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An integrated representation of spatial and temporal relationships between evolving regions

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Abstract. The study of relationships between evolving regions within GIS still needs the development of operators that integrate the spatial and temporal dimensions. This paper introduces a new approach that combines topological relationships between regions in 2-dimensional space with temporal relationships between convex intervals in time. Resulting relationships are defined and visually presented within a 3-dimensional space that integrates the geographical space as a 2-dimensional space and the time line as the third dimension. Conceptual neighbourhoods are identified and extended by the concept of semi-transitions and transitions. Such a flexible framework presents the advantage of being derived from accepted relationships in both space and time. Its computational implementation is therefore compatible with current spatial and temporal GIS models.

Key words: GIS, time, space, relationships

1 Introduction

Modelling the spatial and temporal dimensions has been an intense subject of discussion and study for, among others, philosophy, mathematics, geography and cognitive science. In an early example, Platon discussed the philosophical implications of the perception of our environment as a 2-dimensional space in the Parable of the Cavern, and the awareness of the third spatial dimension. At the beginning of the XX Century, Minkowski introduced the concept of a 3-dimensional cone – and the related notion of temporal distance – that denotes the space-time locations an object can reach from a space-time location under the constraint of a maximum speed (Minkowski 1908) (where a space-time location denotes a point in a three-dimensional space whose dimensions are the two spatial dimensions and the temporal dimension). Similarly, Hägerstrand made an explicit use of time as a variable for the representation of human daily activities within a space-time prism (Hägerstrand 1953).

Nowadays GIS represents an important solution for the study and analysis of real world phenomena. One of the most important functional properties of GIS is its capability to support the exploration of spatial relationships. However, there is a need for an explicit integration of the temporal dimension within GIS in order to support the exploration and analysis of spatio-temporal relationships. Over the past years, the development of spatio-temporal GIS models has been an important subject of research (Langran 1992; Frank 1994; Peuquet 1994; Worboys 1994; Böehlen et al. 1999). However, the integration of time within GIS still requires the development of formal models that support spatio-temporal reasoning.

Current algebras are mainly oriented to either the temporal or spatial dimensions. In time, James Allen's algebra defined 13 binary relationships between convex temporal intervals (Allen 1984). In space, several models identify binary topological relationships between regions in 2-dimensional spaces (Pullar and Egenhofer 1988; Egenhofer 1991; Randell et al. 1992; Clementini et al. 1993; Cui et al. 1993). However, no integrated algebra has been proposed, to the best of our knowledge, for a description of relationships between regions located in space and time.

The objective of the research described in this paper is the representation of relationships between regions located in space and time. We propose a model based on a representation of 2-dimensional topological relationships and temporal relationships in a 3-dimensional space. We take advantage of the fact that current topological models within GIS are often oriented to 2-dimensional applications, providing then an open avenue for considering time as the additional third dimension. From the cross-product of topological and temporal relationships, we identify a minimal set of relationships in a so-called 3-dimensional temporal space. Such a model allows the representation and computational study of independent trajectories in space and time. In order to complete the concepts of conceptual neighbourhoods commonly applied in the spatial and temporal dimensions, we introduce the concepts of semi-transition and transition to describe the evolution of relationships between evolving regions.

The remainder of this paper is organised as follows. Section 2 briefly reviews current approaches to the representation of temporal and topological relationships. Section 3 discusses and develops an integrated representation of topological and temporal relationships within a 3-dimensional temporal space. Section 4 introduces a framework for the representation of conceptual neighbourhoods, semi-transitions and transitions in a 3-dimensional temporal space. Section 5 presents a case study that illustrates the potential of our approach. Finally Sect. 6 draws some conclusions and perspectives.

2 Relationships in space and time

This section briefly introduces the temporal and topological relationships used for the development of our model. Let's introduce some basic temporal hypotheses. We assume time to be continuous; T is the set of measured times isomorphic to the set of real numbers, I is the set of time intervals. Let i be a time interval of I, $i = [t_1, t_2]$ where $t_1, t_2 \in T$ and $t_1 < t_2$. Relationships between these temporal intervals are defined using Allen's temporal operators that define mutually exclusive relationships between time intervals {*equals*, *before*, *meets*, *overlaps*, *during*, *starts*, *finishes*} and their converses {*after*, *met*, *overlapped*, *contain*, *started*, *finished*}, respectively (converse does not apply for *equal* which is a symmetric operator) (Allen 1984).

Firstly, we propose to represent these temporal relationships using a square matrix of order 2 where the rows represent the beginning and ending instants of the first interval, respectively, and the columns the beginning and ending instant of the second interval, respectively. The value of the *i*th line and *j*th column of the matrix, denoted Val(*ij*), is defined as follows:

$$\mathrm{Val}(ij) = \begin{cases} 1 & \text{if row}(i) > \text{col}(j) \\ 0 & \text{if row}(i) = \text{col}(j) \\ -1 & \text{if row}(i) < \text{col}(j) \end{cases}$$

where row(*i*) is the input value of the *i*th line (the beginning instant of the first temporal interval if $i = 1$, the ending instant of the first temporal interval if $i = 2$), col(*j*) the input value of the *j*th column (the beginning instant of the second temporal interval if $j = 1$, the ending instant of the second temporal interval if $j = 2$).

Relationships between convex intervals are expressed using a square matrix of order 2 as summarised in Fig. 1. Let us remark that this matrix approach differentiates between the temporal relationships *before* and *after* (this is not the case for other approaches based on an empty versus non-empty relationships of temporal interval boundaries and interiors).

