

# Demographic change in models of endogenous economic growth. A survey

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**Abstract** While in exogenous growth models demographic variables are linked to economic prosperity mainly via the population size, the structure of the workforce, and the capital intensity of workers, endogenous growth models and their successors also allow for interrelationships between demographic variables and technological change. However, most of the existing literature considers only the interrelationships based on population size and its growth rate and does not explicitly account for population aging. The aim of this paper is (a) to review the role of population size and population growth in the most commonly used endogenous economic growth models, (b) discuss models that also allow for population aging, and (c) sketch out the policy implications of the most commonly used endogenous growth models and compare them to each other.

**Keywords** Demographic change · Technological change · Economic growth

## 1 Introduction

During the last decades, all industrialized countries had to face declines in birth rates, while survival rates continued to improve, allowing people to reach older ages. As a consequence, populations in industrialized countries started to age and eventually—as

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declining mortality rates and migration are not able to compensate the fall in fertility—they will decline (see for example [United Nations 2007](#); [Eurostat 2009](#)). While these developments will happen for sure, it is not clear, how they will affect the overall economic performance of the countries under consideration. The interrelations between long-run economic growth on the one hand and the population size, its growth rate as well as population aging on the other hand are therefore of central interest (see for example [Bloom et al. 2008, 2010](#)).

We review different models explaining medium-run (corresponding to the transitional path) and long-run (corresponding to the steady-state equilibrium or the balanced growth path) economic growth and discuss their predictions when the underlying demographic structure is changed. The results crucially depend on the models used. In exogenous growth models demographic variables are linked to economic growth via population size, the structure of the workforce and the capital intensity of workers. An increase in population size will foster capital dilution, while changes in the decomposition of the workforce will affect aggregate productivity.

Endogenous growth models allow for additional channels through which demographic changes can affect the overall economic performance. These include the role of the number of scientists determining the pace of technological change, changes in the demand for innovative goods as well as changes in the human capital endowment and therefore the productivity of workers.

Semi-endogenous growth models have been mainly designed to get rid of the positive relation between population size and economic growth (scale effect) as evident in endogenous growth models of the first generation since such an effect is not supported by empirical evidence (cf. [Jones 1995](#)). However, this extension has come at the price that economic development again depends on exogenously given parameter values and long-run growth cannot be affected by economic policy. This is a feature that this type of growth models shares with exogenous growth models which is also the reason why they are called semi-endogenous.

Therefore, [Peretto \(1998\)](#) paved the way for another type of scale free endogenous growth models, where the policy interventions are effective. Nevertheless, another shortcoming of most scale free endogenous growth models still exists because they predict a positive relationship between population growth and per capita output growth, which cannot be observed in empirical studies (cf. [Brander and Dowrick 1994](#); [Kelley and Schmidt 1995](#)). Recent attempts in modeling long-run economic growth by [Dalgaard and Kreiner \(2001\)](#) and [Strulik \(2005\)](#) have addressed this issue and their models allow for a negative correlation between population growth and per capita output growth.

The aim of our paper is to provide an overview of the role of varying population size, its growth rate and its age structure for per capita output growth in selected endogenous growth models. Our purpose is to give a short and concise description of the underlying models' features with respect to medium-run as well as with respect to long-run (steady state) growth and to assess its interrelations with the demographic variables. Furthermore, we aim to point out policy implications that these different models offer.

Our paper complements [Gruescu \(2007\)](#) who concentrates on the models of [Solow \(1956\)](#) and [Lucas \(1988\)](#) and also provides a short overview regarding the sensitivity

of endogenous and semi-endogenous growth models with respect to changes in the size and the growth rate of the population.

For notational convenience we will use uppercase letters as aggregates over the whole population, while lowercase letters refer to per-capita variables. Furthermore, we denote growth rates of a variable by  $g$  in equilibrium and by  $g^*$  in the steady state. If some variables  $x_1$  and  $x_2$  grow at different rates, we will differentiate between these rates by denoting the growth rates as  $g_{x_1}$  and  $g_{x_2}$ , respectively.

In Sect. 2 we review selected models of endogenous growth. In Sect. 3 we present selected semi-endogenous growth models, while in Sect. 4 we review models that integrate horizontal and vertical innovations. We focus on extensions of the baseline models that implement demographic variables in more detail, in particular those who analyze population aging. For a short overview of the effects of the role of demographic change in exogenous growth models see [Prettner and Prskawetz \(2010\)](#). Finally, Sect. 5 summarizes the results and draws some conclusions.

