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Shear modulus of several kinds of Japanese bamboo obtained by flexural vibration test

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Abstract The vibrational properties of Japanese bamboo were examined. To obtain the Young's modulus and shear modulus, a flexural vibration test and a longitudinal vibration test were conducted. The Young's modulus with vibration in the R-direction was smaller than that measured in the longitudinal vibration test E_l. This was due to the shift of the neutral axis to the outer layer. On the other hand, the Young's modulus with vibration in the T-direction was close to E_l. Hence, an adequate Young's modulus should be used for each use of bamboo. The shear moduli of the LR and LT planes of bamboo were similar to those of beech. There were high correlations between shear moduli of the LR and LT planes and density.

Key words Bamboo · Shear modulus · Variation in R-direction · Vibration test · Young's modulus

Introduction

In recent years, demand for Japanese bamboo has decreased because of increased imports of bamboo, bamboo charcoal, and bamboo products and the ready availability of substitutes for bamboo. Furthermore, production of Japanese bamboo has been decreasing due to the aging of fellers and the difficulty of finding successors. As a result, bamboo is being underutilized nationwide and it is invading into forests and around private houses.[1] Various uses for bamboo such as bamboo charcoal, bamboo vinegar, structural composite panels, and flooring made of bamboo are being developed.

When building a structure made of bamboo, a shear modulus is indispensable for simulating strength properties with, for example, the finite element method. However, few data have been obtained on the shear modulus, although Young's modulus, bending strength, and shear strength have been measured.[2] Therefore, we measured the shear modulus of several kinds of Japanese bamboo with the flexural vibration test.

Density and Young's modulus increase from the inner layer to the outer layer in bamboo. The results of the flexural vibration test can be affected by this layer structure like the glued laminated timber made of different wood species: when the load is applied in the direction perpendicular to adhesive layers, the Young's modulus is larger than the average of each lamina because lamina with smaller and larger Young's moduli are used for the inner and outer layers, respectively. Hence, the effect of the layer structure particular to bamboo on the measurements of the flexural vibration test was also investigated.

Theory

Position of neutral axis

Here, the simple three-layer model made of rectangular bars for bamboo shown in Fig. 1 is considered. The Young's modulus in the inner layer, the middle layer, and the outer layer are $E_0 = E_1/k$ $(k > 1)$, E_1, $E_2 = kE_1$, respectively. The thickness and width of each layer are h and b, respectively.

Assuming that the neutral axis is at $y = \lambda$, from the balance of stress in a cross section,

$$E_0\int_{-\frac{3}{2}h-\lambda}^{-\frac{1}{2}h-\lambda} ydA + E_1\int_{-\frac{1}{2}h-\lambda}^{\frac{1}{2}h-\lambda} ydA + E_2\int_{\frac{1}{2}h-\lambda}^{\frac{3}{2}h-\lambda} ydA = -bh(h+\lambda)E_0 - bh\lambda E_1 + bh(h-\lambda)E_2 = 0, \quad (1)$$

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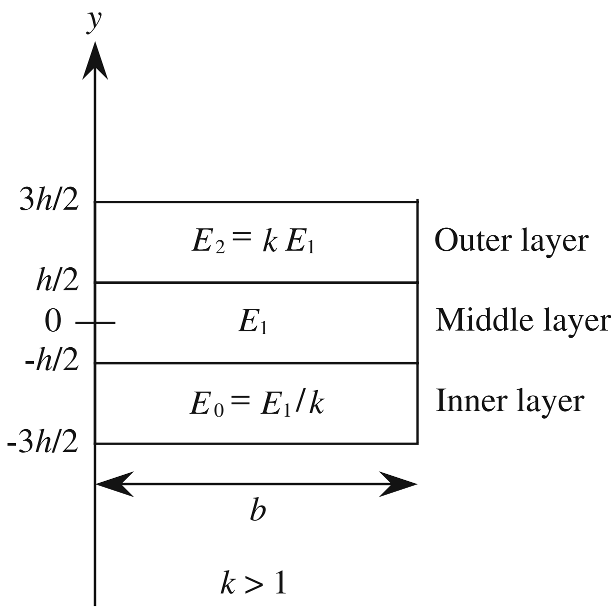

Fig. 1. Simple three-layer model for bamboo

where A is the cross section area.

