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ORIGINAL PAPER

Evaluation of empirical approaches in estimating the deformation modulus of rock masses

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Abstract

The rock mass deformation modulus is an important parameter for analysis of the mechanical behaviour rock structures. Due to high cost, time consuming activity and difficulties in interpretation of in-situ measurements, a number of empirical methods have been developed to estimate the deformation modulus on the basis of classification systems. However, due to a large number of empirical equations, the practical rock engineers have encountered the question which empirical relationship provides the most reliable estimation of the deformation modulus. This paper combines a review of empirical equations and statistical analyses based on the case studies from Iranian geography. Results of ninety-nine plate jacking tests from three dams and hydropower projects were used to evaluate the predictive performance of these empirical methods. Statistical analyses show that the Hoek and Diederichs (Int J Rock Mech Min Sci 43:203–215, 2006) and Ajalloeian and Mohammadi (Bull Eng Geol Environ 73:541–550, 2014) relationships provide the most precise and accurate estimation of the deformation modulus based on the in-situ measurements.

Keywords Deformation modulus · Rock mass classification · Empirical relationships · In-situ measurements

Introduction

Deformability is known as one of the most important characteristics that controls the mechanical behaviour of rock masses. It is characterised by a modulus which describes the relation between the applied load and the resulting deformation. As the behaviour of rock mass is not elastic, the term deformation modulus is commonly used, which is defined as the ratio of stress to corresponding strain including elastic and inelastic behaviour (Ulusay and Hudson 2007).

Due to the discontinuous nature of the rock mass, the deformability measured in laboratory experiments on small rock samples cannot be representative of the whole rock mass from which samples are taken. Therefore, the measured modulus in these laboratory experiments is expected to be significantly higher (Bieniawski 1978a; Isik et al. 2008a).

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Several in-situ methods have been developed for measurement of the deformation modulus, such as plate bearing, plate jacking, Goodman jack, flat jack, pressuremeter and dilatometer tests. There are several sources of inaccuracy in these approaches and different methods usually do not provide the same deformation modulus (Palmström and Singh 2001; Panthee et al. 2016).

The plate bearing test is one of the most common in-situ tests for measurement of the deformation modulus. The plate bearing test involves applying a load to the rock surface and measuring the resulting deformations at the surface. The calculated deformation modulus based on the surface measurements generally gives much lower values due to the damage of rock near the surface, deflection of the loading plate and closure of the gap between plate and the rock mass (Hoek and Diederichs 2006; Palmström and Singh 2001; Sharma et al. 1989).

In the dilatometer test, as the volume of the tested area is too small and tensile stresses are involved in the borehole, the calculated deformation modulus is usually 2-3 times lower (Bieniawski 1978b; Rocha 1974). Goodman jack is another borehole test where the measured deformation modulus value needs to be corrected due to contact angle between the loading platen and the borehole surface as well as the deformation of

