



# Study on elevation effect of blast wave propagation in high side wall of deep underground powerhouse

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## Abstract

In the excavation for underground powerhouse, the blasting vibration leads to dynamic disturbance for construction safety and stability, which means such vibration should be monitored and mitigated. In this paper, the Baihetan Hydropower Station is involved, and the blasting vibration is monitored along either the vertical or the horizontal direction. The monitoring results of the distribution of blasting vibration on the high sidewall shows the vibration velocity at some positions far away from the detonation source is greater than that near the source. Such abnormal elevation amplification phenomenon is different from the distribution of blasting vibration on the slope of open-pit mine. To address this phenomenon, the dynamic finite element simulation is conducted. The comparison between the simulation results and monitoring data validates the proposed solution. Furthermore, the propagation law of blasting vibration velocity of high side wall is proposed, and the local elevation amplification effect of blasting vibration velocity is clarified. Finally, the modified Sadovskii formula is presented, and the reliability and accuracy of the modified formula for the vibration velocity of particles in the direction of elevation are verified and improved through data comparison.

**Keywords** Underground powerhouse · High side wall · Blasting vibration · Elevation effect

## Introduction

In recent years, China has built and developed a number of giant hydropower projects with installed capacity of more than 3,000,000 kW in the Jinsha River, Yalong River, and Dadu River Basin in Western China. Many large or super large underground caverns are used as main hydraulic structures in these hydro power stations (Miller and Gill 1989), and the scale is also rare in the world. The increasing demand for installed capacity raises the space of underground caverns, which means larger span, higher side wall, and more complex geo-structure

around the excavation. However, the rock mechanics problems due to the significant geological features of buried depth have further increased the difficulty of engineering construction (Lu et al. 2011; Shirzadegan et al. 2016; Yan et al. 2015).

In the process of blasting excavation in the underground powerhouse of a hydropower station, after the explosives detonated in boreholes, a part of the energy will be transformed into surrounding rock mass in the form of blasting vibration which propagates in the rock (Lu and Hustrulid 2003; Wu et al. 2004). When blasting vibration reaches a sufficient intensity, all kinds of damage phenomena will be generated (Gad et al. 2005; Khandelwal and Saadat 2015; Yang et al. 2013). The analysis of blasting vibration intensity, magnitude of seismic force and its destructiveness to the engineering structure is one of the most critical issues in the study of blasting seismic effects of engineering structures (Lu et al. 2012b; Newmark and Hall 1982). In the area with elevation changes, the study of elevation amplification effect is a very important research content.

Many scholars have studied the elevation effect for rock slopes. It is found that there is a certain degree of amplification in the propagation of blast wave in elevation direction, which is usually called the elevation amplification effect (Graizer

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2009; Marrara and Suhadolc 1998; Song et al. 2000). Havenith et al. 2003 used the 2D and 3D finite element method to analyze the rock slide slopes and found that in the low-velocity zone of the rock surface, the amplification of the vibration wave has a great relationship with the localization of the strain. Jiang et al. 2014 used numerical model calculations to show that the blasting vibration velocity is dominated by the attenuation trend with the increase of horizontal distance and elevation difference for the same slope monitoring point. At the same level, the blasting vibration velocity mainly decreases with the increase of slope gradient, but there is a phenomenon that the elevation magnification effect is dominant. It can be seen that the research object of blasting vibration elevation effect is mainly concentrated in slopes (Song et al. 2009), but there is little research on the elevation effects of the high side walls of deep underground powerhouses.

Previous studies have pointed out that the main factor affecting the elevation amplification effect of open pit slope is the shape characteristics. Obviously, there is a great difference between the constraint conditions at the end of the open pit slopes and the side wall of the underground caverns. For the open pit slopes, the constraint model is usually simplified as a cantilever beam, but for the side wall of the underground caverns, the upper and lower ends of the surrounding rock are subjected to double constraints of roofs and floors. Therefore, the internal mechanism and distribution characteristics of elevation amplification effect of the side wall of underground cavern will be significantly different from that of open pit slope. Through the modification of Sadovskii equation, many researchers have derived some formulas that are applicable to reflect the elevation amplification effect of open pit slope, but formulas for the high side wall of underground cavern are not available.

This paper starts with the interesting phenomena found in the blasting vibration test of the underground powerhouse of Baihetan hydropower station in China. According to the basic mechanical properties of rock mass, combined with the actual blasting parameters and the monitoring data of blasting vibration in the side wall elevation direction, a 3D model (Gaspari et al. 2010) is established to simulate and calculate the blasting vibration velocity of rock mass in elevation direction. By comparing, refining, and expanding the vibration velocity of the actual measuring point, the propagation curve of the blast wave in the elevation direction of the high side wall of the deep burial underground powerhouse is obtained. Then, modified Sadovskii empirical formula considering the elevation factor was fitted by regression analysis of a large number of measured data. It provides theoretical support for the prediction of blasting vibration velocity of high side walls of underground powerhouses in elevation direction, and as well as giving suggestions for blasting construction and blasting vibration control of underground powerhouses.

## Field monitoring of blasting vibration in the Baihetan Hydropower Station

### Engineering background

The Baihetan Hydropower Station is under construction and the normal water storage level is 825 m after completion. The storage capacity is  $2.06 \times 10^9 \text{ m}^3$  and the initial installed power-generating capacity is 16,000,000 kW. The generating set of the hydropower station is set in the underground powerhouse, with eight sets of 1,000,000 kW hydroelectric generating sets on both sides. It will be the second largest hydropower station in China, after the Three Gorges hydropower station, and it will also be the largest underground hydropower project in the world. The size of its main power house is  $439.0 \text{ m} \times 32.2 \text{ m} \times 78.5 \text{ m}$  (length  $\times$  width  $\times$  height) which is the largest underground powerhouse in the world. The maximum span of the top arch excavation is 15 m, the excavation height is about 84.1 m, and the buried depths of the powerhouse on the left and right bank are 350 m and 550 m, respectively. The excavation volume of the deep rock mass in the underground cavern group exceeds  $25,000,000 \text{ m}^3$ , the total length of the underground cavern group is 217 km. The underground powerhouse project has a large span, a large depth of burial, and a complex geological structure. The difficulty of its excavation is the highest among all hydropower projects in the world (Li et al. 2019). Figure 1 shows the view and location of Baihetan Hydropower Station.

The intact rocks with neither large-scales faults nor joints, which are categorized into level III, account for 70% of the surrounding rocks. The orientation of the maximum in-situ stress matches the axis of the cavern. This means the plastic deformation due to in situ stress is not likely to take place near the cavern. Therefore, it can be expected that the cavern can maintain stable after excavation.

### Blasting vibration test

Blasting vibration test is the main method to study, monitor and control the influence of blasting vibration on underground structures (Lai et al. 2015; Ma et al. 2013). According to the actual site conditions, the blasting vibration test is carried out in the cavity of the main powerhouse of the hydropower station. Figure 2 is a front view of the dimensions of the underground powerhouse of the hydropower station and the layout of the measuring points. MP1~MP10 are the deployment points for the blasting test instruments. In the process of blasting excavation in the ladder section of the powerhouse, the blasting holes are drilled by the crawler dive drills. The diameter of the hole is 90mm, the depth of the hole is 4.0~6.2 m, the hole spacing and the row distance of the main blasting hole is 2 m ~ 3 m. The 70 mm emulsion explosive is continuously charged in the blasting hole, and the single hole charge



**Fig. 1** View and location of Baihetan Hydropower Station

is adjusted according to the blasting conditions, mainly between 10 kg and 26 kg. Due to the requirement to control the vibration velocity of the particles of the side wall, a one-hole and one-stage network is used in the main powerhouse. Holes were detonated by non-electrical millisecond detonators of Ms3, Ms5, Ms7, Ms9, Ms11, Ms13, and Ms15, respectively.

