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Comparative analysis of application efficiency of two iterative multi objective linear programming methods (MP method and STEM method)

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Abstract In this paper we consider a production plan optimization problem for a company that produces textile products. The problem is solved using two iterative methods: a new method based on the cooperative game theory (MP method) and the well-known STEM method. Their application efficiency and the solutions obtained are compared. For this purpose we use four groups of criteria: (1) the general characteristics of the method (2) the criteria from the standpoint of the decision makers, (3) the criteria from the perspective of the analysts, and (4) the 'economic' criteria. The analysis indicates that both methods are highly efficient for solving this kind of production plan optimization problems. However, the decision-makers preferred the MP method.

Keywords Multi-objective linear programming · MP method · STEM method · Interactive methods · Production plan optimization

1 Introduction

Multi-objective programming (MOP) is the most studied area of operations research. Since the beginning of 1970th, when the first papers on MOP appeared, many methods

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for solving such kind of problems have been developed. Indeed, interactive multi-objective linear programing methods are among the most popular approaches to solving various economic problems. The most important reviews of the MOP and related methods are given in Roy (1971), Mac-Crimmon (1973), Cohon and Marks (1975), Bell et al. 1977, Star and Zeleny (1977), Hwang and Masud (1979), Ho (1979), Despontion and Spronk (1979), Zionts (1980), Chankong and Haimes (1983), Yu (1985), Steuer (1985), Fandel and Spronk (1985), Lai and Hwang (1996), Figueira et al. (2005), and Perić (2008).

Different MOP methods do not have equal application efficiency, either from the standpoint of the decision makers or from the perspective of the analysts. The problem of MOP application efficiency has been preoccupying the attention of numerous researchers [see: Agarwal (1973), Dyer (1973), Cohon and Marks (1975), Wallenius (1975), Wallenius and Zionts (1976), Tell (1976), Karwan and Wallace (1980), Schomaker (1980a, b, c), Rietveld (1980), Khairullah and Zionts (1980) and Triantaphyllow (2000), Perić (2008)]. Their findings on application efficiency vary depending on the nature of the problem tackled.

One of the most important issues in evaluating the efficiency of MOP methods is the selection of criteria for their evaluation. Different criteria were used in the papers listed above to evaluate the efficiency of various methods in solving different MOP problems. For that purpose, Agarwal (1973) used Geoffrion's method and STEM method to solve the optimization problem of a city transport network. In their analysis of application efficiency they used the following criteria: (1) the quality of the obtained results, (2) decision-maker's preference for the method and (3) the difficulty in solving the problem.

Dyer (1973) tested Geoffrion's method using 9 subjects as decision-makers. The criteria applied to evaluate its efficiency were: (1) the difficulties involved in the application of the method and (2) the confidence of the decision-maker in the obtained solution.

Masud (1978) evaluated the efficiency of Linear goal programming, STEM and SEMOPS in solving a linear operational production planning model with 4 objective functions, 18 variables and 48 constraints. Two criteria were used to evaluate the efficiency of the methods: (1) the quality of the obtained results and (2) the difficulty of use.

Wallenius presented a comparative evaluation of the efficiency of three interactive methods: Geoffrion's method, the STEM method and an Unstructured approach: a trial and error model. These methods were applied to solve a production planning problem with three objective functions, 25 variables, and 19 constraints. The methods were evaluated according to the following criteria: (1) confidence of the decision-maker in the best compromise solution, (2) ease of use of the method, (3) ease of understanding the method's logic, (4) usefulness of information provided to assist the decision-maker, (5) convergence rate measured by the number of cycles and total time of model resolution and (6) CPU processor time.

Hwang and Masud (1979) listed nine different criteria and divided them into three groups for the evaluation of the efficiency of MOP methods, while Perić (2008) used 13 criteria, distributed among 4 classes, to evaluate the application efficiency of six multi-criteria linear programming methods in solving four manufacturing problems.

The majority of the developed methods employ interactive multi-objective linear programming (MOLP) approach. Interactive MOLP methods have k ($k > 1$) linear objective functions and m linear constraints, and the decision-maker(s) actively participate in the whole process of problem solving. The application of MOLP methods produces a single non-dominated (efficient) solution which is accepted by the decision-maker(s): the preferred solution.

Matejaš and Perić (2014) developed a new interactive iterative MOLP method based on the cooperative game theory (MP method). The authors state that the proposed method provides numerous advantages and support their claim by several simple examples. In this paper we compare the application efficiency of the MP method and the STEM method on a real, practical example of the optimal plan determination problem using four groups of criteria: the general characteristics of the method, criteria from the analyst's point of view, criteria from the perspective of the-decision maker, and the 'economic' criteria. The goal of this paper is to investigate the application efficiency of two interactive MOLP methods (STEM and MP method) which can be applied to optimize processes involving more than one-decision maker. We chose the STEM method since it is an interactive method for solving MOLP problems which accommodates the involvement of multiple decision-makers in the problem-solving process. Moreover, its application efficiency has been investigated in the literature (see Agarwal (1973), Masud (1978), Perić (2008) etc.). The MP method is also an interactive method designed to solve MOLP problems with the participation of multiple decision-makers. However, the application efficiency of this method has not yet been investigated in the literature. There are many different approaches to solve MOLP problems. One of the most recent papers that should be mentioned is Filatovas et al. (2017), where authors introduce a new approach based on evolutionary algorithm R-NSGA, which use a local search strategy to solve multi-objective programming problems.

