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Profit allocation games in supply chains

Petr Fiala

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Abstract The paper considers a supply chain where a number of agents are connected in some network relationship. Game theory is a very powerful framework for studying decision making problems, involving a group of agents in a supply chain. Allocation games examine the allocation of value among agents connected by a network. The ongoing actions in the supply chain are a mix of cooperative and non-cooperative behavior of the participants. The paper proposes a two-stage procedure for profit allocation based on combination of non-cooperative and cooperative game approaches. In the first stage, retailers meet customer price-dependent stochastic demand and seek to maximize total profit from the sale. Retailers are trying to align goals with producers on a contract basis and share the total profit with them. In the second stage, the cooperating producers allocate individual profits.

Keywords Supply chain · Game theory · Allocation · Cooperation · Non-cooperation

1 Introduction

Supply chain is defined as a decentralized system with layers of suppliers, manufacturers, distributors, retailers and customers where material, financial and information flows connect participants in both directions. A supply chain is a complex and dynamic supply and demand network of agents, activities, resources, technology and information involved in moving a product or service from supplier to customer. Most supply chains are composed of independent units with individual preferences. Each unit will attempt to optimize his own preference. Behavior that is locally efficient can be inef-

P. Fiala (✉)
Faculty of Informatics and Statistics, University of Economics, Prague, Czech Republic
e-mail: pfiala@vse.cz

ficient from a global point of view. There are numerous opportunities to create hybrid models that combine competitive and cooperative behavior.

Supply chain management is now seen as a governing element in strategy and as an effective way of creating value for customers. Supply chain management benefits from a variety of concepts that were developed in several different disciplines as marketing, information systems, economics, system dynamics, logistics, operations management, and operations research. Supply chain management has generated a substantial amount of interest both by managers and by researchers.

An increasing number of companies in the world subscribe to the idea that developing long-term coordination and cooperation can significantly improve the efficiency of supply chains and provide a way to ensure competitive advantage. Supply chain formation is the problem of determining the production and exchange relationships across a supply chain. Complex business negotiations often involve interrelated exchange relationships among multiple levels of production. Agents in the supply chain are characterized in terms of their capabilities to perform tasks. Constraints on the task assignment arise from resource availability, where agents require a common resource to accomplish their tasks.

The evolution of supply chain management recognized that a business process consists of several decentralized firms and that operational decisions of these different entities impact each other's profit, and thus the profit of the whole supply chain. To effectively model and analyze decision making in such multi-agent situation where the outcome depends on the choice made by every agent, game theory is a natural choice. Game theory has become a useful instrument in the analysis of supply chains with multiple agents, often with conflicting objectives. The paper analyzes allocation decisions in supply chains. Equilibrium search in supply chains is a very important problem. Allocation games are used for behavior modeling of supply chains and focus on allocation of resources, capacities, costs, revenues and profits. A profit allocation two-stage procedure for equilibrium in supply chains is proposed, based on combination of non-cooperative and cooperative game approaches.

The rest of the paper is organized as follows. Section 2 presents a literature review of supply chain management and game theory models applicable in this area. Section 3 summarizes the basics of the game theory applicable in the allocation of profit in supply chains. In Sect. 4, the problem formulation and an outline of the procedure are provided. Non-cooperative part of the problem is analyzed in the Sect. 5. A cooperative approach for profit allocation is presented in Sect. 6. A numerical example is calculated in Sect. 7. Section 8 presents conclusions.

2 Literature review

There are many concepts and strategies applied in designing and managing supply chains (see Simchi-Levi et al. 1999). The expanding importance of supply chain integration presents a challenge to research to focus more attention on supply chain modeling (see Tayur et al. 1999). In supply chain behavior is much inefficiency. The so-called bullwhip effect (see Tayur et al. 1999), describing growing variation upstream in a supply chain, is probably the most famous demonstration of system dynamics in supply chains. There are some known causes of the bullwhip effect: information asymmetry, demand forecasting, lead-times, batch ordering, supply shortages and price variations. Information sharing of customer demand has an impact on the bullwhip effect and other inefficiencies in supply chains (Fiala 2005). Researchers in supply chain management now use tools from game theory to help managers to make strategic operational decisions in complex multi-agent supply chain systems.