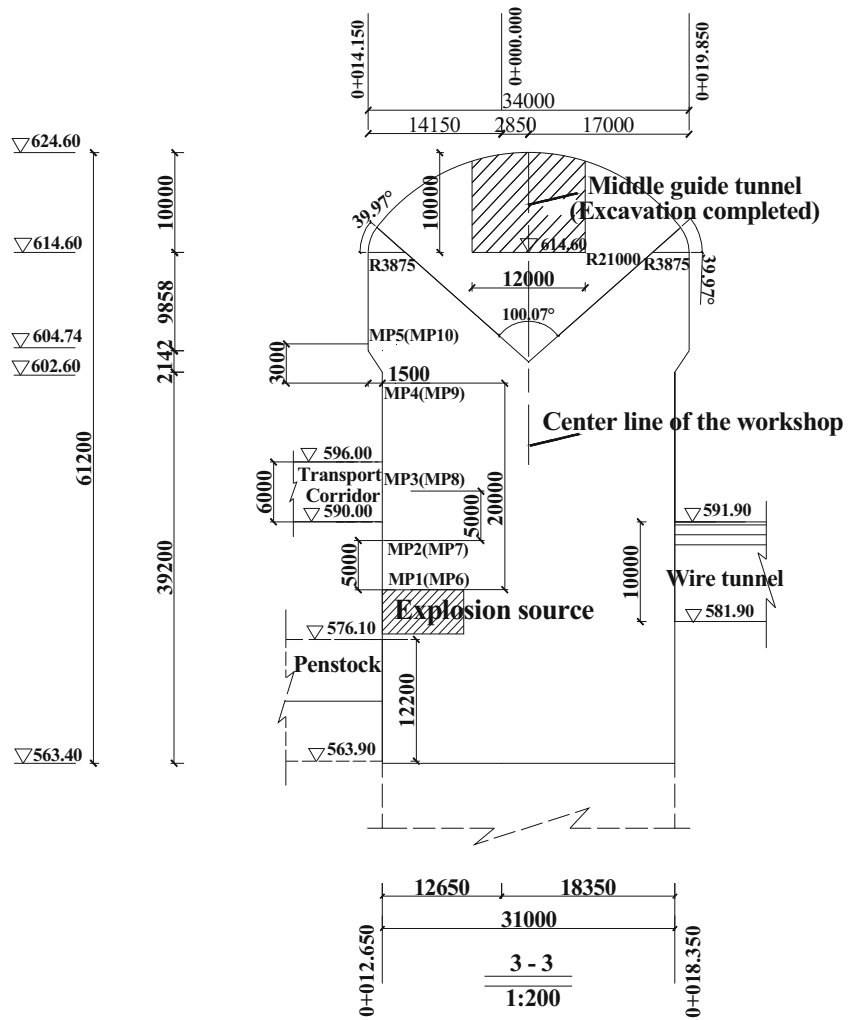
Blasting vibration test system is consisted by L-20 vibration recorders, three-component vibration speed sensors and clients produced by Chengdu Jiaobo Technology Co., Ltd. The recorders and the associated sensors can synchronously measure the vibration speed and frequency of three directions, and record the corresponding time when the speed reaches the peak value. The speed range and frequency range vary from 0.001 to

35.5 cm/s, and from 1 to 500 Hz, respectively. The test precision and reading precision are 5% and 0.1%, respectively.

To study the propagation law of blast wave in the direction of elevation, 10 measuring points are arranged at each test. The vibration velocity sensors of each measuring point are arranged on the same elevation, and two columns of vibration velocity sensors are arranged in the horizontal direction. The sensor in the lowest place is arranged at the bottom of the power house, and the distance between two sensors in the middle is 5 m. Besides, two measuring points are set at the rock anchor beam and 3 m above it. The layout of measuring points is shown in Figs. 3 and 4. The test results of blasting vibration in elevation direction in the main power house are shown in Table 1.



**Fig. 2** A front view of the dimensions of the underground powerhouse of the hydropower station and the layout of the measuring points(unit:mm)



**Discussion**

According to the analysis of the field test data of blasting vibration, it is found that the change processes of  $V_x$  and  $V_z$  do not show a uniform regularity. However, there is sometimes a phenomenon that the vibration velocity at an elevation of 20 m exceeds the vibration velocity at an elevation of 10 m, which means a local amplification effect occurs. However, this phenomenon is not uniform and stable, but it is certainly accidental.

During the propagation of the explosion wave along the elevation direction, through the analysis of  $V_y$ , some interesting rules can be found. The particle vibration has the least restriction in the  $Y$  direction, and the free vibration condition is the best. Therefore, it can be clearly seen that the data of  $V_y$  is larger. In addition, regardless of how the explosive effective load and horizontal distance changes, a uniform regularity can be found: the blasting vibration velocity has an attenuation trend on the whole, but the blasting vibration velocity of the measuring point at an elevation of 20 m is greater than that of the measuring point at an elevation of 10 m, and a local amplification phenomenon appears, as shown in Fig. 5.

From the comparison and analysis of the field measured data, it can be seen that during the blast wave propagation in the elevation direction, the particle vibration velocity does not satisfy the prediction rule of the Sadovskii empirical formula, which means the vibration velocity decreases monotonically with the increase of the distance from the blast source. However, there is a local amplification effect in the middle position, that is, the rock anchor beam. Therefore, due to the general laws of vibration and the particularity of the rock anchor beam in the underground powerhouse, it is assumed that there is an elevation amplification effect in the course of the propagation of blast wave in elevation direction.

**Numerical calculation of elevation effect in underground powerhouse**

**Approximate calculation of ground stress**

When the influence of blasting vibration on underground tunnels is analyzed, if the depth of the underground tunnels is

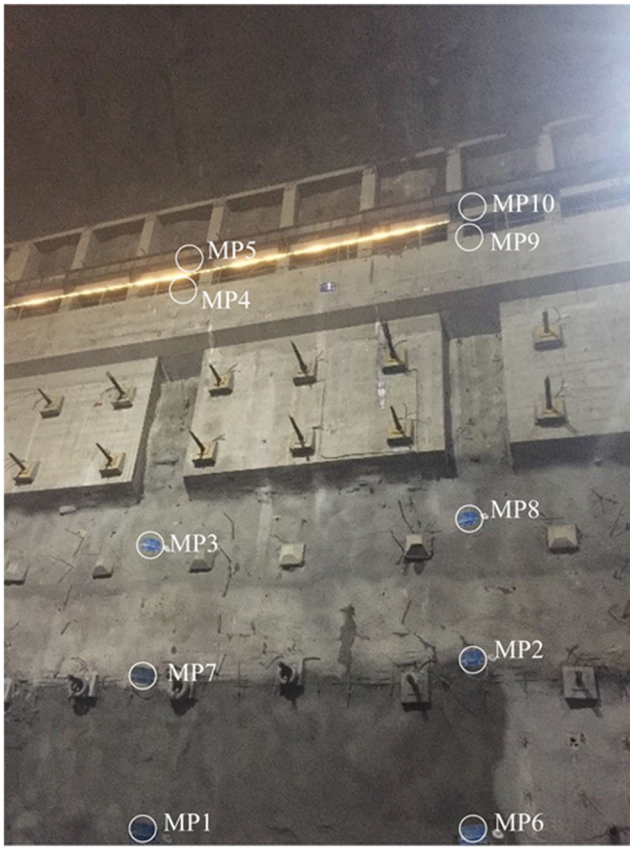


Fig. 3 Instrument layout for blasting vibration test of high side wall of powerhouse

shallow, the impact of ground stress on blasting vibration can be ignored. However, the measured and inversion analysis shows that the maximum principal stress of the underground cavern group on the right bank of the Baihetan Hydropower Station reaches 21.5 MPa. According to the current

classification criteria, the ground stress at the location of the main-transformed cavern is greater than 20 MPa (Li et al. 2012; Xie et al. 2015), and it belongs to the high ground stress region. Therefore, the ground stress involved in the numerical simulation should be determined firstly.

