2, e_3, e_4, e_5\}$, where e_i stand for "dynamic," "economy," "brake," "steering stability" and "smooth-going running," respectively. Let $A = \{e_1, e_2, e_3\} \subset E$ be consisting of the parameters that Mr. X is interested in buying a car. Now all the available information on cars under consideration can be formulated as a fuzzy soft set $(\tilde{F}, A)$ describing "attractiveness of cars" that Mr. X is going to buy.

Table 2 Tabular representation of the fuzzy soft set $(\tilde{F}, A)$

U	$e_1 =$ '*dynamic*'	$e_2 =$ '*economy*'	$e_3 =$ '*brake*'
h_1	0.4	1.0	0.5
h_2	0.6	0.6	0.6
h_3	0.5	0.8	0.8
h_4	0.9	0.5	0.2
h_5	0.3	0.9	0.9

$$\begin{aligned}(F, A) = \{e_1 &= \{(h_1, 0.4), (h_2, 0.6), (h_3, 0.5), (h_4, 0.9), (h_5, 0.3)\},\\ e_2 &= \{(h_1, 1.0), (h_2, 0.6), (h_3, 0.8), (h_4, 0.5), (h_5, 0.9)\},\\ e_3 &= \{(h_1, 0.5), (h_2, 0.6), (h_3, 0.8), (h_4, 0.2), (h_5, 0.9)\}\end{aligned}$$

is a fuzzy soft set over U. Table 2 gives the tabular representation of the fuzzy soft set $(\tilde{F}, A)$.

Definition 4 (see [2]) Let $\mathrm{S} = (\tilde{F}, A)$ be a fuzzy soft set over U, where $A \subseteq E$ and E is the parameter set. Let $\lambda : A \to [0, 1]$ be a fuzzy set in A which is called a threshold fuzzy set. The level soft set of the fuzzy soft set S with respect to the fuzzy set λ is a crisp soft set $L(\mathrm{S}; \lambda) = (F_\lambda, A)$ defined by

$$F_\lambda(a) = L(\tilde{F}(a); \lambda(a)) = \left\{x \in U : \tilde{F}(a)(x) \geq \lambda(a)\right\}, \tag{2}$$

For all $a \in A$.

Generally, there are two kinds of attributes: the benefit type and the cost type. The higher the benefit-type value is, the better it will be. While for the cost type, it is opposite. According to the characteristics of the fuzzy soft set, we propose an ideal solution of interval-valued fuzzy soft set as follows:

$$ideal_{(\tilde{F},A)} = \{(e_1, 1), \ldots, (e_i, 1), (e_{i+1}, 0), \ldots (e_n, 0)\}$$

where $e_1, \ldots, e_i$ denote benefit-type indices and $e_{i+1}, \ldots, e_n$ denote cost-type indices.

2.2 Interval-valued fuzzy soft set

Definition 5 (see [22]) Let U be an initial universe and E be a set of parameters, a pair $(\tilde{F}, E)$ is called an interval-valued fuzzy soft set over $\tilde{P}(U)$, where $\tilde{F}$ is a mapping given by

$$\tilde{F} : E \to \tilde{P}(U). \tag{3}$$

$\forall e \in E$, $\tilde{F}(e)$ is referred as the interval fuzzy value set of parameter e, it is actually an interval-valued fuzzy set of U, where $x \in U$ and $e \in E$, it can be written as: $\tilde{F}(e) = \left\{< x, \mu_{\tilde{F}(e)}(x) >: x \in U\right\}$, here $\tilde{F}(e)$ is the interval-valued fuzzy degree of membership that object x holds on parameter e. If $\forall e \in E, \forall x \in U, \mu^-_{\tilde{F}(e)}(x) = \mu^+_{\tilde{F}(e)}(x)$, then $\tilde{F}(e)$ will degenerate to be a standard fuzzy set and then $(\tilde{F}, E)$ will be degenerated to be a traditional fuzzy soft set.

Example 3 Following Example 2, Table 3 gives the tabular representation of the interval-valued fuzzy soft set $(\tilde{F}, A)$. We can view the interval-valued fuzzy soft set $(\tilde{F}, A)$ as the collection of the following fuzzy approximations:

Table 3 Tabular representation of the interval-valued fuzzy soft set $(\tilde{F}, A)$