Secondly, we introduce some basic topological relationships in a 2-dimensional space. Let us consider Egenhofer's relationships as a reference for our model {*equal*, *touch*, *in*, *contain*, *cover*, *covered*, *overlap*, *disjoint*} (Egenhofer 1991). Let r_1 and r_2 be two regions in a 2-dimensional space, r° denotes the interior of a region of space $r, \partial r$ the boundary of a region of space r. Topological relationships between r_1 and r_2 are expressed as in Fig. 2 where the matrix expresses empty or non-empty pair wise intersections between the interior and boundaries of r_1 and r_2, respectively. The value of the *i*th line and *j*th column of the matrix, denoted Val(*ij*), is defined as follows:

$$\mathrm{Val}(ij) = \begin{cases} 1 & \text{if row}(i) \cap \text{col}(j) = \emptyset \\ 0 & \text{if row}(i) \cap \text{col}(j) = \emptyset \end{cases}$$

$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ *equal*	$\begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix}$ *before*	$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ *after*	$\begin{pmatrix} -1 & -1 \\ 0 & -1 \end{pmatrix}$ *meets*	$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ *met*
$\begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix}$ *overlaps*	$\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ *overlapped*	$\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}$ *during*	$\begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix}$ *contains*	$\begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix}$ *starts*
	$\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$ *started*	$\begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$ *finishes*	$\begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}$ *finished*	

Fig. 1. Temporal relationships between convex intervals

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$
equal	*touch*	*in*	*contain*
$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$
cover	*covered*	*overlap*	*disjoint*

Fig. 2. Topological relationships between two regions in a 2-dimensional space

where row(i) is the input value of the ith line (r_1° if $i = 1, \partial r_1$ if $i = 2$), col(j) the input value of the jth column (r_2° if $j = 1, \partial r_2$ if $j = 2$).

3 2-dimensional and 3-dimensional temporal spaces

A combined representation of topological and temporal relationships is still an open issue from a formal point of view. Some of these difficulties are linked to the problem of perceiving and representing time in space, and formalising space and time. Spatial and temporal relationships provide an appropriate description of properties that apply in their respective dimensions. However, they do not provide significant support to the understanding of binary relationships between temporal regions. We call a temporal region a region located in time, that is, referenced by a convex interval of time. Firstly, topological relationships between temporal regions are valid only during the intersection of the life spans of these regions. Secondly, temporal relationships between these regions do not provide any information about their proximity in the spatial dimension. Temporal animation or mapping techniques can be used to provide a global sense of these relationships. However, these methods are not suited for the evaluation of local relationships. They are also limited to analysis of small spatio-temporal configurations. Several attempts have also been proposed to identify a taxonomy of changes within GIS (Claramunt and Thériault 1995; Hornsby and Egenhofer 1997; Medak 1999; Frank 2000). These studies differentiate between life and motion of spatial entities. However, they are not oriented to the description of spatio-temporal relationships between independent temporal regions.

In a related work we have introduced a model for relationships between temporal regions (Claramunt and Jiang 2000). This proposal maps – and does not project – 2-dimensional topological relationships towards matching topological relationships in a 1-dimensional space, and integrates the time line as the second spatial dimension (Fig. 3). Let us remark that within such a representation, the spatial relationship dimension is mainly qualitative. Distances between regions are not represented. However this is not a limitation as the spatial distance is a measure available in the spatial dimension. This representation of relationships in a 2-dimensional temporal space integrates some visual similarities with the representation of conventional topological relationships.

In order to improve the graphical presentation of our previous model, we propose a 3-dimensional temporal space that combines the temporal dimension with the two spatial dimensions. Our model aims to represent

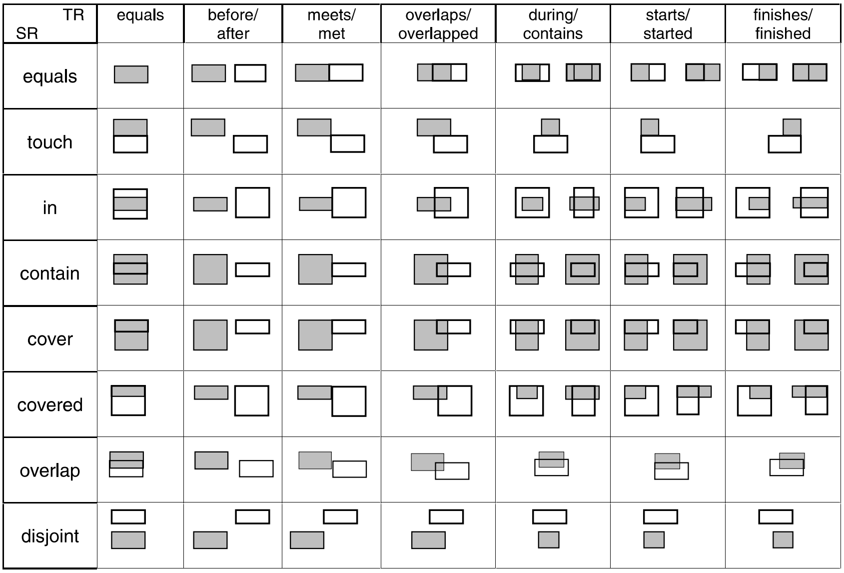

Fig. 3. Visual presentation of relationships in a 2-dimensional temporal space

the minimal combinations of temporal and topological relationships between two temporal regions. A temporal region has previously been defined as a region of space valid for a convex temporal interval. In order to construct a 3-dimensional space, topological relationships are represented in a horizontal plane. The representation of temporal relationships in the vertical axis gives the third dimension. In order to maintain the orientation of time, the temporal dimension is directed. Let us consider two temporal regions located in space and time, denoted by $e_1(r_1, i_1)$ and $e_2(r_2, i_2)$, where r_1 (resp. r_2) represent the region in space where e_1 (resp. e_2) is located and i_1 (resp. i_2) its temporal interval. The spatial extent of such a temporal region can be either static during its temporal interval or a function of time. A temporal region $e_1(r_1, i_1)$ can also mutate to another region $e'_1(r'_1, i'_1)$ with i_1 meets i'_1. We call those regions successive temporal regions (respectively called predecessor and successor). The set of jointly exhaustive and pairwise disjoint relationships, so-called 3-dimensional relationships, between two temporal regions in a 3-dimensional temporal space, is derived from the cross product of topological relationships between r_1 and r_2, and temporal relationships between i_1 and i_2. Figure 4 introduces a graphic presentation of the resulting 3-dimensional relationships that represent the cross product of these topological and temporal relationships. Converse temporal relationships lead to 3-dimensional relationships defined by a rotation of 180 degrees along a horizontal axis.

