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ORIGINAL PAPER

Balance of power in a conflict model

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Abstract
This study provides a microeconomic foundation for the bipolar stability hypothesis in international politics. It extends the well-designed conflict model of Esteban and Ray (Am Econ Rev 101(4):1345–1374, 2011) to include monetary compensation arrangements between the winning and losing groups, presenting a new conflict-related indicator called the balance of power index. The main finding of this study is that societal polarization serves to alleviate rather than exacerbate conflict intensity, which is elucidated by the balance of power index. This new characteristic of polarization is associated with the founding of the bipolar stability hypothesis by Waltz (J Int Affairs 21(2):215–231, 1967), Waltz (Theory of international politics. Addison-Wesley Publishing Company, Reading, 1979) under the economic behavioral model.

Keywords Conflict · Rent-seeking · Monetary compensation · Balance of power · Bipolar stability

JEL Classification D63 · D72 · D74

1 Introduction

Since the latter half of the 20th century, the number of conflicts has been on an upward trend, which has reached an elevated level even in the 21st century. Recently, around 2023, more than 55 armed conflicts took place, with an annual battle-related death toll of over 250,000.[1] The conflict is never a time-specific or localized event; it is a problem that has occurred at many different times and regions. Franzese et al. (2016) show the higher propensity of nations with a prior history of civil war to experience subsequent conflict episodes, indicating the persistence of conflict.

[1] Specific numbers of conflicts and death tolls after the end of World War II are provided by the Uppsala Conflict Data Program (https://ucdp.uu.se/).

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Buhaug and Gleditsch (2008) argue the contagion or spillover effects of conflict, demonstrating that armed conflict in one nation makes neighboring countries more prone to violence. The social conflict not only destructs the short-term productive economic activities during periods of warfare but also lowers the long-run economic growth by impeding both physical and human capital accumulation (Barro 1991; Alesina et al. 1996; Abadie and Gardeazabal 2003).

Contemporary literature on the causes of war and conflict has extensively highlighted various dimensions of ethnicity as underlying factors in social conflicts. In the early stages of this research direction, Easterly and Levine (1997), Fearon and Laitin (2003), and Collier and Hoeffler (2004) focus on ethnic fractionalization, a highly ethnical division into many small groups, as a key determinant of civil conflicts. Montalvo and Reynal-Querol (2005), Montalvo and Reynal-Querol (2010) emphasize another aspect of ethnic polarization into two major clusters of groups as a main driver of the incidence and duration of conflicts. Concurrently, Østby (2008) and Stewart (2010) propose the concept of horizontal inequality (i.e., inequality among culturally defined groups) as a potential risk of conflicts.

While these various factors are addressed, a landmark study encompassing them is presented by Esteban and Ray (2011), which build a behavioral model of conflict that provides a basis for using certain dispersion indices as conflict indicators.[2] They clearly show that the equilibrium level of conflict can be approximated by a linear function of three indices: the Gini-Greenberg index (G), the Herfindahl-Hirschman fractionalization index (F), and the Esteban-Ray polarization index (P).[3] What is particularly interesting about their work is that they successfully explain the causes of conflict by treating these three measures—previously analyzed separately—in a single model with a microeconomic foundation. Furthermore, by explicitly showing how each index and the conflict level are related, they make it possible to facilitate the transition to the subsequent empirical research by Esteban et al. (2012). Using data from 138 countries over 1960–2008, their empirical analysis verifies a strong relationship between conflict and the three indicators of ethnic group distribution.

Founded upon the framework of Esteban and Ray (2011), this study incorporates the role of (two-way) monetary compensation between the winners and losers in conflict. One is monetary payment from the losing to the winning parties, which can be regarded as war reparations. It is a well-known historical fact that victorious nations often impose stringent postwar reparations on defeated nations, sometimes provoking the recurrence of another war and conflict. The other way of compensation runs from winning to losing groups. This direction of monetary compensation is, of course, limited in practice because voluntary redistribution will be *ex post* suboptimal for the winner.[4] Yet, it closely links to the pivotal concept of *power-sharing*

[2] The original conflict model is found in Esteban and Ray (1999), which provides rigorous proof of the existence and uniqueness of equilibrium. Valsecchi (2010) extends the pure contest model of Esteban and Ray (1999) to include public and private goods.

[3] For the properties of the Gini index, see Thon (1982). For details on the background of the Herfindahl-Hirschman fractionalization index, see Hirschman (1964). For the Esteban-Ray polarization index, see Esteban and Ray (1994) and Kawada et al. (2018).

[4] Khan (2000) provides a possible justification for the compensation scenario from gainers to losers in a rent-seeking contest, while he also concedes its rarity in practice.

in conflict resolution and peacekeeping efforts, first proposed by Lijphart (1968), Lijphart (1977) and Nordlinger (1972). This hypothesis argues that the division of territory, resources, and other sources of conflict among actors, both winners and losers, will lead to the settlement of conflicts. This means that the group that wins the conflict allocates certain resources to the losing group, rather than keeping the blessings of victory to itself.[5] Omgba et al. (2021) mention the fact that, in African countries, the group in power voluntarily redistributes state revenue to other ethnic groups out of control, which may have saved years of peace.[6] One might expect that compensation in the first direction would intensify the conflict, and the latter would mitigate it, but it has not been determined whether this will indeed be the case. To clarify the relationship between the direction of monetary compensation and the level of conflict between the winning and losing groups, this study extends the conflict model of Esteban and Ray (2011) by explicitly introducing these two potential forms of monetary compensation arising during the course of conflicts.

The first result of this paper is that the equilibrium conflict is intensified in a bipolar society when monetary compensation flows from the losing to winning groups. This finding not only supports the main result of Esteban and Ray (2011) but also constitutes a uniqueness of this study. In the original conflict model of Esteban and Ray (2011), the escalation of conflict due to societal polarization is rooted in the competition over public goods. In contrast, this study reveals that the intensifying effect of societal polarization on conflict is present even in purely private environments, provided that monetary transfer from the losers to the winner. The key perspective behind this outcome is the incentive to accept the loser's position in the conflict. When a large-populated group wins the conflict, a given amount of monetary payment required on losing parties will be imposed on the rest of a small number of losers, resulting in a considerable burden of monetary payment per capita. Therefore, in this case, agents have less incentive to remain a loser's position, leading to more intense conflict. On the other hand, when a small group becomes the winner, the losing groups can share a given amount of monetary payment among a large number of losers. In this case, the per capita burden of the losers remains small, making them less fighting. If a society has a small number of large groups, the former scenario is likely to occur. In contrast, the latter situation is more probable in a society comprising numerous small groups.