U	e_1	e_2	e_3
h_1	[0.7, 0.9]	[0.6, 0.7]	[0.3, 0.5]
h_2	[0.6, 0.8]	[0.8, 1.0]	[0.8, 0.9]
h_3	[0.5, 0.6]	[0.2, 0.4]	[0.5, 0.7]
h_4	[0.6, 0.8]	[0.0, 0.1]	[0.7, 1.0]
h_5	[0.8, 0.9]	[0.1, 0.3]	[0.9, 1.0]

$$\tilde{F}(e_1) = \{< h_1, [0.7, 0.9] >, < h_2, [0.6, 0.8] >, < h_3, [0.5, 0.6] >, < h_4, [0.6, 0.8] >, < h_5, [0.8, 0.9] >\}$$
$$\tilde{F}(e_2) = \{< h_1, [0.6, 0.7] >, < h_2, [0.8, 1.0] >, < h_3, [0.2, 0.4] >, < h_4, [0.0, 0.1] >, < h_5, [0.1, 0.3] >\}$$
$$\tilde{F}(e_3) = \{< h_1, [0.3, 0.5] >, < h_2, [0.8, 0.9] >, < h_3, [0.5, 0.7] >, < h_4, [0.7, 1.0] >, < h_5, [0.9, 1.0] >\}$$

The major difference between interval-valued fuzzy set and interval-valued fuzzy soft set is the same as the difference between fuzzy set and fuzzy soft set. Moreover, the interval-valued fuzzy set operations based on the arithmetic operations with membership functions do not look natural. It may occur that these operations are similar to the addition of weights and lengths [10]. The reason for the difficulties is possibly the inadequacy of the parameterization tool of the theory, while the interval-valued fuzzy soft set is free of the difficulties mentioned above.

2.3 Similarity between fuzzy sets

Definition 6 (see [16]): $R^+ = [0, \infty)$; $X = \{x_1, x_2, \ldots, x_n\}$ is the universal set; $\mathcal{F}(X)$ is the class of all fuzzy sets of X; $\mu_A(x_i) : X \to [0, 1]$ is the membership function of $A \in \mathcal{F}(X)$; $A^c \in \mathcal{F}$ is the complement of $A \in \mathcal{F}$. $S : \mathcal{F}^2 \to R^+$, the similarity between fuzzy sets A and B as follows:

$$S(A, B) = \frac{\sum_{i=1}^{n} [1 - |\mu_A(x_i) - \mu_B(x_i)|]}{n}, \tag{4}$$

3 A model based on interval-valued fuzzy soft set for group decision-making problems considering risk attitude under uncertain environment

In order to get the weight vector of the attributes, we construct the score matrix of the final fuzzy soft set in this section. From the score matrix and the given attribute weights information, we establish an optimization model to determine the weights of attributes. For the special situations where the information about attribute weights is completely unknown, we establish another optimization model. By solving this model, we get a simple and exact formula, which can be used to determine the attribute weights. This method is different from the interval-valued intuitionistic fuzzy method proposed by [12]. There exist two mainly different aspects between interval-valued intuitionistic fuzzy set and interval-valued fuzzy soft set to deal with this problem.

(a) Score function:

Definition 7 (see [12]) Let R=$(r_{ij})_{m\times n}$ be the collective interval-valued intuitionistic fuzzy decision matrix. $r_{ij} =< \left[a_{ij}, b_{ij}\right], \left[c_{ij}, d_{ij}\right] >$ is an IVIFN, $\left[a_{ij}, b_{ij}\right]$ indicates the degree that the alternative for the alternative $O_j \in O$ satisfy the attribute u_i, while $[c_{ij}, d_{ij}]$ indicates the degree that the alternative $O_j \in O$ does not satisfy the attribute u_i. $\left[a_{ij}, b_{ij}\right] \subset [0, 1]$, $\left[c_{ij}, d_{ij}\right] \subset [0, 1]$, $b_{ij} + d_{ij} \leq 1, i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n.$

Then we call $S = (s_{ij})_{m\times n}$ the score matrix of $R = (r_{ij})_{m\times n}$, and s_i is score for each alternative O_i, such that

$$s_i = \sum_{i=1}^{m} s\left(r_{ij}\right) = \sum_{i=1}^{m} \frac{1}{2}\left(a_{ij} - c_{ij} + b_{ij} - d_{ij}\right), \quad i = 1, 2, \ldots, m, j = 1, 2, \ldots n, \quad (5)$$

Definition 8 (see [7]): Let $f_{ij}(i = 1, 2, \ldots n;\ j = 1, 2, \ldots, m)$ be the element of resultant fuzzy soft set. Then we call $C = (c_{ij})_{n\times m}$ the score matrix of the resultant fuzzy soft set and c_i is choice value for each alternative h_i, such that

$$c_{ij} = \sum_{k=1}^{m}\left(f_{ik} - f_{jk}\right),$$
$$c_i = \sum_{j=1}^{m} c_{ij}$$

Based on the choice value formula, we present the overall choice value of each alternative $h_i (i = 1, 2, \ldots, n)$:

$$c_i(w) = \sum_{j=1}^{m} w_j c_{ij}, \quad i = 1, 2, \ldots, n \quad (6)$$

(b) Parameter form:

The interval-valued intuitionistic fuzzy set is inadequacy of the parameterization tool, while the interval-valued fuzzy soft set is adequacy of the parameterization tool. In the interval-valued intuitionistic fuzzy soft set theory, the initial description of the object has an approximate nature. The absence of any restrictions on the approximate description in fuzzy soft set theory makes this theory very convenient and easily applicable in practice. We can use any parameterization we prefer: with the help of words and sentences, real numbers, functions, mappings, and so on. For example, the “environment” is a parameter in interval-valued intuitionistic fuzzy set, while the “in good environment” or “in bad environment” is a parameter in interval-valued fuzzy soft set.