John von Neumann and Oskar Morgenstern (1944) is the classic work upon which modern game theory is based. Since then, an extensive literature on game theory was published. For example, Myerson's book (1997) provides a clear and thorough examination of the models, solution concepts, results, and methodological principles of non-cooperative and cooperative game theory. Game theory models situations where players make decisions to maximize their own utility, while taking into account that other players are doing the same, and that decisions made by players, impact others utilities. There is a broad division of game theory into two approaches: the cooperative and the non-cooperative approach. These approaches, though different in their theoretical content and the methodology used in their analysis, are really just two different ways of looking at the same problem.

The field of supply chain management has seen, in recent years, a wide variety of research papers that employ game theory to model interaction between players. Cachon and Netessine (2004) provide an excellent survey and state of art especially non-cooperative game techniques. The concept of using non-cooperative agents to formulate allocation mechanisms in a game theoretical setting is closer to the classical market concept than solutions employing cooperative strategies. Most non-cooperative allocation strategies in distributed systems consist of following steps:

- The formulation of utility functions for the system participants.
- The formulation of best response strategies.
- The existence of Nash equilibrium is proved in the system of multiple agents.
- Efficiency is measured compared to achievable welfare.

Nagarajan and Sošić (2008) review the existing literature on applications of cooperative games to supply chain management. They also deal with certain methodological issues when modeling supply chain problems. The paper focuses on applications in supply chains with two central questions of cooperative games:

- What are feasible outcomes and how the players in a coalition allocate the outcomes?
- What are stable coalitions?

Allocation mechanisms are based on different approaches such as negotiations, auctions, Shapley values, etc.

A large number of papers have been published that proposed analyze mechanisms for supply chain coordination. Mechanisms based on non-cooperative game theory usually propose establishment of coordinating contracts. A retailer can usually collect demand information easier than a producer and he has a better motivation for optimally determining sales quantities and prices. There are many types of contracts. The basic type is a wholesale price contract. With a wholesale price contract (Lariviere 1999) the supplier charges the retailer w per unit purchased. The producer knows exactly what retailer will order at every wholesale price and bears no responsibility for the product. All uncertainty regarding the producer profit is foisted onto the retailer. The wholesale price contract coordinates the chain only if the producer earns a non-positive profit. So the producer clearly prefers a higher wholesale price. As a result, the wholesale price contract is generally not considered a coordinating contract. The richer contracts differ from wholesale price contracts by allowing the producer to assume some of the risk arising from stochastic demand.

As an example we introduce buy back contracts as an extension of wholesale price contracts. With a buy back contract (Pasternack 1985) the producer charges the retailer w per unit purchased, but pays the retailer b per unit remaining at the end of the season. The retailer should not profit from left over inventory, so assume $b \leq w$. There is assumed that a returns policy on the decentralized chain introduces no additional cost beyond that incurred by the centralized system.

Quantity flexibility contracts define terms under which the quantity a retailer ultimately orders from the producer may deviate from a previous planning estimate (Tsay 1999; Lariviere 1999). Backup agreements (Eppen and Iyer 1997) state that if a retailer commits to a number of units for the season, the producer will hold back a fraction of the commitment and the retailer can order up to this backup quantity at the original purchase price after observing early demand. Option contracts (Barnes-Schuster et al. 2002) specify that in addition to a firm order at a regular price, the retailer can also purchase options at an option price at the beginning of the selling season. Price protection (Lee et al. 2000) states, that the supplier pays the retailer a credit applying to the retailer's unsold goods when the wholesale price drops during the life cycle. Chen and Cheng (2012) presented price-dependent revenue sharing mechanism.

