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ORIGINAL ARTICLE

The distance decay effect and spatial reach of spillovers

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Abstract
This paper quantifies and graphically illustrates the distance decay effect and spatial reach of spillover effects derived from a spatial Durbin (SD) model with parameterized spatial weight matrices. Building on attributes of the concept of spatial autocorrelation developed by Arthur Getis, we adopt a distance-based negative exponential spatial weight matrix and parameterize it by a decay parameter that is different for each spatial lag in this model, both of the regressand and of all regressors. The quantification and illustration are applied to the spatially augmented neoclassical growth framework, which we estimate using data for 266 NUTS-2 regions in the EU over the period 2000–2018. We find distance decay parameters ranging from 0.233 to 2.224 and spatial reaches ranging from 700 to more than 1500 km for the different growth determinants in this model. These wide ranges highlight the restrictiveness of the conventional SD model based on one common spatial weight matrix for all spatial lags.

Keywords Regional economic growth · Growth spillovers · Regional proximity · Distance decay

JEL Classification C21 · C23 · O47 · R12

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1 Introduction

As the world economy becomes increasingly integrated, there is growing evidence that economic growth is correlated across space. This pattern is clearly visible in the data, and although it is increasingly recognized in empirical studies (Moreno and Trehan 1997; López-Bazo et al. 2004; Ertur and Koch 2007; Ramajo et al. 2008), there is no consensus in the literature on the magnitude and the spatial reach of observed growth spillovers. This lack of consensus is highlighted in a recent article by Rosenthal and Strange (2020) whose title raises the pressing question: "How close is close." Their answer draws on a range of research on agglomeration effects in economics and regional science, yet without providing a clear research methodology on how to estimate spillover effects.

To address this question, we propose a novel approach to determine growth spillovers within the spatially augmented neoclassical growth framework. This approach draws on the work of Arthur Getis regarding the concept of spatial autocorrelation, which we "translate" into present-day spatial econometrics, and the methodology of Tan (2023) to parameterize the spatial weight matrix with a different parameter for each determinant that captures the rate at which interactions between economies decay in terms of distance. Our contribution is to introduce a novel approach to quantify and visualize the spillover effects of each determinant based on distance, while considering the uncertainty associated with the parameter estimates, including the distance decay parameter that defines the accompanying spatial weight matrix.

We illustrate the power of this approach by estimating spillover effects in GDP per capita growth for EU NUTS-2 regions over the period from 2000 to 2018. There is extensive work in the literature that has tried to estimate the magnitude of growth spillovers. Early work on spillovers used regional dummies (Easterly and Levine 1997) or control variables that are averaged across nearby countries (Ades and Chua 1997). Moreno and Trehan (1997) are among the first to use a spatial econometric model to empirically test whether growth spillovers work through the regressand, the error term and/or the income regressor. At that time, they labeled the coefficient of the spatially lagged regressand, which reflects per capita growth in neighboring economies, as a spillover effect. More recent work has used various approaches to measure spillovers at the sub-national level and analyze their spatial reach (Bottazzi and Peri 2003; Funke and Niebuhr 2005; Rodríguez-Pose and Crescenzi 2008). There is also an extensive body of literature on spillovers between urban areas (Glaeser et al. 1992; Henderson et al. 1995). While this literature has provided empirical evidence regarding the existence of growth spillovers, the results regarding their magnitude are inconclusive (Funke and Niebuhr 2005; Ramajo et al. 2008; Benos et al. 2015; Márquez et al. 2015). One reason is that these authors have either used indirect ways to account for growth spillovers, such as the trade-off between national growth and greater regional equality in economic outcomes (Gardiner et al. 2011), or have attempted to estimate growth spillovers directly, using econometric specifications and spatial weight matrices which impose restrictions on the extent of distance decay and the corresponding spatial reach of spillovers.

Our analysis avoids these problems by linking the magnitude of growth spillovers to distance and allowing the rate of distance decay to differ per growth determinant. We illustrate the indirect or spillover effects of each growth in terms of distance, slope, magnitude and level of significance. In addition to the existence and importance of spillover effects consistent with previous studies, our findings confirm substantial variations in their magnitudes due to differences in the rate of distance decay between growth determinants. These findings complement previous studies in the literature on regional economic growth based on the spatially augmented versions of the neoclassical growth model (López-Bazo et al. 2004; Ertur and Koch 2007, 2011; Elhorst et al. 2010).

The setup of this paper is as follows. In Sect. 2 we link our approach to attributes of the concept of spatial autocorrelation developed by Arthur Getis. In Sect. 3 we present the spatially augmented neoclassical model of economic growth and its empirical model in the form of a spatial Durbin (SD) model that we use for our analysis. In Sect. 4 we introduce the parameterizations of the spatial weight matrices and show their relationship with the direct and spillover effects of the growth determinants in the SD model. In Sect. 5 we describe the data, report and discuss the estimation results, plot the spillover effects for the different growth determinants and examine the robustness of the results to changes in the model specification. Finally, Sect. 6 concludes.

2 Arthur Getis: the concept of spatial autocorrelation

In a survey article to the Handbook of Applied Spatial Analysis (Fischer and Getis 2010), Arthur Getis summarizes the development of the concept of spatial autocorrelation over the past decades and highlights its main uses and attributes within the literature (Getis 2010). This article also provides detailed references to all key papers on this topic, which need not be repeated here. In this section we review these main uses and attributes, translate them into present-day spatial econometrics and only provide limited references to some of Getis key papers, which reflect his thoughts on the topic. More details on the development of this literature can be found in the aforementioned survey and in Getis (2008).

Specifically, Getis (2010, pp. 257–259) underscores the following attributes and uses of spatial autocorrelation, which he indicates 'should convince all of those who deal with georeferenced data that an explicit recognition of the concept is basic to any spatial analysis':

1. Proper specification *[to avoid misspecification]* requires that any spatial association is subsumed with the model proper.
2. A thorough understanding of the effects of regressor variables on a dependent variable requires that any spatial effects in both dependent and independent variables are quantified.

3. Spatial autocorrelation statistics are usually designed to test the null hypothesis that there is no relationship among realizations of a single variable, but the tests may be extended to consider spatial relations between variables.
4. Measures of spatial autocorrelation will change in certain known ways when the configuration of spatial units changes.
5. A focus on a single spatial unit's effect on other units and vice versa.
6. Measures of spatial association can identify the parameters of distance decay (for example, the parameters of a negative exponential model).
7. A series of measures of spatial autocorrelation over time sheds light on temporal effects.
8. If the goal is to avoid, as much as possible, spatial autocorrelation in the sample, then a reasonable sample design would benefit from a study of spatial autocorrelation in the region where the sample is to be selected.
9. Before engaging in many types of spatial analysis, it is necessary to make the assumption that spatial stationarity exists.
10. A means of identifying spatial clusters.
11. A means of identifying outliers, both spatial and non-spatial.

Translated into present-day spatial econometrics, these attributes are a plea for the SD model in which the spatial weight matrices take an exponential form and negatively depend on a distance decay parameter. This parameter should also be allowed to differ for each spatial lag in the model, including the spatially lagged regressand and the spatially lagged regressors.

The standard SD model, which has received much attention in applied spatial econometric studies thanks to the work of LeSage and Pace (2009), covers the first two attributes. According to these authors, the cost of ignoring spatial lags in the regressand and the regressor variables, when relevant, is high since the coefficients of the remaining variables may then be biased. By contrast, ignoring a spatial lag in the error term, if relevant, will only result in a loss of efficiency.

Regarding the third attribute, several spatial autocorrelation test statistics have been proposed and used in the applied literature to motivate the use of spatial econometric models. Getis himself did important work in this research area (Getis and Ord 1992; Ord and Getis 1995). A common test statistic is Moran's I. However, when applied to the regressand in raw form, the null hypothesis that it is spatially uncorrelated generally needs to be rejected because this statistic does not control for potential spatial lags in the regressor variables. Theoretically, it is possible that a standard linear regression without any spatial lags is sufficient because the regressor variables may also be spatially correlated in such a way that they fully cover the spatial correlation in the regressand. In this regard, Anselin and Rey (2014) label Moran's I as a "non-constructive test in that the alternative is diffuse, and not a specific (focused) model'' (p. 107).

Another commonly used approach to motivate the use of spatial econometric models is to apply the robust Lagrange multiplier tests developed by Anselin et al. (1996). These tests analyze whether the linear regression model estimated by OLS should be extended to include a spatial lag in the regressand or the error term, known as, respectively, the spatial autoregressive (SAR) model and the spatial error (SE) model. However, these tests also do not control for potential spatial lags in the regressor variables. When estimating the SD model, which includes spatial lags in the regressor variables, this potential misspecification can be avoided. Furthermore, since the OLS, SAR and SE models are special cases of the SD model (LeSage and Pace 2009), it can also be tested using Wald or likelihood ratio (LR) ratio tests whether the SD model simplifies to one of these models (Elhorst 2014; Juhl 2021). Once the SD model (or one of these simpler models) has been estimated, one is not done yet. The researcher should also test the residuals for any remaining spatial dependence. The cross-sectional dependence (CD) test of Pesaran (2015) can be used if panel data are available. In contrast to traditional cross-sectional dependence tests in the literature, this test does not require any pre-specified spatial weight matrix and hence it also fulfills the fourth attribute. It is also well-suited for the typical panel data setting in the empirical spatial econometric literature, where the number of observations in the cross-sectional domain dominates the number of observations over time. If the CD test applied to the residuals of the SD model still points to any remaining spatial dependence, only then further adjustments may be necessary to find a proper model.

