

Open versus closed loop capacity equilibria in electricity markets under perfect and oligopolistic competition

S. Wogrin · B. F. Hobbs · D. Ralph ·
E. Centeno · J. Barquín

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Abstract We consider two game-theoretic models of the generation capacity expansion problem in liberalized electricity markets. The first is an open loop equilibrium model, where generation companies simultaneously choose capacities and quantities to maximize their individual profit. The second is a closed loop model, in which companies first choose capacities maximizing their profit anticipating the market equilibrium outcomes in the second stage. The latter problem is an equilibrium problem with equilibrium constraints. In both models, the intensity of competition among producers in the energy market is frequently represented using conjectural variations. Considering one load period, we show that for any choice of conjectural variations ranging from perfect competition to Cournot, the closed loop equilibrium coincides with the Cournot open loop equilibrium, thereby obtaining a ‘Kreps and Scheinkman’-like result and extending it to arbitrary strategic behavior. When expanding the model framework to multiple load periods, the closed loop equilibria for different conjectural variations can diverge from each other and from open loop equilibria. We also present and analyze alternative conjectured price response models with switching conjectures. Surprisingly, the rank ordering of the closed loop equilibria in terms of consumer surplus and market efficiency (as measured by total social welfare) is ambiguous. Thus, regulatory approaches that force marginal cost-based bidding in

S. Wogrin (✉) · E. Centeno · J. Barquín
Instituto de Investigación Tecnológica, Escuela Técnica Superior de Ingeniería (ICAI),
Universidad Pontificia Comillas, 28015 Madrid, Spain
e-mail: sonja.wogrin@iit.upcomillas.es

B. F. Hobbs
Department of Geography and Environmental Engineering, and Environment, Energy,
Sustainability and Health Institute, The Johns Hopkins University, Baltimore, MD 21218, USA

D. Ralph
Cambridge Judge Business School and Electricity Policy Research Group,
University of Cambridge, Cambridge CB2 1AG, UK

spot markets may diminish market efficiency and consumer welfare by dampening incentives for investment. We also show that the closed loop capacity yielded by a conjectured price response second stage competition can be less or equal to the closed loop Cournot capacity, and that the former capacity cannot exceed the latter when there are symmetric agents and two load periods.

Keywords Generation expansion planning · Capacity pre-commitment · Noncooperative games · Equilibrium problem with equilibrium constraints (EPEC)

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1 Introduction

In this paper we compare game-theoretic models that can be used to analyze the strategic behavior of companies facing generation capacity expansion decisions in liberalized electricity markets. Game theory is particularly useful in the restructured energy sector because it allows us to investigate the strategic behavior of agents (generation companies) whose interests are opposing and whose decisions affect each other. In particular, we seek to characterize the difference between open and closed loop models of investment.

Open loop models extend short-term models to a longer time horizon by modeling investment and production decisions as being taken at the same time. This corresponds to the open loop Cournot equilibrium conditions presented in [22], the Cournot-based model presented in [31], and the model in [6]. However, this approach may overly simplify the dynamic nature of the problem, as it assumes that expansion and operation decisions are taken simultaneously.

The reason to employ more complicated closed loop formulations is that the generation capacity expansion problem has an innate two-stage structure: first investment decisions are taken followed by determination of energy production in the spot market, which is limited by the previously chosen capacity. A two-stage decision structure is a natural way to represent how many organizations actually make decisions. One organizational subunit is often responsible for capital budgeting and anticipating how capital expenditures might affect future revenues and costs over a multi-year or even multi-decadal time horizon, whereas a different group is in charge of day-to-day spot market bidding and output decisions. This type of closed loop model is in fact an equilibrium problem with equilibrium constraints (EPEC), see [21, 29], arising when each of two or more companies simultaneously faces its own profit maximization problem modeled as a mathematical program with equilibrium constraints (MPEC). In the electricity sector, MPECs, bilevel problems, and EPECs were first used to represent short-run bidding and production games among power producers, e.g., [2, 4, 16, 32, 35]. EPECs belong to a recently developed class of mathematical programs that often arise in engineering and economics applications and can be used to model electricity markets [27]. For methods to solve EPECs, i.e., diagonalization, the reader is referred to [17, 18, 21].

Solving large-scale closed loop models can be very challenging, sometimes even not tractable. Therefore, in real-world applications there is a strong incentive to resort to easier open loop models, simply because the corresponding closed loop model cannot be solved (yet). In this paper, our results indicate that when practical considerations motivate adoption of easier, less complicated open loop models, the results may be very different from (possibly more realistic) closed loop formulations.

For space reasons, many of the proofs and other details are omitted, but are available in the full version of the paper [34].

1.1 Review of literature

Several closed loop approaches to the generation capacity expansion problem have been proposed. The papers most relevant for our paper are [22] and [20]. With their paper [20], Kreps and Scheinkman (K–S) tried to reconcile Cournot's [8] and Bertrand's [3] theory by constructing a two-stage game, where first firms simultaneously set capacity and second, after capacity levels are made public, there is price competition. They find that when assuming two identical firms and an efficient rationing rule (i.e., the market's short-run production is provided at least cost), their two-stage game yields Cournot outcomes. For a more detailed review of the K–S literature, the reader is referred to [34]. Our results extend this literature by considering generalizations of K–S-like models to conjectural variations other than competitive (Bertrand) as well as multiple load periods or, equivalently, stochastic load.

In [22] the authors present and analyze three different models: an open loop perfectly competitive model, an open loop Cournot model and a closed loop Cournot model. Each considers several load periods which have different demand curves and two firms, one with a peak load technology (low capital cost, high operating cost) and the other with a base load technology (high capital cost, low operating cost). They analyze when open and closed loop Cournot models coincide and when they are necessarily different. Moreover, they demonstrate that the closed loop Cournot equilibrium capacities fall between the open loop Cournot and the open loop competitive solutions. Our paper differs by considering a range of conjectural variations between perfect competition and Cournot. Our formal results are for symmetric agents but they extend to asymmetric cases. We derive certain equivalency results that can also be extended to asymmetric firms. Moreover, in our models we consider a constant second stage conjectural variation rather than a situation in which the conjectural variation switches to Cournot when rival firms are at capacity. We consider this alternative conjectural variation in Sect. 4.5.

In addition to [23], there are other works that have formulated and solved closed loop models of power generation expansion. In [31] we find a closed loop Stackelberg-based model, where in the first stage a leader firm decides its capacity and then in the second stage the followers compete in quantities in a Cournot game. Centeno et al. [6] presents a two-stage model representing the market equilibrium, where the first stage is based on a Cournot equilibrium among producers who can choose continuous capacity investments and computes a market equilibrium approximation for the entire model horizon and a second stage discretizes this solution separately for each year. In [13] the authors present a linear bilevel model that determines the optimal investment decisions

of one generation company considering uncertainty in demand and in competitors' capacity decisions. In [28] the author applies a two-stage model in which firms choose their capacities under demand uncertainty prior to competing in prices and presents regulatory conclusions. An instance of a stochastic static closed loop model for the generation capacity problem for a single firm can be found in [19], where investment and strategic production decisions are taken in the upper level for a single target year in the future, while the lower level represents market clearing.

Existing generation capacity expansion approaches in the literature assume either perfectly competitive [13] or Cournot behavior [6,31] in the spot market. The proposed open and closed loop models of this paper extend previous approaches by including a generalized representation of market behavior via conjectural variations, in particular through an equivalent conjectured price response. This allows us to represent various forms of oligopoly, ranging from perfect competition to Cournot. Power market oligopoly models have been proposed before based on conjectural variations [5] and conjectured price responses [10], but only for short term markets in which capacity is fixed.

In the case of electricity markets, production decisions undertaken by power producers result from a complex dynamic game within multi-settlement markets. Typically, bids in the form of supply functions are submitted in two or more successive markets at different times prior to operation, where the second and successive markets account for the commitments made in previous markets. Conjectural variations models can represent a reduced form of a dynamic game as pointed out in [11]. This kind of reinterpretation has been proposed by several authors. For example, the two stage forward contracting/spot market Allaz–Vila game can be reduced to a one stage conjectural variations model [23]. For further references of the conjectural variation literature, the reader is referred to [34]. Therefore, conjectural variations can be used to capture very complex games in a computationally tractable way. This is a major reason why many econometric industrial organization studies estimate oligopolistic interactions using model specifications based on the assumption of constant conjectural variations [25]. Our discussion of these references is only to state that in general conjectural variations can represent more complicated games. We do not consider here the problem of estimating or calculating conjectural variations, which can be a very complicated process and would depend on the nature of the particular game that is reduced.