According to Xu et al. 2017, the vertical ground stress is calculated according to the depth of the cavern, and its value is about 10.3 MPa. According to the stress analysis formula of plane stress state in elasticity mechanics (as shown in Eq. (1)), the axial horizontal stress of the cavern can be calculated as 21.35 MPa, and the horizontal stress perpendicular to the axial direction of the cavern is 10.45 MPa.

$$\sigma_{\pm\alpha} = \frac{\sigma_1 + \sigma_3}{2} \pm \frac{\sigma_1 - \sigma_3}{2} \cos 2\alpha \tag{1}$$

### Equivalent load application method

According to the principle of force balance, the peak pressure of the blast wave applied to the wall of the borehole is equivalently applied to the core of the blast hole (Lu et al. 2012a). After the blasting load is approximately equivalent, it acts on the plane determined by the center line of the blasting hole of the same row and the hole axis. Assume that the force generated by the explosive on a single blast hole wall is  $P_0$ , the radius of the hole is  $r_0$ , and the distance between the two adjacent holes is  $a$ , then according to the equilibrium principle of force, the effect of the pressure  $P_0$  exerted on the blast hole wall shown in Fig. 6a is the same as that of the equivalent force on the center line of the blasting hole shown in Fig. 6b, and thus the equivalent load  $P_e$  on the whole surface can be obtained as follows.

$$P_e = \frac{2r_0}{a} P_0 \tag{2}$$

In the formula,  $P_0$  is the blast load acting on the single blast hole wall.  $r_0$  is the radius of blast hole. When the main-transformed cavern is blasting,  $r_0$  is 0.045 m.  $a$  is the distance between the holes (taken as 3.0 m).

After selecting the loading mode of blasting load, it is also necessary to determine the magnitude of the peak pressure of the detonation gas acting on the wall of the blast hole to finally determine the equivalent pressure. In the case of uncoupled charge, the initial stress peak acting on the rock wall under C-J detonation conditions is calculated according to Formula (3) (Saif et al. 2017).

The initial stress peak value is

$$P_0 = \frac{\rho_e D^2}{2(\gamma + 1)} \left(\frac{d_c}{d_b}\right)^{2\gamma} \left(\frac{l_c}{l_b}\right)^\gamma \cdot n \tag{3}$$

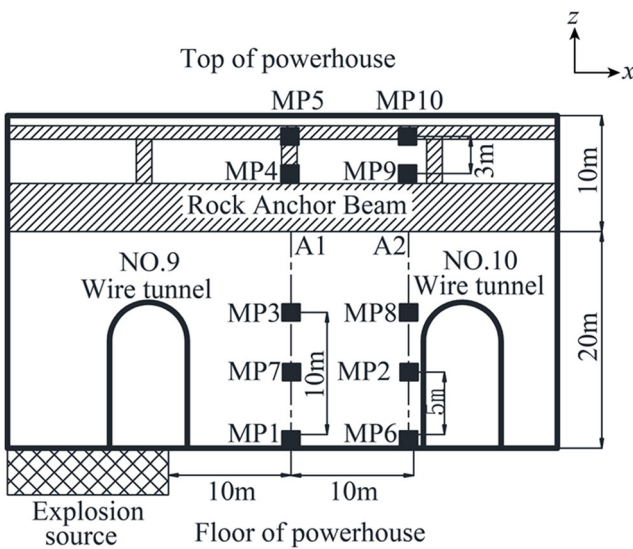


Fig. 4 Schematic layout of measuring points for blasting vibration of high side wall in powerhouse

**Table 1** Summary of test results of blasting vibration in elevation direction in the powerhouse

No.	Quantity of explosive (kg)	Horizontal distance (m)	Elevation (m)	$V_x$ (cm/s)	Dominant frequency (Hz)	$V_y$ (cm/s)	Dominant frequency (Hz)	$V_z$ (cm/s)	Dominant frequency (Hz)
1	26	10	0	9.62	238.0	9.91	81.2	8.70	368.8
		10	5	5.59	139.0	7.27	5.1	6.56	137.1
		10	10	4.96	114.0	5.02	44.4	4.16	157.8
		10	20	4.93	57.7	5.19	15.3	4.99	12.1
		10	23	3.53	215.1	4.80	29.9	3.97	202.2
2	26	20	0	4.38	54.9	3.90	26.2	3.25	43.9
		20	5	3.10	128.2	3.08	126.6	3.33	33.5
		20	10	2.64	187.5	2.57	260.4	2.41	146.5
		20	20	1.96	56.1	2.71	27.0	1.45	485.8
		20	23	0.78	113.0	2.50	285.2	1.35	83.4
3	22	10	0	6.40	326.1	7.23	75.7	5.60	190.6
		10	5	6.10	27.7	6.09	11.3	6.69	220.8
		10	10	3.42	223.2	5.31	167.4	5.63	151.2
		10	20	3.71	161.6	5.30	51.9	4.30	111.6
		10	23	4.37	75.6	5.20	66.0	2.19	151.2
4	22	20	0	3.27	173.6	4.20	15.7	5.75	66.0
		20	5	2.85	93.8	2.16	82.2	3.40	156.2
		20	10	1.25	93.8	1.77	85.2	2.32	75.6
		20	20	0.46	114.3	1.87	42.2	1.11	63.3
		20	23	0.90	76.8	1.76	76.8	1.80	80.8
5	19	10	0	8.71	313.0	7.06	60.4	5.70	90.1
		10	5	7.41	176.0	6.05	293.0	4.61	85.2
		10	10	5.10	313.0	5.00	334.8	7.71	88.4
		10	20	2.90	112.6	5.03	21.7	4.50	74.8
		10	23	4.40	156.0	4.90	187.5	2.43	156.2
6	19	20	0	4.89	99.7	3.90	195.3	2.72	58.6
		20	5	2.03	234.0	1.98	76.8	1.88	99.7
		20	10	1.70	162.0	1.77	97.7	1.59	104.2
		20	20	1.20	217.8	1.85	41.5	2.1	34.6
		20	23	1.31	123.0	1.60	293.0	0.64	260.4

In the formula:  $\rho_e$  is the explosive density, taking 1200 kg/m<sup>3</sup>.  $D$  is the explosive detonation speed, taken as 3600 m/s. The  $d_b$  and  $d_c$  is the diameter of the blast hole and the diameter of the blasting cartridge, which is 0.09 m and 0.07 m, respectively.  $l_b$  and  $l_c$  is the length of the charge and the length of the blast hole, taking 3.5 m and 5.0 m, respectively.  $\gamma$  is 3.0 and  $n$  is 10. According to the charge parameters of blasting holes in underground powerhouse, it can be obtained by using the Formula (3) that the equivalent pressure equals to 12.3 MPa.

