J Geogr Syst (2011) 13:31–48
DOI 10.1007/s10109-010-0131-7

ORIGINAL ARTICLE

Morphological similarities between DBM and a microeconomic model of sprawl

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Received: 18 February 2009 / Accepted: 18 August 2010 / Published online: 7 September 2010

Abstract We present a model that simulates the growth of a metropolitan area on a 2D lattice. The model is dynamic and based on microeconomics. Households show preferences for nearby open spaces and neighbourhood density. They compete on the land market. They travel along a road network to access the CBD. A planner ensures the connectedness and maintenance of the road network. The spatial pattern of houses, green spaces and road network self-organises, emerging from agents individualistic decisions. We perform several simulations and vary residential preferences. Our results show morphologies and transition phases that are similar to *Dieletric Breakdown Models* (DBM). Such similarities were observed earlier by other authors, but we show here that it can be deducted from the functioning of the land market and thus explicitly connected to urban economic theory.

Keywords Urban sprawl · Open space · Neighbourhood externalities · Road network · Dielectric breakdown · Fractal

JEL Classification C61 · C63 · D62 · R21 · R40

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1 Introduction

We propose a theoretical urban growth model where households choose to locate among the cells of a 2D grid space, following a *Cellular Automata* (CA)-like dynamics. Land parcels can either be developed into housing or roads, or remain in agricultural use. The individual choice of households drives the growth pattern of the city, while the development of the road network follows in order to connect each resident to the *Central Business District* (CBD) given exogenously. The city self-organises and leads to spatial patterns where houses, roads, and green spaces are mixed-up.

Our model accounts for three essential features of urban sprawl patterns and processes. First, the mixing of green and developed land uses within the commuting area of a city is a key characteristic of sprawl. Empirical research has shown that households value the spatial arrangement of residential density and green spaces in their neighbourhood. Households trade-off those neighbourhood amenities with commuting costs when buying land or a house (see e.g. Anderson and West 2006; Cavailhès et al. 2006; Cheshire and Sheppard 1995; Irwin 2002). Second, urban sprawl is often defined as a scattered pattern resulting from a set of uncoordinated individual actions (see e.g. Galster et al. 2001). It is thus important to use a dynamic approach to represent the sequence of individual decisions, and not to be constrained by exogenous macro attributes (planning zones, exogenous density gradient, etc.). Third, the shape of the road network affects the commuting costs of households and influences the pattern of residential development, e.g. by orientating growth in particular directions or leading to ribbon-type developments.[1]

The patterns resulting from our simulations show morphological similarities with *Dieletric breakdown models* (DBM) proposed in physics. While such a similarity was previously pointed out by several authors, and used to build urban diffusion models, the similarity is more surprising here since our model is entirely grounded on microeconomic theory without a priori connections with DBM. Our contribution is therefore to show that this morphological property holds with microeconomic foundations, thus filling in a gap between urban economic theory and theoretical urban models only inspired by physical diffusion processes.

In the following Sect. 2, we shortly review abstract models that are based on a self-organising framework and more particularly the DBM. Section 3 then presents the economic rationale of our model, its cellular environment, and microeconomic formulation. In Sect. 4, we provide simulation results obtained by varying households preferences. We compute measures (fractal dimensions) to show transition phases, and we discuss the correspondence between our morphologies and DBM structures. Section 5 concludes and offers some perspectives.

[1] To our knowledge, however, the literature has not yet clarified whether the shape of the network is mainly an exogenous or an endogenous factor of sprawl.

2 Self-organising cities

2.1 Cellular automata, fractals and cities

Cellular Automata (CA) and *fractals* have been used widely in urban geography for representing the spatial development of cities. In essence, CA use local interactions to let urban morphologies emerge through time, while fractals represent the self-similar structures of cities through scales. Many different strategies, sometimes combining CA and fractals, have been used by researchers over the past 20 years. We distinguish three categories of models.

A first group favours abstract models for studying the interactions between different groups of populations. These approaches primarily emphasise the application of the *self-organisation* concept to cities (see e.g. Schelling 1971; Couclelis 1985; Phipps 1989). In a second set of articles, urban growth is modelled more explicitly by referring directly to physical (or biological) models. They emphasise the ability of such models to generate structures and reproduce the morphological evolution of cities (see e.g. Batty and Longley 1986; Batty 1991; Frankhauser 1991; Makse et al. 1995). A third type of models then introduce rules into the previous ones in order to describe, in a heuristic way and at an aggregated level, the spatial interactions between different land uses (see e.g. White and Engelen 1993, 1994).

A common concern of these approaches is the absence of microeconomic foundations. Caruso et al. (2007) and Cavailhès et al. (2004) attempt to bridge this gap. The first within a CA setting, the second within a fractal setting. The first is dynamic, the second static but consider a road network (exogenous). Both models include open-space externalities. All together, therefore, they share important ingredients for further understanding urban sprawl. As we shall see, the model presented here is a first attempt to combine both approaches in a dynamic framework with an emerging road network.

2.2 DLA, DBM and cities

In parallel to these developments, other concepts have been proposed to simulate the formation of clusters in a 2D space and have then been applied to urban development. One example in physics is the *Diffusion-Limited Aggregation* (DLA) model, proposed by Witten and Sander (1981). Starting from an exogenous seed, clusters are created by the sequential addition of particles, which stick to the generated cluster after a random walk. Such a random walk can be described formally by a master equation, describing the change in the probability to find a particle at a given site. DLA processes generate clusters with a fractal dimension around 1.7.

Combining the DLA model with electric fields, Niemeyer et al. (1984) introduced the *Dielectric Breakdown Model* (DBM) to simulate electric discharge patterns within solids, liquids, and gases, and to explain the formation of the branching, self-similar Lichtenberg figures. Pietronero and Wissman (1984) established a formal link between the DLA model and the DBM. Both models have been applied in physics [e.g., discharges in non-homogeneous material by Peruani et al. (2003), surface thermodynamics by Bogoyavlenskiy and Chernova (2000)], chemistry, or biology (see e.g. Chikushi and Hirota 1998; Li et al. 1995).