$$\therefore \lambda = \frac{E_2 - E_0}{E_0 + E_1 + E_2} h = \frac{k - \frac{1}{k}}{k + \frac{1}{k} + 1} h. \tag{2}$$

Comparison of Young's moduli measured in the flexural vibration test and longitudinal vibration test

From the bending theory of glued laminated timber,

$$EI = \sum_{\mathrm{i}=1}^{3} E_{\mathrm{i}} I_{\mathrm{i}}$$

where I is the moment of inertia of the cross section.

$$\begin{aligned}
\therefore E\int_{-\frac{3}{2}h-\lambda}^{\frac{3}{2}h-\lambda} y^2 dA &= E_0\int_{-\frac{3}{2}h-\lambda}^{-\frac{1}{2}h-\lambda} y^2 dA + E_1\int_{-\frac{1}{2}h-\lambda}^{\frac{1}{2}h-\lambda} y^2 dA \\
&\quad + E_2\int_{\frac{1}{2}h-\lambda}^{\frac{3}{2}h-\lambda} y^2 dA \\
&= \frac{bh^3}{12}(13E_0 + E_1 + 13E_2) + 2bh^2\lambda(E_0 - E_2) \\
&\quad + bh\lambda^2(E_0 + E_1 + E_2) \\
&= \frac{bh^3}{12}\left(13k + \frac{13}{k} + 1\right)E_1 + 2bh^2\lambda\left(\frac{1}{k} - k\right)E_1 \\
&\quad + bh\lambda^2\left(k + \frac{1}{k} + 1\right)E_1.
\end{aligned} \tag{3}$$

$$\begin{aligned}
\therefore E &= \frac{\frac{h^2}{12}\left(13k + \frac{13}{k} + 1\right) + 2h\lambda\left(\frac{1}{k} - k\right) + \lambda^2\left(k + \frac{1}{k} + 1\right)}{\frac{9}{4}h^2 + 3\lambda^2} E_1 \\
&= \frac{(K+7)^2(K+1)}{9(7K+13)(K-1)} E_1 \left(K = k + \frac{1}{k} > 2 \text{ when } k > 1\right).
\end{aligned} \tag{4}$$

The Young's modulus from the longitudinal vibration test E_l is the average of the Young's modulus of each layer. Hence,

$$\begin{aligned}
E - E_l &= E - \frac{E_0 + E_1 + E_2}{3} \\
&= -\frac{4(5K+11)(K+1)(K-2)}{9(7K+13)(K-1)} E_1 < 0\ (K > 2).
\end{aligned} \tag{5}$$

Because the position of the neutral axis cannot be known in the tests, $\lambda = 0$ is used in analyzing the data of flexural vibration tests. When the value of λ in the left side of Eq. 3 is 0,

$$\begin{aligned}
E &= \frac{\frac{h^2}{12}\left(13k + \frac{13}{k} + 1\right) + 2h\lambda\left(\frac{1}{k} - k\right) + \lambda^2\left(k + \frac{1}{k} + 1\right)}{\frac{9}{4}h^2} E_1 \\
&= \frac{(K+7)^2}{27(K+1)} E_1.
\end{aligned} \tag{6}$$

$$\therefore E - E_l = -\frac{4(2K+5)(K-2)}{27(K+1)} E_1 < 0\ (K > 2). \tag{7}$$

Shear stress distribution in the cross section

Shear stress in the cross section of a beam is expressed by the following equations.

$$\tau = \frac{F}{b}\frac{\sum E_{\mathrm{i}}\int y_{\mathrm{i}} dA}{\sum_{\mathrm{m}=1}^{3} E_{\mathrm{m}} I_{\mathrm{m}}}, \tag{8}$$

where F is the shear stress of the length–thickness plane.

i) $-\frac{3}{2}h \le y \le -\frac{1}{2}h$

$$\begin{aligned}
\sum E_{\mathrm{i}}\int y_{\mathrm{i}} dA &= E_0\int_{-\frac{3}{2}h-\lambda}^{y-\lambda} y dA \\
&= \frac{1}{2k} bE_1\left(y + \frac{3}{2}h\right)\left(y - \frac{3}{2}h - 2\lambda\right).
\end{aligned} \tag{9}$$