## 2 Endogenous growth models

### 2.1 Models with increasing returns to capital

One of the first attempts to endogenize long-run economic growth was undertaken by [Romer \(1986\)](#) who introduced firm specific knowledge as a production factor. There is learning by doing in the sense that employing capital is assumed to have positive intertemporal knowledge spillovers implying increasing returns to scale with respect to the per capita stock of physical capital in the aggregate production function. This ensures that long-run per capita output growth is possible without relying on exogenous technological change. Such a specification is equivalent to having increasing returns to scale with respect to the capital stock, which has been one of the main critiques regarding the [Romer \(1986\)](#) model.

The central result of the model is that in equilibrium the stock of knowledge grows without bound and so does per capita output. If the number of agents increases, the stock of knowledge would also increase and growth would be fostered. This points out that demography—as represented by population size—is positively related to long-run economic growth. However, population growth is not allowed for and the age structure of the population is ignored. Policymakers are able to intervene in the [Romer \(1986\)](#) framework by implementing policies that increase capital (which is equivalent to knowledge) accumulation.

[Futagami and Nakajima \(2001\)](#) investigate the effects of population aging via introducing life cycle savings decisions into the [Romer \(1986\)](#) framework.<sup>1</sup> Additionally, they assume that individuals live for two periods: In the first period they have to work and earn wages, whereas in the second period they are retired and can only consume out of their savings carried over from the previous period. [Futagami and Nakajima](#)

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<sup>1</sup> To be precise, [Futagami and Nakajima \(2001\)](#) consider the capital stock instead of the stock of knowledge to be associated with increasing returns to scale in the aggregate production function.

(2001) derive an expression for the steady state growth rate of the economy,  $g^*$ ,<sup>2</sup> depending on longevity,  $T$ , the interest rate,  $r$ , the parameters  $\rho$  and  $\theta$ , describing the discount rate and the retirement age respectively, and the technological level of the economy,  $A$ :

$$g^* = A - \alpha A \left[ \frac{\rho}{1 - e^{-\rho T}} \frac{1 - e^{-(r-g)\theta}}{(r-g)\theta} \frac{(e^{(r-g-\rho)T} - 1)}{r - g - \rho} \right]. \quad (1)$$

Since the growth rate of population is set equal to zero, the growth rate of per capita output coincides with the growth rate of total output. We denote such a growth rate by  $g$  without any subscript from now on. Implicitly differentiating equation (Eq. 1) with respect to longevity yields

$$\frac{dg^*}{dT} = \frac{A \partial s(g^*, T) / \partial T}{1 - A \partial s(g^*; T) / \partial g^*}, \quad (2)$$

where  $s(\cdot, \cdot)$  denotes savings as a function of the steady state growth rate and longevity. Futagami and Nakajima (2001) show that, evaluated in equilibrium, the numerator as well as the denominator are negative. Consequently, the growth rate of aggregate output increases as longevity rises. The explanation is that an increase in  $T$  is associated with a longer retirement period, therefore people would have to save more during their working life to afford consumption, when old. Since the production function exhibits increasing returns to scale with respect to the capital stock, the increase in savings promotes output growth.

Regarding the growth-related effects of demographic change, the results carry over from the Romer (1986) model. An interesting feature of the model by Futagami and Nakajima (2001) is, however, that a rise in the retirement age would slow down economic growth. The intuition behind this result is straightforward: As the retirement period of individuals gets shorter, they will have to save less in order to afford consumption when old. This in turn decreases capital accumulation and hence economic growth. However, the result crucially hinges on the fact that the Romer (1986) model has been used as a basis for the analysis. If the Romer (1990) is used instead, this result would have changed (see below). Therefore the policy implication of reducing the retirement age to foster growth, although valid in this particular model, has to be interpreted with caution.

There also have been attempts to introduce an OLG structure according to Blanchard (1985) into the model of Romer (1986) by Alogoskoufis and van der Ploeg (1990) and Saint-Paul (1992). The presence of overlapping generations slows down economic growth because it negatively affects capital accumulation (see also Heijdra 2009, chapter 16.4.3).

<sup>2</sup> Note that in Romer (1986), on which Futagami and Nakajima (2001) is based, there are no transitional dynamics.