Urban geographers took interest into these physical models because of their ability to represent cluster formation and fractal growth. The DLA model was applied to simulate urban growth by Benguigui (1995a,1998) or in a percolation version by Makse et al. (1998). DBM principles were applied to cities by Batty (1991) or Andersson et al. (2002). Batty and Longley (1994, Chap. 8) discuss extensively the analogy of DLA and DBM with cities and more particularly the ability of DBM to simulate cities of various fractal dimensions.

In order to show how the main control parameter of the DBM affects the fractal dimension of the output morphologies, we need to formalise the model. In DBM, the electrodynamic Laplace equation, describing the spatial variation of the electric potential Φ, is transformed into a discrete equation in order to compute the electric potential for each cell at a given simulation time step. The probability of a site i', in contact with an already generated discharge, to be selected and added to the discharge pattern in the next stage depends on the following potential:

$$p_{i \to i'} = \frac{(\Phi_{i'})^{\eta}}{\sum_{i'} (\Phi_{i'})^{\eta}} \tag{1}$$

After having selected the site i', a link is created between i' and the existing discharge, and the potential in i' falls to zero. The parameter η plays an important role for the shape of the generated patterns. As shown by Sanchez et al. (1993), $\eta = 0$ results in a compact cluster, i.e a fractal dimension of 2. Conversely, linear/ribbon-like structures are observed when $\eta > 4$, with fractal dimensions closer to 1. In Fig. 1, we illustrate patterns and transitions that occur from such a variation of η using examples from Bogoyavlenskiy and Chernova (2000). Actually, this is an extended DBM used for studying the relationship between surface thermodynamics and crystal morphology. We use this example on purpose because it results in less dendritic structures which are more similar with the outputs of our model (which is also more complex) (see Sect. 4).

Refining the analysis of Sanchez et al. (1993), one can show that the DBM results in a continuum of forms, with no abrupt transitions. Mathiesen et al. (2008) reported a set of fractal dimensions obtained by varying η from 1 to 4 and showed that the fractal dimension is gradually decreasing from 1.71 (i.e. $\eta = 1$, corresponding to

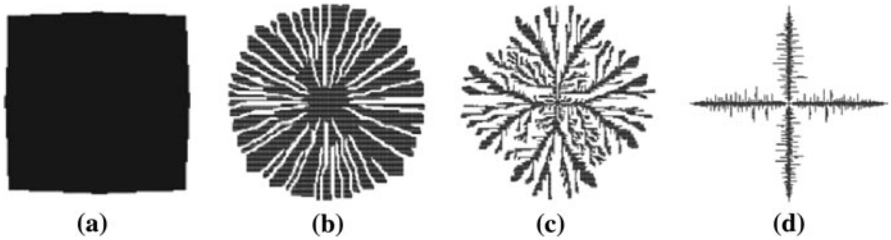

Fig. 1 Patterns of a complex diffusive growth model when varying η (*Source* Bogoyavlenskiy et al. 2000). **a** $\eta = 0.02$, **b** $\eta = 0.10$, **c** $\eta = 1.00$, **d** $\eta = 4.00$

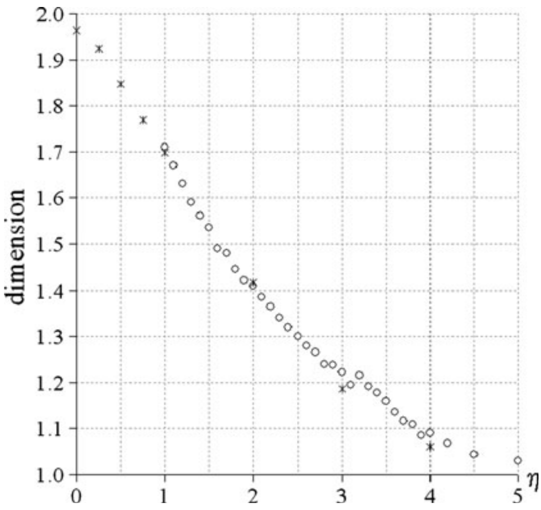

Fig. 2 Fractal dimensions with varying η in a DBM. *Circles* are mean values redrawn from Mathiesen et al. (2008) Fig. 4, and crosses are mean values from Batty and Longley (1994), p. 299

DLA) to 1.10 (i.e. close to linear development). Batty and Longley (1994) also discussed the change in η in terms of a continuum and reported fractal dimensions. We display their measures together with dimensions from Mathiesen et al. (2008) in Fig. 2.

2.3 Urban sprawl interpretation of DBM

Within an urban context, Batty (1991) and Batty and Longley (1994) interpret η as a parameter describing different planning controls. $\eta > 1$ would lead to idealistic geometric city forms, $\eta = 1$ would represent uncontrolled fractal growth, and $\eta < 1$ would be another form of planning. However, this interpretation remains rather loose in absence of a clear microscopic link between the functioning of the model and the decision making of urban agents (residents, planners, etc).

In our view, this lack of micro-foundations is the first, and probably the most important, of two shortcomings of DLA and DBM applications to urban processes. The second problem is that DLA and DBM can only generate connected aggregates. It thus seems difficult to apply them to urban sprawl which by definition is a scattered pattern. This led Benguigui et al. (2001) to propose an approach where residential leapfrogging is introduced. Our model also includes the possibility of leapfrogging. Any reference to DLA-DBM analogies will therefore be based solely on the morphology of the road network, which of course continuously connects all parts of a settlement.

Treating the first problem is more ambitious and requires to state a link between DBM processes and agents' behaviour. We all know that land is an immobile good and that residential choice is therefore a matter of trading-off different geographically located characteristics. Batty and Longley (1994, p. 281) take a first step in that direction when they mention that the adjacency constrain in the DBM represents the scale economies of connected spatial clusters and balances the highest space potential that one can find near the tips of the DBM branches.