ii) $-\frac{1}{2}h \le y \le \frac{1}{2}h$

$$\begin{aligned}
\sum E_{\mathrm{i}}\int y_{\mathrm{i}} dA &= E_0\int_{-\frac{3}{2}h-\lambda}^{-\frac{1}{2}h-\lambda} y dA + E_1\int_{-\frac{1}{2}h-\lambda}^{y-\lambda} y dA \\
&= -\frac{bh}{k}(h+\lambda)E_1 + \frac{1}{2}b\left(y + \frac{1}{2}h\right)\left(y - \frac{1}{2}h - 2\lambda\right)E_1.
\end{aligned} \tag{10}$$

iii) $\frac{1}{2}h \le y \le \frac{3}{2}h$

$$\begin{aligned}
\sum E_{\mathrm{i}}\int y_{\mathrm{i}} dA &= E_0\int_{-\frac{3}{2}h-\lambda}^{-\frac{1}{2}h-\lambda} y dA + E_1\int_{-\frac{1}{2}h-\lambda}^{\frac{1}{2}h-\lambda} y dA + E_2\int_{\frac{1}{2}h-\lambda}^{y-\lambda} y dA \\
&= -\frac{bh}{k}(h+\lambda)E_1 - bh\lambda E_1 + \frac{1}{2}bk\left(y - \frac{1}{2}h\right)\left(y + \frac{1}{2}h - 2\lambda\right)E_1.
\end{aligned} \tag{11}$$

Materials and methods

Specimens

Mosochiku (*Phyllostachys pubescens* Masel, 4 years old) and madake (*Phyllostachys reticulate* C. Koch, 4 years old) from Gifu Prefecture and hachiku (*Phyllostachys nigra* Munro var. *Henonis* Stapt, 3 years old) from Shizuoka Prefecture were used as specimens. The three kinds of bamboo were cut from October to January when bamboo does not contain a lot of sugar. The specimens were conditioned and tested at 20°C and 65% relative humidity. Specimens with dimensions of 5–10 mm (R) (thickness of each specimen) ×5, 10, 15, 20, 25, and 30 mm (T) ×200, 250, or 300 mm (L) (decided by internode length) were made. For hachiku, the dimension in the T-direction was 15 mm.

Flexural vibration test

Free–free flexural vibration tests were conducted to measure the Young's modulus and shear modulus of the LR and LT planes. The test beam was suspended by two threads at the nodal positions of the free–free vibration corresponding to its resonance mode. The vibrations in the R-direction and T-direction were excited by impacting the LT and LR planes of a specimen at one end with a small wooden hammer, respectively. Specimens did not fall down in vibrating in the T-direction. Motion of the beam was detected by a microphone at the other end. The signal was processed through a fast Fourier transform digital signal analyzer to yield high-resolution resonance frequencies. The Young's modulus and shear modulus were calculated with the Goens-Hearmon regression method[3,4] based on Timoshenko's bending theory[5] (called TGH method in this study). By this method, the Young's modulus E_{TGH} and shear modulus G can be obtained at the same time.

Longitudinal vibration tests

Longitudinal vibration tests were also conducted to measure the Young's modulus without the effect of shear deflection. Theoretically, the Young's modulus by this method is equal to that derived by the TGH method for vibration in the T-direction and the average of the inner, middle, and outer layers of bamboo.

Longitudinal vibration was excited by impacting one end of a test beam in the L-direction with a small wooden hammer. Motion of the beam was detected by a microphone at the other end. The resonance frequencies were measured as described above.

Young's modulus E_l is calculated by the following equation.

$$E_l = 4 f_1^2 l^2 \rho, \tag{12}$$

where f_1, l, and ρ are the resonance frequency of the first mode, length, and density of a specimen, respectively.

Results and discussion

Figure 2 shows the data on the shape of the bamboo specimens used for this study. Each point in Figs. 2 and 5–8 is the average of three specimens made from the same internode.

Validity of approximating the cross section of a specimen as a rectangle

The cross section of a specimen is shown in Fig. 3. The dimension in the R-direction is constant a. When a specimen was vibrated in the T-direction, the moment of inertia of the cross section was $ba^3/12$, but this value was very complicated when it was vibrated in the R-direction. Hence, the cross section of the specimen was regarded as a rectangle.

Figure 4 shows the effect of the dimension in the T-direction on the Young's modulus and the shear modulus. Specimens were cut from the same internode. The cross section with the larger dimension in the T-direction becomes different from a rectangle in appearance. Coefficients of variation are listed in Table 1. These values were so small that the specimens could be considered as rectangular bars.