This paper, in addition to Introduction (Sect. 1) and References, consists of four sections. In Sect. 2, the basic concepts of the MOLP model, the new MP method and the STEM method are presented. In Sect. 3, we present the Case study of the application of the MP method and the STEM method in solving the production plan optimization problem, while in Sect. 4 we analyse the application efficiency of the proposed methods. Conclusions present the important findings on the application efficiency of the MP and STEM methods and proposals for future research.

2 Multi-objective linear programming model

Let $\mathrm{c}_k \in \mathrm{R}^n,\ \ k = 1, 2, \ldots, K,\ \ \mathrm{b} \in \mathrm{R}^m$ and $\mathrm{A} \in \mathrm{R}^{m \times n}$ be the given vectors and matrix, respectively. The general MOLP can be stated in the following way

$$\max_{\mathrm{x} \in \mathrm{S}} z_k(\mathrm{x}), \quad k = 1, 2, \ldots, K, \tag{1}$$

where $z_k(\mathrm{x}) = \mathrm{c}_k^T \mathrm{x},\ \ \mathrm{S} = \{\mathrm{x} \in \mathrm{R}^n : \mathrm{x} \geq 0,\ \mathrm{Ax} <=> \mathrm{b}\}$. Here $<=>$ denotes any combination of three possible types of given linear constraints: $\leq,\ \ \geq$ or $=$.

Thus, the model (1) contains K linear objective functions and m constraints with nonnegative variables.

By solving the model (1) in such a way that each of the objective functions is separately maximized on the set **S,** we obtain the marginal solutions of this model. Since the objective functions in MOLP models are mutually conflicting, the values of objective functions will be significantly different for marginal solutions.

Decision-makers (DMs) almost certainly will not be able to choose any of the obtained marginal solutions, but they will look for a compromise solution which will satisfy their preferences towards objective function values.

In order to find the preferred efficient solution, we can use a number of standard multi-objective programming (MOP) methods [see Hwang and Masud (1979)]. However, those methods have different efficiency and give different solutions, so the problem of choosing the appropriate method is always topical.

2.1 A new iterative method for solving MOLP models (MP method)

A new iterative method for solving MOLP problems with any number of DMs was proposed in Matejaš and Perić (2014). This method is based on the idea of the cooperative game theory (Osborne 2004, pp 239–270) and it enables decision-makers to be significantly involved in the process of obtaining the preferred efficient solution.

As we have seen before, a multi-objective programming problem represents the situation where several (K) DMs (*players*) optimize their utilities, or one player optimizes several different objectives, at the same time and on the same constraint set (*budget*). We frequently encounter such situations in practice. Each utility (or objective) is given by the objective function $z_k(\mathrm{x}), \quad k = 1, 2, \ldots, K$. If the analytic form of the budget and the objective functions is linear then we have a multi-objective linear programming (MOLP) problem (1). In Matejaš and Perić (2014) an efficient method (MP-method) for solving these problems is presented. Here we present a brief overview of the method.

There are two sets defined in the MP-method,

$$\mathrm{D} = \left\{\mathrm{x} \in \mathrm{R}^n : \mathrm{x} \geq 0,\ z_k(\mathrm{x}) \geq d_k, \quad k = 1, 2, \ldots, K\right\}, \tag{2}$$

$$\mathrm{D}_\lambda = \left\{\mathrm{x} \in \mathrm{R}^n : \mathrm{x} \geq 0,\ z_k(\mathrm{x}) \geq \lambda d_k, \quad k = 1, 2, \ldots, K\right\}, \quad \lambda \geq 0, \tag{3}$$

where d_k is the aspiration level which the player P_k wants to achieve ($z_k(\mathrm{x}) \geq d_k$). At any point $\mathrm{x} \in \mathrm{D}$ all the players achieve their aspirations fully, while at any point $\mathrm{x} \in \mathrm{D}_\lambda$ they achieve their aspirations to the relative extent of at least λ. The method is stated in a very simple form: find the largest λ for which $\mathrm{D}_\lambda \cap \mathrm{S} \neq \emptyset$ (geometric form) or equivalent,

$$\begin{aligned} &\max_{(\mathrm{x},\lambda)\in \mathrm{G}} \lambda, \\ &\text{where } \mathrm{G} = \left\{(\mathrm{x}, \lambda) \in \mathrm{R}^{n+1} : \mathrm{x} \in \mathrm{S},\ \lambda \geq 0,\ z_k(\mathrm{x}) \geq \lambda d_k, \quad k = 1, 2, \ldots, K\right\}, \end{aligned} \tag{4}$$

(analytic form), which is a standard linear programming (LP) problem. The optimal solution λ^* shows to which (minimum) relative extent all the players can realize their aspirations. For x^* being the optimal point, the indicator

$$\lambda_k = \frac{z_k(\mathrm{x}^*)}{d_k}, \quad k = 1, 2, \ldots, K \tag{5}$$

shows to which extent the player P_k can realize his own aspiration. Thus, the indicators measure the reality of players' aspirations and can be used to improve the solution, if unsatisfactory, in the subsequent iterations (see Matejaš and Perić (2014) for details).

2.2 The STEM method

The STEM method is one of the first interactive methods to solve MOLP problems. It was proposed by Benayoun et al. (1971).