3 Game theory background

This section summarizes some of the basic non-cooperative and cooperative concepts of the game theory that are applied in the proposed approach for profit allocation in supply chains.

The non-cooperative theory of games is strategy oriented; it studies what one may expect the players to do. The non-cooperative theory is a "micro" approach in that it focuses on precise descriptions of what happens.

An n-player non-cooperative game in the normal form is a collection

$$\begin{aligned} \{N = \{1, 2, \ldots, n\}\,; X_1, X_2, \ldots, X_n; \pi_1 (x_1, x_2, \ldots, x_n)\,, \\ \pi_2 (x_1, x_2, \ldots, x_n)\,, \ldots, \pi_n (x_1, x_2, \ldots, x_n)\}\,, \end{aligned} \tag{1}$$

where N is a set of n players; X_i, $i = 1, 2, \ldots, n$, is a set of strategies for player i; $\pi_i (x_1, x_2, \ldots, x_n)$, $i = 1, 2, \ldots, n$, is a pay-off function for player i, defined on a Cartesian product of n sets X_i, $i = 1, 2, \ldots, n$.

Decisions of other players than player i are summarized by a vector

$$\mathbf{x}_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1} \ldots, x_n)\,. \tag{2}$$

A vector of decisions $\left(x_1^0, x_2^0, \ldots, x_n^0\right)$ is a Nash equilibrium of the game if

$$x_i^0\left(\mathbf{x}_{-i}^0\right) = \underset{x_i}{\operatorname{argmax}}\, \pi_i\left(x_i, \mathbf{x}_{-i}\right) \quad \forall i = 1, 2, \ldots, n. \tag{3}$$

A Nash equilibrium is a set of decisions from which no player can improve the value of his pay-off function by unilaterally deviating from it.

Stackelberg games are strategic games with 2 players. They are also called leader-follower games. The leader plays first, anticipating the decision of the follower, and the follower has no other choice than to act optimally as anticipated by the leader. Such games generally reach a compromise situation, called the Stackelberg equilibrium.

The leader's optimal decision, denoted x_1^0, is computed recursively from the knowledge of the follower's optimal response function $x_2^0\left(x_1\right)$:

$$x_1^0 = \underset{x_1}{\operatorname{argmax}}\, \pi_1\left(x_1, x_2^0(x_1)\right) \quad \text{and} \quad x_2^0 = x_2^0\left(x_1^0\right). \tag{4}$$

When the demand is stochastic than the newsvendor model can be applied. The newsvendor model is not complex, but it is sufficiently rich to study important questions in supply chain coordination. In a standard newsvendor problem the price is assumed to be fixed but the problem is to analyze contracts for supply chain coordination with price-dependent stochastic demand.

Cooperative game theory looks at the set of possible outcomes, studies what the players can achieve, what coalitions will form, how the coalitions that do form divide the outcome, and whether the outcomes are stable and robust.

When modeling cooperative games is advantageous to switch from the game in normal form to the game in the characteristic function form. The characteristic function of the game with a set of n players N is such function $v\left(S\right)$ that is defined for all subsets $S \subseteq N$ (i.e. for all coalition) and assigns a value $v\left(S\right)$with following characteristics:

$$v\left(\emptyset\right) = 0, \tag{5}$$

$$v\left(S_1 \cup S_2\right) \geq v\left(S_1\right) + v\left(S_2\right), \tag{6}$$

where S_1, S_2 are disjoint subsets of the set N.

The pair (N, v) is called a cooperative game of n players in the characteristic function form.