Flexural Young's modulus of bamboo

Figure 5 shows the ratio of Young's modulus by TGH method to that by longitudinal vibration test at various heights. As expected from Eq. 7, the Young's modulus in vibrating in the R-direction was smaller than E_l. On the other hand, the Young's modulus in vibration in the T-direction was close to E_l. These results mean that an adequate Young's modulus should be used for each use of bamboo because the direction of load on the bamboo may differ for each purpose.

Shear modulus

In the TGH method, a shear stress distribution factor s is needed. The Young's modulus E_{TGH} and shear modulus G are calculated as $E_{\mathrm{TGH}} = q$ and $G = sE_{\mathrm{TGH}}/p$, where p and q are the slope and intercept in the Goens-Hearmon liner regression, respectively.[4] Hence, G is affected by s. In some cases, the value of s for a specimen with a heterogeneous cross section is different from that (1.18; from Nakao et al.[6]) for a specimen with a homogenous cross section.[7,8] Although the cross section is homogeneous in the T-direction, it is

Table 1. Coefficients of variation of vibrational properties of specimens with various dimensions in the T-direction

Bamboo	Coefficient of variation (%)				
	E_{TGH}/ρ (R)	E_{TGH}/ρ (T)	E_l/ρ	G_{LR}/ρ	G_{LT}/ρ
Mosochiku	2.7	2.1	1.2	4.3	4.7
Madake	5.8	2.2	2.6	7.6	2.4