In the STEM method, each iteration (cycle) contains two phases: (1) calculation phase and (2) decision phase. In the calculation phase in the p cycle we should find a feasible solution which is the "closest" to the ideal objective function value $z_k{}^*$ ($k = 1, 2, \ldots, K$) by solving the following linear programming model:

$$\min_{(\mathrm{x},\lambda)\in \mathrm{S}_p} \lambda, \tag{6}$$

where $\mathrm{S}_p = \{(\mathrm{x}, \lambda) \in \mathrm{R}^{n+1} : \mathrm{x} \in \mathrm{S}; \lambda \geq \left|z_k^* - z_k(\mathrm{x})\right| \cdot \pi_k,\ \lambda \geq 0,\ k = 1, 2, \ldots, K\}$, $\pi_k = \frac{\alpha_k}{\sum_{k=1}^{K} \alpha_k}$, $\alpha_k = \frac{z_k^* - z_k^{\min}}{z_k^*}\left[\frac{1}{\sqrt{\sum_{j=1}^{n} (c_{kj})^2}}\right]$, if $z_k^* > 0$, $\alpha_k = \frac{z_k^{\min} - z_k^*}{z_k^{\min}}\left[\frac{1}{\sqrt{\sum_{j=1}^{n} (c_{kj})^2}}\right]$, if $z_k^* < 0$.

In the *decision phase*, the obtained compromise solution $\mathbf{x_p}$ is presented to the DMs who compare their objective function z_k with the ideal objective function value $z_k{}^*$. If some of the objective functions are satisfied, the decision-makers must lower the level of the satisfied objective function in the amount which will enable the improvement of the unsatisfactory objective functions in the next step of the method. The decision-maker gives Δz_k as the amount of acceptable alleviation.

For the next iterative cycle the feasible set is modified to $\mathrm{S}_{p+1} = \{(\mathrm{x}, \lambda) \in \mathrm{S}^p; z_k(\mathrm{x}) \geq z_k(\mathrm{x}^p) - \Delta z_k; z_i(\mathrm{x}) \geq z_i(\mathrm{x}^p),\ k \neq i;\ \ k, i = 1, \ldots, K\}$. $\pi_k = 0$ is determined and then starts the calculation phase of the $p+1$ cycle. In the calculation phase the analyst can solve several linear programming problems with the feasible $\mathbf{S_p}$ taking Δz_k inputs so that $0 < \Delta z_k^1 < \Delta z_k^2 < \cdots < \Delta z_k^c$. In this way a large number of efficient solutions can be obtained. The set of efficient solutions is presented to the DMs. From those solutions the DMs can choose the preferred solution [Hwang and Masud (1979)].

3 Case study

3.1 Setting the problem

The production planning problem involves determining the type and quantity of products to be manufactured in a given period of time. If the company aims to accomplish more than a single goal in the planning period, the problem can be mathematically represented as a multi-objective programming problem which, in turn, can be solved using multi-objective programming methods.

When solving MOLP problems, it is necessary to define objectives, decision variables, criteria functions, constraints and the parameters of criteria functions and constraints.

A textile manufacturing company plans to produce thirty different products (labeled with „i") in the period of one year. In this case, the quantity of i-product ($i = 1, 2, \ldots, 30$) in the production program are taken as the decision variables (labeled with x_i). The company set the following production programme optimization objectives for the given period: (1) production volume maximization, (2) total profit maximization, and (3) total revenue maximization. The following optimization criteria emerge from the defined goals: (1) production volume in pieces, (2) total profit in monetary units, and (3) total revenue in monetary units (Perić and Babić 2009). To form objective functions we need to use the parameters of net sale price in monetary units (labeled with c_{i3}) and net profit per product in monetary units (labeled with c_{i2}) that are fixed in the planning period by the assumption. The data needed to form the objective functions of the MOLP problem outlined above are given in Table 1 (Perić and Babić 2009).

The indicator 'profit per product' is calculated on the basis of the planned production program structure, planned retail prices, and planned costs. It is assumed that changes in the structure of the production program will not significantly affect this indicator.

The company has six capacity constraints (machines and materials) and thirty-three market constraints. The parameters of manufacturing time and machine capacity in minutes for the machine constraints: Cutting, Sewing and Finishing units (labelled with a_{i1}, a_{i2}, a_{i3}, b_1, b_2 and b_3, respectively) are given in Table 2. All the parameters in the model are fixed in the given period.

The parameters of *the needed quantity of material per product and the maximal quantity of purchase in kg* for the material constraints: *"A", "B" and "C"* (labelled with g_{i1}, g_{i2}, g_{i3}, d_1, d_2 and d_3, respectively) are given in Table 3. All the parameters in the model are fixed in the given period.

Company's market constraints are expressed through maximal and minimal sales restrictions. So the company can sell at most 500,000 pieces of products 1, 2, 3, 6, 7, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, and 30, 345,000 pieces of product 4,575,000 of product 5,172,500 pieces of product 8,230,000 pieces of product 15,345,000 pieces of product 26,230,000 pieces of product 27,300,000 pieces of product 28, and 264,500 pieces of product 29. The company has to produce at least 115,000 pieces of product 6; 172,500 pieces of product 13 and 115,000 pieces of product 16 since it has contracts with existing customers (Perić and Babić 2009).