A particular allocation policy, introduced by Shapley (1953) has been shown to possess the best properties in terms of balance and fairness. So called Shapley vector is defined as

$$\mathbf{h} = \left(h_1, h_2, \ldots, h_n\right), \tag{7}$$

where the individual components (Shapley values) indicate the mean marginal contribution of i-th player to all coalitions, which may be a member. Player contribution to the coalition S is calculated by the formula:

$$v\left(S\right) - v\left(S - \{i\}\right). \tag{8}$$

A complicating factor is that with the increasing number of n players is rapidly increasing number of coalitions and complexity of their production. Shapley value for the i-th player is calculated as a weighted sum of marginal contributions according to the formula:

$$h_i = \sum_S \left\{ \frac{(|S| - 1)!\ (n - |S|)!}{n!} . [v(S) - v(S - \{i\})] \right\}, \tag{9}$$

where the number of coalition members is marked by symbol $|S|$ and the summation runs over all coalition $i \in S$.

A biform game is a combination of non-cooperative and cooperative games, introduced by Brandenburger and Stuart (2007). It is a two-stage game: in the first stage, players choose their strategies in a non-cooperative way, thus forming the second stage of the game, in which the players cooperate. The biform game approach can be used for modeling general buyer–supplier relationships in supply chains. First, suppliers make initial proposals and take decisions. This stage is analyzed using a non-cooperative game theory approach. Then, suppliers negotiate with buyers. In this stage, a cooperative game theory is applied to characterize the outcome of negotiation among the players over how to distribute the total surplus. Each supplier's share of the total surplus is the product of its added value and its relative negotiation power.

4 Problem and solving formulation

The problem is formulated as a supply chain with layers of suppliers, producers, retailers and customers. Suppliers form a layer with m agents and provide m types of resources to producers. The layer of producers is represented by n agents. These agents produce one type of product. The production is characterized by consumption of m resources to produce one unit of the final product. Each production agent is characterized by its available production resources. The resource capacity constraints compare the total availability of resources in the production layer with total consumption of resources to produce total number of q units of products. Producers send the products to retailers. Retailers meet price-dependent stochastic demand of customers. This problem is solved by two-stage procedure based on combination of no-cooperative and cooperative game approaches.

The first stage solves problems by price-dependent stochastic demand of customers:

- How to get maximal profit from customers.
- How to allocate the maximal profit between retailers and producers.

The problems are solved by non-cooperative manner. A Stackelberg game is formulated between the layer of producers and the layer of retailers as a newsvendor problem with pricing. Retailers seek to maximize total profit from the sale and try to align goals with producers on a contract basis and share the total profit with them. The maximization of the profit is by the resource capacity constraints. The equilibrium point (p^0, q^0) is given by values of total number of q production units and optimal price p.

A specific buyback contract is used for coordination. The layer of producers as leader proposes the wholesale price w and the buyback price b. The layer of retailers as follower accepts the prices to coordinate the system. The allocation of the total profit between retailers and producers is given by splitting parameter $\lambda (0 \le \lambda \le 1)$. The value of the parameter λ is negotiated by retailers and producers.

In the second stage, producers address the following issues:

- How the determine the optimal coalition structure.
- How to allocate the profit among the members of the optimal coalition.

The problems are solved by cooperative manner. These agents compete to be members of a coalition and are willing to cooperate to produce products and sell them to customers through retailers. The optimal coalitions are determined according to the maximal profit with respect the resource capacity constraints for the coalition.

The maximal profit is allocated among the members of the coalitions by Shapley values. Shapley value has been shown to possess the best properties in terms of balance and fairness.

5 First stage: non-cooperative problem

We consider a supply chain in one-period setting in which the layer of producers sells a product to the layer of retailers facing stochastic demand from consumers. We assume that stochastic demand u has a continuous distribution $F(u)$ with density $f(u)$. The demand distribution and cost information are common knowledge. Define the failure rate function of the u distribution as

$$g(u) = \frac{f(u)}{1 - F(u)}, \tag{10}$$

and the generalized failure rate function as

$$h(u) = ug(u). \tag{11}$$

Assume the demand distribution has strictly increasing generalized failure rate property (IGFR). Many distributions have the IGFR property, including the uniform, the normal, the exponential, the gamma, and the Weibull.