It is generally believed that the magnitude of blasting load action time is between 10<sup>-6</sup>s and 10<sup>-1</sup>s, and the pressure action time of the detonation gas is 10<sup>-3</sup>s and 10<sup>-1</sup>s. The load duration curve is approximately triangular, as shown in the Fig. 7. In this paper, the total action time of pressure is 10 ms, the rising time of positive pressure is 2.3 ms, and the descending time is 7.7 ms.

### Constitutive model

According to the study by Yang et al. 1996, the result of kinematic hardening model is closer to the results of rock mass blasting dynamic response. It can more accurately reflect the dynamic response characteristics of the surrounding rock under explosion. Using this constitutive model, the plastic kinematic hardening constitutive model has the following formula:

$$\sigma_y = \left[ 1 + \left( \frac{\dot{\varepsilon}}{C} \right)^{\frac{1}{p}} \right] (\sigma_0 + \beta E_p \varepsilon_{eff}^p) \quad (4)$$

In the formula, the  $\beta$  parameter can be adjusted according to the using conditions. When  $\beta = 0$ , the formula is the plastic

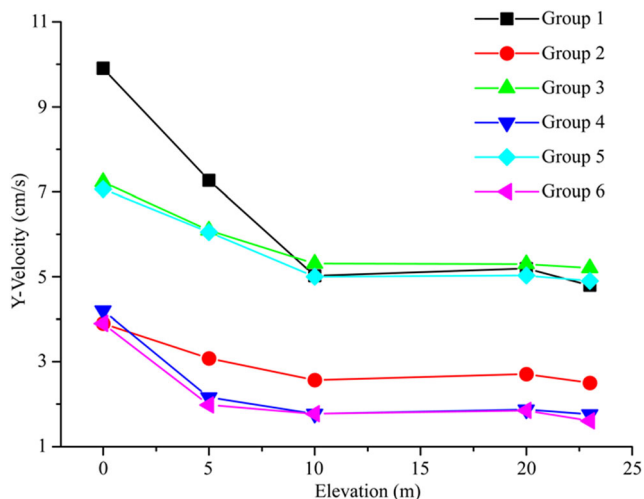


Fig. 5 Blasting vibration propagation

kinematic hardening model. When  $\beta = 1$ , the constitutive model is the equivalent reinforcement model, and the expression of the enhanced modulus  $E_p$  is as follows.

$$E_p = \frac{E_t E_{tan}}{E - E_t} \tag{5}$$

In the formula,  $E_t$  is the Young’s modulus and  $E_{tan}$  is the tangent modulus.

The expression for effective plastic strain  $\varepsilon_{eff}^p$  of rock mass is as follows.

$$\varepsilon_{eff}^p = \int_0^t d\varepsilon_{eff}^p \tag{6}$$

$$d\varepsilon_{eff}^p = \sqrt{\frac{2}{3}} d\varepsilon_{ij}^p \tag{7}$$

In the formula,  $\varepsilon_{ij}^p$  is the plastic strain partial component of rock mass.

**Numerical calculation model**

According to the actual blasting excavation of the underground main power house of Baihetan hydropower station, the single side of high side wall and the upper dome structure of the main power house are selected as the finite element simulation object. The finite element calculation model is established using ANSYS/LS-DYNA dynamic finite element software (Shi et al. 2015). The model is selected in the middle

Fig. 6 Diagram of equivalent loading of blasting load

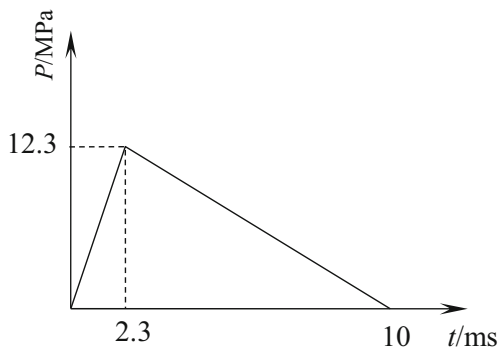
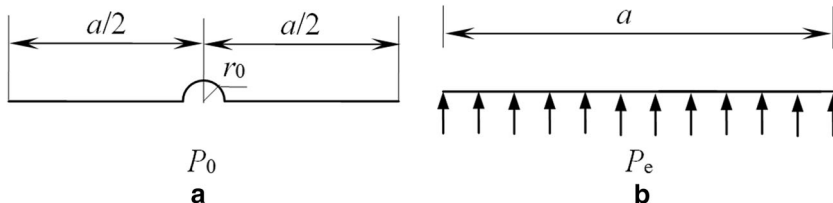


Fig. 7 Time history curve of blasting load

of the actual part of the main power house. The model size is selected based on the actual construction size. The span of the powerhouse is selected according to the half of the actual value, the span of the baseboard is taken as 15.5 m, the span of the upper spandrel is 17.0 m, the height of the model is taken as the actual value, and the value is 42 m from the actual excavation baseboard elevation to the vault.

Regarding the boundary conditions may have a greater impact on the simulation results, thus, the length, width and thickness of the model are 160 m, 50 m, and 35 m, respectively, as shown in Fig. 8a. In order to deeply and systematically study the elevation effect of the blasting vibration of the side wall of an underground powerhouse, the simulated excavation process of the baseboard of plant model is the same as the actual excavation process. In order to ensure the reliability of the simulation results, under the condition of the same bottom elevation, taking 5 m as the footage of blasting cycle, it is proceeding forward in the positive direction along Z axis in 5 times. Therefore, a total of five 3D models of underground powerhouse blasting excavation have been established.

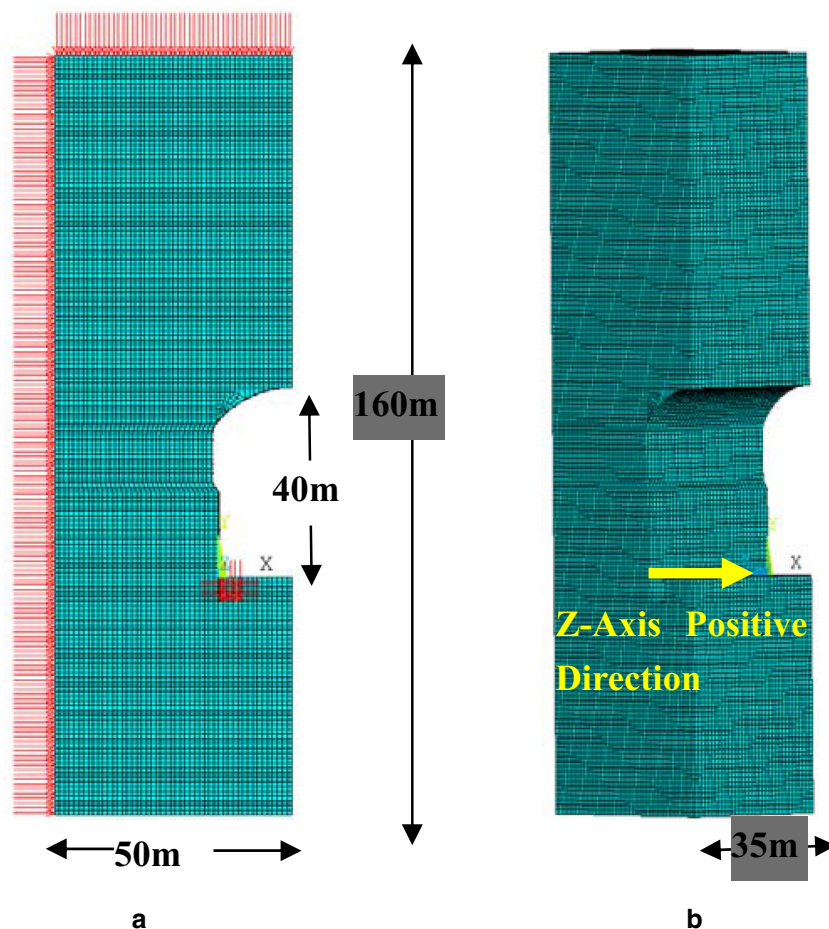
According to the site survey data (Li et al. 2017), the parameters of the underground cavern rock anchor beam can be obtained as shown in Table 2.