This representation has the computational advantage of using well-known topological and temporal relationships. This method is suited to the analysis of spatio-temporal relationships between independent temporal regions, and even to the study of spatio-temporal relationships between successive temporal regions (in fact the application of these concepts to successive

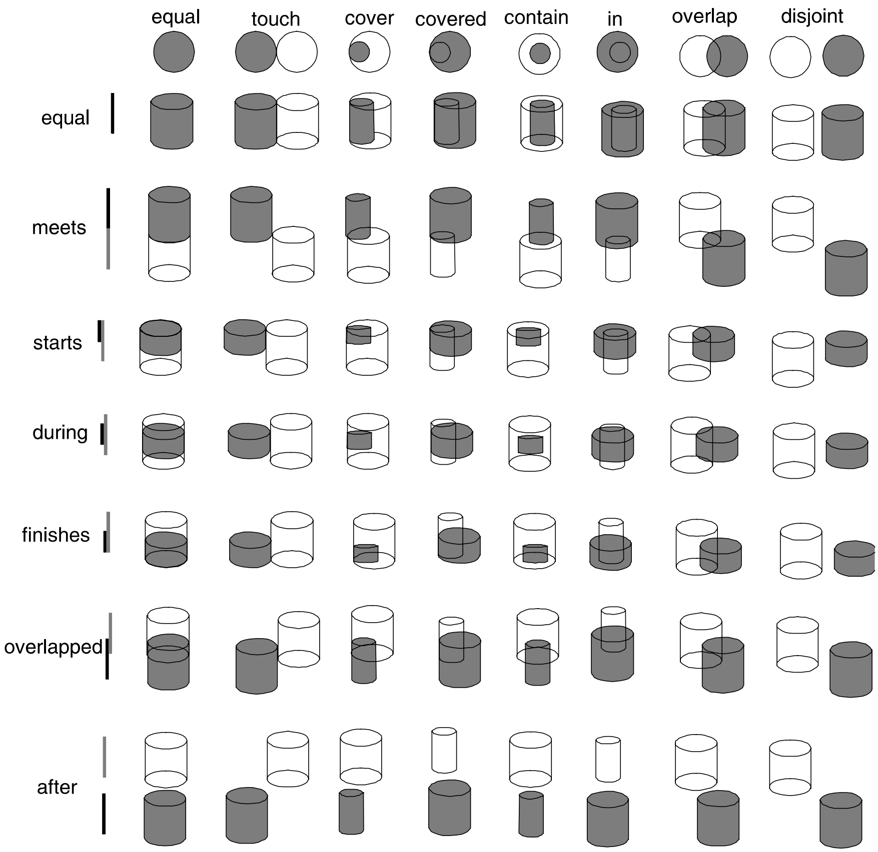

Fig. 4. 3-dimensional relationships between regions in a temporal space

temporal regions restricts possible 3-dimensional relationships to whose defined from the temporal relationships *meet*/*met*).

4 Conceptual neighbourhoods and transitions

4.1 Conceptual neighbourhoods in space and time

Qualitative representations of space and time introduce the notion of conceptual neighbourhoods to identify gradual changes between topological relationships, and to represent similarities between convex temporal intervals (Freksa 1991). This concept has been extended to represent relationships between convex temporal intervals at multiple levels of resolution (Vasilis and Hadzilacos 1999), and binary relationships between cyclic intervals (Hornsby et al. 1999). Conceptual neighbourhoods have also been applied to the spatial dimension in order to identify similarities between topological relationships (Egenhofer 1991; Egenhofer and Al-Taha 1992; Galton 1995; Muller 1998), cardinal relationships (Freksa 1992) approximate distances

(Sharma and Flewelling 1996), spatial configurations (Bruns and Egenhofer 1996), and orientation between line segments (Schlieder 1995).

Topological relationships that are conceptual neighbourhoods can express a gradual transition of relationships between two objects located in space (i.e., two regions). Figure 5 summarises possible transitions of topological relationships (Egenhofer and Al-Taha 1992). For example, if two regions *touch* for a time interval, and both continue to exist after the end of this time interval, then the topological relationships between these two regions for the immediate following time interval are either *touch*, *disjoint* or *overlap*.

Similarly, the gradual transition between two temporal regions can be studied (i.e., two temporal intervals). Continuous transitions of temporal relationships between two temporal regions can be analysed from the study of the possible relationships between the beginning and ending instants of their respective time intervals. Freksa's conceptual neighbourhoods are defined as a direct transition in the represented domain, or in other words, as "a single movement of an interval's start or end point from the other interval's boundary into its interior or exterior, or vice-versa, moving the start or end point from the interior or exterior onto the boundary" (Freksa