Andersson et al. (2002) also pointed the lack of micro-principles in DLA and DBM and take a step further by proposing a model, based on *Markov Random Field*, embedded with economic rationale. Their *Unwilling Neighbour* model generates outputs that are similar to DLA by displaying two counteracting forces: a nearest neighbour attraction force, favouring edge growth, and a longer distance interaction force where development density inhibits further development. The reasoning is that the first corresponds to the benefits of being attached to roads and service infrastructures, while the second represents the disadvantage of higher land prices in more densely developed areas.

Still the link with decision making is not explicit in the formulation and detached from microeconomic reasoning. For example, in a CA setting, Caruso et al. (2007) showed that the price mechanism is different. In fact, residents would bid higher for low-density sites providing more open space at the limit of a city. Then it is because landowners are many and uncoordinated that residents can pocket a surplus of utility by locating in those sites. Another example is the role of the *temperature of the system* leading to noise and interpreted as information deficiencies but without a formal link with models of choice in imperfect information contexts.

3 Self-organisation with economic agents: the S-GHOST model

We develop an urban growth model called *S-GHOST*, i.e. *Self-Generating Housing Open-Space and Transportation*. The purpose of this model is to investigate the development of a city as emerging from individual decisions of identical economic agents. Our starting point is residential microeconomics, independently of physical spatial diffusion processes. Resemblances with physical models will only be analysed *ex-post* using appropriate fractal measures.

We particularly aim at further understanding sprawl patterns. Therefore, following urban economic theory, we model a land market, include the valuation of open-space externalities within residential location, and assume that households commute to a *Central Business District* (CBD). The model is dynamic and the road network is endogenous. Our simulations reveal how houses, roads, and green (agricultural) spaces self-organise around the CBD through time.

3.1 Environment

We consider a closed 2D space with a set of cells i. A pointwise CBD is located exogenously at the centre of the lattice. Two preexisting orthogonal roads intersect at the CBD. Initially, the rest of the grid is occupied by farmland. Each cell has three possible (mutually exclusive) states: residential, road, or agricultural (or undeveloped), which we respectively denote by j, k and l.

The grid is gradually filled in by residences and roads. Development conversions are irreversible (costs would be too high).

3.2 Agents behaviour and the growth of the city

Land belongs to absentee landowners who individually choose to rent their land parcels to the highest bidder, either a resident or a farmer, hence determining land occupancy. Farmers produce foodstuff under constant returns to scale and sell this output on the world market. As a by-product, farmers generate green amenities (open space, landscape, etc.) to the benefits of neighbouring residents.

Households arrive sequentially in the city and choose their location by maximising a utility function subject to a budget constraint. Each migrant chooses freely its location, considering the commuting cost to the CBD and the quality of the neighbourhood. Households enjoy two kinds of neighbourhood externalities: open space/agricultural amenity and local public goods. These are respectively negative and positive functions of neighbourhood density. The rent of residential plots is determined by the competition on the land market and capitalises the effects of the two externalities plus the accessibility to the CBD.

As households migrate into this growing city, agricultural cells are progressively turned into residences. A local public authority creates a connected road network so as to provide all new households with an access to the CBD.

The arrival of new migrants may change the neighbourhood qualities for residents who settled earlier. At each time step, a resident can therefore move and bid for another cell of the existing city in order to maximise his/her utility. This competition leads to an adjustment of all residential rents until all the inhabitants obtain the same utility. This is a *short-run equilibrium*, which in our model is instantaneously reached at each time step.

As residents arrive, the city expands and consequently the commuting costs of the newcomers tend to increase. Households' utility thus progressively decreases with time. Migration stops when the incentive to migrate into the city disappears, that is when households' utility equals the utility in the 'rest of the world'. This is the *long-run equilibrium* of an open-city system.

3.3 Microeconomics

Households' preferences are formalised with a standard Cobb-Douglas utility function $U = kZ^{\delta}H^{\alpha}E^{\beta}S^{\gamma}$. The arguments of the function are as follows: Z, a non-residential composite good made up of every market goods except housing; H, housing, E, open(green)-space externalities, and S, local public goods externalities. k is a constant used to simplify the writing of following equations without affecting the reasoning.

Each household maximises U under a budget constraint: $Y - \theta d = Z + SR$, where Y is the income, θ the unitary commuting cost, d the distance to the city centre and R the unitary land rent (the composite good is taken as the *numeraire*, i.e. $p_Z = 1$).

The parameters $\delta \in [0, 1]$, and $\alpha = 1 - \delta$ indicate respectively the preference for the composite good and for housing, whereas $\beta \geq 0$ and $\gamma \geq 0$ are respectively the preference parameters for open-space and for social externalities.

The first-order conditions for a constrained optimum yield the demand functions of Z and S as well as the following 'indirect utility function':

$$V = (Y - \theta d)R^{-\alpha}E^{\beta}S^{\gamma} \tag{2}$$

Households arrive one by one. At each time step t, a new migrant evaluates the indirect utility (2) provided by each agricultural cell and picks up the cell l that maximises this indirect utility function, considering the commuting cost θd_l^t to the city centre, the land rent R_l^t of the residential plot, the neighbourhood open-spaces E_l^t (i.e. the density of agricultural cells within a given radius around l observed in $t-1$), and the local public goods S_l^t (i.e. the density of residential cells in the same neighbourhood observed in $t-1$).

Let ρ_l be the density of residences within a given neighbourhood around each undeveloped cell l. We define the neighbourhood externalities as $E_l^t = e^{-\rho_l^{t-1}}$ and $S_l^t = e^{\sqrt{\rho_l^{t-1}}}$, thus respectively decreasing and increasing with local density, both at a decreasing marginal rate.

The maximum amount a migrant would accept to pay for a given cell (i.e. the reservation bid rent) is such that it provides the same level of utility he/she would obtain in 'the rest of the world'. However, because several agricultural cells provide the same utility, each newcomer puts the landowners in competition and can obtain a decrease in the rent to be actually paid. The rent drops down to Φ, the level of the agricultural rent (see Caruso et al. 2007, for a more detailed microeconomic explanation).