We define the following quantities: q retailer's total order quantity; c producer's unit production cost; p retail price. The setting can be characterized as a newsvendor problem.

5.1 Centralized solution

Centralized solution is a benchmark for the decentralized supply chain. The centralized chain is considered as an integrated firm that controls production and sales to customers. The profit of an integrated firm for stocking level q is

$$\pi(q) = (p - c)q - p \int_0^q F(u)du. \tag{12}$$

The problem is concave in q and the optimal solution is given by

$$q^0 = F^{-1}\left(\frac{p-c}{p}\right). \tag{13}$$

The maximum system profit $\pi(q^0)$ is completely determined by the production level q^0.

Decentralized solution can be improved by contracting. The contract coordinates the chain if it induces the choice of the centralized system's optimal stocking level q^0. The approach is based on a specific buy back contract for the price-dependent stochastic demand.

5.2 Buy back contracts

The retailer's profit is

$$\pi_R(q) = (p-w)q - (p-b)\int_0^q F(u)du. \tag{14}$$

The producer acts as a Stackelberg leader and anticipates how the retailer will order for any wholesale price. The supplier anticipates a demand curve $q(w)$ and the profit

$$\pi_P(w) = (w-c)q(w) = (w-c)F^{-1}\left(\frac{p-w}{p}\right). \tag{15}$$

The retailer still faces a newsvendor problem. The retailer's problem is concave in q and the optimal solution is given by

$$q(w,b) = F^{-1}\left(\frac{p-w}{p-b}\right). \tag{16}$$

No returns or full returns are suboptimal. An intermediary policy results in chain coordination. The supplier offers a set of coordination contracts $(w(\varepsilon), b(\varepsilon))$ for $\varepsilon \in (0, p-c\rangle$ where

$$w(\varepsilon) = p - \varepsilon, b(\varepsilon) = p - \frac{\varepsilon p}{p-c}. \tag{17}$$

For all $\varepsilon \in (0, p-c\rangle$,

$$\text{p} > w(\varepsilon) > b(\varepsilon), \text{ and} \tag{18}$$

$$\frac{p-w(\varepsilon)}{p-b(\varepsilon)} = \frac{p-c}{p}. \tag{19}$$

The retailer orders the integrated chain quantity

$$q\,(w\,(\varepsilon)\,,b\,(\varepsilon))=q^{0}, \tag{20}$$

and system profit is equal to the integrated chain profit $\pi(q^0)$.

Retailer's profit is increasing in ε

$$\pi_R(w(\varepsilon),b(\varepsilon))=\frac{\varepsilon}{p-c}\pi(q^0). \tag{21}$$

Supplier's profit is decreasing in ε

$$\pi_S(w(\varepsilon),b(\varepsilon))=\left(1-\frac{\varepsilon}{p-c}\right)\pi(q^0). \tag{22}$$

5.3 Price-dependent stochastic demand

Little work has been done on the combined problem of supply chain coordination with price-dependent stochastic demand (Yao et al. 2006). The contracts proposed for coordination with price-independent stochastic demand are not applicable for coordination of supply chains with price-dependent stochastic demand.

We will analyze the multiplicative form of price-dependent stochastic demand

$$D\,(p,u)=y\,(p)\,u, \tag{23}$$

a function of p and u, where u is a random variable independent of p and $y(p)$ is continuous, nonnegative, twice differentiable function. The expectation of D is specified by a function $y(p)$ for any given price p:

$$E\,[D\,(p,u)]=y\,(p)\,. \tag{24}$$

The flows in the supplier–retailer supply chain with stochastic price-dependent demand are captured in Fig. 1. Material and unit financial flows are represented by continuous and dash lines, respectively.