## 2.2 Human capital models

An alternative framework that allows for endogenous economic growth was proposed by Lucas (1988) focusing on human capital accumulation as the engine of growth. Human capital at the individual level is broadly defined as the general skill level  $h(t)$ . Total human capital  $H(t) = h(t)L(t)$ , where  $L$  indicates the size of the labor force,<sup>3</sup> is assumed to follow a linear technology that depends on the time spent in education,  $1 - u(t)$ , the current educational technology,  $E$ , and a depreciation factor,  $\delta$ :

$$\dot{H} = E(1 - u)H - \delta H. \quad (3)$$

The remaining time  $u(t)$  people spend in the production of the final output  $Y$  which is produced with capital  $K$  and effective labor  $H_e = uH$  defined as the skill weighted hours of work in production. As shown in Gruescu (2007, p. 81), assuming a CES utility function with an intertemporal elasticity of substitution equal to  $\frac{1}{\sigma}$ , a discount rate equal to  $\rho$  to describe individual consumption, a population growth rate of  $n$  and a Cobb Douglas type aggregate production function, the steady state growth rate of per capita output is given by

$$g_y^* = \frac{1}{\sigma}(E - \delta - \rho + n). \quad (4)$$

It follows that income per capita is positively associated with educational productivity  $E$ . Obviously, the lower the population growth rate the higher educational productivity needs to be in order to allow for positive per capita income growth. Increasing educational productivity may therefore be seen as a policy tool to sustain economic growth under conditions of population decline.

Gruescu (2007, p. 108ff) introduces population aging into the Lucas model via modeling the dependency ratio  $D = \frac{N-L}{L}$  where  $N$  refers to the population size and  $L$  to the size of the workforce. The steady state growth rate of per capita output changes to

$$g_y^* = \frac{1}{\sigma}(E - \delta - \rho + n - g_{(1+D)}) \quad (5)$$

where  $g_{1+D}$  denotes the growth rate of the dependency ratio. The comparison to Eq. 4 shows that an increase in the dependency ratio—as it will occur as a result of population aging—depresses economic growth. As shown in Gruescu (2007, p. 116f), when the growth rate of the dependency ratio is positive, more time is spent in education and less time in production. Intuitively, people try to counteract the negative effect of aging on economic growth by investing more in education in order to increase future consumption and income.

More detailed demographic structures have recently been introduced in models of human capital by e.g. de la Croix and Licandro (1999). Instead of assuming a continuous investment into education over the whole life, the authors more realistically assume that agents decide on the length of schooling before starting work. The steady

<sup>3</sup> In most of the models presented here the population size is assumed to be equivalent to the labor force. If this is not the case, it will be mentioned explicitly.

state growth rate is shown to first increase and then decrease with increasing life expectancy. I.e. for countries with low life expectancy the relation between economic growth and life expectancy is positive but it may turn negative for advanced countries with high life expectancy. An extension of this model is presented in Boucekkin et al. (2002) where changes in longevity and fertility are investigated.

### 2.3 Horizontal innovations

Starting with the Romer (1990) model, technological change as the main driving force behind economic growth, has been endogenized. Technological improvements are considered as increases in the overall amount of different varieties that can be produced, i.e. the amount of blueprints discovered in the R&D sector of the economy. This type of technological progress is referred to as horizontal innovation. Aggregate output is produced with labor and intermediate goods according to the production function

$$Y = L_Y^\alpha \int_0^A x_i^{1-\alpha} di, \quad (6)$$

where  $L_Y$  refers to workers employed in the final goods sector,  $x$  is the amount of a certain variety  $i \in \mathbb{R}$  used as intermediate input in final goods production,  $A$  refers to the technological frontier which now evolves endogenously and  $1 - \alpha$  is the intermediate share of output. It is easily seen that a growing technological frontier is the central driving force behind long-run economic growth. Its behavior over time is characterized by the differential equation

$$\dot{A} = \delta L_A A, \quad (7)$$

where  $\delta$  represents productivity of researchers, and  $L_A$  is the amount of workers employed in the R&D sector<sup>4</sup> which can also be referred to as scientists. This equation ensures that there are no diminishing returns to scale with respect to the production of new ideas. To put it differently, an increase in  $A$  contributes to the productivity of researchers such that the development of new blueprints does not become ever more complex. With these ingredients, Romer (1990) derives the following expression for the growth rate of an economy

$$g^* = \frac{\delta L(1 - \alpha) - \rho}{\sigma + (1 - \alpha)}, \quad (8)$$

where  $L = L_Y + L_A$  is the total labor force,  $\rho$  represents the time preference rate of individuals and  $\sigma$  denotes the inverse of the intertemporal elasticity of substitution in an individual's utility function. This equation states that the growth rate of

<sup>4</sup> We see that the growth rate of technology is constant if the amount of labor in R&D production stays constant as well.

the economy positively depends on the productivity of researchers, the patience of individuals, and the population size. In order to come up with a balanced growth path where all variables grow at a constant rate, Romer (1990) assumes that the population size  $L$  is fixed. The positive relation between the per capita growth rate of the economy and the population size is called “scale effect”, which is present in all these types of endogenous growth models. The age structure of the population is not considered in the Romer (1990) framework.