1.2 Open loop versus closed loop capacity equilibria

We consider two identical firms with perfectly substitutable products, each facing either a one-stage or a two-stage competitive situation. The one-stage situation, represented by the open loop model, describes the one-shot investment operation market equilibrium. The closed loop model, which is an EPEC, describes the two-stage investment-operation market equilibrium and is similar to the well-known K–S game [20]. Considering one load period, we find that the closed loop equilibrium for any strategic market behavior between perfect competition and Cournot yields the open loop Cournot outcomes, thereby obtaining a K–S-type result and extending it to any strategic behavior between perfect competition and Cournot. As previously mentioned,

Murphy and Smeers [22] have found that under certain conditions the open and closed loop Cournot equilibria coincide. Our result furthermore shows that considering one load period, all closed loop models assuming strategic spot market behavior between perfect competition and Cournot coincide with the open loop Cournot solution. In the multiple load period case we define some sufficient conditions for the open and closed loop capacity decisions to be the same. However, this result is parameter dependent. When capacity is the same, outputs in non-binding load periods are the same for open and closed loop models when strategic spot market behavior is the same, otherwise outputs can differ.

When the closed loop capacity decisions differ for different conjectural variations, then the resulting consumer surplus (CS) and market efficiency (ME) (as measured by social welfare, the sum of CS and profit) will depend on the conjectural variation. It turns out that which conjectural variation results in the highest efficiency is parameter dependent. In particular, under some assumptions, the closed loop model considering perfect competition in the energy market can actually result in lower market efficiency, lower CS and higher prices than Cournot competition. This surprising result implies that regulatory approaches that force marginal cost-based bidding in spot markets may decrease ME and consumer welfare and may therefore actually be harmful. For example, the Irish spot market rules [26] require bids to equal short-run marginal cost. Meanwhile, local market power mitigation procedures in several US organized markets reset bids to marginal cost (plus a small adder) if significant market power is present in local transmission-constrained markets [24]. These market designs implicitly assume that perfect competition is welfare superior to more oligopolistic behavior, such as Cournot competition. As our counter-example will show, this is not necessarily so.

In [14], the authors have arrived at a similar result, however, they only examine the polar cases of perfect or Cournot-type competition. In our work we generalize strategic behavior using conjectural variations and look at a range of strategic behavior, from perfect competition to Cournot competition and we also observe that an intermediate solution between perfect and Cournot competition can lead to even larger social welfare and CS.

The results obtained are suggestive of what might occur in other industries where storage is relatively unimportant and there is time varying demand that must be met by production at the same moment. Examples include, for instance, industries such as airlines or hotels.

This paper is organized as follows. In Sect. 2 we introduce and define the conjectured price response representation of the short-term market and provide a straight-forward relationship to conjectural variations. Then, in Sect. 3 we formulate symmetric open and closed loop models for one load period and establish that our K–S-like result also holds for arbitrary strategic behavior ranging from perfect to Cournot competition. This is followed by Sect. 4, which extends the symmetric K–S-like framework to multiple load periods. We furthermore analyze alternative models in which the second stage conjectural variation switches depending on whether rivals' capacity is binding or not, instead of being constant. In Sect. 5 we first show that the closed loop capacity yielded by a conjectured price response second stage competition can be less or equal to the closed loop Cournot capacity, and that the former capacity cannot exceed the latter for symmetric agents and two load periods. Also in that section we show by

example that under the closed loop framework, more competitive behavior in the spot market can lead to less market efficiency and CS. Finally, Sect. 6 concludes the paper.

2 Conjectural variations and conjectured price response

We introduce equilibrium models that capture various degrees of strategic behavior in the spot market by introducing conjectural variations into the short-run energy market formulation. The conjectural variations development can be related to standard industrial organization theory [12]. In particular, we introduce a conjectured price response parameter that can easily be translated into conjectural variations with respect to quantities, and vice versa if we consider demand to be linear.

First, we consider two identical firms with perfectly substitutable products, for which we furthermore assume an affine price function $p(d)$, i.e., $p(d) = (D_0 - d)/\alpha$, where d is the quantity demanded, $\alpha = D_0/P_0$ is the demand slope, $D_0 > 0$ the demand intercept, and $P_0 > 0$ is the price intercept. Demand d and quantities produced q_i, q_{-i} , with i and $-i$ being the indices for the market agents, are linked by the market clearing condition $q_i + q_{-i} = d$. Hence, we will refer to price also as $p(q_i, q_{-i})$.

Then we define the conjectural variation parameters as $\Phi_{-i,i}$. These represent agent i 's belief about how agent $-i$ changes its production in response to a change in i 's production. Therefore:

$$\Phi_{-i,i} = \frac{dq_{-i}}{dq_i}, \quad i \neq -i, \quad (1)$$

$$\Phi_{i,i} = 1. \quad (2)$$

And hence using (1)–(2) and our assumed $p(q_i, q_{-i})$, we obtain:

$$\frac{dp(q_i, q_{-i})}{dq_i} = -\frac{1}{\alpha} \sum_{-i} \Phi_{-i,i} = -\frac{1}{\alpha} \left(1 + \sum_{-i \neq i} \Phi_{-i,i} \right) \quad (3)$$

As we are considering two identical firms in the models of this paper, we can assume that $\Phi_{i,-i} = \Phi_{-i,i}$ which we define as Φ and therefore relation (3) simplifies to:

$$\frac{dp(q_i, q_{-i})}{dq_i} = -\frac{1}{\alpha} (1 + \Phi) \quad (4)$$

Let us define the conjectured price response parameter θ_i as company i 's belief concerning its influence on price p as a result of a change in its output q_i :

$$\theta_i := -\frac{dp(q_i, q_{-i})}{dq_i} = \frac{1}{\alpha} (1 + \Phi) \geq 0, \quad (5)$$

which shows how to translate a conjectural variations parameter into the conjectured price response and vice versa. The nonnegativity of (5) comes from the assumption that the conjectural variations parameter $\Phi \geq -1$. Throughout the paper we will formulate

the equilibrium models using the conjectured price response parameter as opposed to the conjectural variations parameter, because its depiction of the firms' influence on price is more convenient for the derivations, as opposed to a firm's influence on production by competitors.

As has been proven in [9], this representation allows us to express special cases of oligopolistic behavior such as perfect competition, the Cournot oligopoly, or collusion. A general formulation of each firm's profit objective would state that $p = p(q_i, q_{-i})$, with the firm anticipating that price will respond to the firm's output decision. We term this the conjectured price response model. If the firm takes p as exogenous (although it is endogenous to the market), the result is the price-taking or perfect competition (and $\Phi = -1$), similar to the Bertrand conjecture [20] under certain circumstances. Then the conjectured price response parameter θ_i equals 0, which means that none of the competing firms believes it can influence price (and $\Phi = -1$). If instead $p(q_i, q_{-i})$ is the inverse demand curve $D_0/\alpha - (q_i + q_{-i})/\alpha$, with q_{-i} being the rival firm's output which is taken as exogenous by firm i , then the model is a Nash-Cournot oligopoly. In the Cournot case, θ_i equals $1/\alpha$, which would translate to $\Phi = 0$ in the conjectural variations framework.

We can also express collusion (quantity matching, or 'tit for tat') as $2/\alpha$, which translates to $\Phi = 1$, as well as values between the extremes of perfect competition and the Cournot oligopoly. Some more complex dynamic games can be reduced to a one stage game with intermediate values for Φ (or θ_i respectively). For example, Murphy and Smeers [23] show that the Allaz-Vila [1] two stage forward contracting/spot market game can be reduced to a one stage game assuming $\Phi = 1/2$ (or $\theta = 1/(2\alpha)$). The two stage Stackelberg game can also be reduced to a conjectural variations-based one stage game as shown in [7].

As mentioned above, in the case of electricity markets, production decisions undertaken by power producers result from a complex dynamic game within multi-settlement markets. Conjectural variations models (such as the used in the lower level) can also be used as a computationally tractable reduced form of a dynamic game [11].

3 Generalization of the K-S-like single load period result to arbitrary oligopolistic conjectures

In this section we consider two identical firms with perfectly substitutable products, facing either a one-stage or a two-stage competitive situation. The one-stage situation is represented by the open loop model presented in 3.1 and describes the one-shot investment-operation market equilibrium. In this situation, firms simultaneously choose capacities and quantities to maximize their individual profit, while each firm conjectures a price response to its output decisions consistent with the conjectured price response model. The closed loop model given in 3.2 describes the two-stage investment-operation market equilibrium, where firms first choose capacities that maximize their profit while anticipating the equilibrium outcomes in the second stage, in which quantities and prices are determined by a conjectured price response market equilibrium. We furthermore assume that there is an affine relation between price and demand and that capacity can be added in continuous amounts.