Table 1 Net sale price and profit per product. *Source*: (Perić and Babić 2009)

Product (i)	Net sale price in mon. Units (c_{i3})	Profit in mon. Units (c_{i2})	Product (i)	Net sale price in mon. Units (c_{i3})	Profit in mon. Units (c_{i2})
1	3.50	0.60	16	0.75	0.06
2	3.30	0.54	17	0.98	0.12
3	3.60	0.56	18	2.77	0.45
4	1.80	0.25	19	1.37	0.29
5	1.60	0.25	20	1.58	0.31
6	0.80	0.12	21	2.65	0.45
7	0.70	0.08	22	2.20	0.36
8	0.70	0.08	23	1.55	0.20
9	0.80	0.08	24	1.39	0.25
10	3.60	0.72	25	3.95	0.96
11	3.80	0.54	26	1.45	0.16
12	3.99	0.66	27	1.35	0.11
13	0.78	0.07	28	1.55	0.15
14	0.75	0.07	29	1.50	0.14
15	0.75	0.07	30	3.20	0.09

Table 2 Manufacturing time and machine capacity in minutes. *Source:* (Perić and Babić 2009)

Prod. (i)	Cutting unit (a_{i1})	Sewing unit (a_{i2})	Finishing unit (a_{i2})	Prod. (i)	Cutting unit (a_{i1})	Sewing unit (a_{i2})	Finishing unit (a_{i2})
1	0.90	18.40	2.20	16	0.10	3.30	0.50
2	0.60	19.20	2.21	17	0.70	5.20	0.70
3	1.80	18.70	2.80	18	1.00	21.20	1.80
4	0.70	6.70	0.90	19	0.80	12.90	2.70
5	0.90	7.10	0.50	20	0.70	12.60	1.40
6	0.30	4.00	0.80	21	0.80	18.50	2.40
7	0.30	7.20	0.80	22	0.60	10.10	1.60
8	0.20	5.00	0.50	23	0.60	17.50	1.80
9	0.20	4.60	0.60	24	0.30	15.90	1.80
10	0.30	14.40	2.30	25	1.40	19.50	2.40
11	2.30	11.90	2.00	26	0.50	15.60	4.20
12	0.60	26.40	2.60	27	0.20	5.90	3.80
13	0.20	3.10	0.60	28	0.30	4.40	1.60
14	0.20	3.20	0.50	29	0.30	4.40	1.60
15	0.10	3.40	1.00	30	1.90	13.50	2.00
				Avail. mach. capac.	3,136,320 (b_1)	54,711,360 (b_2)	8,363,520 (b_3)

Table 3 The needed quantity of material per product and the maximal quantity of purchase in kg. *Source:* (Perić and Babić 2009)

Prod. (i)	Material			Prod. (i)	Material		
	„A" (g_{i1})	„B" (g_{i2})	„C" (g_{i3})		„A" (g_{i1})	„B" (g_{i2})	„C" (g_{i3})
1	0.016	0.548	–	16	–	0.069	–
2	–	0.597	–	17	–	0.037	–
3	0.477	0.020	–	18	1.200	–	–
4	0.343	–	–	19	0.012	0.362	–
5	0.286	–	–	20	0.050	–	0.290
6	–	0.134	–	21	0.051	–	0.263
7	–	0.075	–	22	–	0.358	–
8	–	0.056	–	23	–	–	0.291
9	–	0.012	0.083	24	0.012	0.143	–
10	–	0.647	–	25	0.012	0.205	–
11	–	0.006	0.683	26	–	0.599	–
12	–	–	0.684	27	0.017	0.114	–
13	0.452	–	–	28	–	0.114	–
14	–	0.009	0.081	29	0.131	–	–
15	–	0.048	–	30	0.210	–	–
					538,000 (d_1)	523,000 (d_2)	179,000 (d_3)
					Maximum quantity to be purchased		

3.2 Multi-objective linear programming model

Let $x_i =$ the quantity of i product in pieces ($i = 1, \ldots, 30$), $z_1 =$ the objective function of the total production in pieces, $z_2 =$ the objective function of the net-profit in monetary units (m.u) $z_3 =$ the objective function of the total revenue in m.u.

The multi-objective linear programming model for determining the optimal production plan for a 1-year period is given in the following form (Perić and Babić 2009):

A. Objective functions

$$\max_{x \in S} \left\{ z_1 = \sum_{i=1}^{30} x_i, \quad \sum_{i=1}^{30} c_{i2} x_i, \quad \sum_{i=1}^{30} c_{i3} x_i \right\} \tag{7}$$

where

$$S = \left\{ \begin{array}{l} x = (x_1, x_2, \ldots, x_{30}): \\ \sum\limits_{i=1}^{30} a_{i1} x_i \le b_1, \quad \sum\limits_{i=1}^{30} a_{i2} x_i \le b_2, \quad \sum\limits_{i=1}^{30} a_{i3} x_i \le b_3, \quad \sum\limits_{i=1}^{30} g_{i1} x_i \le d_1, \quad \sum\limits_{i=1}^{30} g_{i2} x_i \le d_2, \quad \sum\limits_{i=1}^{30} d_{i3} x_i \le d_3, \\ x_1, x_2, x_3, x_6, x_7, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}, x_{21}, x_{22}, x_{23}, x_{24}, x_{25}, \\ x_{30} \le 50,0000, \quad x_4 \le 345,000, \quad x_5 \le 575,000, \quad x_8 \le 172,500, \quad x_{15} \le 230,000, \quad x_{26} \le 345,000, \\ x_{27} \le 230,000, \quad x_{28} \le 300,000, \; x_{29} \le 264,500, \quad x_6 \ge 115,000, \quad x_{13} \ge 172,500, \quad x_{16} \ge 115,000, \\ x_1, \ldots, x_{30} \ge 0 \text{ and integer.} \end{array} \right\}$$

Table 4 Values of variable and objective function. *Source*: Author's calculations by using Excel Solver