**Comparative analysis between numerical calculation and actual measurement of blasting vibration velocity**

Through the post-processing program, the peak velocity of the measuring point, which is the same as the actual position in Fig. 4, is extracted and compared with the peak vibration



**Fig. 8** Front view and lateral view of the calculation model for blasting excavation in powerhouse. **a** Front view. **b** Lateral view



velocity of the same measuring point on the site and the calculated value of the theoretical formula, as shown in Fig. 9.

For the measurement data, blast wave propagation direction is the  $Z$ -direction in Table 1, the direction of vibration is  $Y$  direction. But, as for the numerical simulation results, the propagation direction is the  $Y$  direction, and the vibration direction is the  $X$  direction. Therefore, the particle vibration velocity studied and extracted in the numerical simulation are all values of the  $X$ -direction.

The first set of measured data is the average of the blasting vibration velocity of the three groups of elevation measuring points that are 10 m away from the blasting source. To further improve the accuracy and reliability of numerical simulation data, five numerical models of the first layer excavation are selected as targets in the blasting test. The blasting vibration

velocity in the elevation direction of the measuring point at the same position in the blasting test is selected, and the simulation results are compared with the measured data, as shown in Fig. 9a.

The second set of measured data is the average of the blasting vibration velocity of the four groups of elevation measuring points that are 20 m away from the blasting source. The numerical model also selects the blasting vibration velocity in the elevation direction of the measuring point at the same position in the blasting test, and compares the simulation results with measured data and theoretical calculations, as shown in Fig. 9b.

An interesting phenomenon was found from the analysis and comparison. When the two sets of data have different horizontal distances, the vibration velocity of the 20 m elevation point is higher than that of the 10 m elevation point. It can be seen that the propagation law of blasting vibration velocity in the side walls of underground powerhouses is not a simple attenuation process in elevation direction, but there is a certain degree of amplification effect.

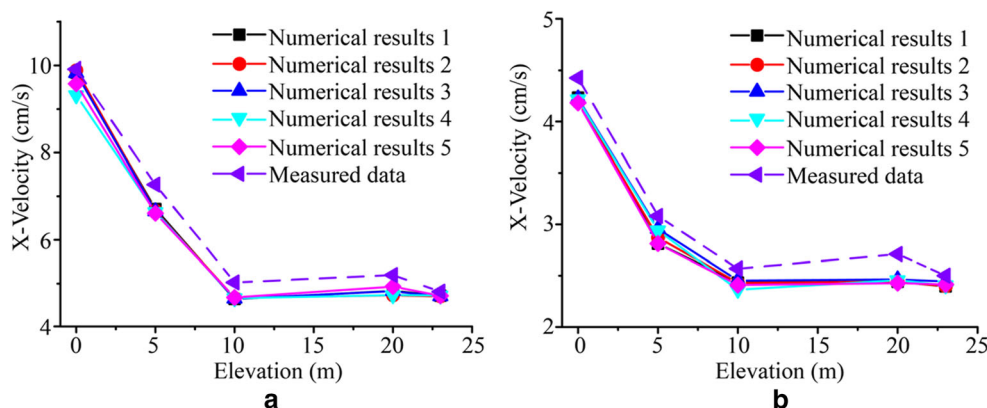
To uncover the propagation law of blasting vibration velocity in the elevation direction during the excavation of underground powerhouse, as well as to avoid the randomness

**Table 2** Rock mass physical and mechanical material

$\rho$ /(kg/m <sup>3</sup> )	$E$ /GPa	$\mu$	$\sigma_t$ /MPa	$\sigma_c$ /MPa	$E_t$ /MPa
2750.0	50.0	0.23	6.0	100.0	8.0



**Fig. 9** Comparison of blasting vibration velocity between measured data and calculated results at different horizontal distances from blasting source. **a** Horizontal distance of 10 m. **b** Horizontal distance of 20 m



and occasionally of the data due to the lack of samples, two columns of elevation measuring points of 10 m and 20 m distance from the blasting source are taken as the objects and the five measuring points in the two cases in Fig. 9 are, respectively, expanded to 52 measuring points.

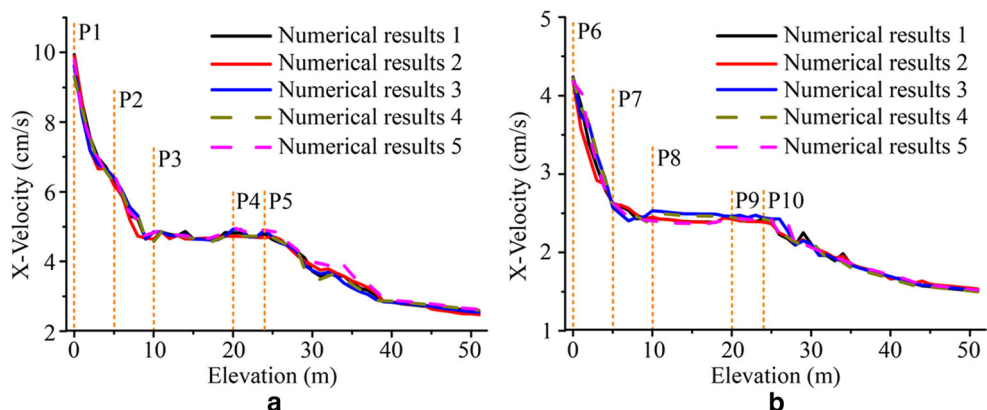
In Fig. 10, P1~P10 correspond to the locations of actual site measuring point MP1~MP10, and also correspond to the location and value of the selection points in numerical calculation in Fig. 9. The purpose is to rule out the influence of other factors and use the elevation change as the only variable. According to the results, the vibration velocity of all the measuring points of each model in the elevation direction is extracted and organized into a graph. The calculation result at a horizontal distance of 10 m from the blasting source is shown in Fig. 10a, and the calculation result at a horizontal distance of 20 m is shown in Fig. 10b.