The main contribution of this section is Theorem 1, in which we show that for two identical agents, one load period and an affine non-increasing inverse demand function, the one-stage model solution assuming Cournot competition is a solution to the closed loop model independent of the choice of conjectured price response within the perfect competition-Cournot range. When the conjectured price response represents perfect competition, then this result is very similar to the finding of [20]. As a matter of fact, Bertrand competition boils down to perfect competition when there is no capacity constraint [30], in which case our results would be equivalent to the K–S result. However, considering that we have a capacity limitation, Bertrand competition and perfect competition are not equivalent. For this reason our results are not exactly K–S, however, taking into account the similarities, they are K–S-like. Thus, Theorem 1 extends the 'Kreps and Scheinkman'-like result to any conjectured price response within a range. Later in the paper, however, we show that this result does not generalize to the case of multiple load periods.

Throughout this section we will use the following notation:

- q_i denotes the quantity [MW] produced by firms $i = 1, 2$.
- x_i denotes the capacity [MW] of firms $i = 1, 2$.
- d denotes quantity demanded [MW].
- p [€/MWh] denotes the clearing price. Moreover $p(d) = (D_0 - d)/\alpha$ where D_0 and α are positive constants, and P_0 denotes D_0/α .
- t [h/year] corresponds to the duration of the load period per year.
- β [€/MW/year] corresponds to the annual investment cost.
- δ [€/MWh] is the variable production cost.
- θ is a constant in $[0, 1/\alpha]$, that is the conjectured price response corresponding to the strategic spot market behavior for each i , see (5).

Furthermore we will make the following assumptions:

- Both cost parameters, δ, β , are nonnegative.
- The investment cost plus the variable cost will be less than the price intercept times duration t , i.e., $\delta t + \beta < P_0 t$, which is an intuitive condition as it simply states that the maximum price P_0 is high enough to cover the sum of the investment cost and the operation cost. Otherwise there is clearly no incentive to participate in the market.
- The same demand curve assumptions are made as in Sect. 2.
- We consider one year rather than a multi-year time horizon, and so each firm attempts to maximize its annualized profit.

3.1 The open loop model

In the open loop model, every firm i faces a profit maximization problem in which it chooses capacity x_i and production q_i simultaneously. When firms simultaneously compete in capacity and quantity, the open loop investment-operation market equilibrium problem consists of all the firms' profit maximization problems plus market clearing conditions that link together their problems by $d = D_0 - \alpha p(q_i, q_{-i})$. Conceptually, the resulting equilibrium problem can be written as (6)–(7):

$$\forall i \begin{cases} \max_{x_i, q_i} & t(p(q_i, q_{-i}) - \delta)q_i - \beta x_i \\ \text{s.t.} & q_i \leq x_i \end{cases} \tag{6}$$

$$d = q_i + q_{-i}, \quad d = D_0 - \alpha p(q_i, q_{-i}) \tag{7}$$

In (6) we describe i 's profit maximization as consisting of market revenues $tp(q_i, q_{-i})q_i$ minus production costs $t\delta q_i$ and investment costs βx_i . The non-negativity constraints can be omitted in this case.¹

Although (6)'s constraint is expressed as an inequality, it will hold as an equality in equilibrium, at least in this one-period formulation. That $x_i = q_i$ for $i = 1, 2$ will be true in equilibrium, can easily be proven by contradiction. Let us assume that at the equilibrium $x_i > q_i$; then firm i could unilaterally increase its profits by shrinking x_i to q_i (assuming $\beta > 0$), which contradicts the assumption of being at an equilibrium.

In this representation the conjectured price response is not explicit. Therefore we rewrite the open loop equilibrium stated in (6)–(7) as a mixed complementarity problem (MCP) by replacing each firm's profit maximization problem by its first order Karush–Kuhn–Tucker (KKT) conditions. The objective function in (6) is concave for any value of θ in $[0, 1/\alpha]$.² Then, due to linearity of $p(d)$, (6)–(7) is a concave maximization problem with linear constraints, hence its solutions are characterized by its KKT conditions. Therefore let \mathcal{L}_i denote the Lagrangian of company i 's corresponding optimization problem, given in (6) and let λ_i be the Lagrange multiplier of constraint $q_i \leq x_i$. Then, the open loop equilibrium problem is then given in (8)–(9).

$$\forall i \begin{cases} \frac{\partial \mathcal{L}_i}{\partial q_i} = tp(q_i, q_{-i}) - t\theta q_i - t\delta - \lambda_i = 0 \\ \frac{\partial \mathcal{L}_i}{\partial x_i} = \beta - \lambda_i = 0 \\ q_i \leq x_i \\ \lambda_i \geq 0 \\ \lambda_i(x_i - q_i) = 0 \end{cases} \tag{8}$$

$$d = q_i + q_{-i}, \quad d = D_0 - \alpha p(q_i, q_{-i}) \tag{9}$$

Due to the fact that $\lambda_i = \beta > 0$, the complementarity condition yields $x_i = q_i$ in equilibrium. In this formulation we can directly see the conjectured price response parameter θ in $\frac{\partial \mathcal{L}_i}{\partial q_i}$. Solving the resulting system of equations yields:

$$q_i = \frac{D_0 t - \alpha(\beta + \delta t)}{t(\alpha\theta + 2)} \quad \forall i \tag{10}$$

¹ For completeness, let us consider the explicit non-negativity constraint $0 \leq q_i$ in the optimization problem (6) and let us define $\mu_i \geq 0$ as the corresponding dual variable. Then, due to complementarity conditions arising from the KKT conditions, we can separate two cases, the one where $\mu_i = 0$ and the other where $\mu_i > 0$. The first case will lead us to the solution presented in the paper, and case $\mu_i > 0$ will lead us to a solution where $\mu_i = t(\delta - P_0)$. Considering that we assumed $P_0 > \delta$, this yields a contradiction to the non-negativity of μ_i . Hence, this cannot be the case and therefore we omit the non-negativity constraint.

² Taking the first derivative of the objective function in (6) with respect to q_i yields: $tp(q_i, q_{-i}) - t\theta q_i - t\delta$. Then, the second derivative is $-2t\theta$, which is smaller or equal to zero for each value of θ in $[0, 1/\alpha]$, which yields concavity of the objective function.

$$p = \frac{D_0t\theta + 2(\beta + \delta t)}{t(\alpha\theta + 2)}. \tag{11}$$

In the open loop model we have not explicitly imposed $q_i \geq 0$, however, from (10) we obtain that the open loop model has a non-trivial solution (i.e., each quantity is positive at equilibrium) if parameters are chosen such that $D_0t - \alpha(\beta + \delta t) > 0$ is satisfied. This condition is equivalent to $\delta t + \beta < P_0t$ using the fact that $\alpha = D_0/P_0$ and $D_0 > 0$, which has already been stated in the assumptions above.

A special case of the conjectured price response is the Cournot oligopoly. In order to obtain the open loop Cournot solution, we just need to insert the appropriate value of the conjectured price response parameter θ , which for Cournot is $\theta = 1/\alpha$. This solution is unique [22]. Then (10)–(11) yield:

$$q_i = \frac{D_0t - \alpha(\beta + \delta t)}{3t} \quad \forall i \tag{12}$$

$$p = \frac{D_0t + 2\alpha(\beta + \delta t)}{3t\alpha}. \tag{13}$$

3.2 The closed loop model

We now present the closed loop conjectured price response model describing the two-stage investment-operation market equilibrium. In this case, firms first choose capacities maximizing their profit anticipating the equilibrium outcomes in the second stage, in which quantities and prices are determined by a conjectured price response market equilibrium. We stress that the main distinction of this model from the equilibrium model described in Sect. 3.1 is that now there are two stages in the decision process, i.e., capacities and quantities are not chosen at the same time. Then we present Theorem 1 which establishes a relation between the open loop and the closed loop models for the single demand period case.

3.2.1 The production level—second stage

The second stage (or lower level) represents the conjectured-price-response market equilibrium, in which both firms maximize their market revenues minus their production costs, deciding their production subject to the constraint that production will not exceed capacity. The argument given above shows, at equilibrium, that this constraint binds if there is a single demand period. These maximization problems are linked by the market clearing condition. Thus, the second stage market equilibrium problem can be written as:

$$\forall i \begin{cases} \max_{q_i} t(p(q_i, q_{-i}) - \delta)q_i \\ s.t. & q_i \leq x_i \end{cases} \tag{14}$$

$$d = q_i + q_{-i}, \quad d = D_0 - \alpha p(q_i, q_{-i}), \tag{15}$$

As in the open loop case, p may be conjectured by firm i to be a function of its output q_i . Using a justification similar to that in the previous section, we can substitute

firm i 's KKT conditions for (14) and arrive at the conjectured price response market equilibrium conditions, as shown in [34].