The second and more remarkable result is that, in the presence of monetary compensation from the winning to losing groups, the equilibrium conflict level is mitigated in a bipolar society.[7] The economic intuition behind this finding can be constructed as just a reversal of the above conjecture. When a large group wins the conflict, a given amount of monetary resources reserved for the losing groups will

[5] Grossman (2004) also presents the related argument that limiting the prerogatives of the winner in the constitutional contest expands the range of parameters within which a self-enforcing constitution can be designed, thereby avoiding civil conflicts effectively.

[6] Beyond African countries, the economic power-sharing policy has also been implemented in countries such as Colombia, Iraq, Malaysia, and the Philippines. (Hartzell and Hoddie 2003).

[7] Gardeazabal (2011) empirically shows that linguistic polarization reduces the level of conflict, using panel data of 250 municipalities located in the Basque Country over 18 years (1989–2006).

be divided by the rest of a small number of losers. Therefore, in this case, agents can expect to receive greater monetary compensation on a per capita basis if they lose the conflict, leading to less resource investment in the conflict. Conversely, when a small group becomes the winner, a given amount of monetary wealth prearranged for the losing parties should be split into a number of remaining losers. In this case, individuals worry about their lower per-capita money and have less incentive to remain a loser, resulting in more intense conflict. The fundamental conclusion is, therefore, that the bipolar society consisting of two major groups contributes to reducing conflict and hence having stabilization in the presence of monetary compensation from the winning to the losing group. By presenting a new conflict-related indicator of *the balance of power index*, which stems from monetary compensation, this paper establishes the bipolar stability hypothesis presented by Waltz (1967), Waltz (1979) in a microeconomic framework.[8]

The remainder of this paper is organized as follows. Section 2 constructs the conflict model. Section 3.1 discusses the existence and uniqueness of equilibrium in this model. The main result is presented in Section 3.2. Section 3.3 analyzes the validity of the approximation assumption behind the main result. Section 4 concludes. All proofs are relegated to the appendices.

2 A model of conflict

This section extends the rent-seeking model of Esteban and Ray (2011) to one in which private goods are distributed among the winning and losing groups. In this paper, a conflict is viewed as an anarchic situation where several ethnic groups compete with each other to seize control of the government budget. The conflict is modeled as a static game with complete information. The subsequent sections describe each component of this conflict game: player, strategy, and payoff function.

2.1 Player

Consider a situation in which society comprises at least two ethnic groups, each consisting of individuals. First, let $m \in \{2, 3, 4, ...\}$ denote the number of groups engaging in a conflict. For each group $i \in \{1, ..., m\}$, let $N_i \in \mathbb{N}$ denote the number of individuals in group i. Then the total population of this society, which equals the number of players in the conflict game, is defined by $N \equiv \sum_{i=1}^{m} N_i$. In addition, for each group $i \in \{1, ..., m\}$, the population share of group i is denoted by $n_i \equiv N_i/N$ so that $\sum_{i=1}^{m} n_i = 1$.

[8] In his bipolar stability hypothesis, Waltz (1967), Waltz (1979) contends that a system dominated by two powers is the most stable. This idea is based on the fact that people are less likely to misidentify their hostile groups and that accidental conflicts are less likely to occur in societies where two large groups are dominant. On the other hand, Deutsch and Singer (1964) advocate a multipolar stability hypothesis. They argue that as the system moves away from bipolarity to multipolarity, the frequency and intensity of conflicts are expected to diminish.

2.2 Strategy

Players simultaneously choose the amount of resources to contribute to the nonproductive activities of conflict. Individuals can influence their group's probability of winning by changing the amount of resources they spend. Specifically, the model assumes that the probability of each group winning the conflict is determined by the relative amount of resources each group spends. Let $r_{ik} \in \mathbb{R}_+$ denote the contribution of resources by individual k belonging to group i. For each group $i \in \{1, ..., m\}$, the aggregate resource contributions of group i is $R_i \equiv \sum_{k \in i} r_{ik}$. In this case, the total resources wasted by the whole society are $R \equiv \sum_{i=1}^{m} R_i$, where R is defined as the conflict intensity. Let p_i be the probability that group i wins the conflict and suppose that

$$p_i = \frac{R_i}{R}$$

for all $i \in \{1, ..., m\}$ if $R > 0$.[9] If $R = 0$, any probability $\tilde{p}_i \in [0, 1]$ can be assigned where $\sum_{j=1}^{m} \tilde{p}_j = 1$. Thus, each individual's payoff function defined below must be discontinuous. However, this poses no problem for the existence of equilibrium (Esteban and Ray 1999).

2.3 Payoff function

Individuals earn a positive payoff from the consumption of public and private goods. Normalizing the budget to unity, let $\lambda \in [0, 1]$ denote the allocated portion for society-wide public goods. The remaining proportion, $1 - \lambda$, is designated for divisible private goods. In this context, national religion and official language can be envisaged as examples of public goods, whereas territorial land and natural resources serve as examples of private goods.

Every group has a different preference over public goods (i.e., every group has a distinct mix of public goods they prefer the most), with identical preferences within the same group. Using the private good as the numeraire, let $u_{ij} \in \mathbb{R}_{++}$ denote the payoff that an individual in group i obtains from one unit of the public good that group j prefers the most. To guarantee that agents acquire higher utility from their choice of public goods, it is assumed that $u_{ii} > u_{ij}$ for all $i, j \in \{1, ..., m\}$ with $i \neq j$. Then, individuals in group i attain payoff λu_{ii} by consuming λ units of the public good when group i wins the conflict. Conversely, if the other group j wins the conflict, they receive payoff λu_{ij} from the consumption of public goods.

All individuals derive identical payoffs from private goods, but the per capita benefit decreases with group size. Esteban and Ray (2011) assume that private goods are entirely consumed by the winning group. In this study, however, the level

[9] This specification of the contest success function follows Tullock (1980), a workhorse model of rent-seeking conflict. Hirshleifer (1989) alternatively postulates that each group's success probability is a function of the *difference* between the group's resource contributions, not their *ratio*. These two frequently used functional forms are formally axiomatized by Skaperdas (1996).

of conflict is characterized under which a certain compensation of private goods is allowed between winners and losers.