Solution	Values of variable	Values of objective function		
		z_1	z_2	z_3
$\max_{x \in S} z_1$	$x_3 = 500{,}000$, $x_4 = 345{,}000$, $x_6 = 115{,}000$, $x_8 = 172{,}500$, $x_9 = 500{,}000$, $x_{10} = 322{,}190$, $x_{13} = 208{,}580$, $x_{14} = 500{,}000$, $x_{15} = 230{,}000$, $x_{16} = 500{,}000$, $x_{21} = 368{,}821$, $x_{24} = 500{,}000$, $x_{27} = 133{,}626$, $x_{28} = 300{,}000$, $x_{29} = 264{,}500$, $x_{30} = 264{,}500$	7,142,645 (100% of z_1*)	1,361,995 (79% of z_2*)	9,287,307 (90% of z_{3*})
$\max_{x \in S} z_2$	$x_3 = 500{,}000$, $x_5 = 522{,}309$, $x_6 = 115{,}000$, $x_{10} = 458{,}209$, $x_{12} = 69{,}444$, $x_{13} = 172{,}500$, $x_{15} = 230{,}000$, $x_{16} = 115{,}000$, $x_{17} = 220{,}375$, $x_{21} = 500{,}000$, $x_{24} = 500{,}000$, $x_{25} = 500{,}000$, $x_{29} = 264{,}500$	4,167,337 (58% of z_1*)	1,728,671 (100% of z_2*)	9,655,347 (94% of z_3*)
$\max_{x \in S} z_3$	$x_3 = 500{,}000$, $x_4 = 345{,}000$, $x_6 = 115{,}000$, $x_8 = 172{,}500$, $x_9 = 500{,}000$, $x_{10} = 322{,}190$, $x_{13} = 208{,}580$, $x_{14} = 500{,}000$, $x_{15} = 230{,}000$, $x_{16} = 500{,}000$, $x_{21} = 368{,}821$, $x_{24} = 500{,}000$, $x_{25} = 500{,}000$, $x_{27} = 133{,}626$, $x_{28} = 300{,}000$, $x_{29} = 264{,}500$, $x_{30} = 91{,}218$	5,551,435 (78% of z_1*)	1,637,435 95% of z_2*)	10,260,245 (100% of z_3*)

3.3 The model solving

The presented model is firstly solved by application of a linear integer programming method maximizing each of the three objective functions separately on the given set of constraints **S**. The obtained solutions are presented in Table 4.

From Table 1 it is obvious that by maximizing function z_1 we obtain a value which significantly differs from the value of that function when we maximize functions z_2 and z_3, respectively. Also, maximizing the other two objective functions results in a significant difference in values of the single objective functions. This reveals a conflict between the objective functions and the need to apply a multi-objective programming method when solving this model (Perić and Babić 2009). Here we present a procedure for determining an optimal production program by application of the MP and STEM methods to solve the multi-objective programming model.

The application of STEM and MP requires an active participation of decision-makers in the problem-solving process. Since we use the data from the paper by Perić and Babić (2009) for the purpose of analyzing the applicability of the method, the authors of this paper have been involved in the problem-solving process as decision-makers. Each of the authors represented one of the objective functions that he wanted to maximize under a given set of constraints.

Decision-makers are aware that the conflict between the objectives precludes maximum value realization of its objective function, which means that they must accept a compromise solution with a lower objective function value. They are also aware that they must find a compromise solution because it is in their common interest. The application of the multi-objective programming method should help decision-makers to understand the decision-making process better. At each step they should know which the decision maker(s) should reduce their aspirations so that the dissatisfied decision-maker(s) can improve the fulfilment of their objective function(s), ultimately leading to a compromise solution that is the best for all decision-makers and for the company as a whole.

3.3.1 Solving the problem by the MP method

The process of solving a production program optimization problem by applying the MP method *starts* by informing the DM_k ($k = 1, \ 2, \ 3$) on the maximal and minimal values of the objective functions.

$$4{,}167{,}377 \le z_1 \le 7{,}142{,}645 \tag{8}$$

$$1{,}361{,}995 \le z_2 \le 1{,}728{,}671 \tag{9}$$

$$9{,}287{,}307 \le z_3 \le 10{,}260{,}245. \tag{10}$$

After informing the decision-makers on the highest and lowest value of their objective function, the decision-makers determine the initial acceptable value of their objective functions. In the first stage, the decision-makers have determined the following acceptable values for their objective functions: $z_1 = 7{,}100{,}000$, $z_2 = 1{,}700{,}000$, $z_3 = 10{,}200{,}000$. At this stage it is normal that each decision-maker aims to realize the value of his objective function that approaches its maximum. The DMs know that they can hardly realize the determined acceptable level in the first step. Once the process has been iterated several times, which requires an active participation of the DMs and negotiations in the process of problem solving, the final acceptable level of the objective function values should be realized (Perić et al. 2017).

In the *second stage* of the first step of the MP method the following integer linear programming model is solved:

$$\max_{(\mathrm{x},\lambda)\in \mathrm{D}_\lambda} \lambda \tag{11}$$

Table 5 The solution of stage 2, step 1. *Source*: Authors' calculations using the Excel Solver software

Solution	λ	z_1	z_2	z_3	λ_1	λ_2	λ_3
I	0.9292	6,597,404	1,579,660	9,980,257,4	0.9292	0.9292	0.9784

Values of variables are omitted from the table, and the indicators λ_k ($k = 1, 2, 3$) are calculated using (5)

Table 6 The solution of stage 2, step 2. *Source*: Authors' calculations using the Excel Solver software

Solution	λ	z_1	z_2	z_3	λ_1	λ_2	λ_3
II	0.9522	6,665,642	1,571,187	9,953,927,1	0.9522	0.9522	0.9954

where

$$\mathrm{D}_\lambda = \left\{ (\mathrm{x}, \lambda) : \mathrm{x} \in \mathrm{S} \cap \left\{ \begin{array}{c} \sum_{i=1}^{30} x_i \geq 7,100,000\lambda \\ \sum_{i=1}^{30} c_{i2} x_i \geq 1,700,000\lambda, \\ \sum_{i=1}^{30} c_{i3} x_i \geq 10,200,000\lambda \end{array} \right\} \right\}$$

.