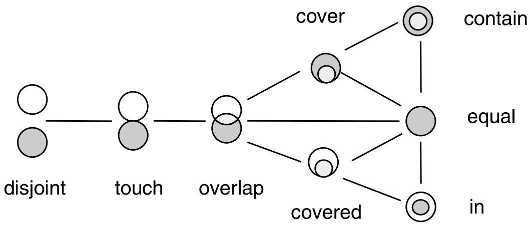

Fig. 5. Conceptual neighbourhoods for topological relationships

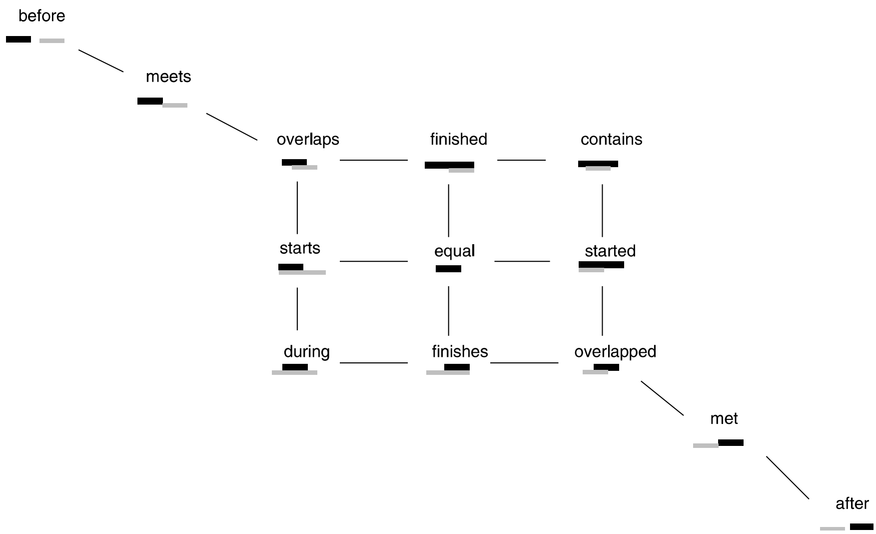

Fig. 6. Conceptual neighbourhoods for temporal relationships

1991). The following figure summarises Freksa's conceptual neighbourhoods.

4.2 Temporal semi-transitions

In order to study the evolution of temporal relationships between a temporal region and two successive temporal regions, we introduce the concept of temporal semi-transition. A temporal semi-transition is defined as a change of temporal relationship between a temporal region and two successive temporal regions. Let us consider a temporal region e_1, e_1 located at (r_1, i_1), and two successive temporal regions e_2 and e_2' respectively located at (r_2, i_2) and (r_2', i_2'), with i_2 *meets* i_2'. Possible temporal semi-transitions between e_1 and (e_2, e_2') are defined as follows:

- If the temporal relationship between the temporal interval i_1 of $e_1 - 1$ and the temporal interval i_2 of e_2 is either *before*, *meets*, *overlap*, starts or *during* then the possible temporal semi-transition between e_1 and (e_2, e_2') is *before* (Fig. 7a).
- If the temporal relationship between the temporal interval i_1 of e_1 and the temporal interval i_2 of e_2 is *after* then possible temporal semi-transitions between e_1 and (e_2, e_2') are either *after*, *met*, *overlapped*, *finished* or *during* (Fig. 7b).
- If the temporal relationship between the temporal interval i_1 of e_1 and the temporal interval i_2 of e_2 is *met* then possible temporal semi-transitions between e_1 and (e_2, e_2') are either *starts*, *equal* or *started* (Fig. 7c).
- If the temporal relationship between the temporal interval i_1 of e_1 and the temporal interval i_2 of e_2 is *overlapped* then possible temporal semi-transitions between e_1 and (e_2, e_2') are either *finished* or *overlap* (Fig. 7d).

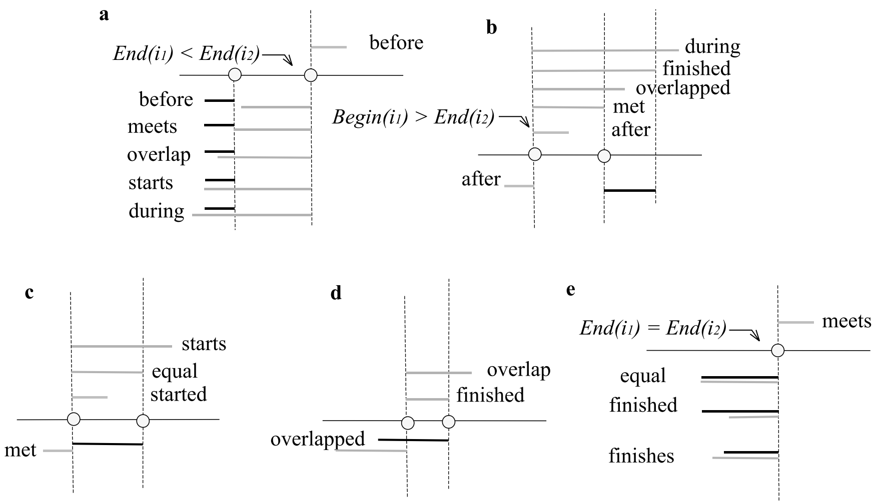

Fig. 7a–e. Temporal semi-transitions

- If the temporal relationship between the temporal interval i_1 of e_1 and the temporal interval i_2 of e_2 is *equal*, *finished* or *finishes* then the possible temporal semi-transition between e_1 and (e_2, e'_2) is *meets* (Fig. 7e).

4.3 Temporal transitions

In order to study the evolution of temporal relationships between two pairs of successive temporal regions, we introduce the concept of temporal transition. A temporal transition is defined as a change of temporal relationship between two temporal regions and their respective successors. Let us consider a first pair of successive temporal regions e_1 and e'_1 respectively located at (r_1, i_1) and (r'_1, i'_1) with i_1 *meets* i'_1; and a second pair of successive temporal regions e_2 and e'_2 respectively located at (r_2, i_2) and (r'_2, i'_2), with i_2 *meets* i'_2. Possible temporal transitions are defined as follows (Fig. 8):

- If the temporal relationship between the temporal interval i_1 of e_1 and the temporal interval i_2 of e_2 is either *equal*, *finishes* or *finished*, then possible temporal transitions between (e_1, e'_1) and (e_2, e'_2) are either *equal*, *starts* or *started* (Fig. 8a).
- If the temporal relationship between the temporal interval i_1 of e_1 and the temporal interval i_2 of e_2 is either *before*, *meets*, *overlap*, *during* or *starts* then possible temporal transitions between (e_1, e'_1) and (e_2, e'_2) are either *before*, *meets*, *overlap*, *contains* or *finished* (Fig. 8b).
- If the temporal relationship between the temporal interval i_1 of e_1 and the temporal interval i_2 of e_2 is either *after*, *met*, *overlapped*, *contains* or *started* then possible temporal transitions between (e_1, e'_1) and (e_2, e'_2) are either *after*, *met*, *overlapped*, *during* or *finishes* (Fig. 8c).