Another advantage of the SD model over other spatial econometric models in empirical research is its flexibility in modeling spillovers, and thus the fifth attribute. The main interest of many empirical researchers is not the parameter estimates of the regressor variables, but the marginal impact of changes they have on the regressand. Two marginal effects stand out: the direct effect of changing the regressor variable of one unit on the regressand of that unit itself, and the cumulative effect of changing the regressor variable of one unit on the regressand of all other units (LeSage and Pace 2009). This cumulative effect is known as the indirect effect, but a more appealing way to refer to it is the synonym *spillover effect*, the description we will use in this paper. Halleck Vega and Elhorst (2015) demonstrate that only models that include spatial lags of the regressor variables are able to produce spillover effects that can take any empirical value relative to the direct effects. By contrast, the popular SAR, SE and combined SAR-SE models are problematic in this respect since they impose restrictions on the magnitude of spillover effects in advance. In the SE model, the spillover effects are zero by construction, and in the SAR and SAR-SE models, the ratio between the spillover and the direct effect is the same for every regressor variable.

Up to now, the sixth attribute of measuring distance decay received relatively little attention in the spatial econometric literature. Most studies adopt one common spatial weight matrix for all spatial lags in the SD model. By parameterizing the distance-based negative exponential spatial weight matrix by a decay parameter that differs for each spatial lag, we also try to give shape to this particular attribute highlighted in Getis' work (Getis and Aldstadt 2004; Getis 2009). The present study illustrates the benefits of this approach in the context of a spatially augmented neoclassical growth framework, which we estimate using annual data for 266 NUTS-2 regions in the EU over the period 2000–2018.

Our approach of estimating the SD model with parametrized spatial weight matrices that differ across spatial lags resembles the underlying logic of multiscale geographically weighted regressions (MGWRs) (Fotheringham et al. 2017, 2024). Both

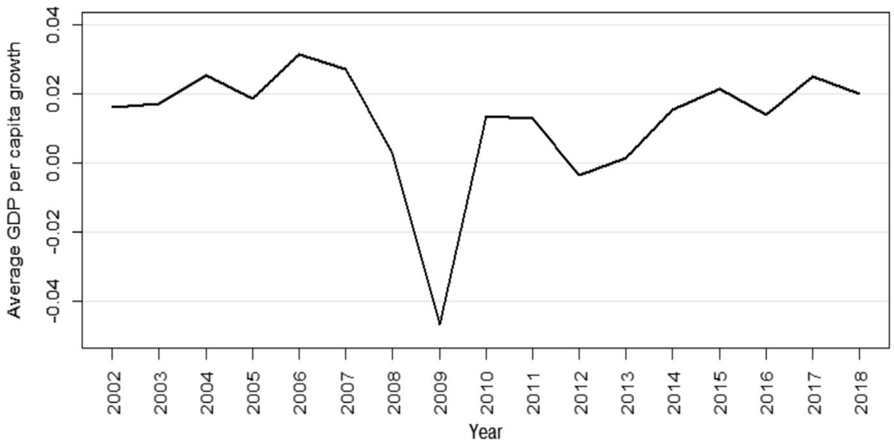

Fig. 1 The average GDP per capita growth rate across all regions over time

approaches recognize that the relationships between the regressand and the regressor variables may operate at different spatial scales and therefore require different spatial weight matrices. The main difference is that MGWR focuses on parameter heterogeneity of the direct effects based on different bandwidths indicating the data-borrowing range of each regressor variable, while the SD model not only focuses on the determination of direct effects but also of spillover effects. This point is also mentioned by Getis (2010, pp. 271–272) when he briefly assesses GWR's strengths and weaknesses.[1]

Using data over a period that covers the financial crisis of 2008–2009 and the resulting Great Recession, followed by the European debt crisis of 2010–2015, we also cover the seventh attribute, as growth rates were relatively high before this recession and relatively low in the period immediately after it. Figure 1 displays the average growth rate of GDP per capita across all regions, which dropped precipitously in 2009 and then recovered gradually. Furthermore, by using data at the sub-national level, which will be characterized by a substantial level of spatial autocorrelation, we also can test whether the proposed SD model is able to cover the eighth attribute by applying the CD test on its residuals.

To test whether the ninth attribute of spatial stationarity is satisfied, we will specify in the next section which restriction on the parameters needs to be verified in the SD model. The last two attributes, the identification of outlier observations and spatial clusters, are also considered in our empirical analysis in Sects. 5.2 and 5.3.

[1] Although existing approaches to estimate MGWRs are so far limited to cross-sectional data, some initial tests conducted based on a cross-sectional version of our data set suggest that the bandwidths of the estimated MGWRs are consistent with the distance decay parameters of our model. However, a proper comparison of the two approaches goes beyond the scope of the present paper.

3 The spatially augmented neoclassical growth framework

The world's evolving income distribution lies at the heart of the economic growth literature. Within this literature, the neoclassical growth framework is the most commonly used framework to understand the pattern of economic growth and the evolution of per capita incomes across countries and regions. The framework originates from theoretical contributions by Solow (1956) and Swan (1956) associating economic growth with the process of capital accumulation under diminishing returns. Following the standard empirical implementation of the neoclassical framework in a panel data context due to Islam (1995) leads to the following regression equation:

$$\Delta \ln y_{i,t} = \beta_1 \ln inv_{i,t} + \beta_2 \ln \left(n_{i,t} + g + q\right) + \beta_3 \ln y_{i,t-1} + \mu_i + \xi_t + \varepsilon_{i,t}, \qquad (1)$$

where ln $y_{i,t}$ denotes the natural logarithm of GDP per capita of economy $i\,(= 1, \dots, N)$ in period $t\,(= 1, \dots, T)$ and $\Delta \ln y_{i,t} = \ln y_{i,t} - \ln y_{i,t-1}$ its growth rate.[2] $inv_{i,t}$ denotes the investment rate whose impact is measured by the parameter β_1. $n_{i,t}$ denotes the rate of population growth, g the rate of technological progress and q the depreciation rate.[3] The combined effect of these three variables is measured by the parameter β_2. $\ln y_{i,t-1}$ is the natural logarithm of the initial level of GDP per capita at the beginning of each time period whose effect is captured by β_3. The specification also includes cross-sectional fixed effects, μ_i, which reflect all time-invariant factors that lead to differences in growth rates across economies, such as geographic and institutional factors. Since growth rates are also affected by common trends, time period fixed effects, ξ_t, are also controlled for. Finally, $\varepsilon_{i,t}$ represents an independently and identically distributed error term for all i with zero mean, variance σ^2 and finite fourth moment. The appendix explains the implications for the parameter estimates of not assuming normality of the error terms.

One important limitation of the standard neoclassical growth framework is the assumption that each economy operates in isolation of others. This assumption seems implausible especially when this framework is applied to sub-national economies between which production factors are highly mobile and technology can be easily transferred (Beugelsdijk et al. 2018). Over the past two decades, awareness of this limitation has increased, leading to increased interest in the influence of an economy's spatial location on its growth rate.

A prominent example is the study of Ertur and Koch (2007). They propose a spatially augmented version of the neoclassical growth model that allows for interaction across economies due to productivity spillovers arising from capital investments. Their model builds on and is supported by a large body of other studies highlighting the importance of technological and knowledge spillovers (e.g., Audretsch and

[2] The presentation of this equation is based on annual data. The index $t-1$ can be replaced by $t-p$ if GDP per capita growth is measured over p years. In that case the growth rate should correspond to an average over this time period.

[3] In line with the common assumptions of the neoclassical growth framework, the rates of technological progress and depreciation, g and q, are not indexed as they are assumed to be common for all economies and time periods. We follow Islam (1995) to assume that $g+q=0.05$. In Sect. 5.4 we investigate what happens if we extend our specification to incorporate endogenous growth determinants.