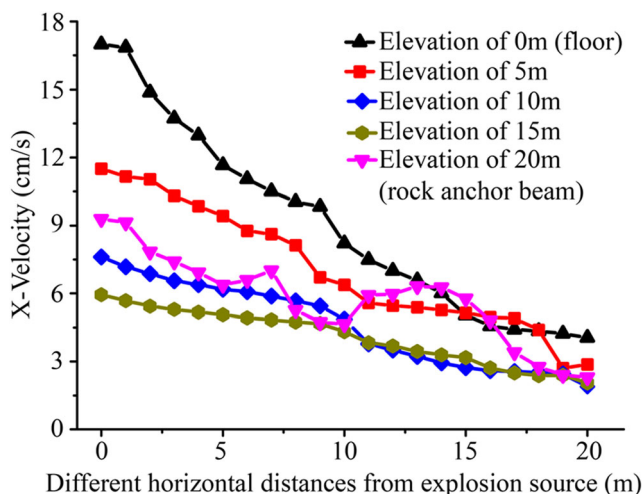
Through the analysis and comparison of the results in Fig. 10, it can be found that (1) the blasting vibration velocity in the elevation direction of the side wall shows a declining trend. The blasting vibration velocity gradually decreases with the increase of the elevation, and the attenuation trend gradually tends to be gentle. (2) Under the overall attenuation trend of the blasting vibration velocity in the elevation direction, there is a gentle or even rebound phenomenon in the position between the elevation of 10 m to 25 m, that is, the local magnification effect. (3) In the comparison results of the two

curve data, the vibration velocity of the measuring point near the rock anchor beam with an elevation of about 20 m is greater than that of the measuring points with elevation between 10 m and 20 m. This reveals that the magnification effect of the rock anchor beam in the local part is more obvious than that in the front part. Thus, we can reasonably speculate that the rock anchor beam should have a certain magnification effect on the particle vibration velocity in elevation direction.

Taking the first layer of the first model as the study object, and the elevation difference was increased by 5 m each time, so the elevation of 0 m, 5 m, 10 m, 15 m, and 20 m is selected as 5 sets of measuring points, each set with 21 measuring points. The elevation of 0 m is the baseboard of the powerhouse and the elevation of 20 m is the rock anchor beam. The blasting vibration velocity in elevation direction is extracted and processed into a curve, as shown in Fig. 11 and the following rules are obtained through analysis and comparison. (1) Except the position of the rock anchor beam, the change of elevation does not affect the trend that the blasting vibration velocity decreases with the increase of the horizontal distance. However, the vibration amplitude of blasting vibration velocity occurs in the elevation direction of rock anchor beam with the increase of the horizontal distance, and the phenomenon of local amplification appears. (2) The blasting vibration velocity in the elevation direction of the same elevation location is

**Fig. 10** Vibration velocity curve in the elevation direction of the numerical simulation at different horizontal distances. **a** Horizontal distance of 10 m. **b** Horizontal distance of 20 m





**Fig. 11** Distribution curve of vibration velocity along horizontal distance at different elevations

decreasing with the increase of horizontal distance. However, the blasting vibration velocity at higher elevation position in the attenuation process exceeds the vibration velocity at lower elevation position. For example, the particle vibration velocity of the elevation of 5 m at the horizontal distance of 15 m~18 m exceeds the particle vibration velocity of the elevation of 0 m, and the contrast between the elevations of 15 m and 20 m also shows this phenomenon, which proves the appearance of the local elevation amplification effect.

## Elevation amplification effect

### Calculation formula considering the elevation effect

There are usually three indicators reflecting the attenuation law of blast wave: vibration velocity, vibration frequency, and duration of vibration. The blasting vibration velocity directly reflects the energy of the blast wave reaching the position of the measuring point. Therefore, for a long time, the blasting vibration velocity has been paid most attention by the scholars and Sadovskii empirical formula is the most commonly used vibration velocity prediction formula based on extensive experiments.

$$V = K \left( \frac{\sqrt[3]{Q}}{R} \right)^\alpha \quad (8)$$

In the formula,  $V$  is the peak velocity of the particle at the measuring point, with unit of cm/s.

$Q$  is the explosive charge, which is the total explosive effective load, and when using millisecond blasting,  $Q$  is the maximum single explosive effective load, with the unit of kilogram.

$R$  is the distance between blasting source and measuring point, with the unit of meter.

$K$  and  $\alpha$  is the coefficient and attenuation index related to geological and topographic conditions between blasting points and measuring points.

A large number of engineering experiments have proved that Sadovskii empirical formula has a high accuracy in predicting the particle blasting vibration velocity of the ground under flat terrain conditions (Yang et al. 2016). However, because the formula does not consider the influence of the height difference between the measuring point and the blasting center, when the topography of the blasting site changes greatly, it is no longer applicable to predict the blasting vibration with the formula. Because  $K$  is a coefficient related to the condition of blasting site, and  $\alpha$  is the attenuation coefficient of blast wave related to geological conditions, many scholars tried to use the changes of the values of  $K$  and  $\alpha$  to reflect the elevation amplification effect.

Zhu and Liu 1988 put forward an improved blasting vibration attenuation formula after accumulating a lot of engineering experience.

$$V = K \left( \frac{\sqrt[3]{Q}}{R} \right)^\alpha \left( \frac{\sqrt[3]{Q}}{H} \right)^\beta \quad (9)$$

In the formula,  $H$  is the elevation difference between the blasting source and the measuring point, with unit of meter. Other parameters are consistent with Formula (8)

Scholars summarized the approximate formula reflecting the elevation effect by analyzing the measured data of multiple blasting vibrations.

$$V = K \left( \frac{\sqrt[3]{Q}}{R} \right)^\alpha H^u \quad (10)$$

In the formula,  $u$  is a coefficient related to height difference and is taken from 0.25 to 0.28. When the height difference is positive, positive value is taken, whereas negative value is taken. The hard rock takes a large value, and the soft rock takes a small value.

Song et al. 2000 conducted on-site monitoring of the blasting vibration of an open-pit iron ore slope. After analyzing the measured data, Eq. (11) is provided to take the height difference into account.

$$V = K \left( \frac{\sqrt[3]{Q}}{R} \right)^\alpha \left( \frac{R}{S} \right)^\beta \quad (11)$$

In the formula,  $R/S$  is the basic coefficient of the influence of height difference, which is obtained through the ratio of the oblique distance  $R$  and the horizontal distance  $S$  from the blasting center to the measuring point.

To further explore the influence mechanism of elevation on blast wave propagation, the functional relation between vibration velocity and other physical parameters (especially

elevation) need to be found through dimensional analysis method which is wildly used in natural science. Based on other studies, the attenuation law of blasting vibration is affected by the quality of explosive, the horizontal distance between the measuring point and the blasting source, and we took the elevation difference into account in this paper. By Buckingham’s theorem ( $\pi$  Theorem), the particle vibration velocity ( $V$ ) of the rock mass can be expressed as follows.

$$V = \phi(Q, \mu', C, \rho, S, H, \alpha, f, t) \tag{12}$$

In the formula,  $Q$  is the explosive mass; dimension  $M$ ;  $\mu'$  is the particle vibration displacement; dimension  $L$ ;  $C$  is the vibration propagation velocity; dimension  $LT^{-1}$ ;  $\rho$  is the density of rock mass; dimension  $ML^{-3}$ ;  $S$  is the horizontal distance between the measuring point and the blasting source; dimension  $L$ ;  $H$  is the elevation difference between the measuring point and the blasting source; dimension  $L$ ;  $a$  is particle vibration acceleration. Dimension  $LT^{-2}$ ;  $f$  is vibration frequency; dimension  $T^{-1}$ ;  $t$  is detonation time. The expression is generally as follows:

$$V = K' \left( \frac{\sqrt[3]{Q}}{S} \right)^\alpha \left( \frac{H}{S} \right)^\beta \tag{13}$$

Based on the above study, it is found that the Sadovskii empirical formula will no longer be accurate for the blasting vibration calculation and prediction for the terrain with elevation effects. Numerous studies also focus on the elevation effect of slope, and there is no reliable theoretical formula for the elevation effect of side wall of the deep underground powerhouse. Therefore, the basic law of the effect of height difference changes on the values of  $K$  and  $\alpha$  is explored through field measurement, and the regression analysis of the values of  $K$  and  $\alpha$  at different height difference is carried out.