Therefore, the indirect utility in each l becomes

$$V_l^t = (Y - \theta d_l^t)\Phi^{-\alpha}(E_l^t)^{\beta}(S_l^t)^{\gamma}. \tag{3}$$

and residents can pocket a utility surplus when migrating into the city.

At each time step, the chosen cell is the one where Eq. 3 is maximum. The cell is denoted by l*. The utility of all households in the city at time t is therefore

$$V^t = \max_l V_l^t = V_{l*}^t \tag{4}$$

The city continues to grow as long as the utility as measured by Eq. 4 exceeds a given threshold $\overline{V}$, i.e. the utility that households can enjoy in 'the rest of the world'. When $V^t = \overline{V}$, we obtain the so-called long-run equilibrium, and city growth is stopped.

3.4 Network generation

Every household should be able to access the CBD via the road network. The road network is gradually generated over time as households migrate into the city. We make the assumption that the generation of road infrastructures follows individual residential decisions (the two preexisting orthogonal roads excepted). A public agent creates new roads based on residential demand. The sequence of action is thus the following:

First, a household decides to migrate into the city since it is possible to find a utility surplus in at least one cell. As explained above, the new household picks up the cell (l^*) where the indirect utility (Eq. 3) is maximised. The cell l^* is thus converted from agricultural to residential use. The conversion is made even if l^* is not yet connected to the existing road network (i.e. roads in $t-1$): sometimes a road adjacent to l^* preexists sometimes not.

Second, the public agent in charge of road infrastructures connects the new resident to the existing road network so that he/she can access the CBD via the network. In case l^* is already connected, the network does not change. Otherwise, the public agent expropriates owners of undeveloped cells in order to create roads and access l^*. The shape of the network results from the application by the public agent of two simple rules: a 'connection rule' and a 'minimum expropriation rule'.

The 'connection rule' states that a new residential cell must have at least one adjacent road cell, and new road cells must be adjacent to an existing road cell. When residential leapfrogs occur (simulations will show that it depends among other factors on neighbourhood size), several new roads cells can be added at each time step to access the new residential place. Several agricultural land owners can thus be expropriated in one shot.

There are often many ways to connect a new cell to the existing network. The public agent minimises the number of new road cells to be created and thus the number of expropriations at each time step. This 'minimum expropriation rule' equivalent to a 'shortest path to the existing network' is obviously not necessarily the shortest path from l^* to the *CBD*. Residents and some planners might see a 'shortest path to CBD rule' as a more desirable option since it would minimise transport costs. However, after conducted simulation tests, it turns out to be inappropriate since it is leading to many duplications of roads and an unrealistic land take by roads.

For consistency, and because households are perfectly informed of decision rules of the public agent, we further assume that new migrants evaluate transport costs (θd in Eq. 3) using the same two rules. Each new resident perfectly knows his/her future travel path even though it is not yet created at the decision moment.

4 Morphological outputs from S-GHOST

S-GHOST was implemented within a Java-based software to undertake simulation experiments. The parameters are the household's income Y, the share of income devoted to housing α, the agricultural rent Φ, the preferences for open-space and public goods externalities β and γ, the size of a cell's neighbourhood $\widehat{x}$, and the unit transportation cost θ.

At each time step, the *ex-aequo* that may arise from Eq. 4 are resolved randomly. The same holds when equivalent minimum expropriation paths can be chosen to connect new cells. Apart from these two tie-breakers, the model is fully deterministic. We verified through simulations that the random tie-breakers do not change qualitatively the results.

We examine below the long-run equilibria that emerge from three series of simulations: varying preferences for the two neighbourhood externalities in turn (neighbourhood extent being $\widehat{x}=3$) and then the distance range of these interactions ($\widehat{x}$).

4.1 Varying preference for open-space externalities

The first series, displayed in Fig. 3, is a situation where residents do not value local public goods, but are more or less interested by green space amenities. The very first example with $\beta = 0$ (Fig. 3a framed) is the standard Alonso model since no externalities are valued. Residents arrange themselves side by side along the two preexisting roads, i.e. the 1D linear model along four directions.

The same structure is obtained as long as $\beta < 0.12$. The valuation of the amenity is not strong enough to perturb the morphology. Beyond this threshold the increasing preference for green amenities leads to dendritic patterns, where houses and roads are separated by undeveloped cells (Fig. 3c–l). This pattern remains stable all over the range shown here and beyond. The form of the network remains qualitatively the same, while gradual changes are only due to an increasing number of residential leapfrogs along this dendritic network as β increases. This gradual change itself stops around $\beta = 2.50$: additional increases of β would lead to similar output since the extent of leapfrogs is limited to the size of the neighbourhood, which is fixed exogenously. We further discuss this limitation with the third simulation series.

Following other authors (see e.g. Benguigui 1995b; Lu and Tang 2004), we use fractal analysis to characterise the shape of the road network. The method is described in Frankhauser (1991), and the results for this first set of simulations are displayed on Fig. 4a. It clearly shows the stability of the network when $\beta > 0.20$. $\beta = 0.13$ is a transition state showing that the pattern gradually quits the linear system before reaching a plateau. Moreover, the fractal dimension of this plateau is around 1.75, reminding dimensions obtained with DLA and DBM (with $\eta = 1$).

4.2 Varying preference for social externalities

We now come to the second series (Fig. 5), where we fix $\beta = 0.25$, i.e. households have some preference for open-space, and vary γ-values. Four qualitatively different phases can be observed along this variation.