3.2.2 The investment level—first stage

In the first stage, both firms maximize their total profits, consisting of the gross margin from the second stage (revenues minus production costs) minus investment costs, and choose their capacities subject to the second stage equilibrium response, which can be written as the following equilibrium problem:

$$\forall i \begin{cases} \max_{x_i} t(p(q_i, q_{-i}) - \delta)q_i - \beta x_i \\ \text{s.t.} \quad \text{Market Equilibrium Conditions for (14), (15)} \end{cases} \tag{16}$$

We know that at equilibrium, production will be equal to capacity. As in the open loop model, this can be shown by contradiction. Since there is a linear relation between price and demand, it follows that price can be expressed as $p = \frac{D_0 - d}{\alpha}$. Substituting $x_i = q_i$ in this expression of price, yields $p = \frac{D_0 - x_1 - x_2}{\alpha}$. Then expressing the objective function and the second stage in terms of the variables x_i yields the following simplified closed loop equilibrium problem:

$$\forall i \begin{cases} \max_{x_i} t \left(\frac{D_0 - x_1 - x_2}{\alpha} - \delta \right) x_i - \beta x_i \\ \text{s.t.} \quad \frac{D_0 - x_1 - x_2}{\alpha} - \theta x_i - \delta \geq 0 \quad : \gamma_i \end{cases} \tag{17}$$

where γ_i are the dual variables to the corresponding constraints. Then, the closed loop equilibrium conditions (assuming a nontrivial solution $x_i > 0$) are:

$$\forall i \begin{cases} t \left(\frac{D_0 - x_1 - x_2}{\alpha} - \delta \right) - tx_i/\alpha - \beta + \gamma_i(-\theta - 1/\alpha) = 0 \\ \frac{D_0 - x_1 - x_2}{\alpha} - \theta x_i - \delta \geq 0 \\ \gamma_i \left(\frac{D_0 - x_1 - x_2}{\alpha} - \theta x_i - \delta \right) = 0 \\ \gamma_i \geq 0 \end{cases} \tag{18}$$

When solving the system of equations given by (18) we distinguish between two separate cases: $\gamma_i = 0$ and $\gamma_i > 0$. The first case, i.e. $\gamma_i = 0$, yields the following solution for the closed loop equilibrium:

$$x_i = \frac{D_0 t - \alpha(\beta + \delta t)}{3t} \quad \forall i. \tag{19}$$

$$p = \frac{D_0 t + 2\alpha(\beta + \delta t)}{3t\alpha} \tag{20}$$

This can be verified without difficulty by checking the market equilibrium conditions, i.e., KKT conditions of (14)–(15); see [34] for details. As in the previous section, the solution is nontrivial due to the assumption that $\delta t + \beta < P_0 t$.

As for uniqueness of the closed loop equilibrium, [22] has proven for the Cournot closed loop equilibrium that if an equilibrium exists, then it is unique. We will investigate uniqueness issues of the closed loop conjectured price response model in future research. Comparing (19) and (20) with the open loop equilibrium (10) and (11) we see that this is exactly the open loop solution considering Cournot competition, i.e. (12) and (13).

Now let us consider the second case, i.e., $\gamma_i > 0$. Then (18) yields the following values for capacities and γ_i :

$$x_i = \frac{D_0 - \alpha\delta}{2 + \alpha\theta} \quad \forall i. \quad (21)$$

$$\gamma_i = \frac{-(D_0 t + \alpha^2(\beta + \delta t)\theta + \alpha(2\beta - t(\delta + D_0\theta)))}{(\alpha\theta + 1)(\alpha\theta + 2)} \quad \forall i. \quad (22)$$

It can be verified [34] that for $\theta \in [0, 1/\alpha]$ the expression of γ_i in (22) is smaller or equal to zero, which is a contradiction to the assumption that $\gamma_i > 0$. Hence the only solution to the closed loop equilibrium is the open loop Cournot solution that results when $\gamma_i = 0$.

3.2.3 Theorem 1

Theorem 1 *Let there be two identical firms with perfectly substitutable products and one load period. Let the affine price $p(d)$ and the parameters needed to define the open loop equilibrium problem (8)–(9) be as described at the start of Sect. 3.*

When comparing the open and closed loop competitive equilibria for two firms, we find the following: The open loop Cournot solution, see (12)–(13), is a solution to the closed loop conjectured price response equilibrium (19)–(20) for any choice of the conjectured price response parameter θ from perfect competition to Cournot competition.

Proof Sections 3.1 and 3.2 above prove this theorem. As in the open loop model, the closed loop model has a non-trivial solution if data is chosen such that $P_0 t > \beta + \delta t$ is satisfied. \square

What we have proven in Theorem 1 is that as long as the strategic behavior in the market (which is characterized by the parameter θ) is more competitive than Cournot, then in the closed loop problem firms could decide to build Cournot capacities. Even when the market is more competitive than the Cournot case (e.g., Allaz–Vila or perfect competition), firms can decide to build Cournot capacities. Hence Theorem 1 states that the Kreps and Scheinkman-like result holds for any conjectured price response more competitive than Cournot (e.g., Allaz–Vila or perfect competition), not just for the case of perfect second stage competition. Theorem 1 extends to the case of asymmetric firms but we omit the somewhat tedious analysis which can, however, be found in [33].

Note that Theorem 1 describes sufficient conditions but they are not necessary. This means that there are cases where Theorem 1 also holds for $\theta > 1/\alpha$. For example

Theorem 1 may hold under collusive behavior ($\theta = 2/\alpha$) when the marginal cost of production (δ) is sufficiently small.³

In the following section we will extend the result of Theorem 1 to the case of multiple demand periods. In particular, under stringent conditions, the Cournot open loop and closed loop solutions can be the same, and the Cournot open loop capacity can be the same as the closed loop capacity for more intensive levels of competition in the second stage of the closed loop game. But this result is parameter dependent, and in general, these solutions differ. Surprisingly, for some parameter assumptions, more intensive competition in the second stage can yield economically inferior outcomes compared to Cournot competition, in terms of CS and total market surplus.

4 Extension of K–S-like result for multiple load periods

In this section we extend the previously established comparison between the open loop and the closed loop model to the situation in which firms each choose a single capacity level, but face time varying demand that must be met instantaneously. This characterizes electricity markets in which all generation capacity is dispatchable thermal plant and there is no significant storage (e.g., in the form of hydropower). We also do not consider intermittent nondispatchable resources (such as wind); however, if their capacity is exogenous, their output can simply be subtracted from consumer quantity demanded, so that d represents effective demand. In particular, this extension will be characterized by Proposition 1. We start this section by introducing some definitions and conditions, followed by the statement of Proposition 1. In the remainder of this section we then introduce the open and the closed loop model for multiple load periods in Sects. 4.1 and 4.2. In Sect. 4.3 we present the proof of Proposition 1. Section 4.4 contains a numerical example of the theoretical results obtained in this section. Finally, in Sect. 4.5 we introduce and briefly analyze alternative conjectured price response models with switching instead of constant second stage behavior, which is arguably more realistic [23], and we show how Proposition 1 can be extended to these alternative types of models.

We adopt the following assumptions:

- We still consider two identical firms and a linear demand function.
- Additionally let us define l as the index for distinct load periods. Now production decisions depend on both i and l . Furthermore let us define the active set of load levels LB_i as the set of load periods in which equilibrium production equals capacity for firm i , i.e., $LB_i := \{l | q_{il} = x_i\}$.
- The conjectured price response θ_l for each i is a constant in $[0, 1/\alpha]$.
- Both cost parameters, δ, β , are nonnegative.
- Moreover, $p_l(d) = (D_{0l} - d_l)/\alpha_l$ where D_{0l} and α_l are positive constants, and P_{0l} denotes D_{0l}/α_l .

³ Let $D_0 = 1, t = 1, \alpha = 1, \beta = 1/2$ and $\delta = 0$, then the open loop Cournot solution is $p = 2/3$, with $x = 1/6$ for each firm. In this case, with these cost numbers, the open loop Cournot equals the closed loop equilibrium with $\theta = 2/\alpha$ (collusion, $\Phi = 1$).

- In addition, let $P_{0l} > \delta$ be true for $l \in LB_i$, which means that the maximum price that can be attained in the market has to be bigger than the production cost, otherwise there would be no investment or production.
- We also assume $P_{0l} \geq \delta$ for $l \notin LB_i$, which is similar to the condition above and guarantees non-negativity, however, it allows production to be zero in non-binding load periods.
- Similarly to the assumption made in the previous section, we also assume that $\sum_{l \in LB_i} P_{0l} t_l > \beta + \delta \sum_{l \in LB_i} t_l$, which states that if maximum price P_{0l} is paid for the durations t_l then the resulting revenue must be more than the sum of the investment cost and the operation cost, otherwise there is no incentive to participate in the market.
- The same demand curve assumptions are made as in Sect. 2.
- We consider one year rather than a multi-year time horizon, and so each firm is maximizing its annualized profit.