Feldman 2004; Autant-Bernard and LeSage 2011). Ertur and Koch (2007) also demonstrate that the empirical counterpart of their spatially augmented version of the neoclassical growth model takes the form of an SD model. This empirical model has not only been applied in a wide range of empirical studies,[4] but it has also been extended in several follow-up studies, including Elhorst et al. (2010), Pfaffermayr (2012), Jung and López-Bazo (2017), Lee and Yu (2016), Fiaschi et al. (2018), Diaz Depena et al. (2019), and Panzera and Postiglione (2022). For a panel of N cross-sectional observations over T time periods, the SD model in vector form reads as

$$\begin{aligned}\Delta Y_t = \theta_0 W(\delta_0)\Delta Y_t + (X_{1t},\dots,X_{Mt})(\beta_1,\dots,\beta_M)' + [W(\delta_1)X_{1t},\dots,W(\delta_M)X_{Mt}](\theta_1,\dots,\theta_M)' \\ + \beta_{M+1}Y_{t-1} + \theta_{M+1}W(\delta_{M+1})Y_{t-1} + \beta_{M+2}\Delta Y_{t-1} + \theta_{M+2}W(\delta_{M+2})\Delta Y_{t-1} + \mu + \xi_t \iota_N + \varepsilon_t,\end{aligned} \tag{2}$$

where $\Delta Y_t = (\Delta \ln y_{1t},\dots,\Delta \ln y_{Nt})'$ denotes an $N\times 1$ vector of the regressand introduced in Eq. (1). $W(\delta_0)\Delta Y_t$ represents the spatial lag of ΔY_t and θ_0 the spatial autoregressive response parameter of this spatial lag. $(X_{1t},\dots,X_{Mt})$ is an $N\times M$ matrix of the regressor variables introduced in Eq. (1) and $[W(\delta_1)X_{1t},\dots,W(\delta_M)X_{Mt}]$ an $N\times M$ matrix of their spatial lags. The impacts of these regressor variables and their spatial lags are measured by the $M\times 1$ vectors $(\beta_1,\dots,\beta_M)'$ and $(\theta_1,\dots,\theta_M)'$, respectively. The spatial weight matrix, symbolized by W, is an $N\times N$ matrix describing the spatial arrangement between each pair of economies i and j, whose elements w_{ij} in this paper are assumed to depend on a distance decay parameter δ_m $(m = 0, 1, \dots, M+2)$. Its functional form is the topic of the next section. Additional regressor variables are Y_{t-1} and $W(\delta_{M+1})Y_{t-1}$ representing the initial levels of GDP per capita in the own and neighboring economies at the start of the observation period, and ΔY_{t-1} and $W(\delta_{M+2})\Delta Y_{t-1}$ representing the time lag of the regressand ΔY_t and its spatial counterpart $W(\delta_0)\Delta Y_t$.[5] As explained above, $\mu = (\mu_1,\dots,\mu_N)^T$ and the set ξ_t $(t = 1,\dots,T)$ denote cross-sectional and time fixed effects, respectively, where ι_N is an $N\times 1$ vector of ones.

Overall, Eq. (2) shows that the GDP per capita growth rate of a given economy depends on the investment rate and the rates of population growth, technological progress and depreciation, both in the given economy and that of its neighbors, which determine the long-run equilibrium or steady-state level of GDP per capita. It further depends on the initial GDP per capita level in both the given and neighboring economies at the start of each time period, which reflects how far each economy is from its long-run equilibrium. Additionally, it depends on its lagged growth rate, as well as the contemporaneous and lagged growth rates of its neighbors.

To find out under which parameter condition the spatially augmented version of the neoclassical growth framework leads to convergence or divergence, we rearrange and express Eq. (2) in terms of GDP per capita levels, to get:

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[4] For detailed surveys of the regional growth literature, see Döring and Schnellenbach (2006), Harris (2011) and Breinlich et al. (2014).

[5] Even though the regressors $W(\delta_0)\Delta Y_t, \Delta Y_{t-1}, W(\delta_{M+1})Y_{t-1}$ and $W(\delta_{M+2})\Delta Y_{t-1}$ share information with Y_{t-1}, they remain unique, which implies that their coefficients are identified. Additional identification requirements in an SD model are discussed in Lee and Yu (2016).

$$\begin{aligned}Y_t = {} & \theta_0 W(\delta_0) Y_t + \left(1+\beta_{M+1}+\beta_{M+2}\right) Y_{t-1} + \left(-\theta_0 W(\delta_0) + \theta_{M+1} W(\delta_{M+1}) + \theta_{M+2} W(\delta_{M+2})\right) Y_{t-1} \\ & - \beta_{M+2} Y_{t-2} - \theta_{M+2} W(\delta_{M+2}) Y_{t-2} + \left(X_{1t},\dots,X_{Mt}\right)\left(\beta_1,\dots,\beta_M\right)' \\ & + \left[W(\delta_1)X_{1t},\dots,W(\delta_M)X_{Mt}\right]\left(\theta_1,\dots,\theta_M\right)' + \mu + \xi_t \iota_N + \varepsilon_t \end{aligned} \tag{3}$$

Assuming row-normalized spatial weight matrices, Yu et al. (2012) show that the sum of the coefficients of the first five terms on the right hand of this equation determines spatial stationarity, i.e., converge or divergence. This yields:

$$\theta_0 + \left(1+\beta_{M+1}+\beta_{M+2}\right) + \left(-\theta_0+\theta_{M+1}+\theta_{M+2}\right) - \beta_{M+2} - \theta_{M+2} = 1 + \beta_{M+1} + \theta_{M+1}. \tag{4}$$

Convergence occurs if the latter sum is smaller than 1, and thus if the coefficients of the initial levels of GDP per capita in the given and neighboring economies are smaller than 0 ($\beta_{M+1} + \theta_{M+1} < 0$). In contrast, divergence occurs if the sum is greater than 1 ($\beta_{M+1} + \theta_{M+1} > 0$). A special case of neither convergence nor divergence occurs when $\beta_{M+1} + \theta_{M+1} = 0$, which Yu et al. (2012) label as spatial co-integration. This corresponds to a situation in which GDP per capita growth rates in different economies fluctuate over the business cycle to a varying extent, but eventually remain on different growth paths during the entire sample period.

4 Parameterization and estimation

Following Arthur Getis' sixth attribute, a negative exponential functional form is used to specify $W(\delta_m)$. Its diagonal elements are set to zero to prevent economies from influencing themselves and its off-diagonal elements are specified by $w_{ij}\left(\delta_m\right) = \exp\left(-\delta_m d_{ij}\right)$, where d_{ij} denotes the geographic distance between each pair of economies *i* and *j*. Although this functional form is commonly used, the novelty of our study is that each distance decay parameter ($\delta_m > 0$) is estimated rather than pre-specified and is allowed to be different for each spatial lag m ($m = 0, 1, \dots, M+2$).[6] Here the values of $m=1$, and $m=2$ refer to the distance decay parameters of the investment rate and the combined rates of population growth, technological progress and depreciation, respectively. Additionally, $m=0$ refers to the distance decay parameter of the spatial lag in the regressand, and $m = M+1 = 3$ and $m = M+2 = 4$ to the distance decay parameters of the initial level of GDP per capita and time-lagged GDP per capita growth rate, respectively. The elements $w_{ij}(\delta_m)$

[6] We limit our discussion to the conventional specification of the SD model, which adopts a common pre-specified spatial weight matrix for each spatial lag. It is important to distinguish the description negative exponential spatial weight matrix from the matrix exponential spatial specification (MESS) proposed by LeSage and Pace (2007). Although the naming suggests similarities to our approach, the spatial multiplier matrix of MESS boils down to $\sum_{s=0}^{\infty} \alpha^s W^s / s!$ compared to $\sum_{s=0}^{\infty} \theta_0^s W^s$ of the conventional SD model. Since $\theta_0 = 1 - \exp(\alpha)$, the estimation results will eventually not differ to any great extent. Although the MESS approach has the computational advantage that the Jacobian in the log-likelihood is zero by construction, it essentially operates like the conventional SD model, as both rely on the same spatial weight matrix for all spatial lags.

of the negative exponential distance decay matrix, after row-normalizing each spatial weight matrix $W(\delta_m)$,[7] read as

$$w_{ij}\left(\delta_m\right) = \frac{\exp\left(-\delta_m d_{ij}\right)}{\sum_{j=1}^{N} \exp\left(-\delta_m d_{ij}\right)}. \tag{5}$$

To draw conclusions regarding the impact of the regressor variables on the growth rate of its own and neighboring economies, we consider their direct and spillover effects, as the parameter estimates alone provide an incomplete picture of the marginal effects in the SD model (LeSage and Pace 2009; Elhorst 2014). The direct effect ($DE_{m'}$) measures the average impact of a change in the m'th regressor variable ($m' = 1, \dots, M+2$) of a given economy on its own growth rate, while the spillover effect ($SE_{m'}$) measures the cumulative effect of changing this regressor variable on the growth rates of all its neighbors:

$$DE_{m'} = \frac{1}{N} tr\left\{ \left(I_N - \theta_0 W\left(\delta_0\right)\right)^{-1} \left(\beta_{m'} I_N + \theta_{m'} W\left(\delta_{m'}\right)\right) \right\}, \tag{6a}$$

$$\begin{aligned} SE_{m'} = & \frac{1}{N} \iota'_N \left\{ \left(I_N - \theta_0 W\left(\delta_0\right)\right)^{-1} \left(\beta_{m'} I_N + \theta_{m'} W\left(\delta_{m'}\right)\right) \right\} \iota_N \\ & - \frac{1}{N} tr\left\{ \left(I_N - \theta_0 W\left(\delta_0\right)\right)^{-1} \left(\beta_{m'} I_N + \theta_{m'} W\left(\delta_{m'}\right)\right) \right\}. \end{aligned} \tag{6b}$$

These effects encompass feedback effects that pass through neighboring regions and eventually circulate across all regions in the sample, including the region that instigated the change in one of the regressor variables.[8] Halleck Vega and Elhorst (2015) demonstrate that only models that at least include spatial lags of the regressor variables ($\theta_{m'}$), such as the SD model, are able to produce spillover effects that can take any empirical value. Parameterizing the spatial weight matrix of every regressor enhances this flexibility. Adopting one common W matrix for each spatial lag can be rather restrictive and lead to incorrect inferences because each regressand–regressor relationship may operate at a different spatial scale. More precisely, if the Ws of all regressors in the SD model are assumed to be the same, i.e., if $\delta_0 = \delta_1 = \cdots = \delta_{M+2}$ in (6b), the spatial reach of their spillover effects will also be the same. While it is true that the impact at a specific distance may remain different for each regressor due to their individual parameters $\beta_{m'}$ and $\theta_{m'}$, the critical point here is that the spatial reach—the distance over which this impact is felt measured by δ_m—will remain uniform for all regressor variables. In Sect. 5.3 we explore this property graphically in the context of our empirical application.