3.1 Problem formulations

Let $U = \{h_1, h_2, \ldots, h_n\}$ be a discrete set of alternatives, consisting of n non-inferior alternatives, and $E = \{e_1, e_2, \ldots, e_m\}$ be the set of attributes. Each alternative is assessed on the m attributes. Let $D = \{d_1, d_2, \ldots, d_k\}$ be the set of k decision makers. The decision problem is to select a most preferred alternative from set U based on the overall assessments of all alternatives on the m attributes. Because the information about the candidates is incompletely known or completely unknown in the partner selection process, the decision maker k cannot easily express a crisp value to candidate h_i with respect to attribute e_j. But the decision maker k can utilize an interval value $a_{ij}^{\tilde{k}}$ to candidate h_i with respect to attribute e_j, where $a_{ij}^{\tilde{k}} = [a_{ij}^{k^L}, a_{ij}^{k^U}]$.

By introducing the risk attitude factor of decision makers, we can transfer an interval value into an exact value. The exact value a_{ij}^{k} is given as [24].

$$a_{ij}^{k} = \overline{a_i^k(j)} + \varepsilon_k \underline{a_i^k(j)} \tag{7}$$

where $\overline{a_i^k}(j)$ represents the middle value of interval value $a_{ij}^{\tilde{k}} = [a_{ij}^{k^L}, a_{ij}^{k^U}]$, and $\overline{a_i^k(j)}$ is computed as $\overline{a_i^k(j)} = \frac{a_{ij}^{k^L} + a_{ij}^{k^U}}{2}$, $\underline{a_i^k(j)}$ represents the width of interval value $a_{ij}^{\tilde{k}}(j) = [a_{ij}^{k^L}, a_{ij}^{k^U}]$, and $\underline{a_i^k(j)} = (a_{ij}^{k^L} - a_{ij}^{k^U})$.

The risk factor ε_k represents the risk attitude of the kth decision maker, and $|\varepsilon_k| \le 0.5$. If the decision maker k is risk averse, then the range of risk factor will be $-0.5 \le \varepsilon_k < 0$. If then decision maker k is risk neutral, then the risk factor $\varepsilon_k = 0$. While the decision maker k is risk preference, then the range of risk factor will be $0 \le \varepsilon_k < 0.5$.

3.2 A model for determining attribute weights

In a multiple attributes decision-making problem, different weights on attributes reflect their varying importance in choosing the optimal alternative. Let $w = (w_1, w_2, \ldots, w_m)^T$ be the attribute weight vector, where $w_j \ge 0$, $j = 1, 2, \ldots, m$ and the weights vector is often normalized to one, that is $\sum_{j=1}^{m} w_j = 1$.

Generally speaking, the incomplete attribute weight information can be expressed as the following forms [6,21]:

(1) A weak ranking: $\{w_{j_1} \ge w_{j_2}\}$, $j_1 \ne j_2$;

(2) A strict ranking: $\{w_{j_1} - w_{j_2} \ge \varepsilon_{j_1 j_2}\}$, $j_1 \ne j_2, \varepsilon_{j_1 j_2} > 0$;

(3) A ranking with multiples: $\{w_{j_1} \ge \alpha_{j_1 j_2} w_{j_2}\}$, $0 \le \alpha_{j_1 j_2} \le 1$, $j_1 \ne j_2$;

(4) A interval form: $\{\beta_j \le w_j \le \beta_j + \varepsilon_j\}$, $0 \le \beta_j < \beta_j + \varepsilon_j \le 1$;

(5) A ranking of differences: $\{w_{j_1} - w_{j_2} \ge w_{j_3} - w_{j_4}\}$, for $j_1 \ne j_2 \ne j_3 \ne j_4$.