Proposition 1 (a) *If the closed loop solutions for different θ between perfect competition and Cournot competition exist and have the same active set of load periods (i.e., firm i 's upper bound on production is binding for the same load periods l) and the second stage multipliers, corresponding to the active set, are positive at equilibrium, then capacity x_i is the same for those values of θ .* (b) *Furthermore, if we assume that the open loop Cournot equilibrium, i.e., $\theta = 1/\alpha$, has the same active set, then the Cournot open and closed loop equilibria are the same.*

Perhaps the most difficult assumption of Proposition 1 is the existence of closed loop equilibria, since in general, EPECs may not have pure strategy equilibria as shown in [15] and example 4 of [18].

4.1 The open loop model for multiple load periods

The purpose of this section is to introduce the open loop model for general θ and multiple load periods and to characterize the equilibrium capacity x_i . Hence, we write the open loop investment-operation market equilibrium as:

$$\forall i \begin{cases} \max_{x_i, q_{il}} \sum_l t_l (p_l(q_{il}, q_{-il}) - \delta) q_{il} - \beta x_i \\ s.t. & q_{il} \leq x_i \quad \forall l \end{cases} \tag{23}$$

$$d_l = q_{il} + q_{-il}, \quad d_l = D_{0l} - \alpha_l p_l(q_{il}, q_{-il}) \quad \forall l \tag{24}$$

As previously mentioned in Sect. 3.1, the non-negativity constraints can be omitted in this case. Similar to Sect. 3.1, we can derive the investment-operation market equilibrium conditions in which we distinguish periods where capacity is binding from when capacity is slack. Details can be found in [34].

For the non-binding load periods $l \notin LB_i$ we can obtain the solution to the equilibrium by solving the system of equations given by the investment-operation market equilibrium conditions, which yields:

$$q_{il} = \frac{D_{0l} - \alpha_l \delta}{2 + \alpha_l \theta_l} \quad \forall i, l \notin LB_i \tag{25}$$

$$p_l = \frac{D_{0l} \theta + 2\delta}{2 + \alpha_l \theta_l} \quad \forall l \notin LB_i. \tag{26}$$

In order to obtain the solution for load levels when capacity is binding, we sum $\frac{\partial \mathcal{L}_i}{\partial q_{il}}$ over all load periods $l \in LB_i$, substitute $q_{il} = x_i$ and use the $\frac{\partial \mathcal{L}_i}{\partial x_i} = 0$ condition. We then express price as a function of capacity ($q_i = x_i$) and we solve the system of equations given by the investment-operations market equilibrium, together with the previously mentioned condition $\frac{\partial \mathcal{L}_i}{\partial x_i} = 0$ for all i , which yields the capacity solution given in (27), as detailed in [34].

$$x_i = \frac{\sum_{l \in LB_i} \left(D_{0l} t_l \prod_{n \neq l \in LB_i} \alpha_n \right) - \prod_{l \in LB_i} \alpha_l \left(\beta + \delta \sum_{l \in LB_i} t_l \right)}{\sum_{l \in LB_i} \left(t_l (2 + \alpha_l \theta_l) \prod_{n \neq l \in LB_i} \alpha_n \right)}, \forall i \tag{27}$$

We know that for $\theta_l \in [0, 1/\alpha_l]$, q_{il} will be a continuous function of x_i and hence from the investment-operations equilibrium conditions we get that λ_{il} will also be a continuous function of x_i . Having obtained capacities x_i , the prices p_l and demand d_l for $l \in LB_i$ follow. We furthermore observe that (10) is a special case of (27) in which we only have one binding load period.

Above it has not been explicitly stated that q_i and x_i are positive variables, but this is satisfied at the equilibrium point, see [34].

4.2 The closed loop model for multiple load periods

Let us now introduce the closed loop problem for multiple load periods and derive an expression for the equilibrium capacity. First, we state the second stage production game for the closed loop game with multiple load periods in (28)–(29) and define Lagrange multipliers λ_{il} for the constraint $q_{il} \leq x_i$.

$$\forall i \begin{cases} \max_{q_{il}} & \sum_l t_l (p_l(q_{il}, q_{-il}) - \delta) q_{il} \\ s.t. & q_{il} \leq x_i \quad \forall l \end{cases} \tag{28}$$

$$d_l = q_{il} + q_{-il}, \quad d_l = D_{0l} - \alpha_l p_l(q_{il}, q_{-il}) \quad \forall l \tag{29}$$

As mentioned in previous sections, we can derive the market equilibrium conditions, assuming that each firm holds the same conjectured price response θ_l in each load period l . θ_l can differ among periods. The complementarity between λ_{il} and $q_i < x_i$ for $l \notin LB_i$ implies that $\lambda_{il} = 0$ for $l \notin LB_i$. Hence we can omit that complementarity condition for those load periods in the market equilibrium formulation, which is given in detail in [34]. Moreover, we assume that multipliers λ_{il} for $l \in LB_i$ will be positive at equilibrium. (If any multipliers are zero, then Proposition 1 may not hold.)

For the non-binding load periods $l \notin LB_i$ we can obtain the solution to the conjectured price response market equilibrium by solving the corresponding system of

equations given by the multiple load period market equilibrium conditions, which yields:

$$q_{il} = \frac{D_{0l} - \alpha_l \delta}{2 + \alpha_l \theta_l} \quad \forall i, l \notin LB_i \tag{30}$$

$$p_l = \frac{D_{0l} \theta + 2\delta}{2 + \alpha_l \theta_l} \quad \forall l \notin LB_i. \tag{31}$$

We cannot yet solve the market equilibrium for the binding load periods $l \in LB_i$ depending as they do upon the x_i 's. Hence we move on to the investment equilibrium problem to obtain those x_i 's, which is formulated below:

$$\forall i \left\{ \begin{array}{l} \max_{x_i} \sum_l t_l p_l (q_{il}, q_{-il}) q_{il} - \sum_l t_l \delta q_{il} - \beta x_i \\ \text{s.t.} \quad \text{Market Equilibrium Conditions for (28), (29)} \end{array} \right. \tag{32}$$

After recalling that $q_l = x_i$ for l belonging to LB_i and then re-arranging terms, we can rewrite the objective function separating the terms that correspond to inactive capacity constraints ($l \notin LB_i$) which do not involve x_i 's at all, and the terms that correspond to the active capacity constraints ($l \in LB_i$). We express price as a function of capacity for $l \in LB_i$ and replace it in the new expression of the objective function, take the derivative of this objective function with respect to x_i , set it to zero and solve for x_i , which yields the closed loop capacity solution given in equation (33). Details can be found in [34].

$$x_i = \frac{\sum_{l \in LB_i} (D_{0l} t_l \prod_{n \neq l \in LB_i} \alpha_n) - \prod_{l \in LB_i} \alpha_l (\beta + \delta \sum_{l \in LB_i} t_l)}{3 \sum_{l \in LB_i} (t_l \prod_{n \neq l \in LB_i} \alpha_n)} \quad \forall i \tag{33}$$

Having obtained capacities x_i , the prices p_l and values for q_{il} for $l \in LB_i$ follow and the solution will be non-trivial, see [34].

4.3 Proof of proposition 1

Proof (Part a) First, we observe that the closed loop capacity, given by (33), does not depend on the conjectured price responses θ_l , for $l = 1, \dots, L$, and in particular this means that two closed loop equilibria with different θ_l 's have the exact same capacity solution as long as their active sets are the same with λ_{il} positive for $l \in LB_i$.

(Part b) Comparing the closed loop capacity (33) with the open loop capacity (27) we note that the open loop capacity does depend on the strategic behavior θ_l in the market whereas the closed loop capacity does not. Moreover we observe that if open loop and closed loop models have the same active set at equilibrium, then their solutions are exactly the same under Cournot competition ($\theta_l = 1/\alpha_l$). If open and closed loop equilibria have the same active set and their θ_l coincide but are not Cournot, then in general their capacity will differ. However, their production q_{il} for $l \notin LB_i$ will be identical, as can be seen by comparing (25)–(26) and (30)–(31). \square

In general, prices will be lower in the second stage under perfect competition than under Cournot competition for periods other than LB_i . In that case, consumers will be better off (and firms worse off) under perfect competition than Cournot competition. The numerical example in Sect. 4.4 illustrates this point. However, this result is parameter dependent as will be demonstrated by the example in Sect. 5.2, where we will show that in some cases Cournot competition can yield more capacity and higher ME than perfect competition. This can only occur for cases where either the binding LB_i differ, or the LB_i are the same but the λ_{il} are zero for some l . We furthermore observe that for one load period, Proposition 1 reduces to Theorem 1, i.e., (19) is a special case of (33).

Proposition 1, like Theorem 1, can be extended to asymmetric firms. As the details are tedious we refer the reader to the proof presented in [33].