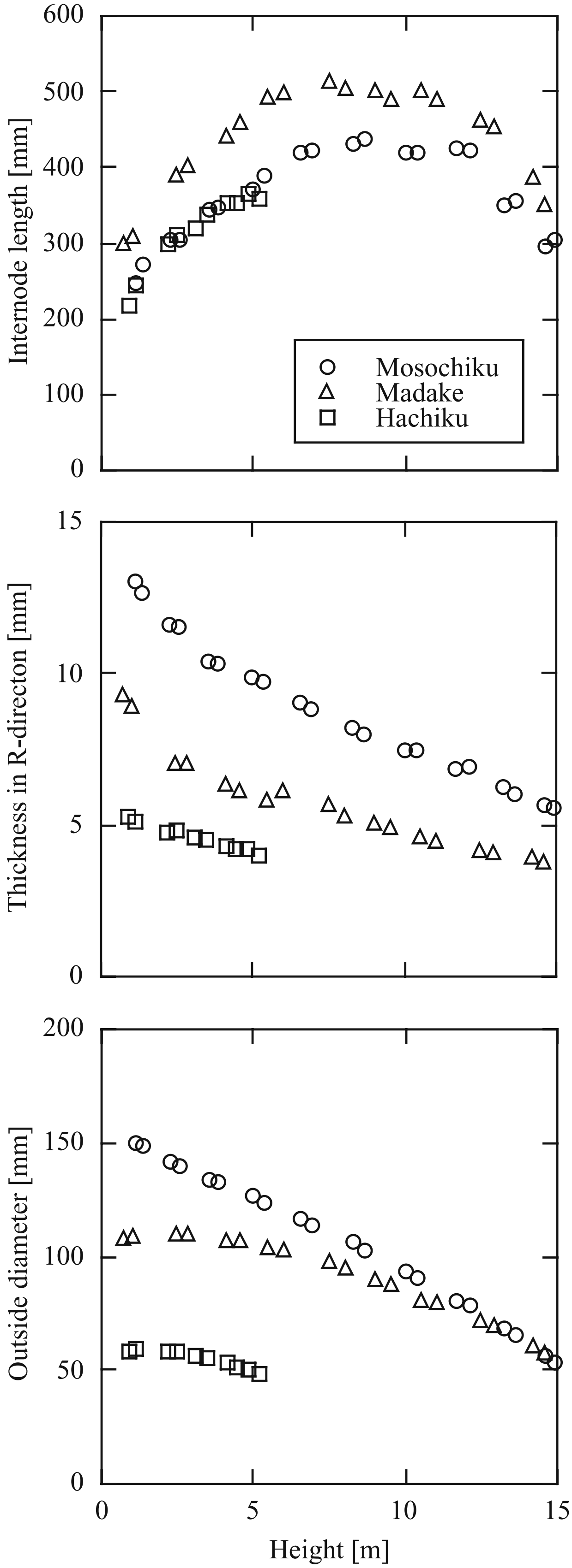

Fig. 2. Collected data on the shape of bamboo

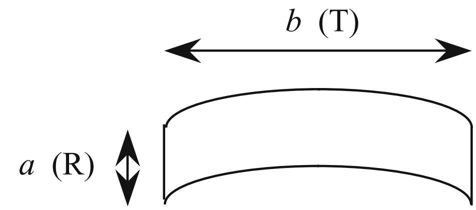

Fig. 3. Shape of the cross section of a bamboo specimen

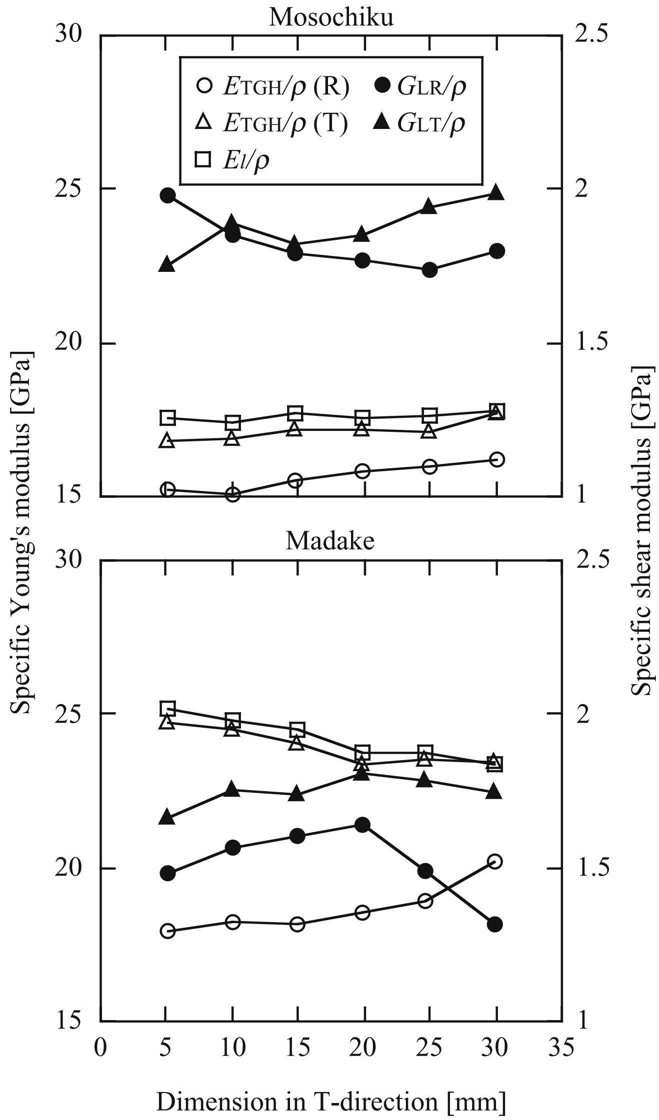

Fig. 4. Effect of dimension in T-direction on vibrational properties. E_{TGH} (R) and E_{TGH} (T) are the Young's moduli vibrating in the R-direction and T-direction, respectively. E_l is the Young's modulus by longitudinal vibration test. G_{LR} and G_{LT} are the shear moduli of the LR and LT planes, respectively. ρ is specific gravity

not homogeneous in the R-direction. Thus, when the shear modulus of the LR plane is calculated, s cannot be 1.18.

The shear stress distribution in the cross section is shown in Fig. 6. The value of $k = 1.6$ ($\lambda = 0.30h$) was derived based on the experimental Young's modulus of the inner, middle,

Table 2. Results of the vibration tests

Bamboo	Density (g/cm^3)	E_{TGH} (R) (GPa)	E_{TGH} (T) (GPa)	E_l (GPa)	G_{LR} (GPa)	G_{LT} (GPa)
Mosochiku	0.872 (0.052)	13.27 (1.21)	15.03 (1.50)	15.25 (1.37)	1.62 (0.15)	1.63 (0.14)
Madake	0.824 (0.031)	15.33 (0.95)	19.99 (0.87)	20.27 (0.79)	1.30 (0.12)	1.47 (0.08)
Hachiku	0.824 (0.022)	16.77 (0.83)	19.10 (0.26)	19.34 (0.35)	1.31 (0.08)	1.65 (0.11)