When γ is null, the situation is visually similar to the dendritic patterns presented previously (Fig. 3d is closest example). When γ increases (Fig. 5a–c), the two externalities gradually neutralise one each other: leapfrogs disappear and the patterns become more and more continuous and linear. Ultimately, the compensating effect of the two externalities lead to an 'Alonso'-like situation as if there were no externalities; this is the second phase starting from $\gamma = 0.11$. The residents line up on the sides of the two preexisting roads in order to minimise the commuting costs, because moving to a lateral location would increase this cost without providing enough social or open-space compensation. This situation persists up to $\gamma = 0.34$ (Fig. 5e) where the intensity of the social amenity leads to lateral streets in order to benefit for more density thus more public goods (third phase). The pattern is made of parallel streets separated by 'green alleys'. Further increase in the social amenity leads to the gradual disappearance of those alleys. From $\gamma = 0.42$, apart from small accidents, lateral streets are closer from each other, and the houses from one street are adjacent to the houses from the next street, leading to a fully compact

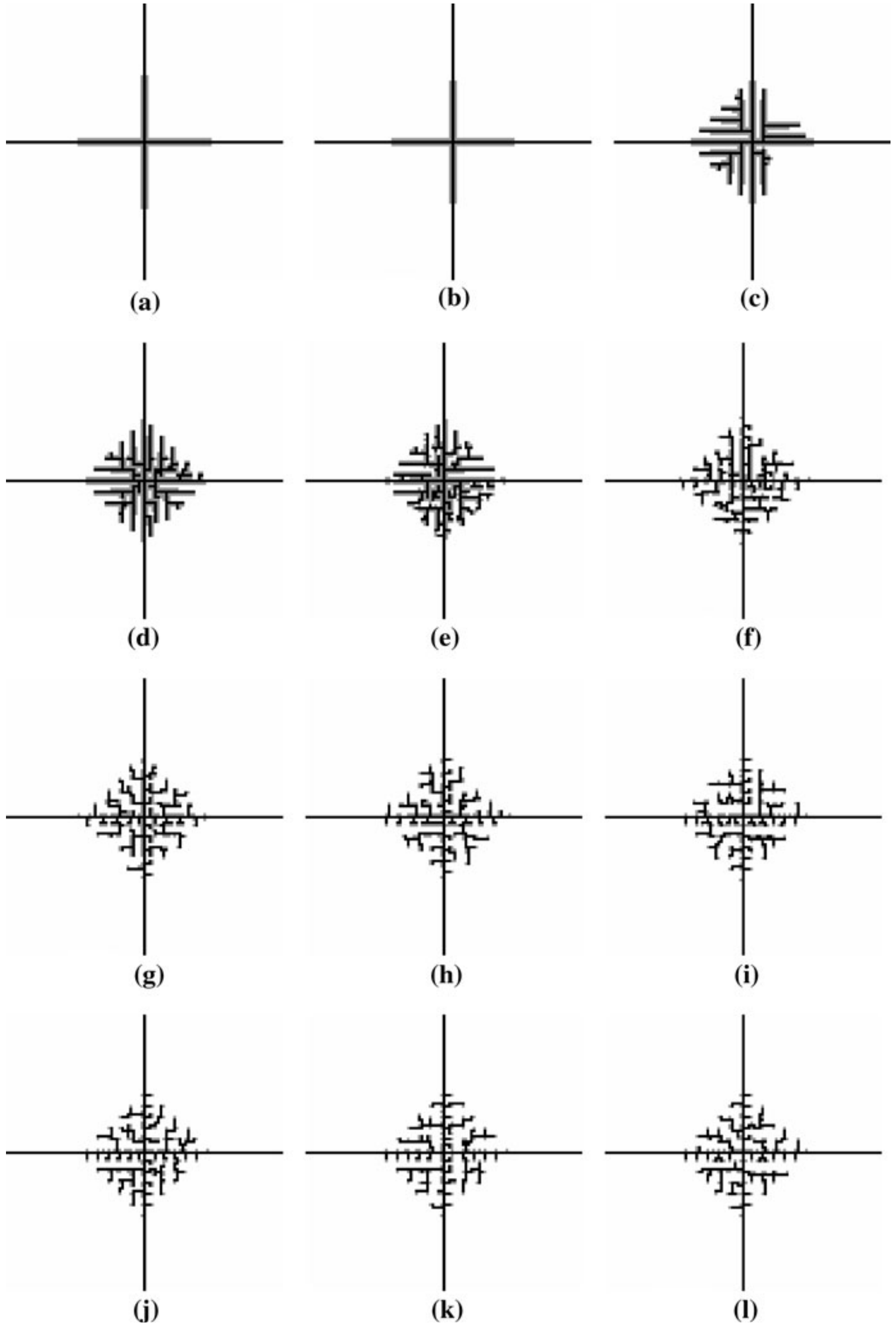

Fig. 3 Long-run equilibria for fixed $\gamma = 0.00$ and varying β. (*Black cells* roads; *grey* residences; *white* agriculture). **a** Alonso $\beta = 0.00$, **b** $\beta = 0.12$, **c** $\beta = 0.13$, **d** $\beta = 0.22$, **e** $\beta = 0.34$, **f** $\beta = 0.70$, **g** $\beta = 1.00$, **h** $\beta = 1.30$, **i** $\beta = 1.60$, **j** $\beta = 1.90$, **k** $\beta = 2.20$, **l** $\beta = 2.50$