4.4 Example with two load periods: LB_i the same for all θ

Let us now consider an illustrative numerical example where two firms both consider an investment in power generation capacity with the following data:

- Two load periods l , with durations of $t_1=3,760$ and $t_2=5,000$ [h/year]
- Demand intercepts D_{0l} given by $D_{0,1}=2,000$ and $D_{0,2}=1,200$ [MW]
- Demand slopes α_l equal to $\alpha_1 = D_{0,1}/250$ and $\alpha_2 = D_{0,2}/200$
- Annualized capital cost $\beta = 46,000$ [€/MW/year]
- Operating cost $\delta = 11.8$ [€/MWh]

Having chosen the demand data for the two load levels such that capacity will not be binding in both periods in any solution, we solve the open loop and the closed loop model and compare results. In Fig. 1 we present the solution of one firm (as the second firm will have the same solution). First we depict the capacity that was built, then we compare production for both load periods and finally profits. Note that for both firms, LB_i will be the same for all θ and will include only period $l = 1$. Later we will present another example where this is not the case, and the results differ in important ways.

As demonstrated in Proposition 1, the closed loop capacity does not depend on behavior in the spot market. However we will see that profits do depend on the competitiveness of short-run behavior, and unlike the single demand period case, are not the same for all θ between perfect competition and Cournot. We refer to the binding load period as 'peak' and to the non-binding load period as 'base'. The closed loop production in the peak load level is the same for all θ , as long as the competitive behavior on the spot market is at least as competitive as Cournot. However, base load production depends on the strategic behavior in the spot market. This can be explained as follows: as long as the strategic behavior in the spot market is at least as competitive as Cournot, peak load outputs are independent of θ because agents are aware that building Cournot capacities will cause the peak period capacity constraint to bind and will limit production on the market to the Cournot capacity. However, given our demand data we also know that capacities will not be binding in the base period and as a consequence outputs will not be limited either. Hence during the base periods the closed loop model will find it most profitable to produce the equilibrium outcomes resulting from the particular conjectured price response.

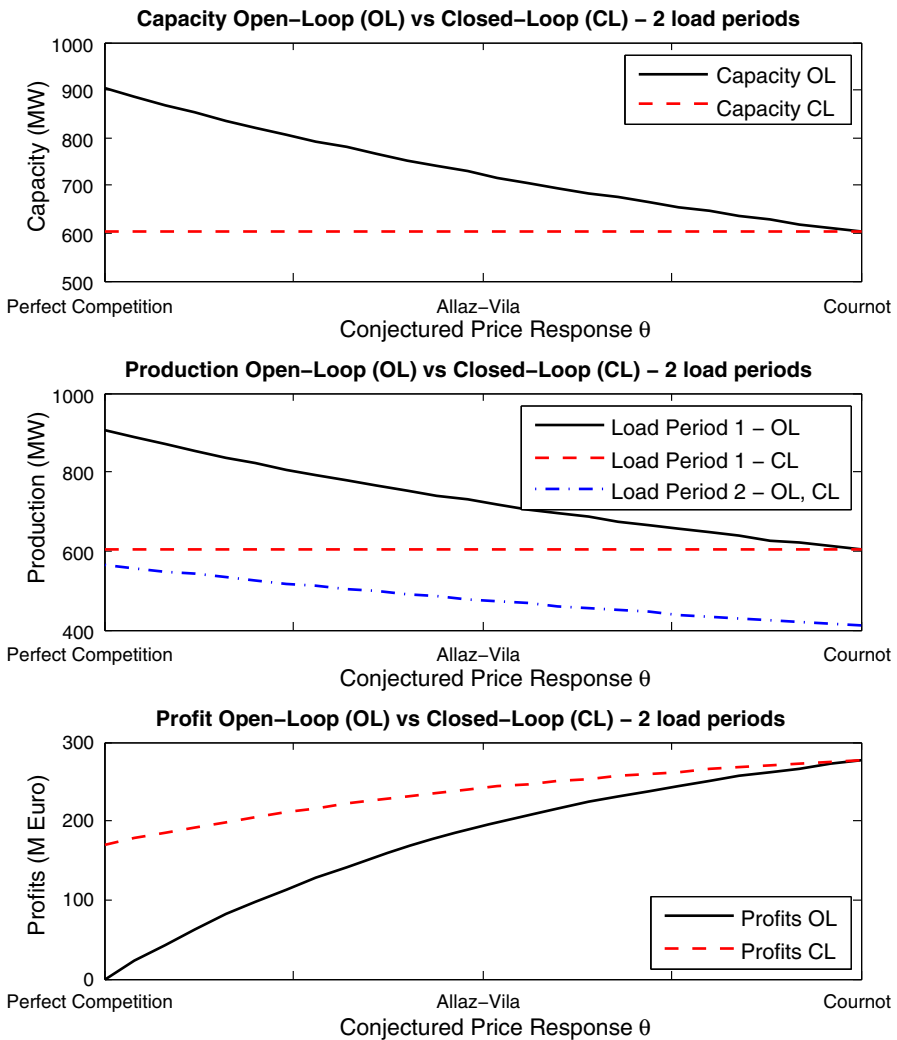


Fig. 1 Built capacity, production and profit of one firm in the two load period numerical example

On the other hand, when considering the open loop model, the capacity (peak load production) does depend on θ . In particular, the open loop capacity will be determined by the spot market equilibrium considering the degree of competitive behavior specified by θ . We observe that for increasing θ between perfect competition and Cournot in the open loop model, less and less capacity is built until we reach the Cournot case, at which point the open and closed loop results are exactly the same. Comparing open and closed loop models for a given θ reveals that while their base load outputs are identical, see Tables 1, 2, capacity and thus peak load production differs depending on θ . Figure 1 also shows that profits obtained in the closed loop model equal or exceed the profits of the open loop model. This gap is largest assuming perfect

Table 1 Closed loop equilibrium solution perfect competition ($\theta_l = 0$), Allaz-Vila ($\theta_l = 1/(2\alpha_l)$) and Cournot ($\theta_l = 1/\alpha_l$) second-stage competition

l	Peak	Base
q_{il} [MW]		
$\theta_l = 0$	602.6	564.6
q_{il} [MW]		
$\theta_l = 1/(2\alpha_l)$	602.6	451.7
q_{il} [MW]		
$\theta_l = 1/\alpha_l$	602.6	376.4
p_l [€/MWh]		
$\theta_l = 0$	99.4	11.8
p_l [€/MWh]		
$\theta_l = 1/(2\alpha_l)$	99.4	49.4
p_l [€/MWh]		
$\theta_l = 1/\alpha_l$	99.4	74.5

Table 2 Open loop equilibrium solution perfect competition ($\theta_l = 0$), Allaz-Vila ($\theta_l = 1/(2\alpha_l)$) and Cournot ($\theta_l = 1/\alpha_l$) second-stage competition

l	Peak	Base
q_{il} [MW]		
$\theta_l = 0$	903.9	564.6
q_{il} [MW]		
$\theta_l = 1/(2\alpha_l)$	723.1	451.7
q_{il} [MW]		
$\theta_l = 1/\alpha_l$	602.6	376.4
p_l [€/MWh]		
$\theta_l = 0$	24.0	11.8
p_l [€/MWh]		
$\theta_l = 1/(2\alpha_l)$	69.2	49.4
p_l [€/MWh]		
$\theta_l = 1/\alpha_l$	99.4	74.5

competition and becomes continuously smaller for increasing θ until the results are equal under Cournot. This means that the further away that spot market competition is from Cournot, the greater the difference between model outcomes.

In standard open loop oligopoly models [12] without capacity constraints, perfect competition gives lower prices and total profits of firms, and greater CS, and market efficiency compared to Cournot competition.⁴ We observe that this occurs for this particular instance of the open and closed loop models, see Tables 2, 3. It can be readily proven more generally that market efficiency, consumer surplus, and average prices are greater for lower values of θ (more competitive second stage conditions) if LB_i are the same for those θ (and multipliers are positive), and capacity is not binding

⁴ Total Profit is defined as $\sum_l t_l(p_l - \delta)(q_{il} + q_{-il}) - \beta(x_i + x_{-i})$. CS is defined as the integral of the demand curve minus payments for energy, equal here to $\sum_l t_l(P_{0l} - p_l)(q_{il} + q_{-il})/2$. Market efficiency (ME) is defined as CS plus total profits.

Table 3 Market efficiency (ME), consumer surplus (CS) and total profit in closed loop solutions

	Perfect competition	Allaz–Vila	Cournot
ME [€]	1.21×10^9	1.19×10^9	1.15×10^9
CS [€]	0.873×10^9	0.681×10^9	0.577×10^9
Total profit [€]	0.342×10^9	0.511×10^9	0.578×10^9

in every l .⁵ However, we will also demonstrate by counter-example that this result does not necessarily apply when LB_i differ for different θ . In particular, in Sect. 5.2 we will present an example in which Cournot competition counterintuitively yields higher ME than perfect competition.