Due to the complexity and uncertainty of decision situation, the information about attribute weights provided by the decision makers is usually incompletely known. Under Xu [20] and Park [12] inspiration, we present an approach to determine the weight of attributes. In the following, we present an approach to determine the weight of attributes.

$$\text{Minimize: } c_i(w) = \sum_{j=1}^{m} w_j c_{ij}$$

$$\text{Subject to: } w = (w_1, w_2, \ldots, w_m)^T \in H, w_j \ge 0, \quad i = 1, 2, \ldots, m, \sum_{j=1}^{m} w_j = 1. \tag{M-1}$$

By solving the (**M-1**) model, we obtain the optimal solution $w^{(i)} = \left(w_1^i, w_2^i, \ldots, w_m^i\right)^T$ corresponding to the alternative h_i. However, in the process of determining the weight vector $w = (w_1, w_2, \ldots, w_m)^T$, we need to consider all the alternatives $h_i (i = 1, 2, \ldots, n)$ as a whole. Thus, we construct weight $W = (w_j^{(i)})_{m \times n}$ of the optimal solutions $w^{(i)} = \left(w_1^{(i)}, w_2^{(i)}, \ldots, w_m^{(i)}\right)^T$ $(i = 1, 2, \ldots, n)$ as:

$$W=\begin{pmatrix} w_1^{(1)} & w_1^{(2)} & \cdots & w_1^{(n)} \\ w_2^{(1)} & w_2^{(2)} & \cdots & w_2^{(n)} \\ \vdots & \vdots & \ddots & \vdots \\ w_m^{(1)} & w_m^{(2)} & \cdots & w_m^{(n)} \end{pmatrix}$$

and we calculate the normalized eigenvector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ of the matrix $(CW)^T(CW)$. Then we can construct a combined weight vector as follows:

$$\begin{aligned} w = W\omega &= \begin{pmatrix} w_1^{(1)} & w_1^{(2)} & \cdots & w_1^{(n)} \\ w_2^{(1)} & w_2^{(2)} & \cdots & w_2^{(n)} \\ \vdots & \vdots & \vdots & \vdots \\ w_m^{(1)} & w_m^{(2)} & \cdots & w_m^{(n)} \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \end{pmatrix} \\ &= \omega_1 w^{(1)} + \omega_2 w^{(2)} + \cdots + \omega_n w^{(n)} \end{aligned} \tag{8}$$

and thus we derive the weight vector $w = (w_1, w_2, \ldots, w_m)^T$ of the attribute $e_j (j = 1, 2, \ldots, m)$.

If the information about attribute weights is completely unknown, we can establish another programming model:

$$\begin{aligned} &\text{Minimize } c(w) = (c_1(w), c_2(w), \ldots, c_n(w)) \\ &\text{Subject to: } \sum_{j=1}^{m} w_j = 1,\, w_j \geq 0, \quad j = 1, 2, \ldots, m \end{aligned} \tag{\textbf{M-2}}$$

where $c_i(w) = \sum_{j=1}^{m} w_j^3 c_{ij}^3$

By linear equal weighted summation method [4,19], the model (**M-2**) can be transformed into a single-objective programming model:

$$\begin{aligned} &\text{Minimized } c(w) = \sum_{i=1}^{n} c_i(w) \\ &\text{Subject to: } \sum_{j=1}^{m} w_j = 1,\, w_j \geq 0, \quad j = 1, 2, \ldots, m \end{aligned} \tag{\textbf{M-3}}$$

To solve this model, we construct the Lagrange function:

$$L(w, \lambda) = c(w) + 3\lambda \left(\sum_{j=1}^{m} w_j - 1 \right) \tag{9}$$

where λ is the Lagrange multiplier.

Differentiating Eq. (9) with respect to $w_j (j = 1, 2, \ldots, m)$ and λ, and setting these partial derivatives equal to zero, the following set of equations is obtained:

$$\begin{cases} \frac{\partial L(w,\lambda)}{\partial w_j} = 3\sum_{i=1}^{n} c_{ij}^3 w_j^2 + 3\lambda = 0 \\ \frac{\partial L(w,\lambda)}{\partial \lambda} = \sum_{j=1}^{m} w_j - 1 = 0 \end{cases} \tag{10}$$

Table 4 The interval-valued fuzzy soft set $(\tilde{G}, E)^{(k)}$

U	e_1	e_2	$\dots$	e_m
h_1	$a_{11}^{\tilde{k}}$	$a_{12}^{\tilde{k}}$	$\dots$	$a_{1m}^{\tilde{k}}$
$\dots$	$a_{21}^{\tilde{k}}$	$a_{22}^{\tilde{k}}$	$\dots$	$a_{2m}^{\tilde{k}}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
h_n	$a_{n1}^{\tilde{k}}$	$a_{n2}^{\tilde{k}}$	$\dots$	$a_{nm}^{\tilde{k}}$

By solving Eq. (10), we get a simple and exact formula for determining the attribute weights as follows:

$$w_j^* = \frac{1}{\sum_{j=1}^{m} \frac{1}{\left|\sum_{i=1}^{n} c_{ij}^3\right|^{1/2}}} \Big/ \left|\sum_{i=1}^{n} c_{ij}^3\right|^{1/2}, \quad j = 1, 2, \dots, m \tag{11}$$

which can be used as the weight vector of attributes. Obviously, $w_j^* \geq 0$, for all j.