To estimate the parameters of Eq. (2) and the corresponding variance–covariance matrix, we use a nonlinear quasi-maximum likelihood (QML) estimator, which does not require normality of the error terms. Heteroskedasticity will be accounted for if

[7] Normalization by rows is a standard requirement to ensure the identification of the slope parameters in a spatial econometric model (Lee and Yu 2016).

[8] Debarsy et al. (2012) also provide formulas to determine these effects over time.

homoskedasticity is rejected. The delta method (Arbia et al. 2020) is used to calculate the significance levels of the direct and spillover effects. The technical details of this estimator—first- and second-order derivatives, the derivation of the information matrix and the variance–covariance matrix, an econometric-theoretical proof that this estimator is asymptotically normal if N goes to infinity, and an overview of the conditions under which the response and distance decay parameters are identified—are described in Tan (2023, Chapter 2). A brief summary is provided in the appendix of this paper.

5 Empirical analysis

5.1 Data

The empirical analysis of our spatially augmented neoclassical growth framework is based on a full-balanced panel of 266 EU NUTS-2 regions across 27 countries over the period 2000–2018 provided by Eurostat's regional database. Conducting the analysis with EU NUTS-2 regions has the advantage of working with harmonized data on GDP and other macroeconomic aggregates, which are not available at a more disaggregated level. We measure economic growth in each region as the change in the natural logarithm of real GDP per capita in constant prices and adjusted for PPP ($\Delta \ln y_t$). This regressand is linked to the current rate of investment spending ($\ln inv_t$) and the combined rate of population growth rate, technological progress and capital depreciation, $\ln\left(n_t + g + q\right)$, as implied by Eq. (1). The correlation coefficients between the four main regressor variables in Eq. (2) amount to 0.30 at the maximum (in absolute value) and their variance inflation factors to 1.03, indicating that multicollinearity is no issue. As part of our robustness analysis, we expand the set of regressor variables to also include regional measures of the tertiary educational attainment of the working-age population ($\ln educ_t$) and the share of employment in science and technology ($\ln sci\&tech_t$). For the construction of parameterized spatial weight matrices, we use the great-circle distance in kilometers between all pairs of regions based on the latitude and longitude coordinates of their centroids.

5.2 Basic results

Table 1 reports the estimation results of our spatially augmented neoclassical growth model for different specifications of the spatial weight matrix or matrices. The estimates in column [1] are based on one common spatial weight matrix for all spatial lags and are representative of a wide range of previous empirical studies. Although many adopt a binary contiguity matrix based on the principle of sharing a common border, one problem is that several EU regions are islands, which become isolated if the contiguity principle is applied to them (Anselin and Rey 2014, pp. 38–40). Since the number of neighbors for the 256 non-island regions in the sample appears to be

5.98 on average, we use a six nearest neighbor matrix in column [1] so that the 10 island regions in our sample can also be included in the analysis.[9]

The estimates in column [2] are based on one common exponential distance decay matrix using a pre-specified value of $\delta = 0.01$. This value of 0.01 has been used in several other studies based on EU regions (Pfaffermayr 2012; Ezcurra and Rios 2020). The estimates in column [3] are based on one common exponential distance decay matrix whose distance decay parameter is estimated rather than pre-specified, using the nonlinear estimation techniques developed by Tan (2023). The obtained estimate of δ in this case when multiplied by 100 is 1.088, which is very close to the pre-specified value of 0.01 in column [2] (when this number is also multiplied by 100). This multiplication is applied so that the optimal value of the distance decay parameter takes value around 1 within the interval (0,10], which from a computational viewpoint performs better (Tan 2023, Ch.2). Finally, in column [4] of Table 1, we report the estimates for our preferred specification where the distance decay parameters of each spatial lag in the model are estimated separately.

We first discuss the results from a statistical point of view. In the next section we also provide an economic interpretation and visualize the spillover effects of the preferred specification.

Comparing the values of the log-likelihood function values (LogL) and the Akaike information criterion (AIC), which corrects for differences in the number of estimated parameters, it appears that as we allow for more flexibility in the spatial weight matrix, this leads to a better fit of the data. When replacing the relatively sparse six nearest neighbor matrix in column [1] with a denser exponential distance matrix in column [2], both statistics improve substantially. When the distance decay parameter in column [3] is estimated subsequently, both statistics improve further, albeit limitedly, as 0.01 in this particular case was already a good guess of the distance decay parameter. Finally, the best fit is obtained when allowing for different instead of one common distance decay parameter.[10] This finding provides compelling empirical evidence that the distance decay parameters associated with each spatial lag are statistically different and therefore are better estimated rather than pre-specified. Indeed the distance decay parameters in column [4] appear to range from 0.233 for the population growth rate to 2.224 for the initial level of GDP per capita.

When running Pesaran's CD-test statistic on the regressand in raw form, we obtain a value of 302.3, indicating that GDP per capita growth rates are strongly spatially autocorrelated. Yet, when applied to the residuals of the models estimated in Table 1, this test statistic drops to values between −0.908 and −0.605, which are

[9] We opted to include the islands in the sample (except for the overseas territories) to acknowledge their existence and avoid an omitted variable bias as discussed in Anselin and Rey (2014). The parameter estimates do not differ significantly (less than 0.005) when estimating the model based on a standard binary contiguity matrix. Log-likelihood or related values are however difficult to compare because the number of observations also differs when islands are excluded.

[10] When conducting an LR test on the LogL value in column [4] relative to column [3], we obtain a test statistic of 21.4 (p value 0.00). This also explains why the AIC increases.

Table 1 Estimation results for different spatial weight matrix specifications

	[1]		[2]		[3]		[4]	
	coeff	*p* value	coeff	*p* value	coeff	*p* value	coeff	*p* value
Coefficient estimates regressors ($\beta_{m'}$)								
$\ln(inv_t)$	0.002	0.23	0.002	0.40	0.001	0.46	0.002	0.45
$\ln(n_t+g+q)$	−0.008	0.07	−0.008	0.06	−0.008	0.05	−0.008	0.01
$\ln(y_{t-1})$	−0.094	0.00	−0.089	0.00	−0.088	0.00	−0.090	0.00
$\Delta\ln(y_{t-1})$	0.014	0.53	0.012	0.63	0.009	0.70	0.008	0.71
Coefficient estimates spatial lags (θ_m)								
$W^*\ln(inv_t)$	0.009	0.00	0.009	0.00	0.010	0.00	0.009	0.00
$W^*\ln(n_t+g+q)$	−0.002	0.62	−0.005	0.32	−0.005	0.31	−0.016	0.00
$W^*\ln(y_{t-1})$	0.066	0.00	0.070	0.00	0.068	0.00	0.087	0.00
$W^*\Delta\ln(y_t)$	0.534	0.00	0.650	0.00	0.631	0.00	0.641	0.00
$W^*\Delta\ln(y_{t-1})$	0.191	0.00	0.194	0.00	0.198	0.00	0.174	0.00
Distance decay estimates (δ_m)								
δ *(common)**100					1.088	0.000		
δ_0*100							1.047	0.000
δ_1*100							1.483	0.340
δ_2*100							0.233	0.244
δ_3*100							0.633	0.000
δ_4*100							2.224	0.001
Direct effects ($DE_{m'}$)								
$\ln(inv_t)$	0.003	0.22	0.003	0.35	0.002	0.41	0.003	0.37
$\ln(n_t+g+q)$	−0.009	0.00	−0.009	0.00	−0.009	0.00	−0.009	0.00
$\ln(y_{t-1})$	−0.092	0.00	−0.087	0.00	−0.087	0.00	−0.090	0.00
$\Delta\ln(y_{t-1})$	0.035	0.02	0.029	0.05	0.029	0.05	0.030	0.04
Indirect/spillover effects ($SE_{m'}$)								
$\ln(inv_t)$	0.021	0.01	0.028	0.02	0.027	0.01	0.027	0.03
$\ln(n_t+g+q)$	−0.014	0.00	−0.028	0.00	−0.026	0.00	−0.059	0.14
$\ln(y_{t-1})$	0.032	0.01	0.035	0.05	0.032	0.05	0.080	0.02
$\Delta\ln(y_{t-1})$	0.407	0.00	0.560	0.00	0.532	0.00	0.478	0.00
LogL	10,919		10,974		10,976		10,986	
AIC	−21,820		−21,931		−21,931		−21,944	
CD test residuals	−0.605		−0.789		−0.802		−0.908	
# Observations	4522		4522		4522		4522	

Regional and time fixed effects are controlled for in all columns

Regionally clustered heteroskedasticity-robust significance values

[1] = Estimates with 6 nearest neighbors matrix

[2] = Estimates with negative exponential matrix but pre-specified distance decay parameter of 0.01

[3] = Estimates with parameterized negative exponential matrix, including the distance decay parameter

[4] = Estimates with parameterized negative exponential matrix but different decay parameters for each spatially lagged variable

all within the confidence interval of $(-1.96, +1.96)$.[11] This indicates that the overall spatial association between the regressand and regressors in these models is properly specified.