Under the extended framework, the payoff from private goods for the typical agent in group i can be written as follows.

$$\begin{cases} y + \frac{\beta(1-\lambda)}{n_i} & \text{if group } i \text{ wins the conflict,} \\ y + \frac{(1-\beta)(1-\lambda)}{1-n_j} & \text{if other group } j \text{ wins the conflict,} \end{cases}$$

where $y \in \mathbb{R}_{++}$ is exogenous secured money regardless of the conflict outcome. The parameter $\beta\,(> 0.5)$ is a key indicator for the direction and magnitude of monetary compensation.[10] Using β, two distinct scenarios are demonstrated: one is a monetary redistribution from the winning group to the rest of the losing groups, and the other is a monetary compensation from the losers to the winner as war reparations. When $0.5 < \beta < 1$, the winner compensates other losers, sharing the private goods between the winning and losing groups in the ratio of β and $1-\beta$, respectively. This situation may be seen as a practice of power-sharing along the economic dimension, e.g., the distribution among competing groups of economic resources such as oil revenues, defined by Hartzell and Hoddie (2003). On the other hand, when $\beta > 1$, the losing groups provide monetary compensation from y in hand to the winner in excess of the fixed amount of the budget, $1-\lambda$.[11] It corresponds to historical cases where the defeated groups is forced to pay war reparations to the winner due to their weak bargaining power. This model will reduce to the original one of Esteban and Ray (2011) when $\beta = 1$ and $y = 0$.

The total per-capita payoff for group i, therefore, can be summarized as $\lambda u_{ii} + y + \beta(1-\lambda)/n_i$ when group i wins the conflict, and $\lambda u_{ij} + y + (1-\beta)(1-\lambda)/(1-n_j)$ when other group j wins.

While each individual can increase the winning probability for one's own group by expending one's own resources, resource contribution also incurs costs. This can be interpreted as both the direct and opportunity costs of the unproductive conflict activities. Let $c(r) \in \mathbb{R}_+$ denote the cost of devoting r units of resources. The cost function $c : \mathbb{R}_+ \to \mathbb{R}_+$ is homogeneous across all individuals, and is specified by the iso-elastic cost function:

$$c(r_{ik}) = \frac{1}{\theta} r_{ik}^{\theta},$$

where $\theta \geq 2$.

Therefore, the overall expected payoff function for individual k in group i, $\pi_{ik} : \mathbb{R}_+^N \to \mathbb{R}$ is summarized as

[10] The domain of β is restricted so that for any individual, the payoff when winning the conflict is strictly greater than the payoff when losing the conflict. This restriction is imposed for the proof of Proposition 1, and especially for the proof of Lemma 1, to hold.

[11] Robinson (2001) assumes that the group in power can extract an exogenous proportion of income from the group out of power, referred to as taxation, which may induce greater conflict.

$$\pi_{ik} = p_i\left(\lambda u_{ii} + y + \frac{\beta(1-\lambda)}{n_i}\right) + \sum_{j\neq i} p_j\left(\lambda u_{ij} + y + \frac{(1-\beta)(1-\lambda)}{1-n_j}\right) - \frac{1}{\theta}r_{ik}^{\theta}$$
$$= \sum_{j=1}^{m} p_j\lambda u_{ij} + p_i\frac{\beta(1-\lambda)}{n_i} + \sum_{j\neq i} p_j\frac{(1-\beta)(1-\lambda)}{1-n_j} + y - \frac{1}{\theta}r_{ik}^{\theta}.$$

In interpreting the first line of this equation, the first term on the right-hand side is the total payoff obtained from public and private goods when own group i wins, multiplied by its probability of winning. The second term on the right-hand side is, for each other group j, the total payoff received when group j wins, weighted by the probability that group j wins. The third term represents the cost associated with the resource contribution.

As in Esteban and Ray (2011), individuals exhibit a positive altruistic interest in the payoffs of other members within the same group. This enables us to analyze various scenarios, from individuals who maximize only their own payoff to those who maximize group payoffs. Let $\alpha \in [0, 1]$ denote the degree of intragroup altruism among the agents. One extreme case, $\alpha = 0$, refers to selfish individuals who care only about their own payoff. The other extreme case, $\alpha = 1$, signifies perfectly altruistic individuals who evaluate the payoff of fellow group members with the same weight as their own payoff. Then, the extended utility function of individual k in group i, $U_{ik} : \mathbb{R}_+^N \to \mathbb{R}$, is defined by

$$U_{ik} \equiv \pi_{ik} + \alpha\sum_{l\neq k\in i}\pi_{il} = (1-\alpha)\pi_{ik} + \alpha\sum_{l\in i}\pi_{il}$$
$$= \sigma_i\left[\sum_{j=1}^{m} p_j\lambda u_{ij} + y + p_i\frac{\beta(1-\lambda)}{n_i} + \sum_{j\neq i} p_j\frac{(1-\beta)(1-\lambda)}{1-n_j}\right] - \frac{1}{\theta}r_{ik}^{\theta} - \alpha\sum_{l\neq k\in i}\frac{1}{\theta}r_{il}^{\theta},$$

where $\sigma_i \equiv (1-\alpha) + \alpha N_i$.

3 Bipolar stability and conflict

3.1 Agents' behavior and equilibrium

The first part of Section 3 formulates individual optimization behavior and discuss the existence and uniqueness of equilibrium. Before solving the utility maximization problem, let us rewrite the expected payoffs in terms of losses relative to the payoff from winning the conflict. For each group $i,j \in \{1, ..., m\}$ with $i \neq j$, group i's individual total loss in the public and private payoff when group j wins is denoted by

$$\Delta_{ij} = \lambda u_{ii} + y + \frac{\beta(1-\lambda)}{n_i} - \left(\lambda u_{ij} + y + \frac{(1-\beta)(1-\lambda)}{1-n_j}\right)$$
$$= \lambda\delta_{ij} + \frac{\beta(1-\lambda)}{n_i} - \frac{(1-\beta)(1-\lambda)}{1-n_j}, \quad (1)$$

where $\delta_{ij} \equiv u_{ii} - u_{ij}$. For each group $i \in \{1, ..., m\}$, $\Delta_{ii} \equiv 0$. (1) shows that the larger β, the larger the payoff losses;

$$\frac{\partial \Delta_{ij}}{\partial \beta} = \frac{1-\lambda}{n_i} + \frac{1-\lambda}{1-n_j} > 0,$$

which may suggest that individuals have a stronger incentive to win the conflict when β is significant. Contrary to that, the resource inputs to the conflict is expected to be less under modest β.