The solution presented in Table 5 was obtained.

None of the decision-makers were satisfied with the achieved value of their objective function.

In the *second step* of the method the decision-makers determine new reduced aspiration levels (values $\lambda = \lambda_1 = \lambda_2 = 0.9292$ suggest decreasing the aspiration level value of the decision makers 1 and 2.). They agreed to determine: d_1 = 7,000,000, d_2 = 1,650,000, d_3 = 10,000,000.

After solving the model (11) with the changed constraints, $z_1 \geq 7,000,000\lambda$ instead of $z_1 \geq 7,100,000\lambda$, $z_2 \geq 1,650,000\lambda$ instead of $z_2 \geq 1,700,000\lambda$, and $z_3 \geq 10,000,000\lambda$ instead of $z_3 \geq 10,200,000\lambda$ the solution presented in Table 6 was obtained.

After the second step, the DM1 and DM2 were not satisfied with the obtained solution, but the DM3 was completely satisfied with the obtained value of his objective function. After that, decision-makers negotiated and agreed to determine new aspirational levels of their objective functions. The values of $\lambda = \lambda_1 = \lambda_2 = 0.9522$ demonstrated that DM1 and DM2 should reduce their aspirational levels to obtain a solution that will be more acceptable to all decision-makers.

Step 3. They agreed to determine: d_1 = 6,900,000, d_2 = 1,600,000, d_3 = 9,900,000. After solving the model (11) with the changed constraints, $z_1 \geq 6,900,000\lambda$ instead of $z_1 \geq 7,100,000\lambda$, $z_2 \geq 1,600,000\lambda$ instead of $z_2 \geq 1,700,000\lambda$, and $z_3 \geq$

Table 7 The solution of stage 2, step 3. *Source*: Authors' calculations using the Excel Solver software

Solution	λ	z_1	z_2	z_3	λ_1	λ_2	λ_3
III	0.9763	6,736,167	1,562,009,8	9,940,604,4	0.9763	0.9763	1.0041

Table 8 The solution of stage 2, step 4. *Source*: Authors' calculations using the Excel Solver

Solution	λ	z_1	z_2	z_3	λ_1	λ_2	λ_3
IV	0.9693	6,591,293	1,580,000	9,996,502,8	0.9693	1.00	1.0097

$9,900,000\lambda$ instead of $z_3 \geq 10,200,000\lambda$ the solution presented in Table 7 was obtained:

The DM1 was not satisfied with the obtained value of the function z_1. The DM1 decreased the acceptance level of the function z_1 to 6,800,000. The decision makers negotiated and agreed to set the lowest level of the function z_2 at 1,580,000. Therefore, the DMs agreed to determine $d_1 = 6{,}800{,}000$, $z_2 \geq 1{,}580{,}000$, and $d_3 = 9{,}900{,}000$.

Step 4. After solving the model (11) with the changed constraints $z_1 \geq 6,800,000\lambda$ instead of $z_1 \geq 7,100,000\lambda$, $z_2 \geq 1,580,000$ instead of $z_2 \geq 1,700,000\lambda$ and $z_3 \geq 9,900,000\lambda$ instead of $z_3 \geq 10,200,000\lambda$ the solution presented in Table 8 was obtained.

Since at this stage the general satisfaction level λ decreased from 0.9763 to 0.9693, a further improvement of the objective function value z_1 was not possible, and the obtained solution was accepted by all DMs. Therefore, the solution process was completed and the preferred solution obtained after only four steps.

We should emphasize that the MP method assumes that the DMs know or can determine the acceptable level of their objective functions. The solution process ensures obtaining the preferred efficient solution that is acceptable to all DMs in the minimal number of steps (Perić et al. 2017).

To demonstrate the high application efficiency of the MP method in solving this problem we will compare it with the application efficiency of the well-known STEM method.

3.3.2 Solving the problem by the STEM method

The process of problem solving by the STEM method involves several steps. First, we solved the following model

$$\min_{(\mathrm{x},\lambda)\in \mathrm{S}_1} \lambda \tag{12}$$

using the calculated $\alpha_1 = 0.07605$, $\alpha_2 = 0.103147$, $\alpha_3 = 0.007656$, $\pi_1 = 0.407$, $\pi_2 = 0.552$, and $\pi_3 = 0.041$, where

$$\mathrm{S}_1 = \left\{ \begin{array}{l} (\mathrm{x}, \lambda) \in \mathrm{R}^{n+1} : \mathrm{x} \in \mathrm{S};\ \lambda \geq |7,142,645 - z_1(\mathrm{x})| \cdot 0.407;\ \lambda \geq |172,8,671 - z_2(\mathrm{x})| \cdot 0.552; \\ \lambda \geq |1,026,0245 - z_3(\mathrm{x})| \cdot 0.0401;\ \lambda \geq 0 \end{array} \right\}$$

.

Table 9 Results after step 2 of the STEM method

Solution	$(\Delta z_1,\ \Delta z_3)$	z_1^*	z_2^*	z_3^*
I	(50,000, 100,000)	6,841,086	1,549,405.7	9,886,265.95
II	(80,000, 200,000)	6,811,087	1,553,130.67	9,897,832.5
III	(100,000, 250,000)	6,791,087	1,555,613,84	9,905,543.15
IV	(150,000, 300,000)	6,741,086	1,561,822.79	9,924,851.09
V	(200,000, 400,000)	6,691,086	1,568,030.67	9,944,128.58
VI	(300,000, 600,000)	6,591,086	1,580,447.33	9,982,714.15
VII	(400,000, 800,000)	6,491,086	1,592,763.97	10,024,598
VIII	(500,000, 1,000,000)	6,391,086	1,605,044.61	10,067,662.1
IX	(600,000, 1,100,000)	6,291,086	1,617,324.62	10,110,708.9
X	(700,000, 1,200,000)	6,191,086	1,627,052.7	10,123,821.1

The following solution was obtained: z_1 =6891,086, z_2 =1,543,198.06, z_3 =9,866,962.