If we then test these residuals for homoskedasticity using the modified Wald test, which is appropriate for our panel dataset, we obtain a p-value so close to zero that homoskedasticity must be rejected in favor of heteroskedasticity. In view of this test result, we calculated regionally clustered heteroskedasticity-robust standard errors and report p-values of the response parameters based on these robust standard errors.

5.3 Interpretation of results and graphing spillover effects

The estimation results of the preferred model in column [4] show a plausible model structure. The coefficient of the lagged GDP per capita growth rate in a given region ($\Delta\ln(y_{t-1})$) is found to be positive but small and statistically insignificant, indicating that recent growth rates are not persistent. By contrast, the coefficients of both the contemporaneous and time-lagged growth rates in neighboring regions ($W^*\Delta\ln(y_t)$ and $W^*\Delta\ln(y_{t-1})$) are found to be much larger and significant. Comparing the magnitudes of the direct and spillover effects for the time-lagged growth rate also reveals a staggering difference. A 1% increase in the growth rate of a region in the previous year will only lead to a 0.030% increase in the current growth rate, whereas the increase would be 0.478% if such an increase occurred in all the neighboring regions.

Looking at the estimates for the investment rate we see a similar picture. Whereas the effect of the investment rate in the region itself and its corresponding direct effect are found to be positive, but small and insignificant, the coefficient of the investment rate in neighboring regions and its corresponding spillover effect are much larger and statistically significant; a 1% increase in the investment rate in neighboring regions is associated with an increase in the GDP per capita growth rate of 0.027%, while such an increase in the region itself is nine times smaller.

Turning to the coefficient estimates of the population growth rate in the own and in neighboring regions, as well as its direct and spillover effects, they all appear to be negative. The difference with the previous two determinants is that only the coefficient in the region itself and its direct effect are significant. The direct effect is -0.009, which implies that if population growth increases by 1%, for example from 1 million to 1.01 million due to an influx of migrants, GDP per capita growth slows down by almost 0.1%.

Finally, looking at coefficient estimates for the initial level of GDP per capita in a given region, we see a strong and significant negative effect on GDP per capita growth (-0.090), suggesting convergent dynamics. Yet one needs to be careful since the initial level of GDP per capita in neighboring regions has a strong and significant positive effect on GDP per capita growth (0.087). The same applies to

[11] Just as in Eqs. (1) and (2), the CD test is based on independently and identically distributed error terms for all i with zero mean. The difference is that the variance σ_i^2 is assumed to be heteroskedastic. The test statistic itself follows a standard normal distribution if N and T go to infinity, which implies that its critical values are ± 1.96 at the 5% significance level.

the corresponding direct (−0.090) and spillover (0.080) effects, which have opposite signs and almost sum to zero. Since we cannot reject the hypothesis that this sum is different from zero, the evidence is rather in favor of spatial cointegration, a situation that is characterized by neither convergence nor divergence over the entire sample period. This finding could be driven by the observation period and the impact of the Great Recession in 2008–2009. Some regions were hit harder than others, while after this recession other regions were able to recover faster.[12]

Although the coefficient estimates and the direct effects may not seem to differ much across the columns of Table 1 at first glance, a different picture emerges when we compare the spillover effects. To illustrate this in more detail, we decompose and plot the spillover effects across regions based on 21 distance classes. If the distance of region i to region j is d_{ij} kilometers, region j is assigned to distance class (d_a, d_b], such that $d_a < d_{ij} \leq d_b$. The specific distance classes, as well as the distribution of the total of $\frac{1}{2}N(N-1) = 35245$ region pairs in each distance class, are shown in Fig. 2. The first classes are based on relatively small intervals given that spillovers are generally believed to decrease rapidly with distance (e.g., López-Bazo et al. 2004).

The following explanation using the time-lagged GDP per capita growth rate as an example is intended to better understand the decomposition presented in these graphs. According to column [4] of Table 1, the spillover effect of this regressor is 0.478. This summary measure, computed based on Eq. (6b), represents the average cumulative effect of changing this regressor in a given region on the regressand of all other 265 regions in the sample, whether near or far. The corresponding graph for this regressor in Fig. 3 decomposes this summary measure across regions based on the above-described distance classes, such that the surface area under the solid line for the time-lagged GDP per capita growth rate adds up to the summary measure of 0.478 reported in Table 1. In addition to the time-lagged GDP per capita growth rate, Fig. 3 also graphs the decomposed spillover effects of the other growth determinants.

In all plots in Fig. 3 the gray-shaded areas indicate the respective 95% confidence intervals, based on the estimation results reported in column [4] of Table 1. To determine these confidence intervals we also account for the uncertainty in the distance decay parameters. If spatial weight matrices are pre-specified, as in the first three columns of Table 1, this type of uncertainty is ignored, as if the researcher does know the right specification of the spatial weight matrix.

The graphs in Fig. 3 show several notable patterns. For the first two distance categories up to 50 km, the spillover effect of the time-lagged GDP per capita growth in neighboring regions is greater than the direct effect in the own region. Whereas the direct effect amounts to 0.030, the spillover effect can be as high as 0.076 in these distance categories. Normally, one would expect the spillover effect to be smaller than the direct effect, even though it is a cumulative effect measured over all other regions in the sample (see Eq. (6b)). This can be explained, though, by the fact that some regions in our sample are located so close to each other geographically that

[12] See also Billé et al. (2023) for a similar finding in Italian regions.

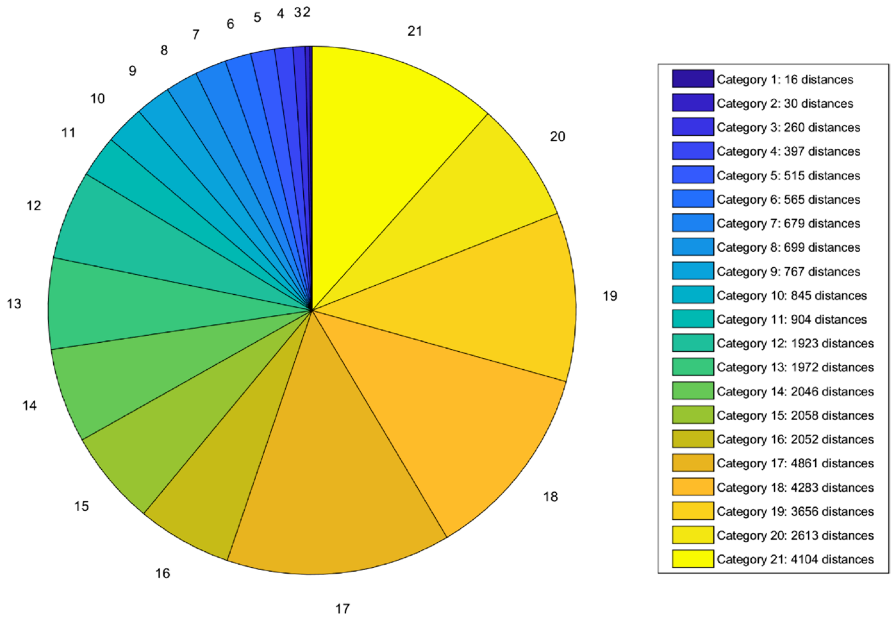

Fig. 2 Pie chart of distance categories between the 266 EU NUTS-2 regions. *Note*: Distance is split up in 21 classes measured in kilometers: 1—(0–25] (too small to be visible in this figure). 2—(25–50]. 3—(50–100]. 4—(100–150]. 5—(150–200]. 6—(200–250]. 7—(250–300]. 8—(300–350]. 9—(350–400]. 10—(400–450]. 11—(450–500]. 12—(500–600]. 13—(600–700]. 14—(700–800]. 15—(800–900]. 16—(900–1000]. 17—(1000–1250]. 18—(1250–1500]. 19—(1500–1750]. 20—(1750–2000]. and 21—> 2000 km

they form a cluster. It concerns neighboring regions around large metropolitan areas such as Brussels, London, Berlin, Prague and the cities of The Hague and Rotterdam (both located in South-Holland). It is to be noted that the situation of having neighboring regions within a distance of 50 km only occurs for a limited number of regions in our sample (0.13%). When regions form such clusters, as documented by Meijers and Burger (2017), they effectively borrow size from each other and benefit from agglomeration effects that are both intra- and inter-regional, which in our model are captured by spillover effects rather than the direct effect.