Figures in parentheses are standard deviations

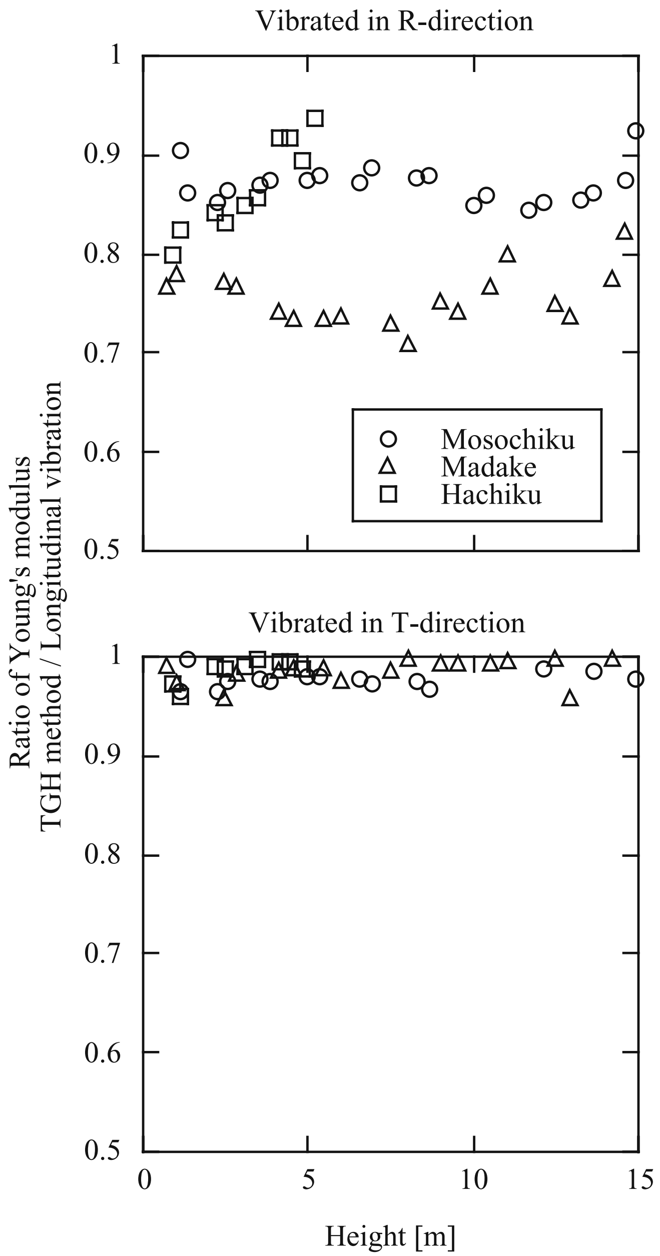

Fig. 5. Ratio of Young's modulus by TGH method (Goens-Hearmon regression method based on Timoshenko's bending theory) to that by longitudinal vibration test

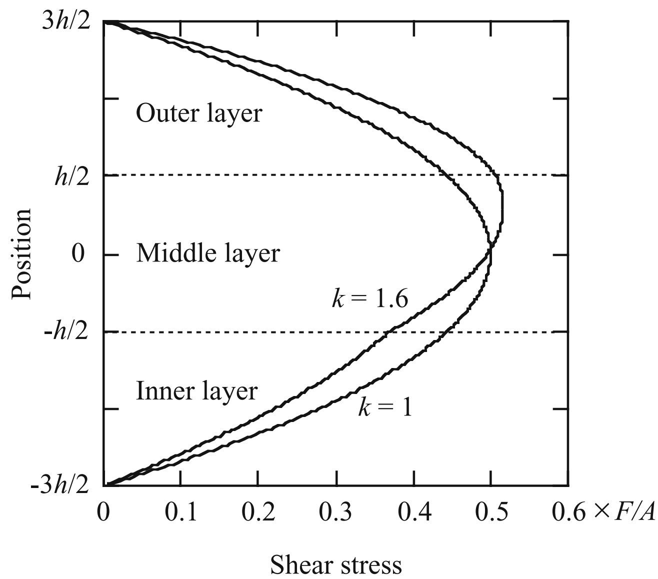

Fig. 6. Shear stress distribution in a cross section. F is in Eq. 8. A is the cross section area

and outer layers of mosochiku and madake.[9,10] Comparing the case of $k = 1.6$ with $k = 1$, the shear stress distributions are similar to each other and the difference in the maximum shear stress is only 4%. Therefore, $s = 1.18$ was used with the TGH method in this study.