Table 1 Review of empirical methods for estimation of the deformation modulus

Researcher	Equation	Description of the study
Conn and Merritt (1970)	$E_{rm} = E_i(0.0231RQD - 1.32)$	Data from 52 plate jacking tests at four different dam sites were used to derive this equation. This equation gives very low or negative estimation of the deformation modulus at low RQD values.
Bieniawski (1978a)	$E_{rm} = 2RMR_{76} - 100$	The data were prepared from different types of deformation modulus measurement techniques at three major engineering projects in South Africa. The proposed equation is valid for $RMR_{76} > 50$ and givea s negative value for RMR_{76} less than 50.
Serafim and Pereira (1983)	$E_{rm} = 10^{\left(\frac{RMR_{76}-10}{40}\right)}$	As the equation proposed by Bieniawski (1978a) was not applicable for $RMR_{76} < 50$, they provided more in-situ data from dam and tunnel projects where $20 < RMR_{76} < 85$. By plotting RMR and E_{rm} in a semi-logarithmic graph, they found a linear relationship which showed good correlation with the Bieniawski (1978a) data.
Gardner (1987)	$E_{rm} = E_i(0.0231RQD - 1.32) \quad RQD > 64\%$ $E_{rm} = 0.15E_i \quad RQD < 64\%$	To resolve the shortcomings of the Coon and Merit (1970) equation at low RQD values, the deformation modulus was suggested to be estimated from the intact rock modulus by using a reduction factor of 0.15. However, this equation was developed using very limited data for RQD < 60%. It was also assumed that for RQD = 100%, the deformation modulus of rock mass is equal to that of the intact rock while RDQ = 100% does not mean that no discontinuity is present in the rock mass.
Nicholson and Bieniawski (1990)	$E_{rm} = 0.01E_i\left(0.0028RMR^2 + 0.9exp\left(\frac{RMR}{22.82}\right)\right)$	By reviewing the equations proposed by Beiniawski (1978a) and Serafim and Pereira (1983) and using their data, they developed an equation which can consider the effect of scale and in-situ principal stresses.
Mehrotra (1992)	$E_{rm} = 10^{\left(\frac{RMR-20}{38}\right)}$	On the basis of 120 uniaxial jacking tests undertaken on ten types of Himalayan rocks at several dam sites in India, an equation was developed between deformation modulus and RMR where the rock masses were classified as poor to fair based on RMR classification.
Grimstad and Barton (1993)	$E_{rm} = 25logQ$	They developed a relationship using the average deformation modulus reported by Bieniawski's (1978a) and also the measured data from several projects reported in the literature. They found that the deformation modulus falls in the range of 10logQ and 40logQ. Their proposed equation is only applicable for Q > 1 (generally hard rocks).
Mitri et al. (1994)	$E_{rm} = 0.5E_i\left(1 - cos\left(\frac{\pi \times RMR}{100}\right)\right)$	They developed an equation between RMR and the ratio of deformation modulus to intact rock modulus. However, no details about the procedure of developing this equation have been provided.
Palmstrom (1995)	$E_{rm} = 5.6RMi^{0.375}$ $RMi = 10^{\left(\frac{RMR-40}{15}\right)}$	By developing an equation between RMi and RMR and inserting this equation into the empirical equation of Serafim and Pereira (1983), a new equation was developed between RMi and the deformation modulus. This equation is valid for RMi > 0.1.
Barton (1996)	$E_{rm} = 10Q_c^{1/3}$ $Q_c = Q \times \frac{\sigma_c}{100}$ σ_c: Uniaxial compressive strength of intact rock.	As in the Q system, the rock matrix compression strength is not considered, he suggested the use of Q_c instead of Q and updated the initial equation of Barton (1995) for estimation of the deformation modulus. The equation was developed by using the data of Bieniawski (1978a) and Serafim and Pereira (1983).
Hoek and Brown (1997)	$E_{rm} = \sqrt{\frac{\sigma_c}{100}}10^{\left(\frac{GSI-10}{40}\right)}$	They found that the Serafim and Pereira's equation (1983) overestimates the deformation modulus for poor quality rocks. They modified the Serafim and Pereira's equation based on practical observations and back analysis of excavation behaviour in poor quality rock masses for $\sigma_c < 100$ *MPa*.
Aydan et al. (1997)	$E_{rm} = 9.7 \times 10^{-6}RMR^{3.54}$	They used the in-situ data of Bieniawski (1978a), Serafim and Pereira (1983), Aydan (1989) and in-situ tests in Japan to develop the equation.
Read et al. (1999)	$E_{rm} = 0.1\left(\frac{RMR}{10}\right)^3$	They found that equations proposed by Serafim and Pereira (1983) and Hoek and Brown (1997) overestimate the deformation modulus at RMR = 100. They used the data set of Serafim and Pereira (1983) to derive their equation.

Table 1 (continued)