This pattern can also be seen for the investment rate where the spillover effect decreases with distance markedly, as does the time-lagged GDP per capita growth. The difference is that it only exceeds the direct effect when it comes to nearby regions up to 25 km. For the spillover effect of the population growth rate, which is negative, we see that its absolute value decreases with distance, gradually reaches a value of zero, and that even for nearby regions in the smallest distance category of 25 km, it is approximately five times as small as the direct effect.

Finally, the spillover effect of the initial level of GDP per capita exhibits a more complex relationship with distance. It is negative at first, then decreases in magnitude with distance, becomes positive around 100 km, increases further up to 350 km and finally falls back to zero over a range of 350 to 1250 km. It shows that nearby regions with high levels of GDP per capita strengthen the convergence effect,

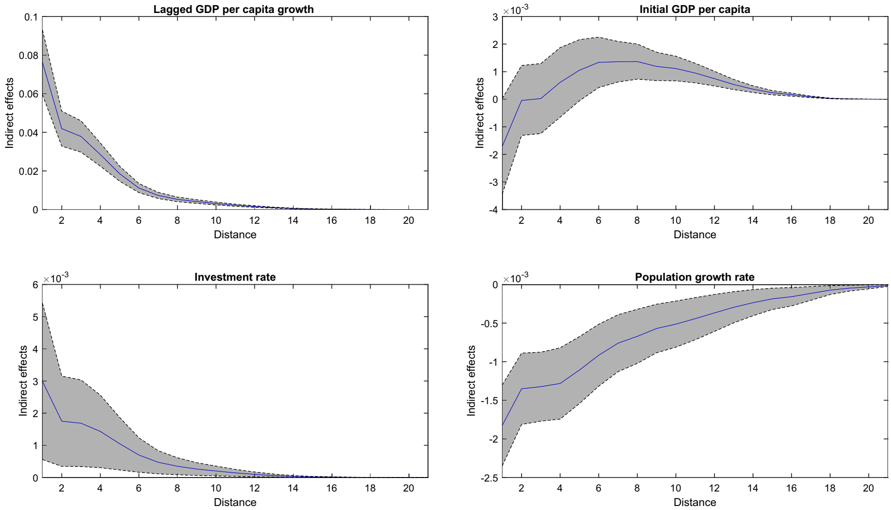

Fig. 3 Spatial spillover effects of the four explanatory variables of GDP per capita growth as a function of distance. *Notes*: The solid lines denote spillover effects and the dotted lines the 95% confidence intervals. Spillover effects are synonymous with indirect effects. Distance is split up in 21 categories based on Fig. 2. The graphs are based on the estimation results reported in column [4] of Table 1

whereas regions with high levels of GDP per capita located farther away and especially in the range of 150 to 500 km weaken the convergence effect. Regions that do already well in terms of growth apparently benefit from richer regions within this particular spatial range, which often concern centrally located regions in the EU though in different countries.

Another notable observation from Fig. 3 is the slope with which the spillover effects decay and their spatial reach is different for different growth determinants. The first is most obvious for the initial level of GDP per capita, which follows a completely different distance decay pattern than the other growth determinants. The second is most obvious for the population growth rate, which turns out to have a spatial reach even beyond 1500 km, whereas the spatial reach of the investment rate does not tend to be greater than 700 km, and of both the growth rate and the initial level of GDP per capita not to be greater than 1250 km. If we had adopted one common spatial weight matrix for all spatial lags in the model, as in the first three columns of Table 1 and constructed the same graphs, their slope and spatial reach would be exactly the same for every growth determinant. More specifically, the graph of the initial level of GDP per capita would change in a downward-sloping graph only, while the spatial reach of the population growth rate would become the same as that of the other growth determinants.

To illustrate this, Fig. 4 graphs the spillover effects of the initial level of GDP per capita and the population growth rate based on the six nearest neighbor matrix and the estimation results reported in column [1] of Table 1. Instead of the inverse U-shaped form in Fig. 3 starting with negative values first, the spillover effects of the initial level of GDP per capita in Fig. 3 start with positive values and indeed are downward-sloping only. Similarly, instead of differing spatial ranges in Fig. 3, the spatial range of both curves in Fig. 4 indeed amounts to the same value of 500 km. Further note that these differences are consistent with the estimated spillover effects reported in the different columns of Table 1. The summary measure of the spillover effects in column [4] is 2.6 times as large for the initial level of GDP per capita and 4.2 times as large for the population growth rate compared to their counterparts in column [1].

We conclude that the sensitivity of the spillover effects to the specification of the spatial weight matrix contrasts with the relative stability of the coefficient estimates and the direct effects seen across the different columns in Table 1. This contrast throws new light on applied research using spatial econometric models. Empirical studies that want to verify whether their results are robust for the specification of the spatial weight matrix, should put more emphasis on the spillover effects rather than the parameter estimates and should consider not only different spatial weight matrices, but also different ones for each spatial lag in their model.

5.4 Robustness checks

Since the empirical literature usually works with different variants of the spatially augmented neoclassical growth framework, in this section we briefly draw attention to three alternative specifications. Their results are reported in Table 2.

Column [1] shows the results when the lagged growth rate is removed from the preferred model. This simpler version has been estimated in several studies, among which the original study of Ertur and Koch (2007). Due to removing this regressor variable, the number of observations increases from 4522 to 4788. The disadvantage of this model run is that the spillover effects caused by lagged and spatially lagged growth rates can no longer be determined, whereas the first graph of Fig. 3 showed that they are more than worth considering.

Column [2] continues with the results when the preferred model is extended to include additional explanatory variables taken from endogenous growth models (Ertur and Koch 2011; Jung and López-Bazo 2017). It concerns the share of the population with tertiary education, as a proxy for regional differences in educational attainment, and the share of employment in science and technology, as a proxy for the share of resources used in research and development. The added value of this extension appears to be limited though. Out of all the additional parameters, six in total, only the coefficient of the share of resources used in research and development appears to be statistically significant. Furthermore, this extension is rejected by the data by comparing the AIC in column [2] of Table 2 with that in column [4] of Table 1.

Column [3] shows the results when the preferred model is not estimated based on annual observations but on three-year overlapping averages, as is more common in this growth literature. Due to taking averages, the number of observations decreases from 4522 to 4256. Given that regressor variables are already averaged, we use the specification without the lagged growth rates. When we compare the results in this column with those in column [1] of the same table, we see that the

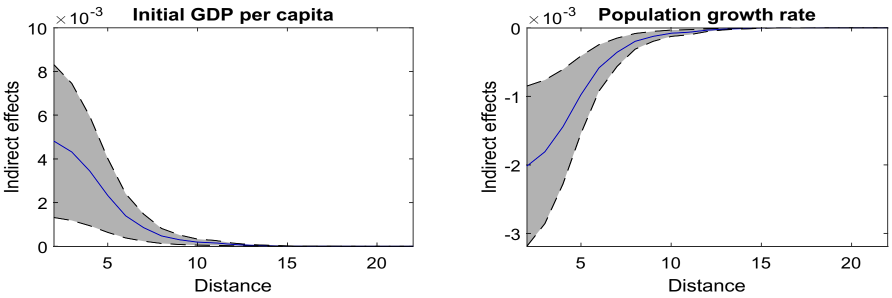

Fig. 4 Spatial spillover effects of two explanatory variables of GDP per capita growth as a function of distance based on one common six nearest neighbors spatial weight matrix. *Notes*: The solid lines denote spillover effects and the dotted lines the 95% confidence intervals. Spillover effects are synonymous with indirect effects. Distance is split up in 21 categories based on Fig. 2. The graphs are based on the estimation results reported in column [1] of Table 1

significance levels of almost all coefficient estimates and marginal effects improve. This approach could thus help to narrow the graphically displayed confidence intervals of the spillover effects.[13]

6 Conclusion

In his contribution to the Handbook of Applied Spatial Analysis, Arthur Getis listed eleven attributes of the concept of spatial autocorrelation. In this paper we translate these attributes into present-day spatial econometrics and estimate the distance decay effect and spatial reach of spillover effects in a SD model. We apply this methodology to study spillovers in GDP per capita growth across EU regions and illustrate these effects and their confidence intervals as a function of distance.

This approach contrasts with the standard practice in empirical studies of routinely reporting for each regressor the direct and spillover effects as two numerical summary measures. Instead, the exposition of the spillover effects based on the graphs developed in this paper constitutes in our view an important step forward in the existing literature. This is because they disentangle the spillover effects as a function of distance, which is one of the major topics in regional science, spatial economics and economic geography. Furthermore, since the spillover effects of the regressors tend to be the main focus of many spatial econometric studies, these graphs may contribute to a better understanding of these effects.

By parameterizing the spatial weight matrix of each spatial lag by a different distance decay parameter, we also show that the spatial reach of the spillover effect of each regressor is no longer the same, which from an empirical viewpoint further

[13] This approach may be considered if outliers are a problem. However, the percentage of residuals greater than three times the standard deviation does not vary more than between 1.19 and 1.53% across the seven regressions in Tables 1 and 2. The largest annualized residual, averaged over all regions in absolute value, of no more than 1.04 occurs in 2009, which is due to the crises described in Sect. 2 and illustrated in Fig. 1.