3.3 Decision algorithm based on risk factor in uncertain environment

Based on above model, we develop a new and practical method considering the risk attitude of decision makers for solving multiple attribute group decision-making problems. The method involves the following steps:

Step 1 For a group decision-making problem, let $U = \{h_1, h_2, \dots, h_n\}$ be a finite set of alternatives, and $D = \{d_1, d_2, \dots, d_k\}$ be the set of decision makers. The decision makers $d_k (k = 1, 2, \dots, k)$ provide their interval-valued fuzzy preferences for each pair of alternatives and construct the interval-valued fuzzy soft sets. The interval-valued fuzzy soft set given by kth decision maker is shown in Table 4: where $a_{ij}^{\tilde{k}} = \left[a_{ij}^{k^L}, a_{ij}^{k^U}\right]$.

Step 2 Transfer an interval-valued fuzzy soft set into a fuzzy soft set for each decision maker by using the Eq. (7).

Step 3 Calculate the similarity coefficients R_{ij} of fuzzy soft sets and form similar matrix R.

According to Definition 6, calculate the similarity between the pth decision maker and qth decision maker as follows:

$$R_{pq} = R\left(M^p, M^q\right) = \frac{1}{m} \sum_{i=1}^{m} R\left(M_i^p, M_i^q\right) \tag{12}$$

Thus, we can get the preference accordance matrix of all decision makers:

$$R = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1n} \\ R_{21} & R_{22} & \cdots & R_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ R_{n1} & R_{n2} & \cdots & R_{nn} \end{bmatrix}$$

where R_{ij} denotes similarity degree between the two fuzzy soft sets. Let b_i denote the row vector sum of similar matrix R, which reflects the deviation of the comments between

ith expert and expert group (including himself). The smaller b_i is, then the higher is the deviation degree between ith decision maker and group.

$$b_i = \sum_{j=1}^{n} R_{ij} \tag{13}$$

According to the proportion of 15 to 30 %, eliminate an expert from experts group, namely the least row vector sum of similar coefficient matrix, and calculate the arithmetic average for the rest. So we can obtain the final evaluate fuzzy soft set $(\tilde{G}, E)$.

Step 4 Calculate the deviation d_{ij} of corresponding elements between final evaluate fuzzy soft set $(\tilde{G}, E)$ and ideal solution $ideal_{(\tilde{G},E)}$ of $(\tilde{G}, E)$ and generate the fuzzy soft set $(\tilde{D}, E)$.

Step 5 Calculate the score matrix C of the fuzzy soft set $(\tilde{D}, E)$. If the information about the attribute weights is partly known, then utilize the model (**M-1**) to obtain the optimal weight vectors $w^{(i)} = \left(w_1^{(i)}, w_2^{(i)}, \ldots, w_m^{(i)}\right)^T$ $(i = 1, 2, \ldots, n)$ corresponding to the alternative $h_i (i = 1, 2, \ldots, n)$, and then construct the weight matrix W. Calculate the normalized eigenvector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ of the matrix $(CW)^T(CW)$ and utilize Eq. (8) to derive the weight vector $w = (w_1, w_2, \ldots, w_m)^T$. If the information about the attribute weights is completely unknown, then we solve the Eq. (11) to determine the attribute weights.

Step 6 Construct the resultant weighted fuzzy soft set $(\tilde{D}, (wE))$ according to fuzzy soft set $(\tilde{D}, E)$.

Step 7 According to the Eq. (6), we can get the relative score of h_i. Then decision is h_k, if $c_k(w) = \min c_i(w)$.

4 Numerical example

In order to demonstrate application of the new method for group decision-making problem, let us suppose there is a core enterprise which wants to select a partner for a subject. The partner selection decision is made on the basis of five main attributes including $e_1 = Cheap, e_2 = DelayDelivery, e_3 = GoodReputation, e_4 = LowRisk$ and $e_5 = GoodQuality$.

There are four partners have been identified as candidates, and four decision makers are responsible for the partner selection problem. The objective here is to select a partner, which can satisfy all attributes in the best way.

Step 1 The evaluation data and the risk attitude of each decision maker are given in Table 5.

Step 2 By introducing the risk attitude factor of decision makers, we can transfer the interval-valued fuzzy soft set into a fuzzy soft set, which is given in Table 6.