The expected profit for centralized solution for any output level q and price p is:

$$\begin{aligned}\pi\,(p,q)&=E\,\{p[\min\,(q,\,D\,(p,u)]-cq\}\\&=E\,\{(p-c)\,q-p\max\,(0;q-D\,(p,u))\}\\&=(p-c)q-py(p)\int_0^{\frac{q}{y(p)}}F(u)du.\end{aligned} \tag{25}$$

The objective is to choose (p^0,q^0) to maximize the expected profit $\pi(p,q)$.

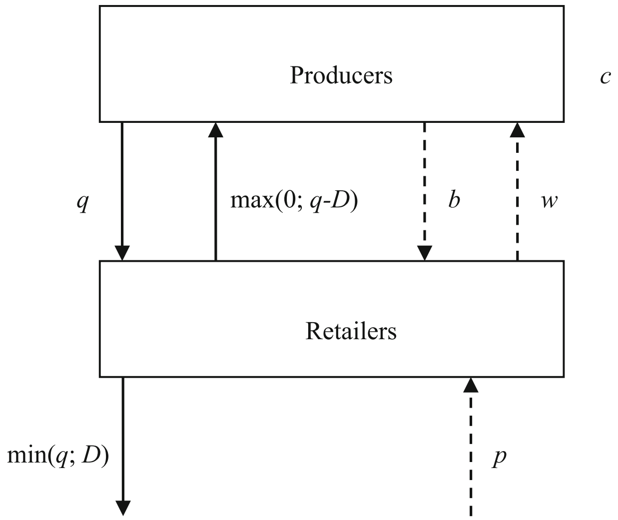

Fig. 1 Producers–retailers supply chain with stochastic price-dependent demand

By fixing price p the problem reduces to standard newsvendor problem without pricing and the optimal level of production

$$q^0 = y(p)F^{-1}\left(\frac{p-c}{p}\right). \tag{26}$$

By substituting it into the expected profit

$$\pi\,(p) = y(p)\left[(p-c)F^{-1}\left(\frac{p-c}{p}\right) - p\int_0^{F^{-1}\left(\frac{p-c}{p}\right)} F(u)du\right]. \tag{27}$$

The problem is now with only one decision variable p and the optimal price p^0 can be obtained by solving

$$\frac{d\pi(p)}{dp} = 0. \tag{28}$$

The assumptions of the existence and uniqueness of the optimal solution (p^0, q^0) are concavity of deterministic part of demand function $y(p)$ and IGFR property of stochastic part of demand function u.

The proposed contract for coordination of the decentralized supply chain is a specific buy-buck contract. The wholesale price w and the buy-buck price b are specified:

$$w = \lambda\,(p-c) + c, \tag{29}$$

$$b = \lambda p, \tag{30}$$

$$\text{where} \quad 0 \le \lambda \le 1. \tag{31}$$

By the setting of the prices w and b the retailer's profit and the supplier's profit for any chosen output level q and price p are

$$\begin{aligned} \pi_R &= E\left\{p[\min(q, D(p,u)] - wq + b\max(0; q - D(p,u))\right\} \\ &= E\left\{(p - w - c)\,q - (p - b)\max(0; q - D(p,u))\right\} \\ &= (1-\lambda)E\left\{(p - c)\,q - p\max(0; q - D(p,u))\right\} = (1-\lambda)\,\pi, \quad (32) \\ \pi_P &= E\left\{(w - c)\,q - b\max(0; q - D(p,u))\right\} \\ &= E\left\{\lambda(p - c)\,q - \lambda p\max(0; q - D(p,u))\right\} = \lambda\pi. \quad (33) \end{aligned}$$

From previous expressions of the retailer's profit and the producer's profit, it is clear that the retailer and the producer solve the same problem as the centralized supply chain and the sum of the retailer's profit and the supplier's profit is equal to the profit of the centralized supply chain. The parameter λ characterizes a splitting of the total profit between the retailer and the supplier.