Table 2 Estimation results of robustness checks

	[1]		[2]		[3]	
	coeff	*p* value	coeff	*p* value	coeff	*p* value
Coefficient estimates regressors ($\beta_{m'}$)						
$\ln(inv_t)$	0.002	0.27	0.002	0.36	0.009	0.00
$\ln(n_t+g+q)$	−0.007	0.06	−0.008	0.02	0.003	0.34
$\ln(educ_t)$			−0.004	0.00		
$\ln(sci\&tech_t)$			0.011	0.00		
$\ln(y_{t-1})$	−0.081	0.00	−0.091	0.00	−0.099	0.00
$\Delta\ln(y_{t-1})$			0.007	0.76		
Coefficient estimates spatial lags (θ_m)						
$W^*\ln(inv_t)$	0.015	0.00	0.015	0.00	0.015	0.00
$W^*\ln(n_t+g+q)$	−0.011	0.02	−0.019	0.00	−0.050	0.00
$W^*\ln(educ_t)$			0.070	0.00		
$W^*\ln(sci\&tech_t)$			−0.007	0.00		
$W^*\ln(y_{t-1})$	0.075	0.00	0.078	0.00	0.091	0.00
$W^*\Delta\ln(y_t)$	0.698	0.00	0.637	0.00	0.760	0.00
$W^*\Delta\ln(y_{t-1})$			0.176	0.00		
Distance decay estimates (δ_m)						
δ *(common)**100	0.965	0.00	1.054	0.00	1.147	0.00
δ_0*100	1.316	0.11	1.296	0.16	1.284	0.03
δ_1*100	0.352	0.12	0.218	0.23	0.366	0.00
δ_2*100			0.163	0.45		
δ_3*100			2.946	0.72		
δ_5*100	0.676	0.00	0.661	0.00	0.687	0.00
δ_6*100			2.213	0.00		
Direct effects (DE_m)						
$\ln(inv_t)$	0.004	0.15	0.003	0.25	0.012	0.00
$\ln(n_t+g+q)$	−0.008	0.00	−0.009	0.00	0.000	0.86
$\ln(educ_t)$			−0.003	0.52		
$\ln(sci\&tech_t)$			0.011	0.04		
$\ln(y_{t-1})$	−0.080	0.00	−0.090	0.00	−0.100	0.00
$\Delta\ln(y_{t-1})$			0.029	0.05		
Indirect/spillover effects (SE_m)						
$\ln(inv_t)$	0.054	0.00	0.042	0.01	0.086	0.00
$\ln(n_t+g+q)$	−0.052	0.05	−0.066	0.15	−0.195	0.00
$\ln(educ_t)$			0.185	0.46		
$\ln(sci\&tech_t)$			0.002	0.93		
$\ln(y_{t-1})$	0.062	0.07	0.057	0.13	0.067	0.01
$\Delta\ln(y_{t-1})$			0.473	0.00		
LogL	10,910		10,991		12,908	
AIC	−21,800		−21,942		−25,795	
CD test residuals	−1.142		−1.083		−0.588	
# Observations	4788		4522		4256	

Table 2 (continued)

Estimates are based on parameterized negative exponential matrices with different decay parameters for each spatially lagged variable. Regional and time fixed effects are controlled for in all columns

Regionally clustered heteroskedasticity-robust significance values

[1] = Model without lagged growth rates

[2] = Model extended to include endogenous growth variables

[3] = Model estimated based on three-year averages rather than annual observations and without lagged growth rates

enhances the flexibility of these effects. This finding highlights the restrictiveness of the SD model based on one common spatial weight matrix for all spatial lags, reflecting the standard in spatial econometric research up to now. In their 2009 spatial econometric textbook, LeSage and Pace (2009, pp. 72–73) presented the spillover effects from first- to ninth-ordered neighbors numerically in an attempt to disentangle the spillover effects across space. However, hardly any study has explored this further. We hope that graphing the spillover effects for each individual regressor, as in this paper, will be followed up in more studies. While we opted to illustrate this methodology based on regional data, it can also be applied to finer micro-datasets, which should allow for a more precise estimation of spillover effects.

Appendix: Brief description of the QML estimator

The log-likelihood (LogL) of the spatially augmented neoclassical growth framework with parameterized spatial weight matrices reads as

$$\ln L(\vartheta) = -\frac{NT}{2}\ln\left(2\pi\sigma^2\right) + T\ln\left|I_N - \theta_0 W\left(\delta_0\right)\right| - \frac{1}{2\sigma^2}e^{*\prime}e^{*}. \tag{7}$$

where $\vartheta = \left(\zeta, \sigma^2, \theta_0, \delta_0. \dots .\delta_{M+2}\right)$ and $\zeta = \left(\beta_1, \dots, \beta_{M+2}, \theta_1, \dots, \theta_{M+2}, \xi_1, \dots, \xi_{T-1}\right)$. The parameter vector ζ captures the response coefficients of the explanatory variables, their spatial lags and the time fixed effects, as specified in Eq. (2) and below shortly symbolized by Z^*. The superscript * is used to denote the demeaned values of the variables for cross-sectional fixed effects. For reasons specified below, the response parameter θ_0 of the lagged dependent variable $W\left(\delta_0\right)\Delta Y^*$ is taken separately in the estimation. Finally, it is assumed that the data are sorted first by time and then by cross-sectional unit.

The QML estimator of ζ and σ^2 can be solved analytically from the log-likelihood function conditional on the remaining parameters:

$$\hat{\zeta}\left(\theta_0, \delta_0, \dots, \delta_{M+2}\right) = \left(Z^{*\prime}Z^*\right)^{-1}Z^{*\prime}S\Delta Y^*. \tag{8}$$

$$\hat{\sigma}^2\left(\theta_0, \delta_0, \dots, \delta_{M+2}\right) = \frac{1}{NT}(S\Delta Y^* - Z^*\zeta)'(S\Delta Y^* - Z^*\zeta). \tag{9}$$

where $S = I_T \otimes \left(I_N - \theta_0 W\left(\delta_0\right)\right)$. By substituting these solutions in (7), the concentrated log-likelihood function of θ_0 and the distance decay parameters δ_m $(m = 0, \ldots, M + 2)$ is obtained

$$\ln L\left(\theta_0, \delta_0, \ldots, \delta_{M+2} | \hat{\zeta}, \hat{\sigma}^2\right) = -\frac{NT}{2} \ln\left(2\pi\hat{\sigma}^2\right) + T \ln\left|I_N - \theta_0 W\left(\delta_0\right)\right| - \frac{NT}{2} \tag{10}$$

where $\hat{\sigma}^2$ is programmed as in (9), and $\hat{\zeta}$ as part of this expression is programmed as in (8). This iterative two-stage setup has the effect that if one or more values of θ_0 and δ_m change, the estimates for $\hat{\zeta}\left(\theta_0, \delta_0. \ldots, \delta_{M+2}\right)$ and $\hat{\sigma}^2\left(\theta_0, \delta_0, \ldots, \delta_{M+2}\right)$ change accordingly in the maximization process.

By demeaning the variables for the cross-sectional fixed effects, the transformed errors e^* become linearly dependent. Consequently, $\hat{\sigma}^2$ will be biased when T is small or fixed. To get an unbiased estimate, Lee and Yu (2010) propose the bias correction (bc) $\hat{\sigma}^2_{bc} = (T/(T-1)\hat{\sigma}^2$. This correction can easily be carried out after the parameters of the model have been estimated.

By making the spatial weight matrices dependent on distance decay parameters, they become stochastic in the sense that they are subject to a margin of error. Gupta (2019) shows that many established estimation methods also work with an exogenous stochastic spatial weight matrix, as long as the sample size N diverges to infinity faster than the row and column sums of the stochastic spatial weight matrices. This condition requires that the distance decay parameters of the negative exponential distance decay matrix are strictly positive (Tan 2023. p. 20). Furthermore, by row-normalizing the exponential distance decay matrices, each pair of parameters (θ_m, δ_m) $(m = 0, \ldots, M + 2)$ is identified, as long as each response parameter θ_m is bounded away from zero. By contrast, if we would allow $\theta_m = 0$, the row and column elements of this response parameter in the information matrix equal zero, as a result of which it is not invertible and the variance covariance matrix not defined.

The response and distance decay parameters can be estimated by maximum likelihood (ML) or quasi-(Q)ML, depending on whether or not the error terms are assumed to be normally distributed. If their distribution is not specified, as in this paper, application of QML will have a downward effect on the significance levels of the parameter estimates, because the asymptotic distribution of the QML estimator is

$$\sqrt{NT}\left(\hat{\vartheta} - \vartheta\right) \rightarrow N(0, \Psi). \tag{11}$$

with

$$\Psi = \lim \frac{T}{T-1}\left(\frac{1}{NT}\Sigma_\vartheta\right)^{-1}\left(\frac{1}{NT}\Sigma_\vartheta + \frac{T-1}{T}\frac{\mu_4 - 3\sigma^4}{\sigma^4}\Omega_\upsilon\right)\left(\frac{1}{NT}\Sigma_\vartheta\right)^{-1}. \tag{12}$$

where μ_4 denotes the fourth moment of the error terms and the variance–covariance matrix Σ_ϑ and the correction matrix Ω_υ are specified in Tan (2023. pp. 42–44). This expression shows that only if the error terms are assumed to be normally distributed, the impact of Ω_υ cancels out. This is because $\mu_4 - 3\sigma^4 = 0$ under this circumstance, yielding $\psi = \lim \frac{T}{T-1} \frac{1}{NT} \Sigma_{\vartheta}^{-1}$. In addition to this, the significance levels of the parameter estimates of the regressors are corrected for heteroskedasticity.