Step 3 Calculate the similarity coefficients R_{ij} of fuzzy soft sets and form the similar matrix R.

$$R = \begin{pmatrix} 1 & 0.9745 & 0.9476 & 0.9584 \\ 0.9745 & 1 & 0.9621 & 0.9659 \\ 0.9476 & 0.9621 & 1 & 0.9767 \\ 0.9584 & 0.9659 & 0.9767 & 1 \end{pmatrix}$$

Table 5 The decision interval-valued fuzzy soft sets for each decision maker

Decision maker	U	e_1	e_2	e_3	e_4	e_5
DM#1 $\varepsilon_1 = -0.35$	h_1	[0.48, 0.69]	[0.32, 0.46]	[0.80, 0.84]	[0.02, 0.05]	[0.94, 0.96]
	h_2	[0.38, 0.62]	[0.34, 0.38]	[0.84, 0.85]	[0.06, 0.08]	[0.95, 0.97]
	h_3	[0.44, 0.57]	[0.38, 0.43]	[0.87, 0.89]	[0.09, 0.12]	[0.95, 0.97]
	h_4	[0.36, 0.49]	[0.48, 0.54]	[0.91, 0.93]	[0.08, 0.11]	[0.98, 1.00]
DM#2 $\varepsilon_2 = 0.00$	h_1	[0.41, 0.72]	[0.33, 0.37]	[0.79, 0.83]	[0.05, 0.07]	[0.95, 0.98]
	h_2	[0.45, 0.59]	[0.36, 0.44]	[0.82, 0.84]	[0.06, 0.08]	[0.96, 0.98]
	h_3	[0.46, 0.65]	[0.36, 0.41]	[0.87, 0.89]	[0.09, 0.11]	[0.97, 0.99]
	h_4	[0.33, 0.43]	[0.41, 0.50]	[0.89, 0.92]	[0.08, 0.10]	[0.98, 1.00]
DM#3 $\varepsilon_3 = 0.25$	h_1	[0.46, 0.66]	[0.33.0.50]	[0.74, 0.78]	[0.03, 0.05]	[0.92, 0.96]
	h_2	[0.43, 0.61]	[0.28, 0.54]	[0.76, 0.80]	[0.04, 0.07]	[0.93, 0.96]
	h_3	[0.39, 0.61]	[0.33, 0.44]	[0.82, 0.85]	[0.08, 0.10]	[0.94, 0.96]
	h_4	[0.35, 0.56]	[0.30, 0.41]	[0.85, 0.88]	[0.06, 0.09]	[0.96, 0.98]
DM#4 $\varepsilon_4 = 0.30$	h_1	[0.50, 0.60]	[0.32, 0.40]	[0.76, 0.80]	[0.04, 0.05]	[0.93, 0.95]
	h_2	[0.47, 0.61]	[0.33, 0.48]	[0.79, 0.82]	[0.04, 0.06]	[0.92, 0.94]
	h_3	[0.39, 0.50]	[0.35, 0.45]	[0.83, 0.86]	[0.08, 0.11]	[0.93, 0.95]
	h_4	[0.41, 0.53]	[0.37, 0.47]	[0.84, 0.88]	[0.09, 0.12]	[0.94, 0.96]

Table 6 A fuzzy soft set by transferring for each DM

Decision maker	U	e_1	e_2	e_3	e_4	e_5
DM#1	h_1	0.5115	0.3410	0.8060	0.0245	0.9430
	h_2	0.4160	0.3460	0.8415	0.0630	0.9530
	h_3	0.4595	0.3875	0.8730	0.0945	0.9530
	h_4	0.3795	0.4890	0.9130	0.0845	0.9830
DM#2	h_1	0.5650	0.3500	0.8100	0.0600	0.9650
	h_2	0.5200	0.4000	0.8300	0.0700	0.9700
	h_3	0.5550	0.3850	0.8800	0.1000	0.9800
	h_4	0.3800	0.4550	0.9050	0.0900	0.9900
DM#3	h_1	0.6100	0.4575	0.7700	0.0450	0.9500
	h_2	0.5650	0.4750	0.7900	0.0625	0.9525
	h_3	0.5550	0.4125	0.8425	0.0950	0.9550
	h_4	0.5075	0.3825	0.8725	0.0825	0.9750
DM#4	h_1	0.5800	0.3840	0.7920	0.0480	0.9460
	h_2	0.5820	0.4500	0.8140	0.0560	0.9360
	h_3	0.4780	0.4300	0.8540	0.1040	0.9460
	h_4	0.5060	0.4500	0.8720	0.1140	0.9560

Then we can obtain

$$b_1 = 3.8805; \quad b_2 = 3.9025; \quad b_3 = 3.8864; \quad b_4 = 3.9010$$

According to the proportion of 15 to 30 %, eliminate an expert from experts group, namely the least row vector sum b_1 of similar coefficient matrix, and calculate the arithme-

Table 7 Tabular representation of the final evaluate fuzzy soft set $(\tilde{G}, E)$