Using these expressions, the expected payoff function for individual k in group i, $\pi_{ik} : \mathbb{R}_+^N \to \mathbb{R}$, can be rewritten as

$$\pi_{ik} = \lambda u_{ii} + y + \frac{\beta(1-\lambda)}{n_i} - \sum_{j=1}^{m} p_j \Delta_{ij} - \frac{1}{\theta} r_{ik}^{\theta},$$

and the extended utility function of individual k in group i, $U_{ik} : \mathbb{R}_+^N \to \mathbb{R}$, is rewritten as

$$U_{ik} = \sigma_i \left[\lambda u_{ii} + y + \frac{\beta(1-\lambda)}{n_i} - \sum_{j=1}^{m} p_j \Delta_{ij} \right] - \frac{1}{\theta} r_{ik}^{\theta} - \alpha \sum_{l \neq k \in i} \frac{1}{\theta} r_{il}^{\theta}. \tag{2}$$

Hence, given the resources devoted by all the other individuals, individual k in group i chooses $r_{ik} \in \mathbb{R}_+$ to maximize (2). The Nash equilibrium of this game is defined by a vector of optimal contributions $r^* = (r_{ik}^*)_{k \in i, i \in \{1,...,m\}}$ such that r_{ik}^* solves the maximization problem for individual k in group i. Solving the individual maximization problem yields the following results for the existence and uniqueness of equilibrium.

Proposition 1 *An equilibrium always exists and is unique. For all $i \in \{1, ..., m\}$ and all $k \in i$, the amount of resources devoted by individual k in group i is strictly positive and completely described by the following first-order condition:*

$$\frac{\sigma_i}{R} \sum_{j=1}^{m} p_j \Delta_{ij} = r_{ik}^{\theta-1}. \tag{3}$$

In particular, within each group, all members make the same contribution, i.e., $r_{ik} = r_{il}$ for all $i \in \{1, ..., m\}$ and all $k, l \in i$.

Proof See Appendix A. □

Proposition 1 guarantees the existence of equilibrium and its uniqueness. It states that the solution to the individual maximization problem is always interior, which means that, in equilibrium, all individuals engaging in the conflict make a positive contribution. It is also confirmed that, in equilibrium, individuals belonging to the same group invest the same amount of resources.

3.2 Main result

On the basis of the existence and uniqueness of the equilibrium in the previous discussion, this section investigates the relationship between the equilibrium conflict and some social indices. The main argument of this study is that the effects of societal polarization on the equilibrium conflict hinge upon the way of monetary compensation. When the losers of conflict compensate the winner, societal polarization escalates the equilibrium conflict intensity. However, when the winner of the conflict compensates for other losers, the equilibrium conflict level lowers in polarized societies consisting of two dominant groups, which is consistent with the bipolar stability theory.

To be consistent with Esteban and Ray (2011), it is defined that the behavioral correction factor of group i as $\gamma_i \equiv p_i/n_i$ for each group $i \in \{1, ..., m\}$. It should be noted that γ_i is an endogenous variable since p_i is determined by the relative amount of resource contributions among groups. However, if one can assume that all individuals contribute the same amount of resources, γ_i would be equal to 1 for all groups, and the probability of winning (p_i) would be equivalent to the group population shares (n_i). Then, the main result of this study can be derived in the following clear expression.

Proposition 2 *Given the approximation assumption that every behavioral correction factor equals one, i.e.,* $\gamma_i = 1$ *for all* $i \in \{1, ..., m\}$, *the average per capita conflict* ($\rho \equiv R/N$) *is a linear function of the four indices* G, P, F, *and* B:

$$\rho^\theta = \left(\frac{R}{N}\right)^\theta \approx \omega_1 + \omega_2 G + \alpha\lambda P + \alpha\beta(1-\lambda)F - \alpha(1-\beta)(1-\lambda)B, \tag{4}$$

where

$$G \equiv \sum_{i=1}^{m}\sum_{j\neq i} n_i n_j \delta_{ij}, \quad P \equiv \sum_{i=1}^{m}\sum_{j\neq i} n_i^2 n_j \delta_{ij}, \quad F \equiv \sum_{i=1}^{m}\sum_{j\neq i} n_i n_j, \quad B \equiv \sum_{i=1}^{m}\sum_{j\neq i} \frac{n_i^2 n_j}{1-n_j},$$

$\omega_1 = (1-\alpha)(1-\lambda)(\beta m - 1)/N$, *and* $\omega_2 = (1-\alpha)\lambda/N$. *In particular, when the population is large enough, i.e.,* $N \to \infty$, *the average per capita conflict is proportional to only the three indices* P, F, *and* B, *provided that group cohesion* $\alpha > 0$.

(i) *When the winning group compensates other losing groups,* $0.5 < \beta < 1$, *societal polarization will mitigate the equilibrium conflict intensity,* $\partial\rho^\theta/\partial B < 0$.
(ii) *When the losing groups compensate the winning group,* $\beta > 1$, *societal polarization will intensify the equilibrium conflict level,* $\partial\rho^\theta/\partial B > 0$.

Proof See Appendix B. □

Proposition 2 states that if the behavioral correction factor can be approximated as 1 for all groups, then the equilibrium average per capita conflict can be expressed by the following four indices: Gini coefficient (G), polarization index (P), fractionalization index (F), and balance of power index (B). Moreover, it finds that the relative importance of these indicators depends on the degree of publicness of the budget (λ), the level of altruism within the group (α), and the monetary distribution among the winner and losers (β). Observing that this finding aligns with Proposition 2 in Esteban and Ray (2011) by setting $\beta = 1$ in equation (4), this study presents more generalized arguments. In what follows, our primary attention is centered on the analysis of the three indices P, F, and B, given that the first and second terms on the right-hand side of equation (4) become negligible under sufficiently large population.