Further technical details of the QML estimator—first- and second-order derivatives, the derivation of the information matrix and the variance–covariance matrix, an econometric-theoretical proof that this estimator is asymptotically normal if N goes to infinity, and an overview of the conditions under which the response and distance decay parameters are identified—are described in Tan (2023, Chapter 2). The data used and programming code developed to generate the results reported in Tables 1 and 2 and Figs. 3 and 4 are made available at spatial-panels.com. The results obtained meet all the identification conditions discussed in this appendix.

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References

Ades A, Chua HB (1997) Thy neighbor's curse: regional instability and economic growth. J Econ Growth 2:279–304

Anselin L, Rey SJ (2014) Modern spatial econometrics in practice. GeoDa Press LLC

Anselin L, Bera AK, Florax R, Yoon MJ (1996) Simple diagnostic tests for spatial dependence. Reg Sci Urban Econ 26(1):77–104

Arbia G, Bera AK, Doğan O, Taşpınar S (2020) Testing impact measures in spatial autoregressive models. Int Reg Sci Rev 43(1–2):40–75

Audretsch DB, Feldman MP (2004) Knowledge spillovers and the geography of innovation. In: Henderson JV, Thisse JF (eds) Handbook of regional and urban economics, vol 2. Elsevier, Amsterdam, pp 2713–2739

Autant-Bernard C, LeSage JP (2011) Quantifying knowledge spillovers using spatial econometric models. J Reg Sci 51(3):471–496

Benos N, Karagiannis S, Karkalakos S (2015) Proximity and growth spillovers in European regions: the role of geographical, economic and technological linkages. J Macroecon 43:124–139

Beugelsdijk S, Klasing MJ, Milionis P (2018) Regional economic development in Europe: the role of total factor productivity. Reg Stud 52(4):461–476

Billé AG, Tommelleri A, Ravazzola T (2023) Forecasting regional GDPs: a comparison with spatial dynamic spatial models. Spat Econ Anal 18(4):530–551

Bottazzi L, Peri G (2003) Innovation and spillovers in regions: evidence from European patent data. Eur Econ Rev 47(4):687–710

Breinlich H, Ottaviano GI, Temple JR (2014) Regional growth and regional decline. In: Aghion P, Durlauf SN (eds) Handbook of economic growth, vol 2. Elsevier, Amsterdam, pp 683–779

Debarsy N, Ertur C, LeSage JP (2012) Interpreting dynamic space-time panel data models. Stat Methodol 9(1–2):158–171
Diaz Depena A, Rubiera-Morollon F, Paredes D (2019) New approach to economic convergence in the EU: a multilevel analysis from the spatial effects perspective. Int Reg Sci Rev 42(3–4):335–367
Döring T, Schnellenbach J (2006) What do we know about geographical knowledge spillovers and regional growth?: a survey of the literature. Reg Stud 40(3):375–395
Easterly W, Levine R (1997) Africa's growth tragedy: policies and ethnic divisions. Quart J Econ 112(4):1203–1250
Elhorst JP (2014) Spatial econometrics: from cross-sectional data to spatial panels. Springer, Berlin
Elhorst JP, Piras G, Arbia G (2010) Growth and convergence in a multiregional model with space–time dynamics. Geogr Anal 42(3):338–355
Ertur C, Koch W (2007) Growth, technological interdependence and spatial externalities: theory and evidence. J Appl Econom 22(6):1033–1062
Ertur C, Koch W (2011) A contribution to the theory and empirics of Schumpeterian growth with worldwide interactions. J Econ Growth 16(3):215–255
Ezcurra R, Rios V (2020) Quality of government in European regions: do spatial spillovers matter? Reg Stud 54(8):1032–1042
Fiaschi D, Lavezzi AM, Parenti A (2018) Does EU cohesion policy work? Theory and evidence. J Reg Sci 58(2):386–423
Fischer MM, Getis A (eds) (2010) Handbook of applied spatial analysis: software tools, methods and applications. Springer, Berlin
Fotheringham AS, Yang W, Kang W (2017) Multiscale geographically weighted regression (MGWR). Ann Am Assoc Geogr 107(6):1247–1265
Fotheringham AS, Oshan TM, Li Z (2024) Multiscale geographically weighted regression: Theory and practice. CRC Press, Boca Raton
Funke M, Niebuhr A (2005) Regional geographic research and development spillovers and economic growth: evidence from West Germany. Reg Stud 39(1):143–153
Gardiner B, Martin R, Tyler P (2011) Does spatial agglomeration increase national growth? Some evidence from Europe. J Econ Geogr 11(6):979–1006
Getis A (2008) A history of the concept of spatial autocorrelation: a geographer's perspective. Geogr Anal 40(3):297–309
Getis A (2009) Spatial weights matrices. Geogr Anal 41(4):404–410
Getis A (2010) Spatial autocorrelation. In: Fischer MM, Getis A (eds) Handbook of applied spatial analysis. Springer, Berlin, pp 255–278
Getis A, Aldstadt J (2004) Constructing the spatial weights matrix: using a local statistic. Geogr Anal 36(2):90–104
Getis A, Ord JK (1992) The analysis of spatial association by use of distance statistics. Geogr Anal 24(3):189–206
Glaeser EL, Kallal HD, Scheinkman JA, Shleifer A (1992) Growth in cities. J Polit Econ 100(6):1126–1152
Gupta A (2019) Estimation of spatial autoregressions with stochastic weight matrices. Econom Theor 35:417–463
Halleck Vega S, Elhorst JP (2015) The SLX model. J Reg Sci 55(3):339–363
Harris R (2011) Models of regional growth: past, present and future. J Econ Surv 25(5):913–951
Henderson V, Kuncoro A, Turner M (1995) Industrial development in cities. J Polit Econ 103(5):1067–1090
Islam N (1995) Growth empirics: a panel data approach. Q J Econ 110(4):1127–1170
Juhl S (2021) The Wald test of common factors in spatial model specification search strategies. Polit Anal 29(2):193–211
Jung J, López-Bazo E (2017) Factor accumulation, externalities, and absorptive capacity in regional growth: evidence from Europe. J Reg Sci 57(2):266–289
Lee LF, Yu J (2016) Identification of spatial Durbin panel models. J Appl Econom 31(1):133–162
LeSage JP, Pace RK (2007) A matrix exponential spatial specification. J Econom 140(1):190–214
LeSage JP, Pace RK (2009) Introduction to spatial econometrics. Chapman and Hall/CRC, Boca Raton
López-Bazo E, Vayá E, Artis M (2004) Regional externalities and growth: evidence from European regions. J Region Sci 44(1):43–73
Márquez MA, Ramajo J, Hewings GJ (2015) Regional growth and spatial spillovers: evidence from an SpVAR for the Spanish regions. Pap Reg Sci 94:S1–S18

Meijers EJ, Burger MJ (2017) Stretching the concept of ‘borrowed size. Urban Stud 54(1):269–291
Moreno R, Trehan B (1997) Location and the growth of nations. J Econ Growth 2(4):399–418
Ord JK, Getis A (1995) Local spatial autocorrelation statistics: distributional issues and an application. Geogr Anal 27(4):286–306
Panzera D, Postiglione P (2022) The impact of regional inequality on economic growth: a spatial econometric approach. Reg Stud 56(5):687–702
Pesaran MH (2015) Testing weak cross-sectional dependence in large panels. Econom Rev 34(6–10):1089–1117
Pfaffermayr M (2012) Spatial convergence of regions revisited: a spatial maximum likelihood panel approach. J Reg Sci 52(5):857–873
Ramajo J, Marquez MA, Hewings GJ, Salinas MM (2008) Spatial heterogeneity and interregional spillovers in the European Union: do cohesion policies encourage convergence across regions? Eur Econ Rev 52(3):551–567
Rodríguez-Pose A, Crescenzi R (2008) Research and development, spillovers, innovation systems, and the genesis of regional growth in Europe. Reg Stud 42(1):51–67
Rosenthal SS, Strange WC (2020) How close is close? The spatial reach of agglomeration economies. J Econ Perspect 34(3):27–49
Solow RM (1956) A contribution to the theory of economic growth. Q J Econ 70(1):65–94
Swan TW (1956) Economic growth and capital accumulation. Econ Rec 32(2):334–361
Tan C (2023) Modelling spatial weight matrices and lags in spatial panel models. Dissertation, University of Groningen. https://doi.org/10.33612/diss.630685888
Yu J, de Jong R, Lee LF (2012) Estimation for spatial dynamic panel data with fixed effects: the case of spatial cointegration. J Econom 167:16–37