Abstracting from market structure considerations, policymakers have different opportunities to intervene because subsidies for savings or for the use of intermediate inputs have growth effects rather than level effects as in exogenous growth models. The most important opportunity to intervene in order to speed up long-run economic growth is to subsidize R&D. This would increase wages for scientists such that the corresponding sector could attract more workers and due to Eq. 7 produce more blueprints.

Futagami et al. (2002) use the Romer (1990) model as a basic framework for analyzing the growth effects of increasing longevity. In contrast to Futagami and Nakajima (2001), who use the Romer (1986) model as starting point, technological change is endogenously determined in this setup. Due to the complexity of the expression for per capita growth,  $g$ , there are no clear implications on its reaction in response to demographic change. The derivatives of  $g$  with respect to longevity,  $T$ , and the retirement age,  $\theta$ , lead to expressions with ambiguous sign. If one wants to analyze demographic aspects more thoroughly in this context, the model would have to be simplified considerably. However, two implications of Futagami et al. (2002) are worth mentioning since they contrast the results of Futagami and Nakajima (2001). First, increases in longevity are not necessarily associated with rising growth rates and second, increases in the retirement age  $\theta$  may not harm long-run economic growth perspectives. The intuition for this result is that the Romer (1990) model, on which Futagami et al. (2002) rely, emphasized the crucial role of the size of the labor force on economic growth. Since increases in longevity mainly increase the number of retirees, its effects on economic growth are therefore limited, while raising the retirement age increases the labor force and thereby countervails the negative effect of lower aggregate savings also triggered by the rise in the retirement age.

Prettner (2009) abandons the representative agent assumption of the Romer (1990) model and introduces an OLG structure according to Blanchard (1985). Each individual has to face a constant age independent risk of death which, due to the law of large numbers, corresponds to the fraction of individuals dying at each instant. The birth rate is assumed to be equal to the death rate such that population growth is ruled out in order to be consistent with the Romer (1990) approach. However, changing the mortality rate and thereby also the birth rate leads to a shift in the age structure of the population and allows to analyze the effects of population aging on economic growth. The following expression for the steady state per capita growth rate is derived

$$g^* = \frac{\lambda L \alpha - \rho - \mu \Omega \sigma}{\alpha + \sigma}, \quad (9)$$

where  $\lambda$  denotes the productivity of researchers,  $L$  is the labor force,  $\alpha$  refers to the intermediate input share in final goods production,  $\rho$  is the rate of pure time preference of individuals,  $\mu$  is the death rate which is equal to the birth rate,  $\Omega \in [0, 1]$  is a constant and  $\sigma$  is a coefficient of relative risk aversion such that the intertemporal elasticity of substitution is  $1/\sigma$ . While the main implications of the Romer (1990) model carry over to this framework, Prettner (2009) shows that a decrease in mortality, which corresponds to population aging, fosters economic growth. The reason is that individuals who live longer, are more likely to postpone consumption into the future and increase their savings. Higher savings will decrease interest rates and consequently returns to R&D investments accruing in the future are discounted less heavily. As a result, investments into R&D are more likely to pay off and an economy with an older population structure features faster per capita growth in the long-run. This approach abstracted from productivity differentials between younger and older workers as well as from considerations with respect to pension systems. Introducing these aspects could be promising for future research regarding the impact of demographic change on economic development.

## 2.4 Vertical innovations

In contrast to Romer (1990), technological progress is referred to as an increase in the quality of varieties in the Grossman and Helpman (1991) framework. It is assumed that there is a continuum of goods, whose quality can be improved infinitely via innovations by a factor of  $\lambda$ . These innovations are introduced by the firms who carry out in-house R&D. The corresponding type of technological progress is referred to as vertical innovation. Research success—which positively depends on R&D intensity—follows a Poisson Process with arrival rate  $\tilde{\tau}$ . Additionally, it is assumed that only the firm which produces the highest quality of a certain good is able to sell it.<sup>5</sup> For these incumbents it is not optimal to carry out R&D for the product line, where they are leaders, but rather to invest in innovations of another variety. The optimal research effort for each firm that is not yet the leader is indeterminate due to the constant returns to labor. Since all product lines yield the same profit, individual firms are indifferent which industry they should target with their R&D efforts. Denoting aggregated R&D efforts of all firms in one product line as  $\iota$ , the law of large numbers ensures that a fraction  $\iota$  of all products in an economy is improved at each instant. The corresponding growth rate of per capita output is  $g = \iota \log \lambda$ .