U	e_1	e_2	e_3	e_4	e_5
h_1	0.5850	0.3972	0.7907	0.0510	0.9537
h_2	0.5557	0.4417	0.8113	0.0628	0.9528
h_3	0.5293	0.4092	0.8588	0.0997	0.9603
h_4	0.4645	0.4292	0.8832	0.0955	0.9737

Table 8 Tabular representation of the fuzzy soft set $(\tilde{D}, E)$

U	e_1	e_2	e_3	e_4	e_5
h_1	0.5850	0.3972	0.2093	0.0510	0.0463
h_2	0.5557	0.4417	0.1887	0.0628	0.0472
h_3	0.5293	0.4092	0.1412	0.0997	0.0397
h_4	0.4645	0.4292	0.1168	0.0955	0.0263

tic average for the rest. So we can obtain the final evaluate fuzzy soft set $(\tilde{G}, E)$ (see Table 7).

Step 4: Calculate the deviation d_{ij} of corresponding elements between final evaluate fuzzy soft set $(\tilde{G}, E)$ and ideal solution $ideal_{(\tilde{G},E)} = (0, 0, 1, 0, 1)$ and generate the fuzzy soft set $(\tilde{D}, E)$ (see Table 8).

Step 5: Calculate the score matrix C of the fuzzy soft set $(\tilde{D}, E)$.

$$C = \begin{pmatrix} 0.2055 & -0.0885 & 0.1812 & -0.1050 & 0.0257 \\ 0.0883 & 0.0895 & 0.0988 & -0.0578 & 0.0293 \\ -0.0173 & -0.0405 & -0.0912 & 0.0898 & -0.0007 \\ -0.2765 & 0.0395 & -0.1888 & 0.0730 & -0.0543 \end{pmatrix}$$

Case 1

If the information about attribute weights is incomplete known, assume that the information about attribute weights, given by decision makers, is shown as follows, respectively:

$$\begin{aligned} &d_1 : w_1 \leq 0.3, 0.2 \leq w_3 \leq 0.5; \\ &d_2 : 0.1 \leq w_2 \leq 0.2, w_5 \leq 0.4; \\ &d_3 : w_3 - w_2 \geq w_5 - w_4, w_4 \geq w_1; \\ &d_4 : w_3 - w_1 \leq 0.1, 0.1 \leq w_4 \leq 0.3. \end{aligned}$$

Then the set H of the known information about attribute weights provided by the decision makers is

$$\begin{aligned} H = \{&w_1 \leq 0.3, 0.2 \leq w_3 \leq 0.5, 0.1 \leq w_2 \leq 0.2, w_5 \leq 0.4, \\ &w_3 - w_2 \geq w_5 - w_4, w_4 \geq w_1, w_3 - w_1 \leq 0.1, 0.1 \leq w_4 \leq 0.3\} \end{aligned}$$

Because the first decision maker is eliminated, then the set H of the known information about attribute weights becomes $\{H' = 0.1 \leq w_2 \leq 0.2, w_5 \leq 0.4, w_3 - w_2 \geq w_5 - w_4, w_4 \geq w_1, w_3 - w_1 \leq 0.1, 0.1 \leq w_4 \leq 0.3\}$.

Use the method (**M-1**) to obtain the optimal weight vectors $w^{(j)} = \left(w_1^{(j)}, w_2^{(j)}, w_3^{(j)}, w_4^{(j)}\right)^T$ $(j = 1, 2, 3, 4)$ corresponding to the alternative $h_j (j = 1, 2, 3, 4)$:

Table 9 Tabular representation of the resultant weighted fuzzy soft set $(\tilde{D}, (wE))$

U	$e_1 w_1$	$e_2 w_2$	$e_3 w_3$	$e_4 w_4$	$e_5 w_5$
h_1	0.1219	0.0580	0.0645	0.0134	0.0035
h_2	0.1158	0.0645	0.0582	0.0165	0.0035
h_3	0.1103	0.0598	0.0435	0.0262	0.0030
h_4	0.0968	0.0627	0.0360	0.0251	0.0020

$$\begin{aligned} w^{(1)} &= (0.066667, 0.2, 0.166667, 0.3, 0.266667)^T \\ w^{(2)} &= (0.066667, 0.1, 0.166667, 0.3, 0366667)^T \\ w^{(3)} &= (0.233333, 0.2, 0.333333, 0.233333, 0)^T \\ w^{(4)} &= (0.266667, 0.1, 0.366667, 0.266667, 0)^T \end{aligned}$$

and construct the weight matrix

$$W = \begin{pmatrix} 0.066667 & 0.066667 & 0.233333 & 0.266667 \\ 0.2 & 0.1 & 0.2 & 0.1 \\ 0.166667 & 0.166667 & 0.333333 & 0.366667 \\ 0.3 & 0.3 & 0.233333 & 0.266667 \\ 0.266667 & 0.366667 & 0 & 0 \end{pmatrix}$$

then

$$(CW)^T(CW) = \begin{pmatrix} 0.0021 & 0.0023 & 0.0054 & 0.0059 \\ 0.0023 & 0.0028 & 0.0067 & 0.0076 \\ 0.0054 & 0.0067 & 0.0187 & 0.0213 \\ 0.0059 & 0.0076 & 0.0213 & 0.0246 \end{pmatrix}$$