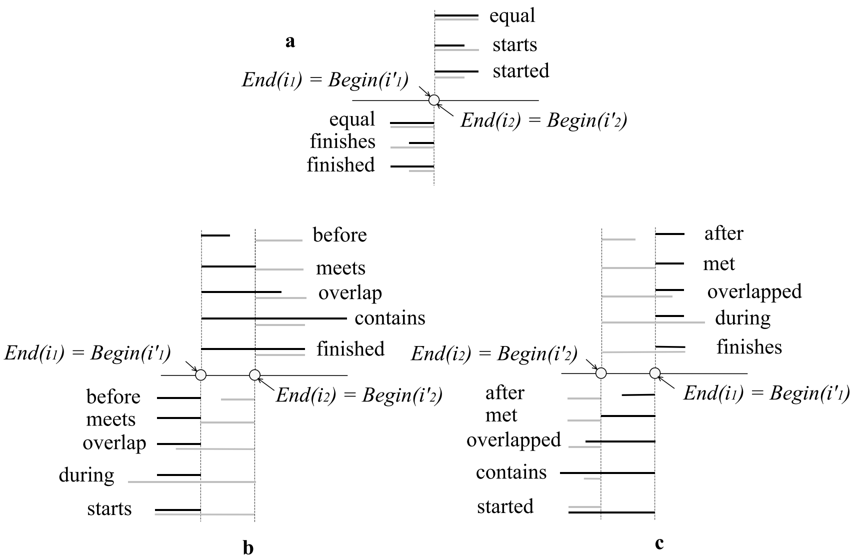

Fig. 8a–c. Temporal transitions

We can remark that temporal transition and temporal semi-transitions are not reflexive and not symmetric relationships.

4.4 Conceptual neighbourhoods for 3-dimensional relationships in a temporal space

We apply the principles of conceptual neighbourhoods to 3-dimensional relationships in a temporal space. Following Freksa's formulation, these conceptual neighbourhoods (also called continuous transitions hereafter) can also be evaluated as a direct transition of the resulting cylinders in the represented 3-dimensional temporal space (Fig. 4). In other words this rule gives "a single movement of either the plane or the cylindrical surface of one cylinder onto the interior of the exterior of the other cylinder". The conceptual neighbourhoods of a given 3-dimensional relationship (*S*, *T*) defined over a topological relationship *S* and a temporal relationship *T* are the 3-dimensional relationships defined by the pairs of topological relationships *S* (resp. the conceptual neighbourhoods of *S*) with the conceptual neighbourhoods of *T* (resp. *T*).

For example, the conceptual neighbourhoods of the 3-dimensional relationship given by the pair of temporal and spatial relationships (*finishes*, *touch*) are the 3-dimensional relationships (*finishes*, *touch*), (*equal*, *touch*), (*during*, *touch*), (*overlap*, *touch*), (*finishes*, *touch*) and (*finishes*, *overlap*) (Fig. 9). The relationship conceptual neighbourhood of 3-dimensional relationships is a reflexive and symmetric relationship.

4.5 Semi-transitions of 3-dimensional relationships in a temporal space

In order to study the evolution of 3-dimensional relationships between a temporal region and two successive temporal regions, we introduce a second rule: there is a semi-transition of 3-dimensional relationships between one temporal region and two successive temporal regions if and only if there is both a continuous transition between their successive topological relation-

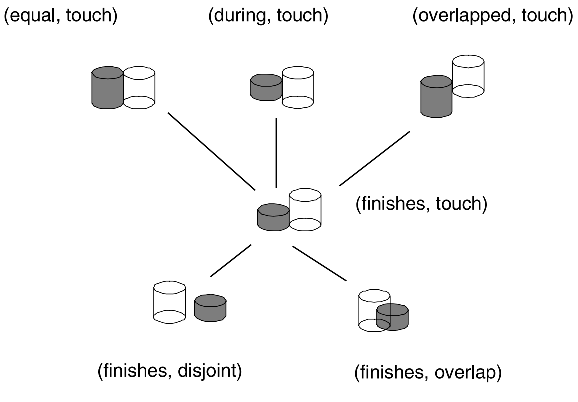

Fig. 9. Conceptual neighbourhoods: example of the 3-dimensional relationship (*finishes*, *touch*)

ships, and a temporal semi-transition between their successive temporal relationships. The possible semi-transitions of 3-dimensional relationships between a temporal region and two successive temporal regions are summarised in Fig. 10. Figure 10 introduces a matrix of matrix that replicates every 3-dimensional relationship identified in Fig. 4. This allows for the identification of the semi-transitions for each 3-dimensional relationship within the large matrix, by "highlighting" corresponding 3-dimensional relationships in each small matrix. For example, the possible semi-transitions of the 3-dimensional relationship given by the pair of temporal and spatial relationships (*met*, *equal*) are given by the 3-dimensional relationships (*equal*, *equal*), (*equal*, *cover*), (*equal*, *covered*),