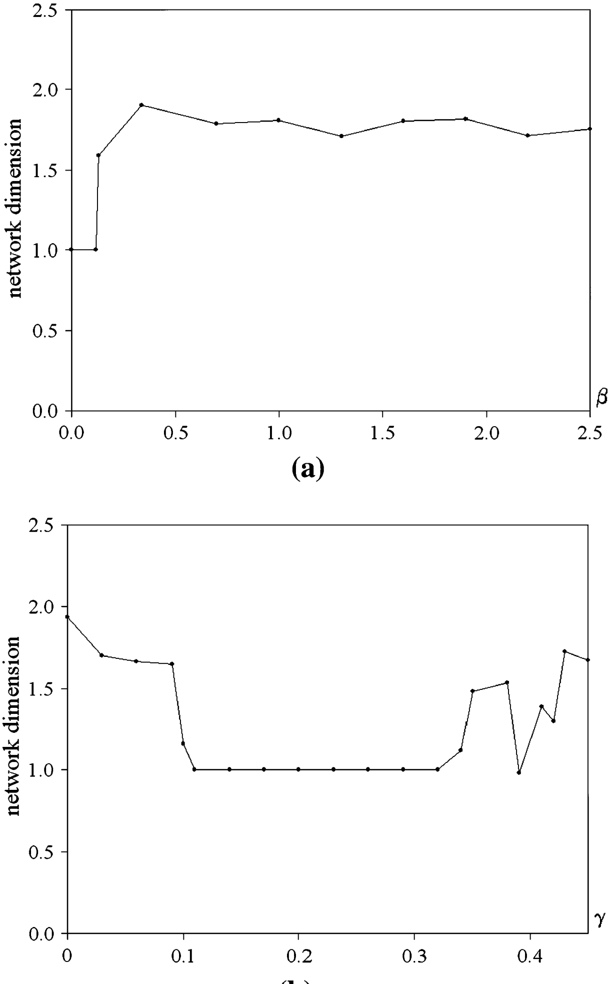

Fig. 4 Fractal dimension of the network when varying the preference for neighbourhood externalities. **a** γ = 0.00 and varying β. **b** β = 0.25 and varying γ

settlement. This is the fourth phase. With $\gamma > 0.45$ (not displayed in figure) the situation is similar though it fills in all four quadrants.

Another result clearly identifiable in this series is that the different quadrants may have different levels of development even at long-run equilibrium. The continuous building of houses along certain roads, being irreversible, prevent further lateral growth in some quadrants (clear examples in Fig. 5e–g). Such morphological lock-ins break the symmetry of the outputs. Interestingly for urban planners, those lock-ins may also generate green squares very close to the CBD (see Fig. 5h–j).

The fractal analysis of the network for this series shows well three of the phases identified above: first, start from DLA-like dendritic structures with gentle decrease

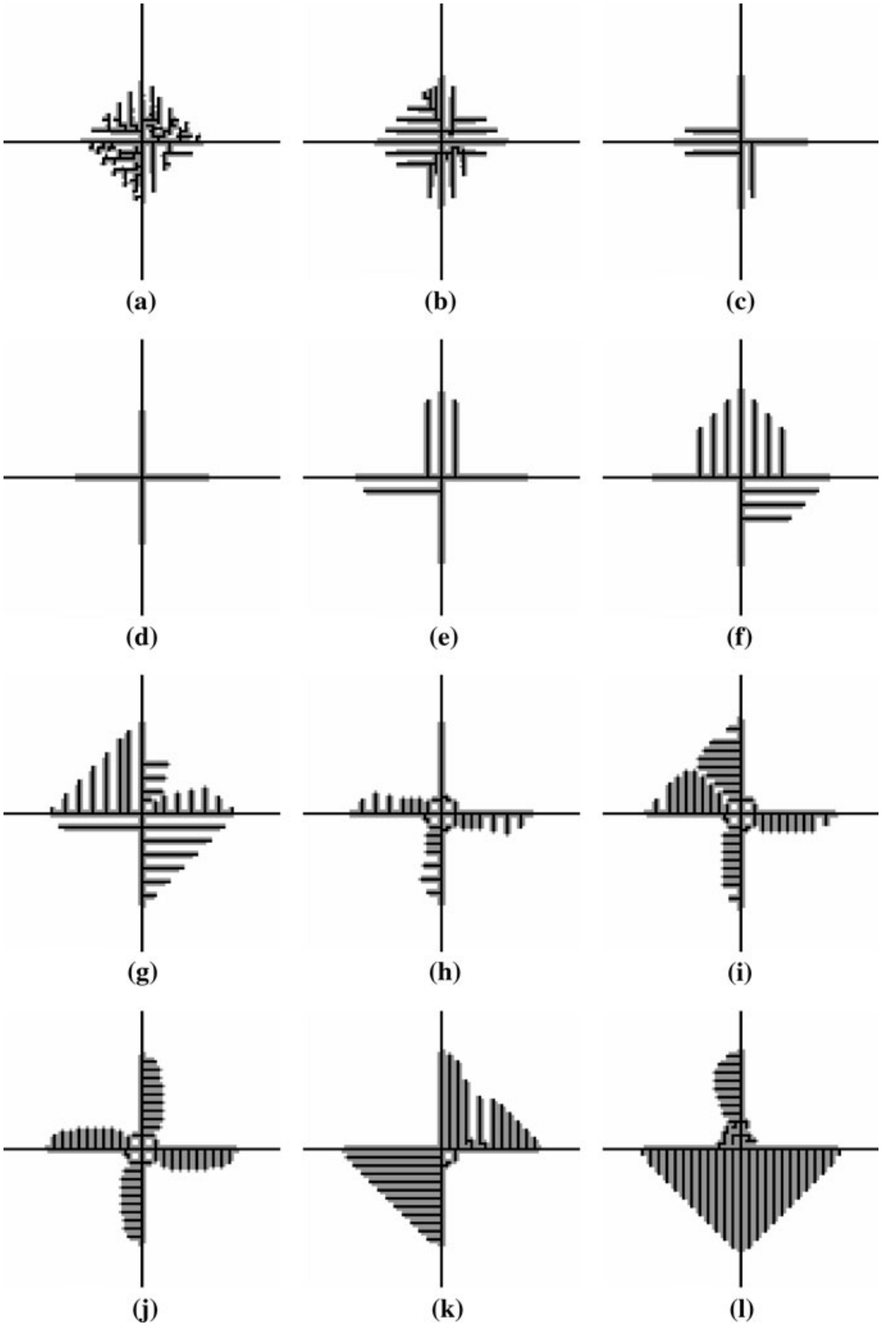

Fig. 5 Long-run equilibria for fixed $\beta = 0.25$ and varying γ. (*Black cells* roads; *grey* residences; *white* agriculture). **a** $\gamma = 0.00$, **b** $\gamma = 0.90$, **c** $\gamma = 0.11$, **d** $\gamma = 0.34$, **e** $\gamma = 0.34$, **f** $\gamma = 0.35$, **g** $\gamma = 0.38$, **h** $\gamma = 0.39$, **i** $\gamma = 0.41$, **j** $\gamma = 0.42$, **k** $\gamma = 0.43$, **l** $\gamma = 0.45$

in fractal dimension; second, cross-like Alonso pattern in the range $0.11 \leq \gamma < 0.34$ (fractal dimension $= 1$); third, from $\gamma = 0.34$ onwards, increased fractal dimension in a rather irregular manner due to accidental lock-ins. At that stage, the gradual absence of green alleys compacts residential use but does not affect the shape of the network in a significant manner.[2]