Combining the Euler equation from the consumer's optimization problem with the expression for the profit flow of firms and imposing the market clearing condition that total labor must either be employed in R&D or in production leads to the following

<sup>5</sup> To be precise: It is not only quality that matters, but the quality-adjusted price. Only the firm with highest quality of its output can charge a quality-adjusted price that allows for positive profits. If the firm with the second highest quality of its output wants to produce, it can charge a higher quality-adjusted price but then nobody would want to buy its good (Bertrand competition). It is optimal for the incumbent to charge exactly this price (limit price) because its profits would be lower if the price were also lower, while another firm would enter the market and thereby erode the profit of the incumbent, if the price were higher.



expression for optimal aggregate R&D intensity:

$$i = \frac{(1 - \frac{1}{\lambda})L}{a_I} - \frac{\rho}{\lambda}$$

where  $L$  denotes the total size of the labor force which is again equivalent to the population size,  $\rho$  is the discount rate of individuals and  $a_I$  represents a firm's labor input coefficient for creating an innovation with probability  $\tilde{i}$ . Putting things together, the steady state per capita output growth rate can be written as

$$g^* = \left[ \frac{(1 - \frac{1}{\lambda})L}{a_I} - \frac{\rho}{\lambda} \right] \log \lambda \quad (10)$$

which tells us that a decrease in the labor requirement for R&D, an increase in the size of innovations and a decrease in the discount rate of individuals boosts growth. Moreover, the population size  $L$  has a growth enhancing effect because a larger population *ceteris paribus* increases the number of scientists. Consequently, the main policy implication carries over from the [Romer \(1990\)](#) model. Subsidizing R&D raises long-run per capita output growth because the R&D sector could then attract more scientists. Population growth is not allowed because it would be inconsistent with a balanced growth path. In contrast, population aging could be considered by following a similar strategy as in [Prettner \(2009\)](#) in case of the [Romer \(1990\)](#) model.

### 3 Semi-endogenous growth models

All endogenous growth models considered so far exhibit a scale effect in the sense that economies with larger populations grow faster. Furthermore, they suggest that an increase in the number of scientists would lead to acceleration of technological progress and thus per capita output growth. [Jones \(1995\)](#) argues that these predictions are not supported by empirical evidence. He proposes a scale invariant model by relying on the [Romer \(1990\)](#) framework but changing Eq. 7 to

$$\frac{\dot{A}}{A} = \delta \frac{L^\lambda}{A^{1-\phi}}, \quad (11)$$

where  $0 < \lambda \leq 1$  accounts for the fact that different researchers may work on similar problems and therefore create the same result twice, i.e. there is a redundancy in R&D if  $\lambda < 1$ , and  $\phi$  is a parameter measuring the strengths of intertemporal knowledge spillovers. If  $\phi < 1$  these spillovers are insufficiently small to prevent innovations from becoming more and more difficult as the technological frontier evolves. With these assumptions, [Jones \(1995\)](#) is able to derive the per capita growth rate of the economy in the steady state as

$$g_y^* = \frac{\lambda n}{1 - \phi} \quad (12)$$

with  $n$  being the growth rate of the population and hence also of the labor force. Consistent with empirical findings, there are no scale effects in the long run, so the size of the labor force does not matter. However, the growth rate of the labor force  $n$ , which is required to be positive, is decisive in determining per capita growth. The faster the population grows, the higher is the long-run per capita growth rate of the economy. Since the steady state growth rate only depends on parameter values, there is no way for policymakers to increase growth in the steady state which is the reason why these models are called semi-endogenous. Several extensions of the original Jones (1995) model have been suggested in the literature, e.g. Kortum (1997) and Segerström (1999). As regards the role of population growth and policy implications, they carry over from Jones (1995).