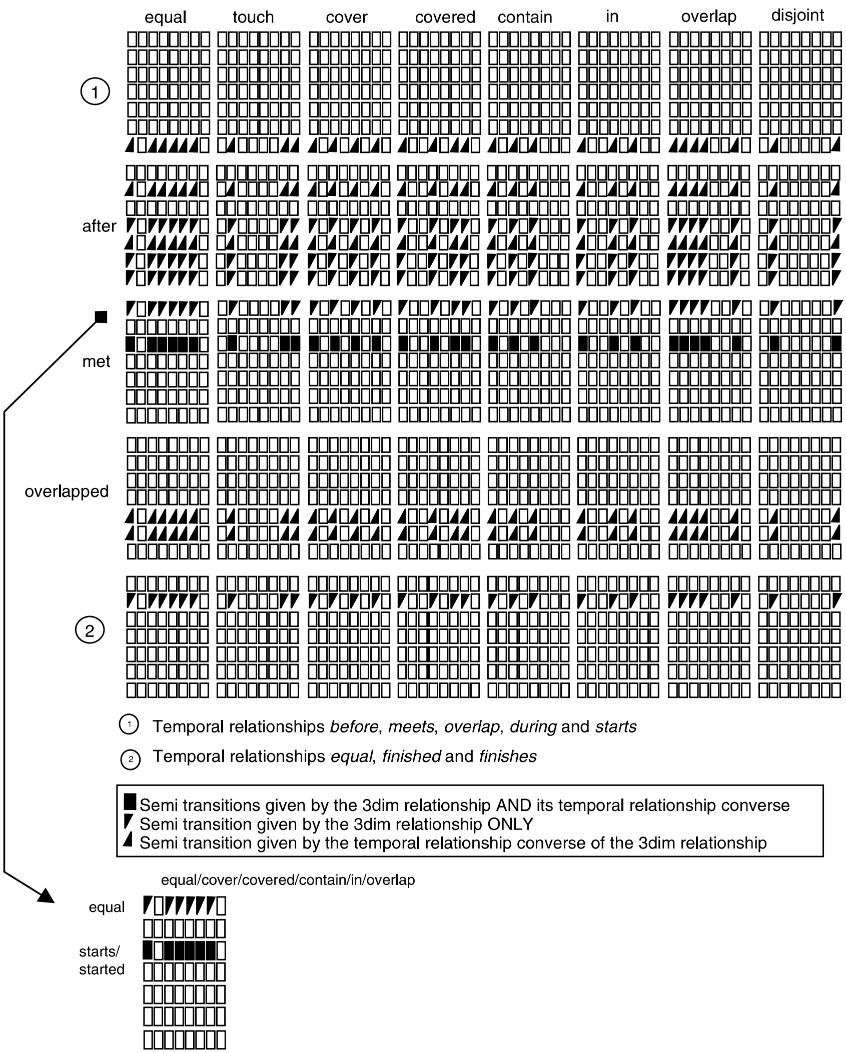

Fig. 10. Semi-transitions of 3-dimensional relationships

(*equal*, *contain*), (*equal*, *in*), (*equal*, *overlap*), (*starts*, *equal*), (*starts*, *cover*), (*starts*, *covered*), (*starts*, *contain*), (*starts*, *in*), (*starts*, *overlap*), (*started*, *equal*), (*started*, *cover*), (*started*, *covered*), (*started*, *contain*), (*started*, *in*) and (*started*, *overlap*).

4.6 Transitions of 3-dimensional relationships in a temporal space

In order to study the evolution of 3-dimensional relationships between two pairs of successive temporal regions, we introduce a third rule: there is a transition of 3-dimensional relationship between two pairs of successive temporal regions if and only if there is both a continuous transition between their successive topological relationships, and a temporal transition between their successive temporal relationships. The possible transitions of 3-dimensional relationships between two pairs of successive temporal regions are summarised in Fig. 11. Figure 11 introduces a matrix of matrix that replicates every 3-dimensional relationship identified in Fig. 4 (following the principles established for Fig. 10). For example, the possible transitions of the 3-dimensional relationship given by the pair of temporal and spatial relationship (*finishes*, *touch*) are given by the 3-dimensional relationships (*equal*, *overlap*), (*equal*, *disjoint*), (*equal*, *touch*), (*starts*, *overlap*), (*starts*, *disjoint*), (*starts*, *touch*), (*started*, *overlap*), (*started*, *disjoint*) and (*started*, *touch*).