The DM2 was not satisfied with the obtained solution while the DM1 and DM3 were satisfied.

In the second step of the STEM method applied to solve the theoretical model (6) we used the following inputs: $\pi_1 = \pi_3 = 0$, $\pi_2 = 1$,

$$(\Delta z_1,\ \Delta z_3) = \left\{\begin{array}{l} (50000,\ \ 100,000)\,;\ (80,000,\ \ 200,000)\,;\ (100,000,\ \ 250,000)\,;\ (150,000,\ \ 300,000)\,; \\ (200,000,\ \ 400,000)\,;(300,000,\ \ 600,000)\,;(400,000,\ \ 800,000)\,;\ (500,000,\ \ 1,000,000)\,; \\ (600,000,\ \ 1,100,000)\,;\ (700,000,\ \ 1,200,000) \end{array}\right\}$$

Therefore, the following models are solved:

$$\min_{(\mathrm{x},\lambda)\in \mathrm{S}_2^c} \lambda, \tag{13}$$

$$\text{where } \mathrm{S}_2^c = \left\{\begin{array}{l} (\mathrm{x},\ \lambda) \in \mathrm{R}^{n+1} : \mathrm{x} \in \mathrm{S};\quad \lambda \geq 1,728,671 - z_2(\mathrm{x}); \\ \quad z_1(\mathrm{x}) \geq 6,891,086 - \Delta z_1; z_3(\mathrm{x}) \geq 9,866,962 - \Delta z_3; \\ z_2(\mathrm{x}) \geq 1,543,198; \lambda \geq 0 \end{array}\right\}.$$

The obtained results are presented in Table 9.

Based on the sensitivity analysis presented in Table 9, the decision-makers should choose the preferred solution. In order to choose the preferred solution, the decision-makers negotiated about each of the 10 solutions offered. DM3 was not too demanding regarding the acceptance of the offered solutions. Most negotiations were conducted between DM1 and DM2. After additional calculations of trade-offs between individual solutions, DM1 and DM2 agreed to accept the preferred solution. They agreed to accept the solution VI with z_1 =6,591,086, z_2 =1,580,447.33 and z_3 =9,982,714.15.

4 Analysis of the obtained results

To analyse the efficiency of the presented methods we solved a production plan optimization problem. For that purpose, we formed a multi objective linear integer programming model with three linear objective functions, 30 variables and 39 linear constraints. All computations were performed using Excel Solver to solve linear integer programming problems. Three decision-makers participated in the problem-solving process, each focusing on one objective function.

The criteria for evaluating the efficiency of the presented methods in solving the production plan optimization problem were divided into four groups: (Perić 2008).

The first group (I) represents the general characteristics of the method:

(1) Number, character and significance of the assumptions associated with the method
(2) Required data (types, quantity, accuracy)
(3) The character of the model that the method solves (linear, nonlinear)
(4) The results of the method (number of efficient solutions, ranking of efficient solutions: yes or no)
The second group of criteria are used to evaluate the efficiency of the method from the analyst's standpoint:
(5) The difficulty of using the method (whether there are difficulties in formulating the mathematical model, whether the mathematical model needs to be supplemented or significantly changed, whether the modification of the mathematical model significantly increases its scope (number of variables and constraints).
The third group (III) evaluate the efficiency of the method from the perspective of the decision maker:
(6) The difficulty of using the method (what information is required from the decision maker and is it difficult to provide)
(7) Clarity of the method
(8) Confidence in the reliability of the method and the solution obtained
(9) Whether the decision-maker is involved in the process of model solving
(10) Can the decision-maker, if he or she participates in the process of model solving, influence the change in the solutions obtained (change the preference)
(11) Whether the method enables the decision-maker to learn about the system being optimized.
The fourth group (IV) of criteria evaluate the methods by applying economic criteria:
(12) The total deviation of the obtained solution from the ideal point, expressed as the sum of the individual deviations of each objective function from the corresponding ideal point
(13) Computer-related technical difficulties (total time of problem solving on the computer) expressed through the time required for model building, data entry into the computer, and total analyst's time spent with the decision-maker(s) to obtain the preferred solution.

Based on the above criteria, the DMs evaluated the efficiency of the presented methods. After completing the process of obtaining the preferred solutions using the

MP and STEM method, the decision-makers evaluated the application efficiency of these methods according to the presented criteria. They rated the methods according to the outlined criteria on a scale from 1 to 5 (1 for the least efficiency according to the default criterion and 5 for the greatest) An overview of the efficiency results of the MP method and the STEM method is shown in Tables 10 and 11, respectively. Individual numerical evaluations of the methods are presented in the last column of the Tables 10 and 11 as DM1 + DM2 + DM3.

Since in process of solving a production plan optimization problem the STEM and MP methods use standard computer software to solve a linear programming problem, we consider that the size of the model (number of variables and constraints) does not affect the application efficiency of these methods, so the results can be generalized for all models with a limited number of variables and constraints.