Calculate the normalized eigenvectors ω of the matrix $(CW)^T(CW)$:

$$\omega = (0.1030, 0.1291, 0.3581, 0.4099)^T$$

Use Eq. (8) and derive the weight vector w:

$$\begin{aligned} w = W\omega &= \begin{pmatrix} 0.0667 & 0.0667 & 0.2333 & 0.2667 \\ 0.2000 & 0.1000 & 0.2000 & 0.1000 \\ 0.1667 & 0.1667 & 0.3333 & 0.3667 \\ 0.3000 & 0.3000 & 0.2333 & 0.2667 \\ 0.2667 & 0.3667 & 0.0000 & 0.0000 \end{pmatrix} \begin{pmatrix} 0.1030 \\ 0.1291 \\ 0.3581 \\ 0.4099 \end{pmatrix} \\ &= (0.2083, 0.1461, 0.3083, 0.2625, 0.0748)^T \end{aligned}$$

Step 6: Construct the resultant weighted fuzzy soft set $(\tilde{D}, (wE))$ according to fuzzy soft set $(\tilde{D}, E)$, which is shown in Table 9.
Step 7: According to Eq. (6), we can obtain

$$c_1(w) = 0.0600; \quad c_2(w) = 0.0488; \quad c_3(w) = -0.0140; \quad c_4(w) = -0.0948.$$

Rank all the alternatives $h_i (i = 1, 2, 3, 4)$ in accordance with the scores $c_i(w) : h_4 \succ h_3 \succ h_2 \succ h_1$, and thus, the most desirable alternative is h_4.

Table 10 Tabular representation of the resultant weighted fuzzy soft set $(\tilde{D}, (wE))$

U	e_1w_1	e_2w_2	e_3w_3	e_4w_4	e_5w_5
h_1	0.0125	0.2059	0.0202	0.0077	0.0099
h_2	0.0119	0.2289	0.0182	0.0094	0.0101
h_3	0.0113	0.2121	0.0136	0.0150	0.0085
h_4	0.0099	0.2225	0.0113	0.0144	0.0056

Case 2

If the information about the attribute weights is completely unknown, we utilize Eq. (11) and get the attribute weights.

$$w^* = [0.0214, 0.5183, 0.0966, 0.1503, 0.2134]$$

Construct the resultant weighted fuzzy soft set $(\tilde{D}, (wE))$ according to fuzzy soft set $(\tilde{D}, E)$, which is shown in Table 10.

According to Eq. (6), we can obtain

$$c_1(w) = -0.0343, \quad c_2(w) = 0.0554, \quad c_3(w) = -0.0168, \quad c_4(w) = -0.0043$$

Then, we rank all the alternatives $h_i (i = 1, 2, 3, 4)$ by using the overall values $c_i(w)(i = 1, 2, 3, 4)$: $h_1 \succ h_3 \succ h_4 \succ h_2$, and thus, the most desirable alternative is h_1.

5 Conclusion

This article puts forward a framework to tackle multi-attribute decision-making problems under uncertain environment using the interval-valued fuzzy soft set. We have investigated MAGDM problems that take account of decision makers' attitude toward risk under uncertain environment. The proposed model aims at handing the situations where the attribute parameter information is expressed by interval-valued fuzzy soft set, and the information about attribute weights is incompletely known or completely unknown.

To determine the attribute weights, an optimization model is established which the attribute weights can be determined. Especially, for the situations where the information about the attribute weights is completely unknown, we establish another optimization model. By solving the model, we get a simple and exact formula, which can be used to determine the attribute weights. According to these models, a method based on interval-valued fuzzy soft set, which considers the decision makers' risk attitude under uncertain environment, is given to rank the alternatives. An illustrative example is developed to demonstrate how to apply the proposed procedure. Numerical example illustrates the benefit of this proposed framework: it is capable for handling incomplete weights. Moreover, this approach is not without limitations as parameterization. In future research, our work will focus on the application of multiple attribute group decision making based on interval-valued fuzzy soft set to other domains.

Acknowledgments Our work is sponsored by National Science Foundation of China (71171209/G011203) and the Ministry of education, humanities social sciences fund plan projects, China (10XJA630010). The authors wish to express their appreciation to the Editors of this journal and the three anonymous reviewers for their constructive comments that significantly improve the quality and presentation of the paper.

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