The extension of the model introduces a new element concerning conflict intensity denoted as B in equation (4), called the balance of power index. Figure 1 depicts the balance of power index for two cases: that of two groups (solid line) and that of three groups, with the third group's population share being 0.1 (dashed line). The horizontal axis indicates the population share of group 1, and group 2's population share is calculated as $n_2 = 1 - n_1$ (in the solid line) and $n_2 = 0.9 - n_1$ (in the dashed line). As indicated by Fig. 1, the index B measures the degree of the balance of power between the top two groups. The balance of power index (B) has properties similar to the polarization index (P) in that B reaches its maximum value when a society comprises two equal-sized groups.[12]

Returning to equation (4), the remarkable outcome for the balance of power index (B) is two-fold. First, when the winner of conflict requires other losers to pay war compensation ($\beta > 1$), societal polarization plays an intensified impact on the level of conflict even under the purely private case ($\lambda = 0$). In the original conflict model of Esteban and Ray (2011), polarization can no longer work on the level of conflicts when $\lambda = 0$, and only the factor intensifying conflict is ethnic diversity, represented by F. However, this model shows that the presence of the balance of power index B allows societal polarization to heighten the conflict level even when only private goods are at stake. The intuitive mechanism underlying this result can be obtained by investigating how the winning group size alters individual private payoffs. When $\beta > 1$, each member of the losing group must contribute $(1 - \beta)(1 - \lambda)/(1 - n_i)$ in monetary compensation to the winning group i. Given a fixed total amount of payment, the larger the size of the losing groups, i.e., the larger $1 - n_i$, the smaller the per-capita payment burden. Conversely, this implies that the increased size of the winner would compel the losers to bear greater monetary compensation, thereby inducing agents to win the conflict. In a polarized society split into two major ethnic groups, it is more probable that such a situation would arise. This explanation confirms that the polarization reinforces the intensity of conflict over private goods, $\partial\rho^\theta/\partial B > 0$.

[12] These indices P and B exhibit a much similar pattern in the case of a pure contest, i.e., a situation in which each group dislikes equally the outcomes of all other groups, represented by $\delta_{ij} = 1$ for all $i, j (\neq i)$. Also, there are particular relations between the balance of power index (B) and the fractionalization index (F): $F = 1 - B$ holds if each group has an equal population size, and $F = B$ holds if the number of groups is two. Otherwise, these indices take different values in general.

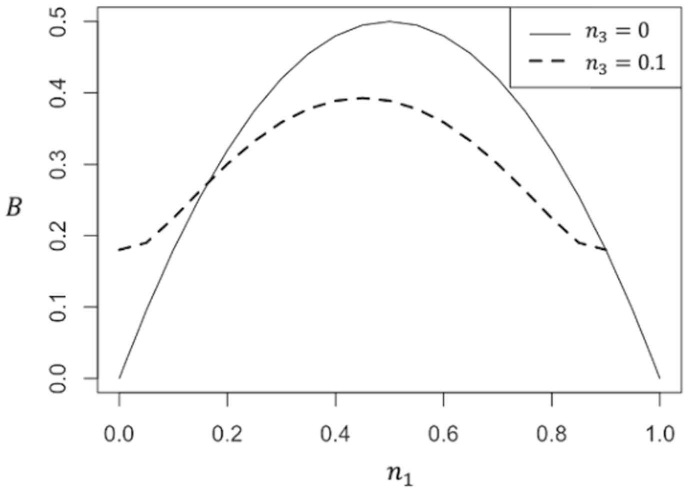

Fig. 1 Balance of Power Index

Second, however, in the presence of monetary compensation from the winner to the losers ($0.5 < \beta < 1$), societal polarization has the potential to rather decrease the equilibrium conflict intensity. The intuition behind this result is just a reversal of the above view that β is strictly greater than one. When $0.5 < \beta < 1$, the losing groups, in turn, receive private goods with the per capita value of $(1-\beta)(1-\lambda)/(1-n_i)$ when group i wins. If the winning group i has a large population, the private good redistribution can be shared among the small population, and the private payoff when losing becomes large. Thus, if the opponent's group size is relatively large, the per capita money compensated by the winner is expected to be large, leading to less incentive to fight. Conversely, if the opponent's group is relatively small, the incentive to win becomes dominant because they are afraid of receiving a small monetary redistribution when losing the conflict. Therefore, once the private good is shared between the winner and losers, the larger the mutual group size, the weaker the conflict. This consideration firmly explains the central result of $\partial\rho^\theta/\partial B < 0$.

The other two indices in equation (4), F and P, correspond to the Hirschman-Herfindahl fractionalization index and the Esteban-Ray polarization index, respectively. The former is widely used to measure the degree of ethnic diversity, which expresses the probability of two randomly chosen individuals belonging to different groups. Thus, F takes a minimum value of 0 when a society comprises only one group and approaches its upper bound of 1 as the number of equal-sized groups tends to infinity. The latter is characterized by a reasonable set of axioms (Esteban and Ray 1994; Kawada et al. 2018). This index, which reflects the characteristics of the societal polarization, attains its maximum value at a symmetric bimodal distribution. Besides, the Esteban-Ray polarization index depends on the inter-group distances (δ_{ij}), thus capturing different dimensions of social structures from the fractionalization index.

The relationship between the polarization index P and the intensity of conflict is positive, basically attributed to the conflict over public goods. Firstly, it is natural that widening inter-group distances leads to more intense conflicts. If the two groups have similar preferences over the public good, they receive a relatively large public payoff even if they lose the conflict against the other group. Thus, they have less incentive to fight each other. However, if the two groups differ greatly in their preferences over the public good, they would suffer a large utility loss when they lose to the other group, generating a stronger incentive to win the conflict. Secondly, however, it is not immediately evident that a bimodal population distribution is more conflictual, as the consumption of public goods is, by nature, independent of group sizes. Equation (2) helps us understand the relationship between the conflict level and group sizes. When individuals have altruistic preferences, i.e., $\alpha > 0$, the larger group entails greater consideration of the peer utility, indicated by the term of σ_i. This suggests larger groups incur greater utility losses when losing a conflict, rendering them more combative. In a bipolar society, either group commands a significant proportion of the population, thereby leading to increased resource inputs in conflicts. These two effects are captured by the polarization index P.

Another indicator in equation (4), the fractionalization index F, is also positively related to the intensity of the conflict. To illustrate this, let us focus on the private payoff from winning a conflict. Recall that the per-capita private payoff will shrink as a group's population gets larger. If a group has a relatively large population, each member of the group receives a lower private payoff even if they win the conflict. Thus, the incentive to win the conflict diminishes for a group with a relatively large population share. Conversely, the incentive to win the conflict increases for a relatively small group because the per capita monetary payoff is expected to be large. Therefore, regarding the private payoff when a group wins, the smaller the group, the more intense the conflict, as expressed by the fractionalization index F.