6 Second stage: cooperative problem

In the considered problem the layer of producers is represented by n agents. The set of production agents is denoted $N = \{1, 2, \ldots, n\}$. These agents compete to be members of a coalition $S \subseteq N$and are willing to cooperate to produce products and sell them to customers through retailers. A coalition S is defined as a subset of the set N of n producers with characteristic vector $\mathbf{e}(S) \in \{0, 1\}^n$ such that

$$\begin{aligned} &e_j(S) = 1, \text{ if } j \in S, \text{ and} \\ &e_j(S) = 0, \text{ otherwise.} \end{aligned}$$

The production is characterized by consumption of m resources. The resource vector $\mathbf{r} = (r_1, r_2, \ldots, r_m)$ represents consumption of m resources to produce one unit of the final product. Each agent is characterized by its available production resources. The availability is defined by an availability matrix $\mathbf{A} = [a_{ij}]$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, where a_{ij} is the amount of resource i available at agent j. The resource capacity constraints for coalition S are given

$$q\mathbf{r} \leq \mathbf{Ae}(S). \quad (34)$$

The cooperative problem is formulated as to maximize profit of producers by a production quantity q and a coalition structure S subject to resource capacity constraints

$$\pi_P = E\left\{(w - c)\,q - b\max(0; q - D(p,u))\right\} \to \max$$

subject to

$$\begin{aligned} &q\mathbf{r} \leq \mathbf{Ae}(S), \\ &q \in \mathrm{R}, \mathbf{e}(S) \in \{0, 1\}^n. \quad (35) \end{aligned}$$

Problem (35) can be solved for given vectors $\mathbf{e}(S)$. The total profit maximization is achieved for the grand coalition, i.e. $\mathbf{e}(S)=\mathbf{1}$. But some smaller coalitions can give maximal profit also.

The maximal profit for producers is denoted by Π_P. For the profit allocation, it is necessary to identify all the coalitions that achieve this maximal profit by testing problem (35) with given maximal profit.

The coalitions with lower potential profit than the maximal get 0. The individual profit Π_{Pi} for the members from the coalitions with the maximal profit is allocated by Shapley values

$$\Pi_{Pi} = \Pi_P \sum_S \left\{ \frac{(|S|-1)!\ (n-|S|)!}{n!} \right\}. \tag{36}$$

Computation of the Shapley value allocation requires computing the solution of the problems (35) for all coalitions $S \subseteq N$. This can be time consuming for large sets of producers.

The procedure of the profit allocation algorithm can be summarized in following steps:

Step 1 Solve the problem (35) with resource capacity constraints for the grand coalition to obtain the price p^0 and the optimal level of production q^0. Compute the maximal total expected profit.

Step 2 Negotiations between retailers and producers how to allocate the total profit is given by splitting parameter $\lambda (0 \leq \lambda \leq 1)$.

Step 3 Set the wholesale price vector w computed by (29) and buyback price b computed by (30).

Step 4 Identify all the coalitions that achieve the maximal profit by testing problem (35) with given maximal profit for producers Π_P.

Step 5 Compute the Shapley value allocation (36) to allocate the expected profit among the producers.

7 A numerical example

The procedure is illustrated by the following numerical example. Consider a simple supply chain with two suppliers providing resources ($m = 2$), three producers ($n = 3$) with availability of resources given by the availability matrix $\mathbf{A}$ that produce a product with consumption of resources given by the resource vector $\mathbf{r}$ and unit cost c:

$$\mathbf{A} = \begin{bmatrix} 0 & 80 & 70 \\ 140 & 0 & 40 \end{bmatrix}, \quad \mathbf{r} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad c = 2.$$