### Blasting vibration velocity regression analysis

According to the years of research and experience accumulated in the blasting, the data of blasting vibration in several elevations in the main power house are tested, and the regression analysis is carried out by the Sadovskii empirical formula. The logarithm of the two sides in Formula (8) is taken respectively in the regression analysis, and then the results are obtained as follows:

$$\lg V = \lg K + \alpha \lg \frac{\sqrt[3]{Q}}{R} \tag{14}$$

Let  $y = \lg V$ ,  $x = \lg \sqrt[3]{Q}/R$ ,  $\beta = \lg K$ , then  $y = \beta + \alpha x$ .

Thus, formula (14) is transformed from a non-linear relationship to a one-dimensional linear relationship, and the values of  $\beta$  and  $\alpha$  are solved by using a least-squares estimate

to find the value of  $K$ . The Formula (15) can be used to determine  $\beta$  and  $\alpha$ .

$$\alpha = \frac{\sum_{i=1}^n (x_i - \bar{x})y_i}{\sum_{i=1}^n (x_i - \bar{x})x_i} \tag{15}$$

$$\beta = \bar{y} - \alpha \bar{x}$$

In the formula,

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i, \quad \bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$$

To evaluate the effect of regression, a correlation coefficient  $R$  was introduced.

$$R = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 \cdot \sum_{i=1}^n (y_i - \bar{y})^2}} \tag{16}$$

The value of  $|R|$  is 0 ~ 1. The closer the value of  $|R|$  is to 1, the stronger the linear relationship between  $y$  and  $x$  is in the equation. When  $|R|$  is greater than 0.8, the regression formula can be considered as trustworthy. Otherwise, the abrupt points in the measured data should be discarded for further regression analysis until the relevant requirements are met.

Through regression analysis, it is found that the correlation and reliability of the fit of Formula (12) is relatively high. Therefore, using the dimension analysis method considering the elevation factor, a regression analysis on the measured data of blasting vibration in the elevation direction of the high side wall of an underground powerhouse is performed to obtain the modified Sadovskii empirical formula considering the elevation factor.

Take the logarithm of both sides of Formula (12) to get the following equation.

$$\ln V = \ln K' + \alpha \ln \left( \frac{\sqrt[3]{Q}}{S} \right) + \beta \ln \left( \frac{H}{S} \right)$$

Let  $Y = \ln V$ ,  $C = \ln K'$ ,  $X_1 = \ln(\sqrt[3]{Q}/S)$ , and  $X_2 = \ln(H/S)$ , then two-variable linear equation with  $X_1$  and  $X_2$  as the random independent variable and  $Y$  as the random dependent variable can be obtained.

$$Y = C + \alpha X_1 + \beta X_2$$

Because the object of study is the elevation effect of vibration velocity, only the  $Y$ -direction vibration velocity data in the blasting vibration field test data is selected. Remove the 6 sets data of bottom plate positions from the 30 sets of measured data because  $H/S$  is not meaningful at this time. The remaining 24 sets of data are subjected to regression analysis

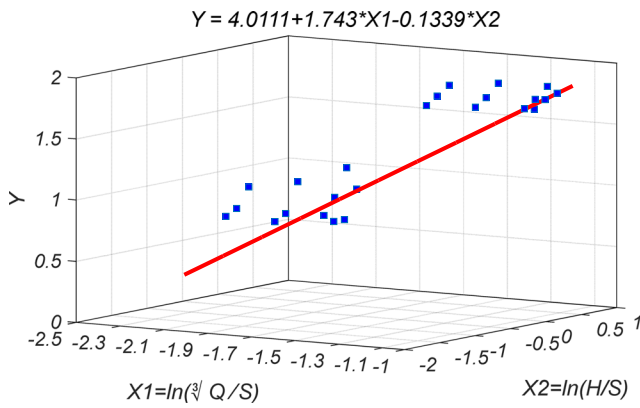


Fig. 12 Regression analysis curve of blasting vibration test data

using MATLAB software, and the modified Sadovskii empirical formula considering the elevation factor is obtained as Eq. (17). And the regression coefficient  $R$  is 0.917.

From Formula (17), it can be observed that the  $\beta$ , which represents the influence coefficient of the elevation difference is  $-0.1339$ , which is less than 0. Therefore, the blasting vibration velocity in the elevation direction is no longer monotonically inversely proportional to the horizontal distance and is proportional to the explosive effective load, and its magnitude is also influenced and determined by the ratio of elevation to horizontal distance. The explosive effective load and horizontal distance are substituted into the correction formula to obtain the relationship between vibration velocity and elevation.

$$V = 55.21 \left( \frac{\sqrt[3]{Q}}{S} \right)^{1.743} \left( \frac{H}{S} \right)^{-0.1339}, \quad R = 0.917 \quad (17)$$

By comparing with the numerical simulation results, it is found that the measured data have the local amplification effect in the range of 10 m ~ 40 m, and the modified formula can accurately and reliably predict the particle vibrational velocity in the elevation direction of the high side wall of the powerhouse without taking into account the influence range of the elevation effect. The prediction result of horizontal distance of

10 m is shown in Fig. 13a, and the prediction result of horizontal distance of 20 m is shown in Fig. 13b.

In order to further improve the accuracy of the modified formula for blasting vibration velocity prediction of the high side wall of powerhouse, the effect of elevation amplification within the range of 10 m to 40 m in elevation should be considered. Take the vibration velocity difference between the numerical results of horizontal distances of 10 m and 20 m and the formula results as the ordinate, and the elevation distance as the abscissa. Regression analysis was used to fit the supplementary formulas of the modified formula after considering the influence of elevation amplification effect, as shown in Formulas (18) and (19). Therefore, the modified formulas that involve the elevation amplification effect at horizontal distance of 10 m and 20 m from the blast source are Formulas (20) and (21), respectively.

$$\begin{aligned} \Delta V_1 = \Delta y &= -0.00519x^2 + 0.24859x - 1.82447, R^2 \\ &= 0.88154 \end{aligned} \quad (18)$$

$$\begin{aligned} \Delta V_2 = \Delta y &= -0.00189x^2 + 0.09199x - 0.66621, R^2 \\ &= 0.85976 \end{aligned} \quad (19)$$