In sum, by adding monetary compensation between groups with conflicting interests to the model, this study has uncovered hidden conflict-related indicator, named the balance of power index, B, which measures the degree of societal polarization. The relationship between the balance of power index and the conflict intensity is critically contingent upon the direction of monetary compensation among the winning and losing groups. When the losers in a conflict have to compensate for the winner, the polarization radicalizes the equilibrium conflict level. However, different characteristics will emerge when the winner of the conflict compensates other losers. In this case, societal polarization effectively reduces the conflict intensity. This distinctive feature of polarization, which leads to a decrease in the conflict level, aligns with the bipolar stability theory proposed by Waltz (1967), Waltz (1979).

3.3 Accuracy of the approximation

The primary findings presented in the previous section are based on a strong approximation assumption of the behavioral correction factor, $\gamma_i \equiv p_i/n_i$.[13] This section focuses on the relationship between γ_i and some other parameters (α, λ, and β) to investigate the validity of the approximation assumption. The subsequent discussions confirm that the extended model can retain similar observations about the variations in behavioral correction factors as derived by Esteban and Ray (2011). Moreover, though limited to analysis in specific cases, the introduction of the new parameter β may improve the precision of the approximation.

Let us first revisit the basic formula for the approximation result (4) as

$$\rho^{\theta} = \sum_{i=1}^{m} \sum_{j \neq i} \gamma_i^{2-\theta} \gamma_j n_i n_j \left[\frac{1-\alpha}{N} + \alpha n_i \right] \left[\lambda \delta_{ij} + \frac{\beta(1-\lambda)}{n_i} - \frac{(1-\beta)(1-\lambda)}{1-n_j} \right], \quad (5)$$

which is equivalent to equation (A12) stated in Appendix B. The true level of conflict is determined by equation (5), which contains the endogenous variable of γ_i. Meanwhile, the approximate value of conflict level is obtained when γ_i is set to 1 in equation (5). Then, our concern is whether the behavioral correction factor γ_i has any systematic relation with some parameters and population distribution.

To answer this question, the study focuses on the case of contests ($\delta_{ij} = 1$) and quadratic cost functions ($\theta = 2$). Recalling that $\sigma_i \equiv (1-\alpha) + \alpha N_i$ and $\rho \equiv R/N$, the first-order condition (3) implies

$$r_i \rho = \left[\frac{1-\alpha}{N} + \alpha n_i \right] \sum_{j=1}^{m} p_j \Delta_{ij} = \left[\frac{1-\alpha}{N} + \alpha n_i \right] \left[(1-p_i)\Delta_i - \sum_{j \neq i} p_j \Delta_j \right], \quad (6)$$

where $\Delta_i \equiv \lambda + \beta(1-\lambda)/n_i$ and $\Delta_j \equiv (1-\beta)(1-\lambda)/(1-n_j)$. When the total population is sufficiently large (i.e., $N \to \infty$) while keeping population proportions in each group constant, (6) can be reduced as

$$r_i \rho = \alpha n_i \left[(1-p_i)\Delta_i - \sum_{j \neq i} p_j \Delta_j \right].$$

Multiplying $n_i \equiv N_i/N$ both sides, and using the fact that $R_i = r_i N_i$ and $p_i = R_i/R$ yields

[13] There can be a discrepancy between the probability of winning p_i and the relative population sizes n_i. For instance, Pareto (1927) and Olson (1965) argue that smaller groups have a higher ratio of p_i to n_i. Conversely, Esteban and Ray (1999) find that in the case of contests, larger groups consistently exhibit higher levels of contention per capita compared to smaller groups. Furthermore, their analysis shows that, with a uniform population distribution over three groups, the groups located at the extremes in preference space tend to allocate more per capita resources.

$$p_i = \frac{\alpha n_i^2 \left[\Delta_i - \sum_{j \neq i} p_j \Delta_j\right]}{\rho^2 + \alpha n_i^2 \Delta_i} \tag{7}$$

for all $i \in \{1, ..., m\}$, where ρ^2 and p_i for all $i \in \{1, ..., m\}$ must solve

$$\sum_{j=1}^{m} \frac{\alpha n_j^2 \left[\Delta_j - \sum_{h \neq j} p_h \Delta_h\right]}{\rho^2 + \alpha n_j^2 \Delta_j} = 1. \tag{8}$$

Equation (8) shows that ρ^2 is linearly homogeneous in α, i.e., $\rho^2 = \alpha \hat{\rho}^2$, where $\hat{\rho}$ is the average per capita conflict when $\alpha = 1$. Substituting this into (7) and dividing through by n_i, an expression for the correction factors is given as

$$\gamma_i = \frac{n_i \left[\Delta_i - \sum_{j \neq i} p_j \Delta_j\right]}{\hat{\rho}^2 + n_i^2 \Delta_i} \tag{9}$$

for all $i \in \{1, ..., m\}$, where $\hat{\rho}^2$ and p_i for all $i \in \{1, ..., m\}$ must solve

$$\sum_{j=1}^{m} \frac{n_j^2 \left[\Delta_j - \sum_{h \neq j} p_h \Delta_h\right]}{\hat{\rho}^2 + n_j^2 \Delta_j} = 1.$$

Equation (9) suggests that γ_i is independent of α, whereas it depends on λ and β as Δ_i and Δ_j are functions of those parameters. Therefore, provided that sufficiently large population, the approximation assumptions do not suffer from serious problems against variations in α, which is consistent with Esteban and Ray (2011).

The more profound issue of the approximation lies in the fact that γ_i is dependent on the two parameters of λ and β that determine the structure of public and private goods, as well as the population distribution over the groups. First, in the case of pure public goods ($\lambda = 1$), $\Delta_i = 1$ and $\Delta_j = 0$. Substituting these into (9) yields precisely the same expression of γ_i as in Esteban and Ray (2011):

$$\gamma_i = \frac{n_i}{\hat{\rho}^2 + n_i^2}.$$

Despite the ambiguity of the sign of $\partial \gamma_i / \partial n_i$, the numerical analysis conducted by Esteban and Ray (2011) allows us to anticipate that there is a tendency to overestimate the true value of conflict when $\lambda = 1$. This is because, with public goods at stake, small groups actually put in fewer resources relative to their population sizes, which is ignored in the approximation process. Second, so long as the entire budget is not allocated to public goods, i.e., $\lambda < 1$, the term Δ_j remains in equation (5), posing difficulties for further analysis. To give a more detailed examination, consider the case of two groups ($m = 2$) with a purely private budget ($\lambda = 0$). In the two-group cases, $1 - n_j$ coincides with n_i, and thus $\Delta_j = (1 - \beta)/n_i$. Putting this into (9) and rearranging gives