Figure 7 shows the shear modulus at various heights. Shear moduli on the LR and LT planes of mosochiku increased with height. The shear modulus on the LT plane of madake increased slightly with height. The shear modulus on the LR plane of madake and shear moduli on the LR and LT planes of hachiku did not change significantly. Roughly speaking, these tendencies corresponded to the height-related density changes as shown in Fig. 8. According to Inokuchi et al.,[9,10] density is affected by the volume fraction of a bundle sheath. A high correlation between shear modulus and density is shown in Fig. 9.

Because the elastic moduli of wood relate to fine structure such as the degree of crystallinity and fibril angle of cellulose,[11–13] a similar relationship may exist in bamboo. When the bamboo sheath is stripped from the culm, the internodal growth is related to the crystallinity.[14,15] The fibril angle of mosochiku decreased remarkably from the first joint to the 9th and remained nearly constant from the 9th to the 30th, and then increased slightly in the 50th joint.[16] The specimens in this study were made from mature bamboo and did not contain extremely low and high positions. The relationship between the elastic moduli and fine structure of bamboo is a problem that needs to be studied in detail.

The results of the vibration tests are shown in Table 2. The shear moduli of bamboo were similar to beech.[17]

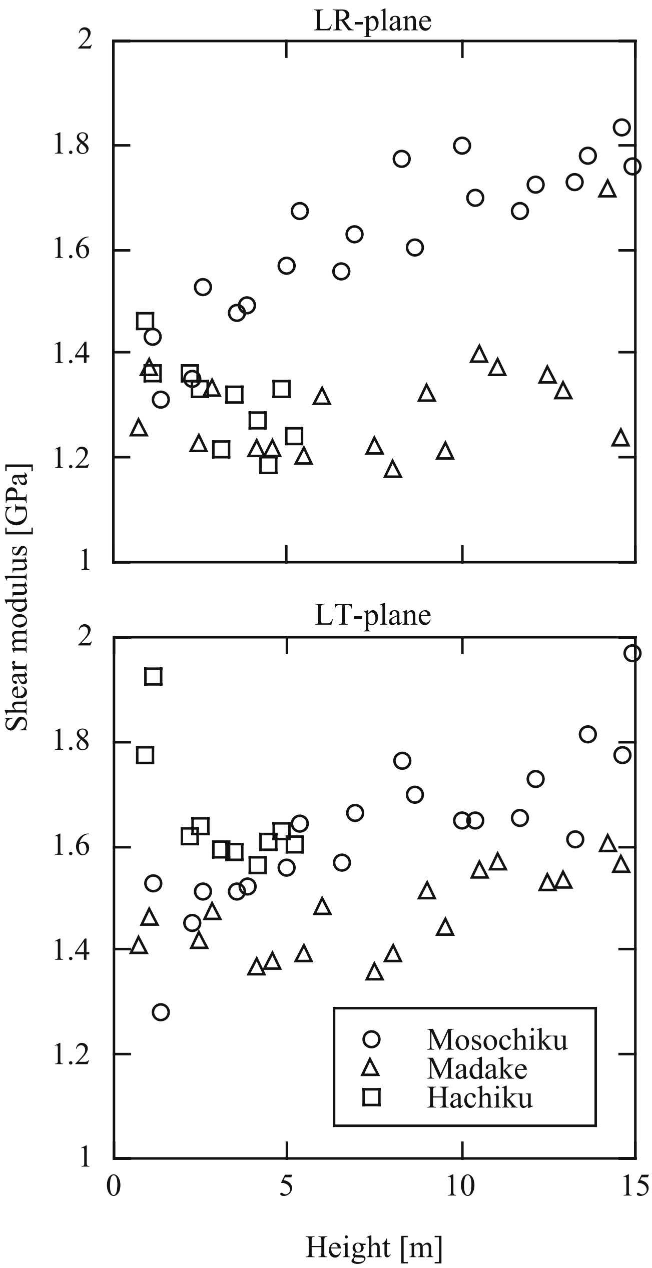

Fig. 7. Shear modulus at various heights

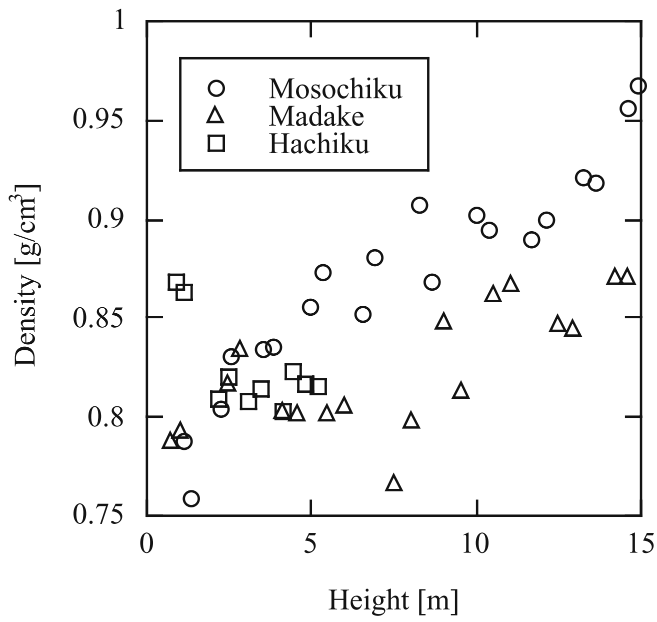

Fig. 8. Density at various heights

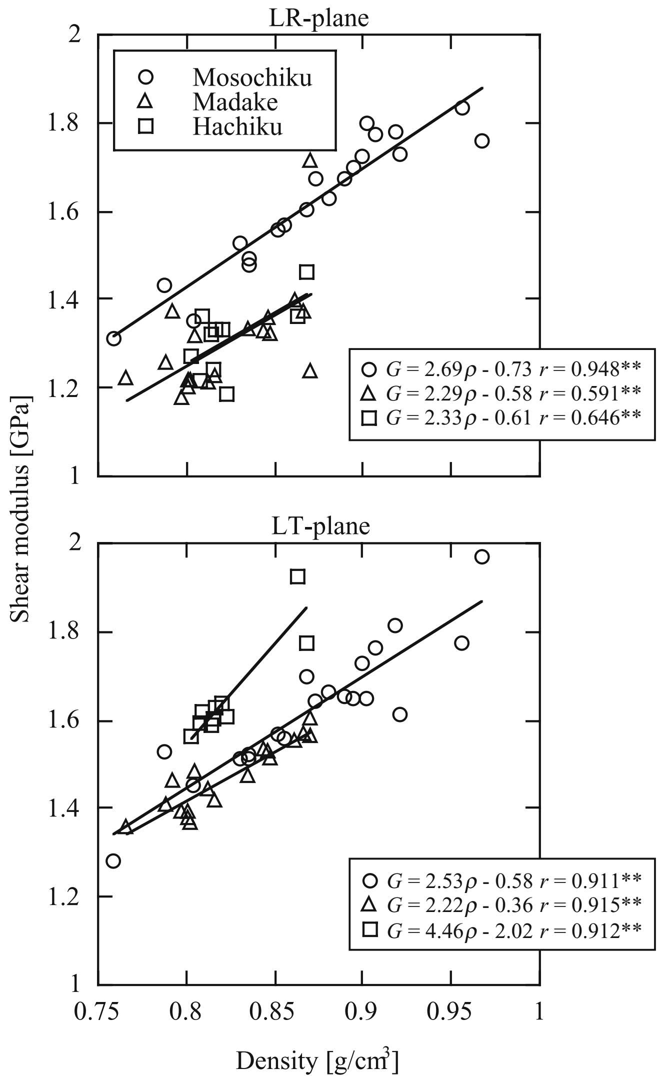

Fig. 9. Relationship between shear modulus and density. G, ρ, and r are shear modulus, density, and the coefficient of correlation, respectively. *Double asterisk*, significant at the 1% level

Conclusions

Vibration properties of bamboo were examined and the following results were obtained:

1. The Young's modulus for vibration in the R-direction was smaller than that measured with longitudinal vibration test E_l. This was because of the shift of the neutral axis to the outer layer. On the other hand, the Young's modulus for vibration in the T-direction was close to E_l.
2. Shear moduli on the LR and LT planes were obtained. They were similar to those of beech.

3. There were high correlations between shear moduli on the LR and LT planes and density.

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