Prettner (2009) abandons the representative agent assumption in the Jones (1995) model and introduces an OLG structure according to Buiter (1988). Each individual faces a constant age independent risk of death  $\mu$ . To be consistent with the central assumption of the Jones (1995) approach, the birth rate  $\beta$  is assumed to be larger than the death rate such that the population grows at the positive rate  $n = \beta - \mu$ . The following expression for the steady state growth rate  $g_y^*$  can be derived<sup>6</sup>

$$g_y^* = \frac{\beta - \mu}{1 - \phi}, \quad (13)$$

where  $\phi \in [0, 1]$  measures intertemporal knowledge spillovers. The central results of the Jones (1995) model carry over to this framework but now it is possible to disentangle population growth between changes in longevity and changes in fertility. While a decrease in fertility—associated with population aging—lowers population growth and thereby the steady state per capita growth rate of the economy, the converse holds true for decreasing mortality—associated with an increase in population growth. Consequently, while population aging has been beneficial for long-run economic growth in the Romer (1990) case, it hampers economic development in the Jones (1995) framework.

A model that allows for a negative correlation between population growth and economic growth is presented in Dalgaard and Kreiner (2001). This model builds on the Romer (1990) approach to set up a scale invariant growth model by endogenizing human capital accumulation. The production function

$$Y_t = \left( \frac{H_t}{A_t} \right)^\alpha \int_{j=0}^{A_t} x_{jt}^\gamma dj Z^{1-\alpha-\gamma} \quad (14)$$

is used, where  $H_t = h_t L_t$  denotes the stock of human capital, with  $h_t$  being average quality of labor and  $L_t$  referring to the size of the labor force,  $A_t$  is the technological frontier,  $x_{jt}$  represents specialized inputs,  $Z$  refers to a production factor of fixed supply, which is normalized to one,  $\alpha$  represents the human capital share of final output,

<sup>6</sup> Note that this expression does not allow for duplication in the research process as compared to the standard Jones (1995) model. However, this does not affect the central results.

$\gamma$  the intermediate input share, and  $1 - \alpha - \gamma$  the fixed input factor share. The first term on the right hand side ensures that, as technology gets more and more complex, the importance of human capital in final goods production increases.

Households have to decide how to allocate their income between consumption and investment, where the latter is divided into human capital investments, whose return are higher future wages, and investments into new ideas, whose return are interest payments on the invested capital. Additionally, it is assumed that governments can subsidize both investments. They finance these subsidies by means of lump sum taxes.

Altogether their model structure leads to laws of motion for the technological level and the average quality of labor as

$$\dot{A}_t = I_t^A, \quad (15)$$

$$\dot{h}_t = \frac{I_t^H}{L_t} - nh_t, \quad (16)$$

where  $I_t^A$  is the fraction of household's income invested in technology,  $I_t^H$  is the fraction of household's income invested in human capital and  $n$  is the population growth rate. The last term in Eq. 16 refers to congestion, meaning that it becomes more and more difficult to sustain a high average human capital level if the number of people to be educated increases. With these assumptions, [Dalggaard and Kreiner \(2001\)](#) are able to derive per capita output growth in the steady state as

$$g_y^* = \frac{1}{\epsilon} \left( (1 + \tau^H) \alpha \left( \frac{\gamma(1 - \gamma)}{\alpha} \frac{1 + \tau^A}{1 + \tau^H} \right)^{\frac{1 - \alpha - \gamma}{1 - \gamma}} \gamma^{\frac{2\gamma}{1 - \gamma}} - \theta - n \right), \quad (17)$$

where  $\tau^H$  are subsidies for human capital investments,  $\tau^A$  are subsidies for technological investments and  $\theta$  is an individuals' discount rate. Equation (17) states that per capita output growth positively depends on investment in human capital, investment in technology, and the intermediate input share, whereas it negatively depends on the individual's discount rate and, remarkably, the growth rate of the population.<sup>7</sup>

To summarize, the following conclusions can be drawn: Long-run per capita output growth is not affected by changes in the population size, but population growth has a negative impact under the utility function used in the model. In contrast to standard semi-endogenous growth models, policymakers are able to intervene via changes in subsidies for human capital as well as technological investments. Population aging is not considered in the [Dalggaard and Kreiner \(2001\)](#) model but could again be introduced as described in [Prettner \(2009\)](#).

<sup>7</sup> This result is, however, not robust against respecifications of the instantaneous utility function. See [Dalggaard and Kreiner \(2001\)](#) for details.