$$\gamma_i = \frac{2\beta - 1}{\hat{\rho}^2 + n_i}.$$

When $\beta = 1$, the exact same expression of γ_i as in Esteban and Ray (2011) follows. Observing that

$$\frac{\partial \gamma_i}{\partial n_i} = -\frac{2\beta - 1}{\left(\hat{\rho}^2 + n_i\right)^2} < 0 \tag{10}$$

indicates the well-known Pareto-Olson argument that smaller groups tend to contribute more intensively. This will cause an underestimation of the actual conflict level when λ is close to zero, as shown in the numerical simulations of Esteban and Ray (2011). More importantly, (10) indicates that the problem of underestimation can be mitigated when $0.5 < \beta < 1$, while it will be more severe when $\beta > 1$. In essence, the extension of the model can influence the sensitivity of underestimation; the fitness between the true and approximated level of conflict can be improved in situations where the winner compensates other losers ($0.5 < \beta < 1$), and the gap between them can be widened in situations where the losers compensate for the winner ($\beta > 1$).

Due to the inclusion of monetary compensation between the winning and losing groups in the original conflict model, somewhat complicated analyses are required in this paper. While the analytical discussions in this section have drawn only special and limited remarks, several results of Esteban and Ray (2011) can be established even within the extended model. First, the behavioral correction factor γ_i is robust against a change in the level of altruism within the group (α). Second, when the conflict is over public goods (with large λ), the approximation result tends to be overestimated than the true value of conflict. On the other hand, it turns out to be underestimated when private goods are at dominant issue (with small λ). The extent of underestimation can be relieved (reinforced) in situations where there is monetary compensation from the winning (losing) to losing (winning) groups.

4 Conclusion

This study incorporates monetary compensation between the winning and losing groups in the well-structured conflict model of Esteban and Ray (2011) and provides a new perspective on the conflict-polarization nexus by establishing the bipolar stability theory proposed by Waltz (1967), Waltz (1979). A series of war and conflict studies focus on societal polarization as a prominent factor contributing to the escalation of conflict. However, in the field of international politics, Waltz (1967), Waltz (1979) presents a contrary hypothesis called bipolar stability theory, which argues that a situation consisting of two great powers is the most stable. His hypothesis suggests that the polarized society does not always lead to an intensification of conflict but can instead bring social stability. The principal result of this study can, at least in part, give a microeconomic foundation for the bipolar stability theory from the perspective of the disputant's incentive.

The key factor leading to this insight lies in the monetary sharing among the winners and losers in a conflict, and the direction of the monetary transfer is crucial in determining the level of the conflict. Under an arrangement in which monetary compensation is paid by the losers to the winners, societal polarization would work to raise the level of the conflict. On the contrary, if losing groups also have access to a given monetary fund and are given certain resources from the winning group, the societal polarization can reduce the level of conflict. The level of conflict is lowest in a polarized society consisting of two major groups when even a partial transfer of money is made from the winning group to the losing groups. This is because small groups that are expected to lose the conflict will have an incentive to lower the resources they spend on the conflict. If they do lose the conflict, but a certain amount is transferred from the winner to the loser, the amount received per capita will be larger because of the smaller number of members in the group. This reduces the resources put into increasing the probability of winning in order to lower the cost of being a loser, which brings about a situation consistent with Waltz's bipolar stability hypothesis that social polarization with two main groups is stable.

In closing the paper, three remaining tasks should be addressed. First, the existence of a self-enforcing monetary transfer among agents in the rent-seeking conflict should be analyzed explicitly in light of the essential literature on voluntary redistribution (e.g., Azam 1995; Bös and Kolmar 2003; Filipovich and Sempere 2008). Second, numerical analyses for rigorous checks of the approximation assumption could be possible. Esteban and Ray (2011) provide several simulation results across various parameter settings to see the plausibility of model predictions. In this study, there needs to be corresponding analyses, especially for factors such as inter-group distances (δ_{ij}), total population sizes (N), and cost elasticities (θ). Third, empirical studies that identify pathways through which polarization mitigates conflict would be useful in testing the new hypotheses presented in this study. If the balance of power index is relevant to the conflict, the estimation results in Esteban et al. (2012) may suffer from an omitted variable bias, which causes their maximum likelihood estimators to be biased and inconsistent. These are topics for future research.

Appendix A

Proof of Proposition 1 The following lemma is useful in proving this proposition.

Lemma 1 *In any equilibrium, there exists some group* $j \in \{1, ..., m\}$ *such that* $R_j > 0$.

Proof of Lemma 1 For the sake of contradiction, let us suppose that in equilibrium, $R_i = 0$ for all $i \in \{1, ..., m\}$. Then, because $R = \sum_{i=1}^{m} R_i = 0$, each group has a success probability given by the arbitrary probability vector $\tilde{p} \in \{(p_1, ..., p_m) \in [0, 1]^m \mid \sum_{j=1}^{m} p_j = 1\}$. Fix this arbitrary probability vector

$\tilde{p} = (\tilde{p}_1, ..., \tilde{p}_m)$. Then, there exists some group $j \in \{1, ..., m\}$ such that $0 \leq \tilde{p}_j < 1$. Let us focus on this group j.

The extended utility of individual k in group j is

$$\sigma_j \left[\lambda u_{jj} + y + \frac{\beta(1-\lambda)}{n_j} \right] - \sigma_j \sum_{s=1}^{m} \tilde{p}_s \Delta_{js}.$$

If k changes the strategy from $r_{jk} = 0$ to $r_{jk} > 0$, then k's extended utility becomes

$$\sigma_j \left[\lambda u_{jj} + y + \frac{\beta(1-\lambda)}{n_j} \right] - \frac{1}{\theta} r_{jk}^{\theta}.$$

Thus, k can increase the extended utility by deviating from $r_{jk} = 0$ to $r_{jk} > 0$ such that

$$-\sigma_j \sum_{s=1}^{m} \tilde{p}_s \Delta_{js} < -\frac{1}{\theta} r_{jk}^{\theta},$$

which contradicts this agent's optimal behavior. □

I now return to the main proof. By Lemma 1, for each $i \in \{1, ..., m\} \backslash \{j\}$ and each $k \in i$, the following problem can be well-defined for any $r_{ik} \in \mathbb{R}_+$. Given any vector of resources expended by all other groups and by the rest of the members of the own group,

$$\max_{r_{ik} \in \mathbb{R}} \quad -\sigma_i \sum_{j=1}^{m} p_j \Delta_{ij} - \frac{1}{\theta} r_{ik}^{\theta}, \tag{A1}$$

$$\text{s.t.} \quad p_j = \frac{R_j}{R} \quad \text{for all } j \in \{1, ..., m\}, \tag{A2}$$

$$r_{ik} \geq 0. \tag{A3}$$

By substituting (A2) into the objective function (A1), this problem can be reduced as follows.