4.5 Conjectured price response models with switching conjectures

In this section we consider and analyze alternative conjectured price response models that are a variant of the previously presented models. In particular, we propose models in which a firm always has a Cournot conjectured price response with respect to the output of a rival in periods when their capacity is binding, and an arbitrary conjectured price response θ between perfect competition and Cournot when the rival's capacity is not binding. This type of model is arguably more realistic because producers in the second stage will recognize the times when rivals are at their capacity constraint and cannot increase output. This argument has been thoroughly discussed in [23].

In general, when solving models with switching conjectures, one has to have in mind that in a multi-player game, some generation companies may have binding capacity, but others might not in the same load period. In this case, the conjecture of the generation company at capacity would be θ and the rival's conjecture would be the Cournot conjecture. Such models are more difficult to solve than the previously presented models as some kind of iterative process has to be adopted and moreover, pure strategy equilibria might not exist. Hence, for the sake of simplicity of this analysis, we assume that a pure strategy equilibrium exists and we furthermore assume that the equilibrium is a symmetric one. Because of space limitations, we omit the formulation of these alternative open and closed loop models and other details, which can be found in [34].

As an aside we note that asymmetric equilibria may exist, even if the firms themselves are symmetric. For an analysis of the case of asymmetric equilibria, which is more complicated and extensive than the case discussed in this section, and numerical examples that show this can happen with the Allaz–Vila conjectural variation, the reader is referred to [23]. Although asymmetric equilibria are of interest and deserve further study, we do not explore these alternative models outside of this section.

Our main result, extending Proposition 1 to models with switching conjectures, is that many different kinds of strategic behavior in the energy market yield the same equilibrium capacity provided the active sets of load periods are the same at equilibrium and the conjecture ranges between perfect and Cournot competition:

⁵ This is proven by demonstrating that for smaller θ , the second stage prices will be lower and closer to marginal operating cost in load periods for those periods that capacity is not binding.

- Closed loop models with switching conjectures in style of [23], as discussed above, under any conjecture (i.e., any conjectured price response θ).
- Open and closed loop models in style of [23] under any conjecture.
- Two closed loop models, one with and the other without switching conjectures, under any conjecture.

However should different conjectures or different ways of reacting to a competitor being at capacity lead to different binding load periods, then installed capacity may also differ (if a pure strategy equilibrium exists at all).

5 Ranking of closed loop equilibria: capacity and market efficiency

In this section we make some observations concerning capacity results in closed loop equilibria. We also discuss the ambiguities that occur regarding social welfare when comparing two closed loop equilibrium solutions with different strategic behavior in the spot market. In Sect. 5.1 we prove that the capacity of a closed loop model with competitive behavior between perfect competition and Cournot can be lower or equal to the closed loop Cournot second stage capacity, depending on the choice of data, and moreover, that it cannot be higher for symmetric players in the two period case. In 5.2 we prove by counter-example that the ranking of closed loop conjectured price response equilibria, in terms of ME (aggregate CS and market surplus) and consumer welfare, is parameter dependent.

5.1 Comparisons of capacity from closed loop equilibria

In this section we analyze the effects of the strategic behavior in the spot market on capacity in the closed loop model. This work is an extension of the work of Murphy and Smeers [23], in which they compare a closed loop Cournot model to a model with an additional forward market stage, i.e., a closed loop Allaz–Vila model with capacity decisions. They find that, depending on the data, the capacity yielded by the closed loop Allaz–Vila model can either be more, less or equal to the capacity given by the closed loop Cournot model in a market with asymmetric players. We extend their results to general conjectural variations considering symmetric companies, and compare our closed loop model with Cournot second stage competition to a closed loop model with arbitrary second stage competition between perfect competition and Cournot. We show that in this comparison the capacity yielded by conjectured price response second stage competition can be less (decreasing) or equal to the closed loop Cournot capacity. Further, we find that the former capacity cannot exceed the latter for symmetric agents and two load periods.

Part (a) of Proposition 1 proves that if two closed loop solutions for different θ between perfect competition and Cournot competition have the same active set of load periods, then capacity is the same for those values. The corresponding numerical example has been presented in Sect. 4.4. This demonstrates that it is possible for two different closed loop models to yield the same capacity. However, from the closed form expression for capacity, given in (33), we also know that when active sets of

load periods do not coincide, then the solutions will generally not be the same. For an example in which the closed loop Cournot second stage capacity is strictly above the capacities yielded by other closed loop models with more competitive strategic behavior, we refer the reader to Sect. 5.2.

For two load periods, the case in which capacity increases with increasing competition (which occurred for an asymmetric case in [23]) cannot happen for symmetric agents. The proof of this result as well as a two load period numerical example of decreasing capacity can be found in [34]. Our result therefore shows that, for the two load period case in [23], asymmetry is a necessary condition in order for the capacity of the closed loop conjectured price response solution to be larger than in the closed loop second stage Cournot equilibrium. We hypothesize that in the case of symmetric agents this might generally be true for multiple load periods as well. However, proving this hypothesis or finding a counterexample is out of the scope of this paper and will be a topic of future research.

5.2 Ambiguity in ranking of closed loop equilibria when LB_i differs for different θ

In this section we show by counter-example that the ranking of the closed loop conjectured price response equilibria, in terms of ME and consumer welfare, is parameter dependent. An interesting result we obtain is that it is possible for the closed loop model that assumes perfectly competitive behavior in the market to actually result in lower ME (as measured by the sum of surpluses for all parties and load periods), lower CS, and higher average prices than when Cournot competition prevails. This counter-intuitive result implies that contrary to regulators' beliefs that requiring marginal cost bidding protects consumers, it actually can be harmful. In [14] the authors have arrived at a similar result comparing perfectly competitive and Cournot spot market behavior, however, they only look at the polar cases of perfect competition or Cournot-type competition. In our paper we generalize strategic behavior using conjectural variations and look at a range of strategic behavior, from perfect competition to Cournot competition and we furthermore observe that an intermediate solution between perfect and Cournot competition can lead to even larger social welfare and CS despite yielding a level of installed capacity intermediate between the perfect competition and the Cournot cases. In particular: *The ranking of conjectured price response equilibria in terms of market efficiency and consumer welfare is parameter dependent.* This occurs because in general the LB_i differ among the solutions. It does not occur when LB_i are the same for all θ and multipliers are positive as proven (and illustrated) in the previous section.

A counter-example: Let us now consider two firms both making an investment in generation capacity using the following data:

- Twenty equal length load periods l , so $t_l = 438$ [h/year] for $l = 1, \dots, 20$
- Demand intercept D_{0l} , obtained by $D_{0l} = 2,000 - 50(l - 1)$ [MW] and demand slope α_l , obtained by $D_{0l}/250$ for $l = 1, \dots, 20$
- Capital cost $\beta = 46,000$ [€/MW/year]
- Operating cost $\delta = 11.8$ [€/MWh]

Table 4 Closed loop equilibrium solution under perfect competition second-stage competition ($\theta_l = 0$) with capacity $x_i = 456.2$ MW

l	1	2	3	4	5	6	7	8	9	10
q_{il} [MW]	456.2	456.2	456.2	456.2	456.2	456.2	456.2	456.2	456.2	456.2
p_l [€/MWh]	135.9	133.0	129.9	126.7	123.3	119.7	115.8	111.8	107.4	102.8
l	11	12	13	14	15	16	17	18	19	20
q_{il} [MW]	456.2	456.2	456.2	456.2	456.2	456.2	456.2	456.2	456.2	456.2
p_l [€/MWh]	97.9	92.7	87.1	81.0	74.5	67.5	59.9	51.6	42.6	32.8

Table 5 Closed loop equilibrium solution under Cournot second-stage competition ($\theta_l = 1/\alpha_l$) with capacity $x_i = 550.6$ MW

l	1	2	3	4	5	6	7	8	9	10
q_{il} [MW]	550.6	550.6	550.6	550.6	550.6	550.6	539.9	524.0	508.2	492.3
p_l [€/MWh]	112.4	108.8	105.1	101.2	97.1	92.7	91.2	91.2	91.2	91.2
l	11	12	13	14	15	16	17	18	19	20
q_{il} [MW]	476.4	460.5	444.6	428.8	412.9	397.0	381.1	365.2	349.4	333.5
p_l [€/MWh]	91.2	91.2	91.2	91.2	91.2	91.2	91.2	91.2	91.2	91.2