#### 4 Growth models with horizontal and vertical innovations

One shortcoming of semi-endogenous growth models in the spirit of Jones (1995) is that they result in long-run economic growth relying on parameter values which cannot be affected by economic policy. The model of Peretto (1998) derives a long-run growth rate that can be influenced by economic policy and gets rid of the scale effect by integrating horizontal and vertical innovations. This is done by assuming that incumbents carry out quality improving and thus productivity enhancing in-house R&D, whereas new firms, who continuously enter markets, perform horizontal innovations by introducing new products. As a consequence, per capita growth of the economy, measured by increasing consumption of an aggregate good consisting of different varieties, has three dimensions:

$$g \equiv \frac{\dot{C}}{C} = g_v + \frac{1}{\epsilon - 1} g_h + x, \quad (18)$$

where  $\epsilon$  refers to the elasticity of substitution between different product varieties. The three dimensions are: a) growth via gains in efficiency (vertical innovations) denoted as  $g_v$ , b) growth via increases in the number of varieties, i.e. increases in the number of firms that enter the market (horizontal innovation) denoted as  $g_h$  and c) growth via increases in production denoted as  $x$ .

While a scale effect is present in quality improving R&D, there is no effect of a larger population size on the long-run rate of entry. This is the case because Peretto (1998) assumes that knowledge spillovers in horizontal innovation are insufficiently low to generate long-run growth, hence only changes in the population growth rate are able to effect steady state entry. However, the scale effect present in quality improving R&D vanishes in the long-run steady state equilibrium because an increase in the population size not only spurs vertical innovations but also horizontal innovations (firm entry) and therefore in-house R&D resources have to be divided among more firms. This eventually leads to a rate of quality improvements that is equivalent to the original rate before the increase in the population size.

In the steady state, where production and the number of varieties grow at the same rate  $g_h = x = n$  with  $n$  denoting population growth, this expression reduces to

$$g^* = \frac{\epsilon}{\epsilon - 1} n + g_v. \quad (19)$$

Finally, after solving for the general equilibrium, Peretto (1998) arrives at the following expression

$$g^* = \frac{\epsilon n}{\epsilon - 1} + \frac{\theta \rho [\alpha \theta (\epsilon - 1) - \beta]}{\beta [1 - \theta (\epsilon - 1)]}, \quad (20)$$

where  $\theta$  is the elasticity of a unit cost reduction of a specific variety with respect to quality improvements,  $\rho$  is the discount rate of individuals and  $\beta$  refers to the productivity of labor in horizontal innovation. By inspection of Eq. 20, it is clear that there are no scale effects, but that population growth positively affects steady state

per capita growth like in semi-endogenous growth models. However, an increase in population growth has ambiguous effects on per capita growth of the economy in the medium-run because on the one hand, it increases entry (horizontal innovation) and output growth but on the other hand the increase in the number of firms slows down productivity growth (vertical innovations). An increase in the population size leads to temporarily faster per capita growth but eventually the economy converges back to the original steady state growth rate. Population aging is not analyzed in this model but could again be introduced as described in [Prettner \(2009\)](#).

[Peretto \(1998\)](#) also analyzes the effects of governmental subsidies on medium- and long-run economic growth perspectives. The central result is that subsidies for vertical R&D increase long-run growth, while subsidies for horizontal R&D decrease long-run growth. This is due to the entry of new firms that shifts resources away from quality improving R&D with higher intertemporal knowledge spillovers to the development of new products with lower intertemporal knowledge spillovers. Several variations and extensions of the [Peretto \(1998\)](#) model exist in the literature (e.g. [Young 1998](#); [Dinopoulos and Thompson 1998](#); [Howitt 1999](#)).

An interesting paper that takes into account the endogenous demographic structure in a model of horizontal and vertical R&D is [Connolly and Peretto \(2003\)](#). Fertility is endogenized as in the [Becker and Barro \(1988\)](#) model allowing for a more realistic demographic structure and thereby making policy experiments with respect to changes in reproduction costs and exogenously given mortality rates possible. [Connolly and Peretto \(2003\)](#) derive the steady state expression for per capita growth as:<sup>8</sup>

$$g = (\chi - 1)n + \theta z, \quad (21)$$

where  $\chi$  denotes the research spillovers from horizontal innovations,  $n$  denotes population growth which pins down firm entry in the long-run and  $\theta$  is the elasticity of production with respect to quality improvement, whose rate is denoted by  $z$ .

Obviously, scale effects do not occur in this case. However, as long as research spillovers from horizontal innovations are larger than one, population growth positively affects per capita output growth. [Connolly and Peretto \(2003\)](#) restrict their attention to this case.

With respect to innovations, there are three possible policies to be implemented, whose effects are investigated numerically by [Connolly and Peretto \(2003\)](#). First, a subsidy for vertical innovations positively affects medium- as well as long-run per capita output growth. Second, a subsidy for horizontal innovations positively affects medium-run but negatively affects long-run per capita output growth, and third, symmetric positive subsidies for both types of research positively affect medium-run per capita output growth, whereas long-run per capita output growth remains unchanged.