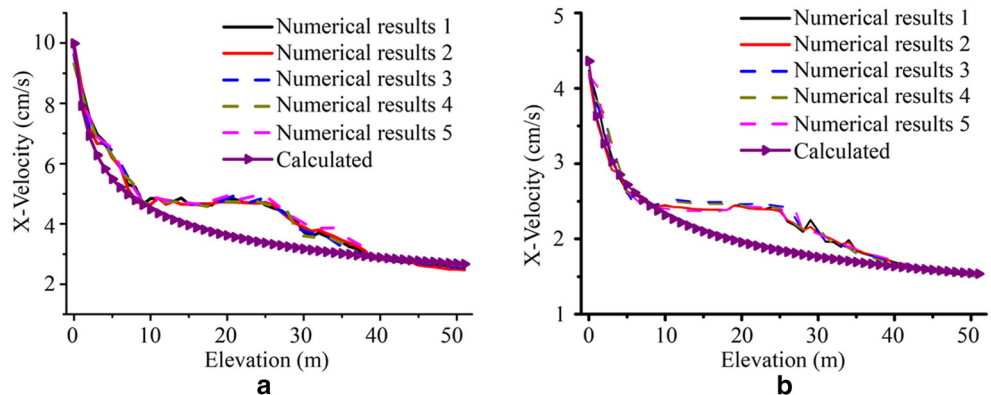
$$\begin{aligned} V_1 &= 55.21 \left( \frac{\sqrt[3]{Q}}{10} \right)^{1.743} \left( \frac{H}{10} \right)^{-0.1339} - 0.00519H^2 \\ &\quad + 0.24859H - 1.82447 \end{aligned} \quad (20)$$

$$\begin{aligned} V_2 &= 55.21 \left( \frac{\sqrt[3]{Q}}{20} \right)^{1.743} \left( \frac{H}{20} \right)^{-0.1339} - 0.00519H^2 \\ &\quad + 0.09199H - 0.66621 \end{aligned} \quad (21)$$

### Discussion

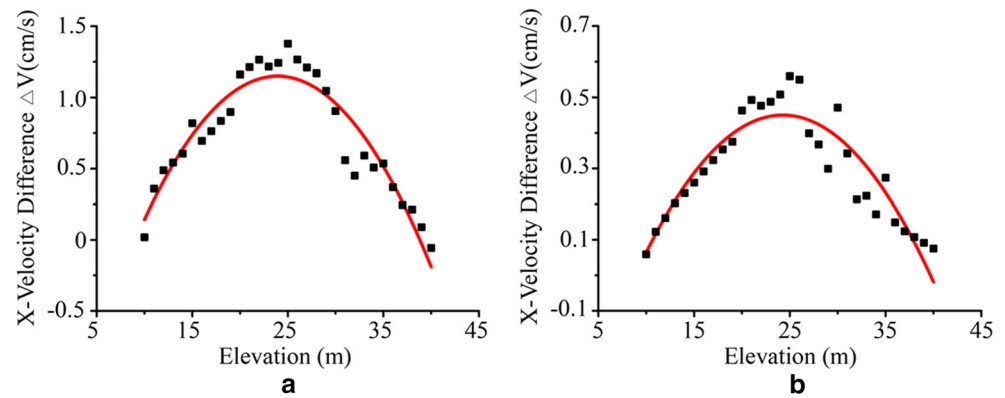
Comparing the measured data of Fig. 5 with the calculated results of Fig. 9. It can be seen that the elevation amplification

Fig. 13 Comparison curve between modified formula and numerical simulation results at different horizontal distance from blasting source. a Horizontal distance of 10 m. b Horizontal distance of 20 m





**Fig. 14** Supplementary regression analysis curves of elevation effects at different horizontal distance. **a** Horizontal distance of 10 m. **b** Horizontal distance of 20 m



effect reflected by the numerical results is consistent with the field tests, and the obtained amplitude and vibration velocity changes are highly consistent. This shows that both field test and numerical calculation indicate that the elevation amplification effect appears in the propagation of blast wave in the elevation direction.

To uncover the propagation law of blast wave in elevation direction, the measuring point data in the numerical model is expanded, measuring point is taken in each meter, and the propagation law curve in elevation direction is refined. It is found that the amplification effect is produced locally in the elevation range of 10 m ~ 40 m, and the most significant position of the amplification effect is at the middle part of the high sidewall, that is, the position of the rock anchor beam.

According to the results of the study on the mechanism of elevation amplification effect by scholars, it is speculated that the specific position (at middle) and the particular geomorphology of rock anchor beam should be the main cause of the elevation amplification effect during the vibration process of the high side wall. First of all, unlike the slope, the two ends of the high side wall of the underground powerhouse are completely restrained and cannot vibrate freely. If a row of points along the elevation of the high side wall is simplified as a rod, the vibration effect at the middle position will be larger than the two ends. Secondly, due to the complete constraint at both ends of the high side wall, it cannot be dissipated in time after the blasting vibration occurs, and the blasting vibration will produce the superposition effect at the middle position. Finally, due to the particularity of the geomorphology of the rock anchor beam, when the blast wave is transmitted to the rock anchor beam, under the action of the “Twhiplash effect” (Yang et al. 2016), the elevation local amplification effect is generated. Therefore, in the process of propagation and attenuation of the blast wave along the elevation direction, the combination of the vibration characteristics of the middle position and the geomorphic characteristics of the rock anchor beam is the mechanism of the local amplification effect of the elevation.

Even though the Sadovskii equation is commonly cited in the analysis for the vibration velocity of particle, the

associated correction is still required, when this model is expected to be utilized for the underground high side walls rather than open-pit slope. However, such correction, especially with the consideration of varying elevation, is rarely proposed in the literatures. Therefore, the fitting data and the measured data have poor consistency, with an average error between 30% and 50%. Therefore, through dimensional analysis considering the elevation factor, the modified Sadovskii empirical formula is fitted by the regression of the field test data. By comparing with the numerical simulation and the field measured data, the reliability of the prediction of the blasting vibration velocity of the high side wall of the powerhouse is verified by using the modified formula, and its reliability is greater than that of the Sadovskii empirical formula and the regression analysis of single variable, as shown in Fig. 12. Hence, considering the influence of local elevation amplification effect, when the elevation range is 10 m ~ 40 m, the influence term of elevation amplification effect of the modified formula should be supplemented to further modify the prediction and calculation accuracy of the modified formula as shown in Figs. 13 and 14. Obviously, according to the fitting results of the same test conditions, the error between the Formulas (9), (10), (11) and the actual measurement data is still more than 30%, which are 30.2%, 45.8%, and 30.7% respectively. And the fitting result of the modified formula, the error is just 4.5%, the correlation and reliability of the fitting formula is relatively high, and the accuracy is significantly improved.

## Conclusion

Even though the impact of elevation on blasting vibration has been commonly analyzed in the project of open-pit slope, such impact in the underground cavern with high sidewall are rarely proposed. The difference of propagation constraints between open-pit slope (i.e., multi-layer bench) and vertical high sidewall indicates the requirement of both analytical solution and in situ monitoring for the impact of elevation on blasting vibration in the underground caverns. The following

conclusions can be drawn from the comparative analysis of the modified Sadovskii empirical formula, the numerical calculation and the field test.

In the aspect of experimental research, it is rare to set measuring points for the ultra-high side walls of underground caverns in terms of elevation. The experimental results present a new and interesting phenomenon, which is the vibration velocity of the rock anchor beam is greater than that of the part with lower elevation, and the local elevation magnification effect appears during the propagation process.

According to the new findings, in view of the significant difference between the blasting vibration response of the high side wall of the underground cavern and the open pit slope in the end constraint conditions, the mechanical calculation model of the distribution characteristics of the blasting vibration direction of the deep underground chamber is established. And the numerical calculation results verify the rule of the field test data. It is found that the blast wave locally produces a magnification effect in the range of 10 m to 40 m in the process of propagation in the elevation direction, and the most significant one located in the middle rock anchor beam.

Based on dimensional analysis considering elevation factors, the modified Sadovskii empirical formula is fitted by regression analysis, and the prediction accuracy of the modified formula is further improved by supplementing the influence term of elevation amplification effect.

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**Data availability** The data used to support the findings of this study are available from the corresponding author upon request.

## Declarations

**Conflicts of interest** The authors declare that there are no conflicts of interest regarding the publication of this paper.

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