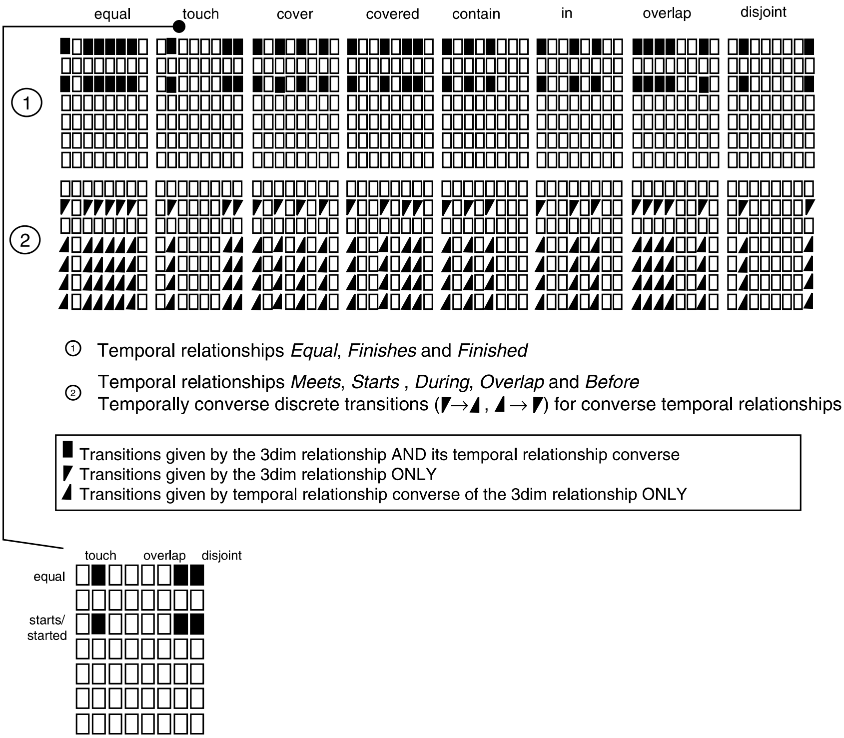

Fig. 11. Transitions of 3-dimensional relationships

5 Case study

The following case study illustrates the potential of 3-dimensional relationships in a 3D temporal space, and the respective roles of conceptual neighbourhoods, semi-transitions and transitions. The case study consists of a small rural region that integrates some land parcels and three pollution regions. A first pollution region continuously expands (P1), its beginning and ending times are known. A second pollution region (P2) mutates to a third pollution region (P3) (P2 and P3 are successive temporal regions). The locations in space of P2 and P3 do not change. The starting and ending times of P2 are known, then the starting time of P3 is also known, not its ending time (as P3 is not terminated at the present time, then P3 is not a fixed temporal region).

Figure 12 illustrates the successive known topological configurations of the case study over the year 2000. Let us remark that parcels 104 and 105 merge into parcel 107 (parcels 104 and 105, 104 and 107 are successive temporal regions), while parcel 106 is split into parcels 109 and 110 (parcels 106 and 109, 106 and 110 are successive temporal regions too). Table 1 gives the temporal intervals of the parcels and the pollution regions. We assume that the present time is the 31-12-2000 and thus we investigate the different configurations of 3-dimensional relationships.

In order to explore the relationships between the pollution parcels and the land parcels, we use 3-dimensional relationships. Figure 13 illustrates 3-dimensional relationships over the year 2000, between P1, and some land parcels. As the way P1 expands within its life span is not completely known, different alternatives are presented where necessary. For example, this figure shows that P1 topologically *touches* 105 for at least a common sub-interval of their respective existence, but two alternatives of the possible evolution of 3-dimensional relationships between P1 and 105 are presented. The same principles apply between P1 and parcels 106 (two alternatives) while the 3-dimensional relationships between P1 and parcels 100 and 108 are well defined on the contrary. Let us mention another peculiarity: during the expansion of P1, the topological relationship between P1 and the land parcel 100 changes from *disjoint* to *overlaps*; by application of the concept of conceptual neighbourhood introduced in Fig. 5, we can derive that there is at least a sub-interval of time of the existence of P1, for which P1 *touches* the land parcel 100. The application of the concept of temporal semi-transition is illustrated by the possible evolution of the 3-dimensional relationship between P1 and 105, towards the 3-dimensional relationship between P1 and 107 (as 107 can be considered as a successor of 105); and the evolution of the 3-dimensional relationship between P1 and 106, towards the 3-dimensional relationship between P1 and 110 (as 110 can be considered as a successor of 106).

Let us study 3-dimensional relationships between P2, P3 and the land parcels that are not always topologically disjoint with P2 and P3 (i.e., 103, 104 and 107) since the start of the year 2000. Topological relationships between P2, P3 and these parcels are well defined as their respective locations in space do not change. The temporal relationships between P2 and the parcels are also well defined as P2 terminates before these parcels. On the other hand, the possible temporal relationships between P3 and the parcels

103 and 107 are given by the possible relationships between one temporal interval (P3) that starts *after* the beginning of a second one (103, 107). Therefore the possible temporal relationships between P3 and 103 or 107 are *during*, *finishes* and *overlapped*.

The 3-dimensional relationships between P2, P3 and the other parcels are derived from the above temporal relationships and some fixed topological relationships (Fig. 15). The concept of temporal transition is illustrated by the possible evolution after the year 2000 of the 3-dimensional relationships between P2 and 104, towards the 3-dimensional relationship between P3 and 107 (Fig. 15). Semi-transitions are illustrated by the possible evolution of the