A further interesting paper that relates to demographic change via endogenous human capital accumulation is [Strulik \(2005\)](#). He builds on the models with horizontal and vertical innovations and implements endogenous educational decisions of households. Aggregate human capital is obtained by multiplying the population size

<sup>8</sup> In doing so they restrict their parameter values such that both types of innovations have to occur in equilibrium. For details see [Connolly and Peretto \(2003\)](#).

with the per capita human capital created by education. It is assumed that newborns enter the economy without education and therefore faster population growth hampers aggregate human capital growth. Eventually, the growth rate of per capita output in the steady state,  $g_y^*$ , can be written as

$$g_y^* = \frac{\xi - \delta - \rho}{\theta - 1} \left( 1 - \frac{1}{\theta + \phi} \right) + \frac{1}{\theta - 1} \left[ m - \frac{\theta - (1 - m)}{\theta + \phi} \right] n, \quad (22)$$

where  $\xi$  is the productivity of education in creating human capital,  $\delta$  is the rate of knowledge depreciation,  $\rho$  is the discount rate of individuals,  $\theta$  measures relative risk aversion in the CRRA instantaneous utility function and determines the intertemporal elasticity of substitution as  $\frac{1}{\theta}$ ,  $\phi$  is a collection of parameters which can be shown to exceed 1 and negatively depends on competition captured by the elasticity of substitution between intermediate input varieties,  $\sigma$ ,  $m$  governs altruism, i.e. for  $m = 1$  the utility function is Benthamite and households maximize utility of all members of the dynasty, whereas for  $m = 0$  the utility function is Millian and households just maximize their own per capita utility and finally,  $n$  denotes population growth. It is immediately clear that the steady state per capita growth rate of the economy decreases in impatience of households and depreciation of human capital and it increases in productivity of education. While the population size does not matter for the steady state growth rate, the effects of population growth are ambiguous, except for two knife-edge parameter assumptions. If  $m = 0$  (Millian preferences) the only effect of population growth on economic growth is the human capital dilution effect which occurs because individuals enter the economy uneducated and therefore population growth reduces per capita human capital. In this case population growth negatively impacts upon per capita output growth in the steady state. In contrast, if  $m = 1$  (Benthamite preferences), there is the additional effect that population growth reduces the effective rate of time preference. In this case population growth positively impacts upon per capita output growth in the steady state.

Altogether the standard way in which policymakers can influence economic growth, namely through R&D subsidies, is not applicable here. However, increases in efficiency of education as well as more competition would be able to spur economic development.

## 5 Conclusions

Low birth rates and increasing survival to higher ages will shape the future demographic structure in most industrialized countries. Whether—through which channels and in which extent—those demographic developments will impinge on economic growth is not yet clear. One approach we followed in this paper is to rely on formal economic growth models and study the relationship between demographic developments and economic growth prospects. More specifically, the aim of our paper was to review selected endogenous economic growth models and illustrate their predictions on the interrelationship between demographic development and economic growth. In particular, we reviewed extensions of those models that explicitly allow for

population aging if such frameworks were available. In addition, we addressed the policy implications of the models under considerations.

We may draw several conclusions from our review. (a) While there have recently been attempts to include a more realistic age structure and more realistic models of survival into exogenous growth models, there is still scope for further research on this topic in endogenous growth models. (b) Whether population growth or population size foster or hamper economic growth strongly depends on the modeling framework. Endogenous growth models of the first generation (Sect. 2) yield a positive association between population size and economic growth. In semi-endogenous growth models (Sect. 3) population growth, instead of population size, positively influences economic growth. In our view, most promising are recent semi-endogenous growth models (see [Dalgaard and Kreiner 2001](#); [Strulik 2005](#)) that allow for a negative association of population growth and economic growth. (c) With respect to models that explicitly account for population aging we again find dependence of the results on the underlying assumptions. Population aging can have positive impacts on economic growth if it triggers additional savings or investments into R&D while leaving the size of the workforce unchanged (see [Futagami and Nakajima 2001](#); [Prettner 2009](#)), it can also have negative impacts if pension schemes are designed such that the size of the workforce decreases relative to the amount of retirees (see [Futagami et al. 2002](#); [Gruescu 2007](#)) or if population aging is driven by declines in fertility that also slow down population growth (see [Prettner 2009](#)).

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