4.3 Varying neighbourhood radii

We performed additional simulations with constant preferences (β and γ) to assess the morphological effect of changing the size of the neighbourhood where the externalities are perceived, i.e. the spatial extent of the local interactions. Variations between long- and short-range interactions are common in urban growth models. One can find a recent example and discussion for CA models in van Vliet et al. (2009). See also the enlarged interaction field in Andersson et al. (2002) mentioned above.

We use two series to illustrate the effect of neighbourhood size change. The first (Fig. 6) is a case where households have no preference for open-space but only for social amenities ($\beta = 0$, $\gamma = 0.5$). The second case (Fig. 7) is a reversed situation, where households have no preference for social amenities but only for open-spaces ($\beta = 0.5$, $\gamma = 0$). In each figure, the top row shows long-run equilibria with increasing neighbourhood size from left to right. The bottom row shows the change through time for a selected case in order to exemplify the relationships between neighbourhood size and the emerging form of the road network.

From the long-run equilibria in Fig. 6, we can see that short-range neighbourhood preferences lead to a compact city while enlarging the neighbourhood leads to a more regular and less dense urban forms that encapsulate large green spaces. The fractal dimension of the network decreases accordingly.

It is striking (especially from cases (c) and (d) of Fig. 6) to obtain green spaces within a city while people prefer to agglomerate both at the local (externalities) and the regional (commuting costs) levels. We see that the simple fact that residents consider a larger living environment makes the more local urban form less dense in this case.

The intermediate steps displayed for the case where $(\hat{x} = 8)$ (Fig. 6e–i) illustrate how green spaces are gradually encapsulated by the residents and the emerging road network. Residents value high density in a 8 cells radius. Up to $t = 50$ they prefer to expand in a cross-like manner. At some moment ($50 < t < 60$), however, when the distance to the CBD is above 8, it is preferable for them to locate in a new parallel road in order to keep, within their neighbourhood, the residents who live in the orthogonal main road, thus keeping a high density benefit. They do not locate in the closest cell from the CBD, i.e. on the diagonal, because it would require a longer travel along preexisting residences, but perpendicularly to the main street. The process is repeated again further away from the centre, finally leading to the stage displayed on Fig. 6c, where regular open-space squares appear as well as roads with

[2] The dimension drops at $\gamma = 0.39$ but the fractal curve adjustment was not robust at that point due to reduced spatial expansion.

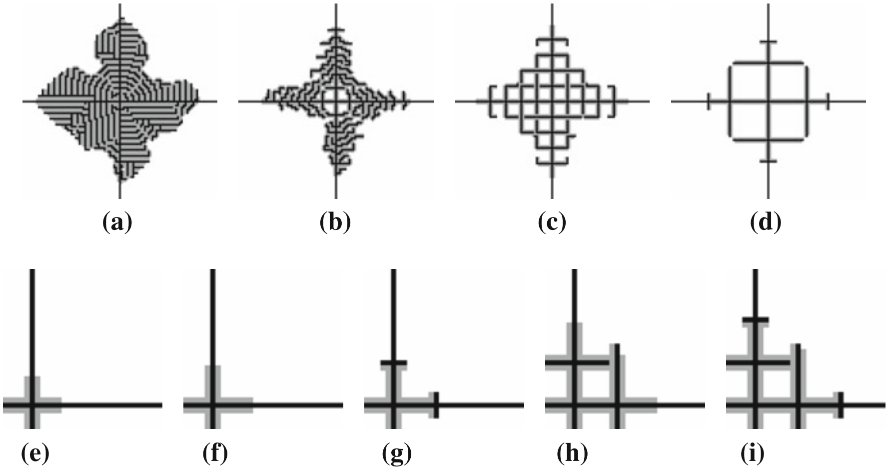

Fig. 6 *Top* long-run equilibria with fixed social and null open-space preferences ($\beta = 0.00$, $\gamma = 0.50$) when varying neighbourhood radius $\hat{x}$ (values in *brackets* indicate network fractal dimension). **a** $\hat{x} = 4\,(1.81)$, **b** $\hat{x} = 7\,(1.66)$, **c** $\hat{x} = 8\,(1.62)$, **d** $\hat{x} = 20\,(1.33)$. *Bottom* intermediate time steps for case (**c**) (zoom around NE quadrant). **e** $t = 40$, **f** $t = 50$, **g** $t = 60$, **h** $t = 220$, **i** $t = 230$. (*Black cells* roads; *grey* residences; *white* agriculture)