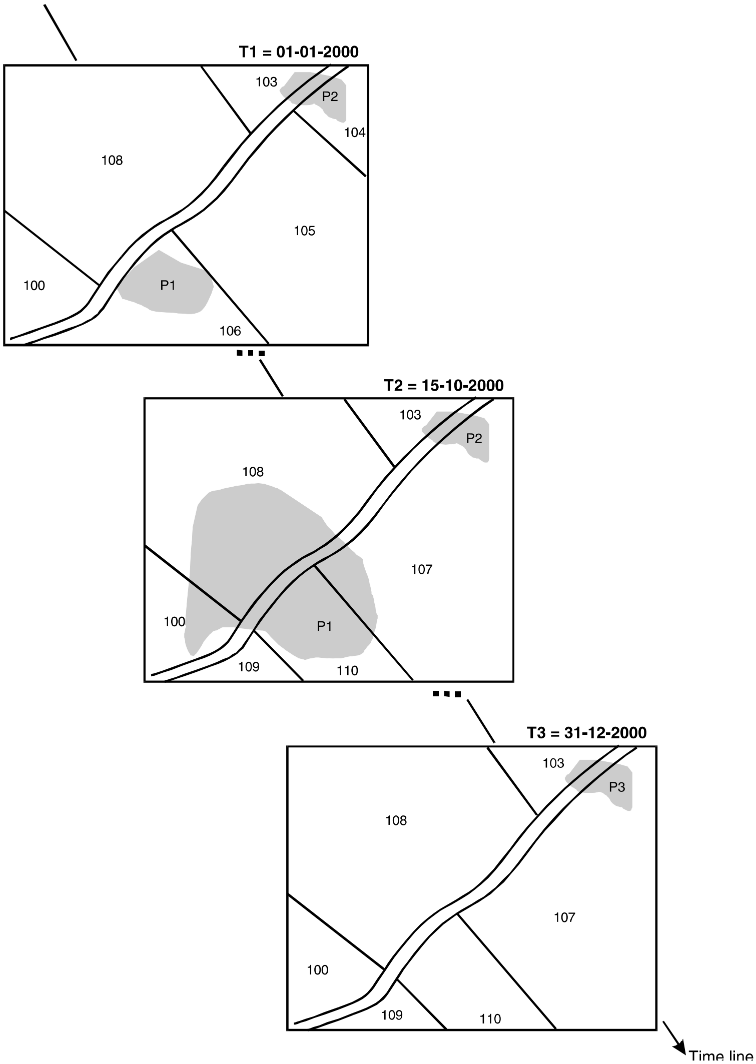

Fig. 12. Case study: evolution of topological configurations

Table 1. Case study: temporal intervals

ID	Start time	End time
100	15-02-1998	–
103	14-01-1998	–
104	30-03-1999	01-07-2000
105	15-03-1999	01-07-2000
106	15-02-1998	15-06-2000
107	01-07-2000	–
108	01-01-1998	–
109	15-06-2000	–
110	15-06-2000	–
P1	15-02-1998	15-10-2000
P2	15-02-1998	15-06-2000
P3	15-06-2000	–

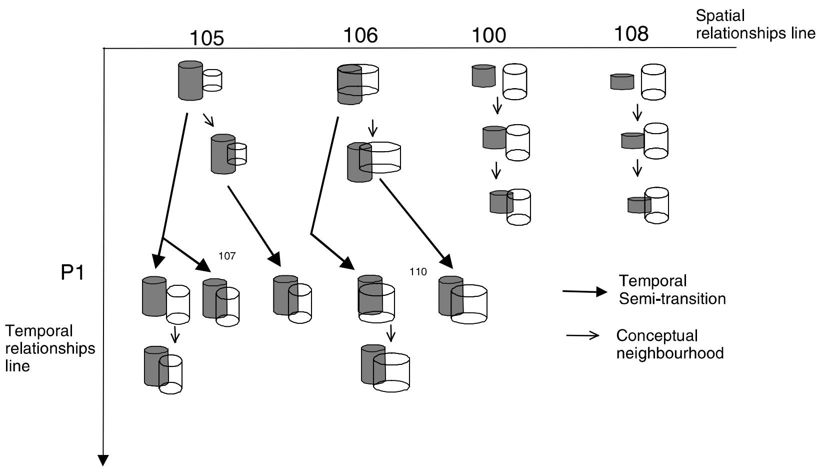

Fig. 13. Evolution of 3-dimensional relationships between P1 and some land parcels over the year 2000

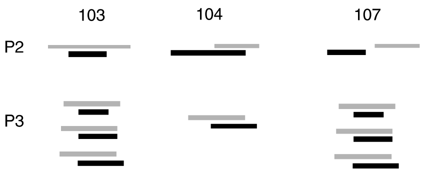

Fig. 14. Possible temporal relationships between P2/P3 and some land parcels

3-dimensional relationship between P2 and 103 towards the 3-dimensional relationship between P3 and 103.

This simplified case study illustrates the application of 3-dimensional relationships to the study of the evolution of a topological configuration. The combined application of conceptual neighbourhoods, semi-transitions and

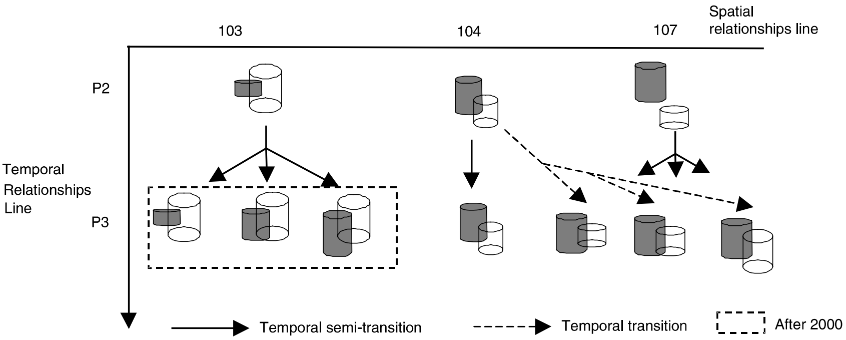

Fig. 15. Possible 3-dimensional relationships between P2/P3 and some land parcels since the start of the year 2000

transitions supports the identification of possible evolutions of 3-dimensional relationships, and the way topological and temporal relationships evolve. These constraints can be used to check the consistency of evolving topological configurations, or even to identify unknown relationships in space and time. Such principles can also be generalised to geographical applications oriented to the exploration of life trajectories in space and time.

6 Conclusion

The representation of relationships in both space and time is a mandatory requirement of many studies related to the study of real-world phenomena. Many applications require a combined representation of topological and temporal relationships in order to analyse trajectories in space and time. This paper proposes an analysis of relationships in both space and time. An integrated model of topological relationships between regions in 2-dimensional spaces and convex temporal intervals is developed. Such a model leads to a representation of these relationships in a 3-dimensional temporal space. We apply the principles of conceptual neighbourhoods, and introduce semi-transitions and transitions between resulting relationships in the 3-dimensional space.

As illustrated by the case study, our algebra is adapted to the study of evolving spatial configurations. Conceptual neighbourhoods and transitions have several advantages as they allow the computational evaluation of unknown relationships and the estimation of the future states of evolving temporal regions. As this model is based on an integration of relationships in both space and time, its implementation is upwardly compatible with current spatial and temporal models. The proposed model is flexible enough to be applied and adapted to different application contexts. The model can also be applied on 3-dimensional topological relationships. The only limitation being in this case the perception of relationships in a 4-dimensional space, not the computational implementation of these operators. Further wok concerns the application of this model to environmental and urban studies.

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