Retailers are confronted with the price-dependent stochastic demand of customers, where a deterministic part is given by the formula

$$y\,(p) = 100 - p^2$$

and stochastic part is given by the random variable x on the interval (0.50, 1.50) with uniform distribution function

$$F\left(u\right)=u-0.50.$$

The centralized unconstrained solution of the total expected profit:

$$\pi\left(p\right)=(100-p^{2})\left[(p-2)F^{-1}\left(\frac{p-2}{p}\right)-p\int_{0}^{F^{-1}\left(\frac{p-2}{p}\right)}(u-0,5)du\right]$$
$$=(100-p^{2})\left(\frac{1,5p-2}{p}\right)\left(\frac{1,5p-2}{2}\right).$$

Solving the equation

$$\frac{d\pi(p)}{dp}=0$$

we get optimal price

$$p^{0}\cong 6.56.$$

and the optimal level of production

$$q^{0}=y(p)F^{-1}\left(\frac{p-c}{p}\right)\cong 68.36.$$

The expected profit

$$\pi\left(p^{0},q^{0}\right)\cong 266.89.$$

Setting the splitting parameter by negotiation between producers and retailers

$$\lambda=0.50,$$

the wholesale price w and the buy-buck price b are specified

$$w=\lambda\left(p-c\right)+c\cong 4.28,$$
$$b=\lambda p\cong 3.28.$$

Allocation of profits of retailers and producers

$$\pi_{R}=\lambda\pi\left(p^{0},q^{0}\right)\cong 133.45,$$
$$\pi_{P}=(1-\lambda)\pi\left(p^{0},q^{0}\right)\cong 133.45.$$

The maximal unrestricted profit for producers Π_P is feasible for the grand coalition $\{1, 2, 3\}$.

The restrictions on the level of production from model (1) are satisfied:

$$q\mathbf{r} \leq \mathbf{Ae}\,(S)\,, \text{ where } \quad \mathbf{e}(S) = \mathbf{1},$$
$$q \leq 170, \quad 2q \leq 180.$$

We have 7 potential coalitions for 3 producers. The maximal profit for producers Π_P is also feasible for coalitions $\{1, 2\}$ and $\{1, 3\}$. Shapley allocation is

$$(2/3\Pi_P, 1/6\Pi_P, 1/6\Pi_P) \cong (88.96, 22.24, 22.24)\,.$$

8 Conclusions and outlook

The aim of this paper is to propose mechanism for profit allocation in supply chains. The proposed procedure comes from the fact that the ongoing actions in the supply chain are a mix of cooperative and non-cooperative behavior of the participants. A combination of non-cooperative and cooperative game approaches is used. In the non-cooperative part, a coordination mechanism based on a specific buy-back contract is applied between producers and customers with price-dependent stochastic demand. The contract has desirable features: full coordination of the supply chain, flexibility to allow any division of the supply chain's profit, and easy to use. The cooperative part is merely focused on two concepts, coalition formations by resource capacity constraints and profit sharing. Profit sharing is carried out on the recognized concept of Shapley value.

The analysis of the simple cases for the approach gives recommendations for more complex real problem. Supply chain structures are typical for a mix of cooperative and non-cooperative behavior of the participants but applications of the proposed approach are not limited only to supply chains. There are problems in design, production, scheduling, inventory, and other situations that exhibit characteristics for analyzing non-cooperative and cooperative behavior. The price-dependent stochastic demand model is suitable for analyzing specific market situations. For example, analyses of electricity markets are suitable for using the proposed procedure that can be combined with models of work (Vasin et al. 2013).

The approach seems to be useful and promising for next research. There are some possible extensions of the approach and some areas for further research. The model can be extended with respect to the multi-level structure, a larger number of products and further quantitative parameters of the model. Next step in extensions can be inclusion of uncertainty in data and characteristics, for example using of fuzzy approaches or interval data. The instruments for analyzing non-cooperative and cooperative parts can be modified for the uncertainty case. For example, the Shapley value for cooperative games was extended to the interval Shapley value for situations where the coalition values are compact intervals of real numbers (Alparslan Gök et al. 2010). In general the approach is suitable for use of other types of games.

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