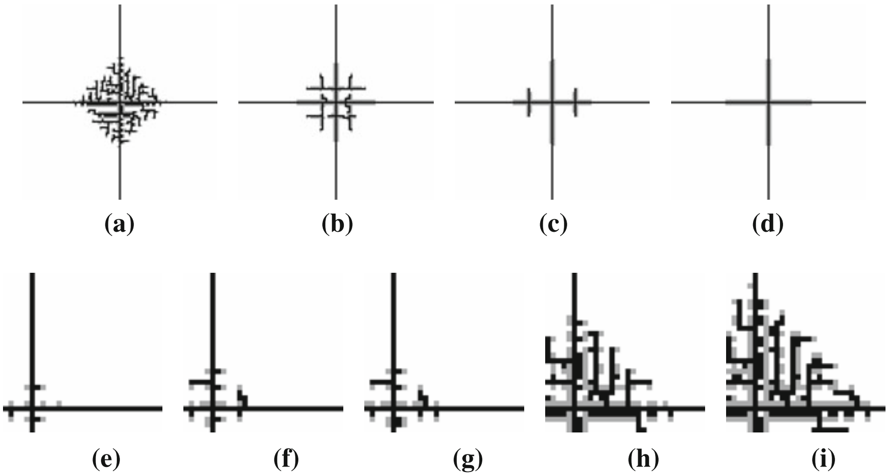

Fig. 7 *Top* long-run equilibria with fixed open-space and null social preferences ($\beta = 0.00$, $\gamma = 0.50$) when varying neighbourhood radius $\hat{x}$ (values in *brackets* indicate network fractal dimension). **a** $\hat{x} = 2\,(1.75)$, **b** $\hat{x} = 6\,(1.41)$, **c** $\hat{x} = 11\,(1.27)$, **d** $\hat{x} = 12\,(1.00)$. *Bottom* intermediate time steps for case (**a**) (zoom around NE quadrant). **e** $t = 10$, **f** $t = 20$, **g** $t = 30$, **h** $t = 170$, **i** $t = 270$. (*Black cells* roads; *grey* residences; *white* agriculture)

an 8 cell-wide interval. Although they would provide a lot of density amenities, the squares left undeveloped cannot be built later on because residents are continuously located along the roads, restricting access to the available cells. This is a good example of how the road system organisation and ribbon-type development can lead to inaccessible periurban agricultural spaces and unnecessary commuting.

The second set of images (Fig. 7) shows maybe less surprising results: the city is shrinking when residents who like low-density environment, consider larger neighbourhoods. Moreover, the change from a small to a large neighbourhood leads to a simplification (homogenisation) of the landscape as in the previous set. This is again shown by the decreasing fractal dimension: from a dendritic form we tend towards a cross-like Alonso form.

Zooming and looking at the dynamics for the first case ($\hat{x} = 2$), one can see how residents leapfrog in order to obtain the lowest density (Fig. 7e–i). The leapfrogs in this case are 2 cells wide, i.e. the size of the neighbourhood. Leapfrogging beyond would make no sense for a household since it would increase commuting without improving neighbourhood amenities. From $t = 40$ to $t = 60$, we can see clearly how several new roads are built to connect the new residences which locate as much as possible away from the others. The whole sequence also shows that leapfrogs can later be filled in and network ramifications added as the city grows.

4.4 Towards a link with the DBM

It was a real surprise for us to find DBM-like patterns from our economic model. Residential choice was modelled independently of physical processes, and very simple rules were added for generating a road network (following residential decisions). Dendritic patterns with dimensions close to 1.75 are observed with high β-values, specifically reminding the DLA structure. More compact structures occur when open-space preferences are less important and compensated by social interactions. One can see morphological analogies between the spectrum of simulations we have undertaken and the patterns from Bogoyavlenskiy and Chernova (2000) shown in Fig. 1.

With a closer look at our model and DBM formulations, the results are in fact less surprising. Indeed, both models consider the spatial diffusion of a network, and a formal link between both models can be envisaged. The choice of the site for a new house depends on the indirect utility function (Eq. 3), the variables E_l and S_l depending on the density ρ_l. First, ρ_l reminds the electric potential which depends on the neighbourhood via the Laplace-operator. Second, β and γ remind the role of η in the DBM. In both cases, structural changes depend on an exponent that weights the local potential or density.

There are substantial differences however. The relationship between the density (potential) and E_l and S_l is non-linear and more complex than in a DBM. It involves other parameters. This combination leads to additional phase transitions. The structural change in the urban morphology that we observed when changing β or γ (and measured with fractal dimensions) are less continuous than in DBM cases when varying η (see Fig. 2). Also the fact that the compensation of the two preferences is made through a cross-like pattern is particular to our model and not independent of both the initial state and the repelling force towards the centre. Finally, our model is not probabilistic. This might explain why our patterns are more path-dependent or include more regular ramifications.

5 Conclusion

We have presented a model simulating the joint expansion of residential areas, road network, and green areas in a metropolitan area. Our purpose was to build a dynamic model strongly grounded on microeconomic foundations and able to represent the main features of urban sprawl. We therefore used an urban economic model framework: a household maximises a utility function under a budget constraint. Utility includes preferences for neighbourhood quality in terms of open-space and public goods. Costs include commuting to a central workplace and residential rent. A land market is modelled and economic equilibrium conditions are satisfied. This economic system, representing residential decision and market, was transferred into a self-organising system and simulated on a theoretical grid space. Simple rules were added to generate a road network and serving residents.

Several simulations were undertaken to show the impact of households preferences on the urban pattern. Various forms were observed, including compact, dendritic, and linear forms with different arrangement of roads, open spaces and residences. We found gradual transition phases in the pattern when gradually changing preferences. We also found more singular outputs like asymmetries and enclaved green spaces that do not correspond to individual wishes or optimal spatial arrangements.

A striking result was to find similarities with dendritic forms obtained with DLA models, and transition phases that are similar to those we can find in a DBM. Several authors have used DBM-like models to represent urban growth but only few have tried to reveal the micro-mechanisms that would make sense for urban processes and subsequently for planning. None has explicitly tackled the link with urban economics. This article therefore fills a gap by showing that the morphological properties of DBM and their ability to represent urban sprawl can hold with microeconomic foundations.

Needless to say that substantial efforts should to be made to fully decipher the analogy and to draw not only mathematical links but also conceptual links between economic and physical notions.

Moreover, several processes and morphologies revealed by the model are puzzling for an economic and geographical understanding of sprawl. Particularly the question of the welfare optimality of the patterns emerging from individual decisions is to be analysed. Subsequently, the need and design of effective planning actions should be questioned (unlocking lock-ins? network design? green belts and zone-based planning?)

Finally, though such an abstract theoretical model can already be used to test generic scenarios, we need to address the question of passing the model to real case studies with proper calibration methods.

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