First we assume perfect competition, i.e., $\theta_l = 0$. We solve the resulting closed loop game by diagonalization [18], which is an iterative method in which firms take turns updating their first-stage capacity decisions, each time solving a two-stage MPEC while considering the competition’s capacity decisions as fixed. The closed loop equilibrium solution assuming perfect competition in stage two is shown in Table 4. Second, we assume Cournot competition in the spot market, i.e., $\theta_l = 1/\alpha_l$. Again we solve the closed loop game by diagonalization, yielding the results shown in Table 5. We observe that under second stage perfect competition, the capacity of 456.2 MW is binding in every load period and prices never fall to marginal operating cost. Moreover, the total installed capacity of 912.4 MW is significantly lower than that installed under Cournot, which is 1,101.2 MW. On the other hand, under Cournot competition, each firm’s capacity of 550.6 MW is binding only in the first six load periods and the firms exercise market power by restricting their output to below capacity in the other fourteen periods. Furthermore considering that the Cournot capacity is well above the perfectly competitive capacity, it follows that during the six peak load periods, perfectly competitive prices will be higher than Cournot prices. Third, we solve the closed loop game assuming Allaz–Vila as an intermediate level of competitiveness between perfect competition and Cournot, i.e., $\theta_l = 1/(2\alpha_l)$. This yields the equilibrium given in Table 6. Finally, the system optimal plan, which is obtained by central planning under a maximization of social welfare objective, is presented in Table 7. As expected, this solution exhibits the highest total installed capacity of 1,651.8 MW and

Table 6 Closed loop equilibrium solution assuming Allaz–Vila second-stage competition ($\theta_l = 1/(2\alpha_l)$) with capacity $x_i = 515.2$ MW

l	1	2	3	4	5	6	7	8	9	10
q_{il} [MW]	515.2	515.2	515.2	515.2	515.2	515.2	515.2	515.2	515.2	515.2
p_l [€/MWh]	121.2	117.9	114.4	110.8	106.9	102.8	98.5	93.9	89.0	83.8
l	11	12	13	14	15	16	17	18	19	20
q_{il} [MW]	515.2	515.2	515.2	514.5	495.5	476.4	457.3	438.3	419.2	400.2
p_l [€/MWh]	78.3	72.3	66.0	59.4	59.4	59.4	59.4	59.4	59.4	59.4

Table 7 System optimal plan solution with capacity $x_i = 825.9$ MW

l	1	2	3	4	5	6	7	8	9	10
q_{il} [MW]	825.9	825.9	825.9	825.9	825.9	825.9	809.9	786.1	762.2	738.4
p_l [€/MWh]	43.5	38.2	32.7	26.8	20.6	14.0	11.8	11.8	11.8	11.8
l	11	12	13	14	15	16	17	18	19	20
q_{il} [MW]	714.6	690.8	667.0	643.1	619.3	595.5	571.7	547.9	524.0	500.2
p_l [€/MWh]	11.8	11.8	11.8	11.8	11.8	11.8	11.8	11.8	11.8	11.8

the lowest prices. We have omitted the open loop results here, however, they can be found in [34].

This closed loop investment game can be viewed as a kind of prisoners' dilemma among multiple companies. An individual company might be able to unilaterally improve its profit by expanding capacity, with higher volumes making up for lower prices. But if all companies do that, then everyone's profits could be lower than if all companies instead refrained from building. (In this prisoners' dilemma metaphor we have not taken into account another set of players that is better off when the companies all build, i.e., the consumers, who enjoy lower prices and more consumption; as a result, overall ME as measured by total market surplus may improve when firms "cheat".)

Standard (single stage) oligopoly models [12] without capacity constraints find that perfect competition gives lower prices and greater ME than Cournot. Considering that standard result, our results seem counter-intuitive, but they are due to the two-stage nature of the game. In particular, less intensive competition in the commodity market can result in more investment and more consumer benefits than if competition in the commodity market is intense (price competition a la Bertrand). In terms of the prisoners' dilemma metaphor, higher short run margins under Cournot competition provide more incentive for the "prisoners" to "cheat" by adding capacity. Note that in order to get these counter-intuitive results, firms do not need to be symmetric, as shown in a numerical example in [33].

Comparing the ME and the CS that we obtain in the perfectly competitive, Cournot, Allaz–Vila and the social welfare maximizing solutions in Table 8, we observe that, surprisingly, apart from the welfare maximizing solution the highest social welfare

Table 8 Market efficiency (ME), consumer surplus (CS) and total profit in closed loop solutions and social welfare maximizing solution

	Perfect competition	Allaz–Vila	Cournot	Social welfare maximum
ME [€]	1.24×10^9	1.30×10^9	1.28×10^9	1.47×10^9
CS [€]	0.621×10^9	0.717×10^9	0.638×10^9	1.436×10^9
Total profit [€]	0.621×10^9	0.584×10^9	0.638×10^9	0.034×10^9

and the highest CS is obtained under the intermediate Allaz–Vila case. Even more surprising is that the capacity obtained under Allaz–Vila competition is lower than the Cournot capacity, but still yields a higher social welfare. This is because the greater welfare obtained during periods when capacity is slack (and Allaz–Vila prices are lower and closer to production cost) offsets the welfare loss during peak periods when the greater Cournot capacity yields lower prices.

Another surprise is that not only ME but also profits are non-monotonic in θ . Both perfect competition and Cournot profits are higher than Allaz–Vila profits; the lowest profit thus occurs when ME is highest, at least under these parameters. However, higher profits do not always imply lower ME, as a comparison of the perfect competition and Cournot open loop cases shows. Cournot shows higher profit, CS, and ME than perfect competition. That is, Cournot is Pareto superior to perfect competition under these parameters because all parties are better off under the Cournot equilibrium.

6 Conclusions

In this paper we compare two types of models for modeling the generation capacity expansion game: an open loop model describing a game in which investment and operation decisions are made simultaneously, and a closed loop equilibrium model, where investment and operation decisions are made sequentially. The purpose of this comparison is to emphasize that when resorting to easier, less complicated open loop models, instead of solving the more realistic but more complicated closed loop models, the results may differ greatly. In both models the market is represented via a conjectured price response, which allows us to capture various degrees of oligopolistic behavior. Setting out to characterize the differences between these two models, we have found that for one load period, the closed loop equilibrium equals the open loop Cournot equilibrium for any choice of conjectured price response between perfect competition and Cournot competition—a generalization of Kreps and Scheinkman-like [20] findings. In the case of multiple load periods, this result can be extended. In particular, if closed loop models under different conjectures have the same set of load periods in which capacity is constraining and the corresponding multipliers are positive, then their first stage capacity decisions are the same, although not their outputs during periods when capacity is slack. Furthermore, if the Cournot open and closed loop solutions have the same periods when capacity constrains, then their solutions are identical. When comparing two closed loop models, one with Cournot behavior and the other with different strategic second stage behavior between perfect

competition and Cournot, then the closed loop conjectured price response capacity can be less or equal than the closed loop Cournot second stage capacity. Moreover the former capacity cannot exceed the latter when there are symmetric agents and two load periods.

We also explore alternative conjectured price response models in which the strategic second stage competition switches to Cournot in load periods in which rivals' capacity is binding. When capacity does not bind, the strategic behavior can range from perfect competition to Cournot. Such alternative models may be more realistic, however, pure strategy equilibria might not exist and they are more difficult to solve.

As indicated in the first numerical example, when having market behavior close to Cournot competition, the additional effort of computing the closed loop model (as opposed to the simpler open loop model) does not pay off because the outcomes are either exactly the same or very similar depending on the data. But if behavior on the spot market is far from Cournot competition and approaching perfect competition, the additional modeling effort might be worthwhile, as the closed loop model is capable of depicting a feature that the open loop model fails to capture, which is that generation companies would not voluntarily build all the capacity that might be determined by the spot market equilibrium if that meant less profits for themselves. Thus the closed loop model could be useful to evaluate the effect of alternative market designs for mitigating market power in spot markets and incenting capacity investments in the long run, e.g. capacity mechanisms, in Sakellaris [28]. Extensions could also consider the effect of forward energy contracting as well (as in Murphy and Smeers [23]). These policy analyses will be the subject of future research.

The second numerical example shows that depending on the choice of parameters, more competition in the spot market may lead to less ME and less CS in the closed loop model. This surprising result indicates that regulatory approaches that encourage or mandate marginal pricing in the spot market in order to protect consumers may actually lead to situations in which both consumers and generation companies are worse off.

In future research we will address the issue of existence and uniqueness of solutions, as has been done for the Cournot case by Murphy and Smeers [22], who found that a pure-strategy closed loop equilibrium does not necessarily exist but if it exists it is unique. We will also address the question concerning under what a priori conditions the active sets of open and closed loop equilibria coincide. There will be further investigation of games in which the conjectural variation is endogenous, resulting from the possibility that power producers might adopt the Cournot conjecture in binding load periods since they may be aware that their rivals cannot expand output at such times. Finally, the games presented here will be extended to multi-year games with sequential capacity decisions, and the effects of forward contracting will be investigated.

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