$$\max_{r_{ik} \in \mathbb{R}} \quad -\sigma_i \sum_{j=1}^{m} \frac{R_j}{R} \Delta_{ij} - \frac{1}{\theta} r_{ik}^{\theta}, \quad \text{s.t.} \quad r_{ik} \geq 0. \tag{A4}$$

Note that the objective function (A4) is class C^1. Because $-\sigma_i \sum_{j=1}^{m} (R_j/R)\Delta_{ij}$ is strictly concave in r_{ik} and $(1/\theta) r_{ik}^{\theta}$ is strictly convex in r_{ik}, the objective function (A4) is strictly concave in r_{ik}. Let $\bar{r}_{ik}$ be a unique optimal contribution satisfying the constraint qualification. Define the Lagrangian as

$$\mathcal{L}(r_{ik}, \lambda) = -\sigma_i \sum_{j=1}^{m} \frac{R_j}{R} \Delta_{ij} - \frac{1}{\theta} r_{ik}^{\theta} + \lambda r_{ik}.$$

By the Karush-Kuhn-Tucker theorem, there exists $\bar{\lambda} \in \mathbb{R}$ such that[14]

$$\frac{\sigma_i}{R} \sum_{j=1}^{m} \frac{R_j}{R} \Delta_{ij} - \bar{r}_{ik}^{\theta-1} + \bar{\lambda} = 0, \tag{A5}$$

$$\bar{\lambda} \bar{r}_{ik} = 0, \tag{A6}$$

$$\bar{r}_{ik} \geq 0, \tag{A7}$$

$$\bar{\lambda} \geq 0. \tag{A8}$$

Case i: $\bar{r}_{ik} = 0$. (A5) gives

$$\bar{\lambda} = -\frac{\sigma_i}{R} \sum_{j=1}^{m} \frac{R_j}{R} \Delta_{ij},$$

which is negative and hence does not satisfy (A8).
Case ii: $\bar{r}_{ik} > 0$. (A6) gives $\bar{\lambda} = 0$. Thus, from (A5), $\bar{r}_{ik}(> 0)$ satisfies

$$\frac{\sigma_i}{R} \sum_{j=1}^{m} \frac{R_j}{R} \Delta_{ij} = \bar{r}_{ik}^{\theta-1}, \tag{A9}$$

Therefore, it is shown that every member of every group other than j must satisfy the interior first-order condition (3). Furthermore, because every group other than j makes positive contributions, the maximization problem for every individual in group j is also well-defined, and hence, they must satisfy (3). Thus, in equilibrium, $R_i > 0$ for all $i \in \{1, ..., m\}$ and (3) is satisfied for all $i \in \{1, ..., m\}$ and all $k \in i$.

Also, since the left-hand side of equation (A9) is common for all $k \in i$, it is true that $\bar{r}_{ik} = \bar{r}_{il} \equiv r_i$ for all $i \in \{1, ..., m\}$ and all $k, l \in i$. Thus, the first-order condition for an individual in group $i \in \{1, ..., m\}$ is given by

$$\frac{\sigma_i}{R} \sum_{j=1}^{m} p_j \Delta_{ij} = r_i^{\theta-1}. \tag{A10}$$

In what follows, let us prove the existence and uniqueness of the equilibrium. Multiplying both sides of (A10) by R_i and using $R_i = r_i N_i$ yield

[14] See Mas-Colell et al. (1995), pp. 959-960.

$$\sigma_i \sum_{j=1}^{m} p_i p_j \Delta_{ij} = N_i r_i^{\theta},$$

and now define $v_{ij} \equiv \sigma_i \Delta_{ij} / N_i$ for all $i \in \{1, ..., m\}$ to obtain the system

$$\sum_{j=1}^{m} p_i p_j v_{ij} = r_i^{\theta} \tag{A11}$$

for all $i \in \{1, ..., m\}$. This is precisely the system described in Proposition 3.1 of Esteban and Ray (1999), with s in place of p and $c(r) \equiv (1/\theta) r^{\theta}$. Under the assumption that $\theta \geq 2$, the proofs of Propositions 3.2 and 3.3 can apply entirely unchanged to show that the system (A11) has a unique solution. □

Appendix B

Proof of Proposition 2 Recall the equilibrium condition (A11):

$$\sum_{j=1}^{m} p_i p_j \frac{\sigma_i \Delta_{ij}}{n_i N} = r_i^{\theta} = \frac{p_i^{\theta} \rho^{\theta}}{n_i^{\theta}}.$$

Multiplying both sides by $p_i^{1-\theta} n_i^{\theta}$ and using the fact that $p_i = \gamma_i n_i$ give

$$\begin{aligned} p_i \rho^{\theta} &= \sum_{j=1}^{m} p_i^{2-\theta} p_j n_i^{\theta-1} \frac{\sigma_i \Delta_{ij}}{N} \\ &= \sum_{j=1}^{m} \gamma_i^{2-\theta} \gamma_j n_i n_j \frac{\sigma_i \Delta_{ij}}{N}. \end{aligned}$$

Adding overall i, the following is obtained;

$$\rho^{\theta} = \sum_{i=1}^{m} \sum_{j=1}^{m} \gamma_i^{2-\theta} \gamma_j n_i n_j \frac{\sigma_i \Delta_{ij}}{N}. \tag{A12}$$

Recall that $\sigma_i = (1 - \alpha) + \alpha N_i$, $\Delta_{ii} \equiv 0$, and $\Delta_{ij} = \lambda \delta_{ij} + \beta(1 - \lambda)/n_i - (1 - \beta)(1 - \lambda)/(1 - n_j)$ for all $j \neq i$. Expanding the terms σ_i and Δ_{ij} and setting all correction factors to their approximate values of 1,

$$\rho^{\theta} \approx \sum_{i=1}^{m} \sum_{j \neq i} n_i n_j \left[\frac{1 - \alpha}{N} + \alpha n_i \right] \left[\lambda \delta_{ij} + \frac{\beta(1 - \lambda)}{n_i} - \frac{(1 - \beta)(1 - \lambda)}{1 - n_j} \right].$$

The expansion of these terms proves the result. □

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