Based on the results presented in Tables 10 and 11 we can conclude that the methods are similar. Both methods are iterative and require active participation of the decision-makers in the resolution process. Both methods achieve the best results when dealing with multiple decision-makers, as by the assumption the decision-makers are not able to estimate the preferences between the objective functions but are capable of recognizing “good solutions”.

However, in addition to the similarities between the two methods, there are differences that give advantage to the MP method. Namely, the MP method is based on the idea of cooperative game theory where the decision-makers are actively involved in solving problems by negotiating at each step of the solving process in order to obtain a preferred solution that satisfies all decision-makers. The MP method at each step reveals which of the decision-makers should reduce his or her aspiration level(s) to enable the improvement of the insufficiently fulfilled objective function level(s).

Since the total score obtained by adding up the decision-makers’ grades for different criteria is higher for the MP method than the STEM method (171 vs.154) we conclude that the MP method has a greater application efficiency in solving of the production plan optimization problem than the STEM method. More precisely, the two methods were graded equally on the criteria belonging to Groups II and IV (15 and 27 respectively). According to the criteria grouped under I and III, the MP method had a higher score (50:36 and 82:60 respectively). Consequently, assigning weights to the criteria groups would not change the final result.

5 Conclusions

In this paper we investigate the application efficiency of the MP and STEM methods in solving a production plan optimization problem of an enterprise engaged in the industrial production of textile products. The MP and STEM methods were applied to a model with three objective functions, 30 decision variables, and 39 constraints. Three decision-makers, who were also the authors of this paper, were actively involved in the optimization problem solving. The standard MS Office Solver software was used to solve the model. Four groups of criteria [(1) general characteristics of the method, (2) the efficiency of the method from analyst’s standpoint, (3) the efficiency of the

Table 10 Overview of the application efficiency of the MP method. *Source*: Authors' descriptions and calculations

Criteria group	Criteria	Descriptive rating of method efficiency	Numeric value from 1 to 5
I	1	The decision maker is unable, due to the complexity of the problem, to indicate a priori information about the preferences, but is capable of identifying the "good" solutions and the relative importance of the criteria functions in the resolution process	3+3+4
	2	Deterministic data	5+5+4
	3	Linear or non-linear problems	5+5+5
	4	One preferred solution accepted by all decision-makers	4+3+4
II	5	The problem-solving analyst using this method is not bound by any additional efforts as it solves a series of standard single-criteria linear (non-linear) programming models	5+5+5
III	6	The method is simple to use. The decision-maker is required to determine and change his or her aspirations in accordance with the solutions that the method gives and in agreement with other decision-makers	5+5+5
	7	The resolution process is understandable to the decision-maker	4+5+5
	8	Decision-makers have a high degree of confidence in the method and the preferred solution that it provides	4+5+4
	9	Decision-makers are actively involved in the process of problem solving	5+5+5
	10	Decision-makers can influence the solution by changing their aspirational levels	4+4+5
	11	The method enables decision-makers to learn about the system being optimized	4+4+4
IV	12	The total deviation of the obtained solution from the ideal point in percent amounts 18.89	5+5+5
	13	The total time of problem-solving on the computer amounts to 3.5 h	3+3+3
Total			171

method from the decision-makers' perspective, and (4) economic criteria] were used to evaluate the efficiency of the MP and STEM methods.

After solving the model using the MP and STEM methods, the efficiency was evaluated on four groups of criteria. The two methods had the same rating on Groups

Table 11 Overview of the application efficiency of the STEM method. *Source* Authors' descriptions and calculations

Criteria Group	Criteria	Descriptive rating of method efficiency	Numeric value from 1 to 5
I	1	The decision maker is unable, due to the complexity of the problem, to indicate a priori information about the preferences, but is capable of identifying the "good" solutions and the relative importance of the criteria functions in the resolution process	3+4+3
	2	Deterministic data	5+5+5
	3	Linear problems	4+4+4
	4	One preferred solution accepted by all decision-makers	4+3+4
II	5	The problem-solving analyst using this method is not bound by any additional efforts as it solves a series of standard single-criteria linear programming models	5+5+5
III	6	The method is simple to use. The decision maker is required to provide information on the amount of reduction of the level of a certain objective function in order to increase the insufficiently satisfied level of other objective functions	5+4+3
	7	The resolution process is understandable to the decision-maker	4+3+3
	8	Decision-makers have a high degree of confidence in the method and the preferred solution that it provides	4 +3+3
	9	Decision-makers are actively involved in the process of problem-solving	4+4+4
	10	Decision-makers can influence the solution by determining the amounts of reduction of their objective function values	3+4+3
	11	The method enables decision-makers to learn about the system being optimized	4+3+3
IV	12	The total deviation of the obtained solution from the ideal point in percent amounts 18.99	5+5+5
	13	The total time of problem solving on the computer amounts to 2.5 h	4+4+4
Total			153

The numeric values presented in Tables 2 and 3 were obtained from the decision makers of criteria II and IV. However, the MP method was evaluated as more efficient according to the criteria in Group I and Group III (50:36 points and 82:60 points, respectively). The MP method has the greatest advantage according the group of criteria that relate to the decision-maker's assessment. These advantages consist of the following:

(a) the MP method is simple to use. The decision-maker is only required to determine and change his or her aspirations in relation to the solutions provided by the method and in agreement with other decision-makers.
(b) Decision-makers are actively involved in a problem-solving process that is understandable to the decision maker.
(c) Decision-makers have a high degree of confidence in the method and the preferred solution that it provides.
(d) Decision-makers can influence the solution by changing their aspirational levels.
(e) The method enables decision-makers to learn about the system being optimized.
(f) Future research could investigate the application efficiency of the MP method in solving nonlinear multi-objective programming problems.

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