Researcher	Equation	Description of the study
Diederichs and Kaiser (1999)	$E_{rm} = (7 \pm 3)\sqrt{Q'}$ $Q' = 10^{\left(\frac{RMR-44}{21}\right)}$	The test data of field modulus measurements presented by Barton (1983) and Bieniawski (1978a) were used to develop an equation for modulus based on rock quality and confinement.
Ramamurthy (2001)	$E_{rm} = E_i \exp(-0.0115 J_f)$ J_f: Joint factor; $J_f = J_n/rn$, J_n: Joint frequency, n: joint inclination coefficient and r: Joint strength coefficient.	Using the data of uniaxial compression tests on jointed specimens from Brown (1970), Einstein and Hirschfield (1973), Arora (1987), Yaji (1984) and Roy (1993), an equation was proposed between the deformation modulus and the joint factor J_f.
Palmstrom and Singh (2001)	$E_{rm} = 7RMi^{0.4}$ $E_{rm} = 8Q^{0.4}$	A data set of 42 plate jacking tests and Goodman jack tests data (with correction factor of 2.5) at eight hydropower projects in India, Nepal and Bhutan and also using the data provided by Clerici (1993) and Thorpe et al. (1980) were used to develop these equations which are valid in the range of $1 < RMi < 30$ and $1 < Q < 30$.
Hoek et al. (2002)	$E_{rm} = \left(1-\frac{D}{2}\right)\sqrt{\frac{\sigma_c}{100}}10^{\left(\frac{GSI-10}{40}\right)}$	The Hoek and Brown (1997) equation was modified by the inclusion of the factor D to consider the effects of blast damage and stress relaxation on the deformation modulus.
Kayabasi et al. (2003)	$E_{rm} = 0.135\left[\frac{E_i\left(1+\frac{RQD}{100}\right)}{WD}\right]^{1.1811}$ WD: weathering degree.	There were 57 plate loading tests data from two dam sites in Turkey used to develop equations based on the simple regression and multiple regression analyses. The equation, which was developed based on simple regression analysis, provided more reliable prediction.
Gokceoglu et al. (2003)	$E_{rm} = 0.001\left[\frac{\left(\frac{E_i}{\sigma_c}\right)\left(1+\frac{RQD}{100}\right)}{WD}\right]^{1.5528}$ $E_{rm} = 0.0736 exp(0.0755RMR)$ $E_{rm} = 0.1451 exp(0.0654GSI)$	There were 115 data (57 data from Kayabasi et al. (2003) and 58 new data) obtained from in situ plate loading and dilatometer tests. These data were used to develop their new equations where the RMR was in the range of 20 to 85.
Zhang and Einstein (2004)	$E_{rm} = E_i 10^{(0.0186RQD - 1.91)}$	Using the data collected by Coon and Merritt (1970), Bieniawski (1978a) and Ebisu et al. (1992), a new equation was proposed. Large scatter in the data for $RQD \geq 70\%$ was observed which may be related to insensitivity of RQD to discontinuity frequency or spacing as well as test methods and directional effects.
Sonmez et al. (2004)	$E_{rm} = E_i(S^{\alpha})^{0.4}$ $S = \exp\left(\frac{GSI-100}{9-3D}\right), \alpha = \frac{1}{2} + \left(\frac{\exp\left(\frac{-GSI}{15}\right)-\exp\left(\frac{-20}{3}\right)}{6}\right)$	They assumed that for $GSI = 100$, the modulus ratios of the rock mass and intact rock should be theoretically identical and by undertaking statistical analysis on the same database of Gokceoglu et al. (2003), an empirical equation was developed.
Ramamurthy (2004)	$E_{rm} = E_i exp\left(\frac{RMR-100}{17.4}\right)$	By proposing an equation between the J_f and RMR, the primary empirical equation of Ramamurthy (2001) was modified.
Sonmez et al. (2006)	$E_{rm} = E_i 10^{\left[\left(\frac{(RMR-100)(100-RMR)}{4000exp\left(\frac{-RMR}{100}\right)}\right)\right]}$	Using the experimental data from Bieniawski (1978a), Serafim and Pereira (1983), Nicholson and Bieniawski (1990) and three averaged values from Sonmez et al. (2004), and found that the deformation modulus estimated by Sonmez et al. (2004) equation is overestimated for lower values of RMR ($RMR < 50$) while yield lower values for higher RMR ($60 < RMR < 80$). Based on these data a new empirical equation was developed.
Hoek and Diederichs (2006)	$E_{rm} = E_i\left[0.02 + \frac{1-\frac{D}{2}}{1+exp\left(\frac{60+15D-GSI}{11}\right)}\right]$ $Erm = 100\left[\frac{1-\frac{D}{2}}{1+exp\left(\frac{75+25D-GSI}{11}\right)}\right]$	Based on data from 423 plate tests, 53 flat jack and 18 back analysis of in situ measurements in China and Taiwan, sigmoid equations were developed to constrain the increase of modulus as the rock becomes more massive. For cases when reliable properties of the intact rock are not available, the second equation was suggested.
Chun et al. (2006)	$E_{rm} = 0.3228\, exp\,(0.0485RMR)$	Pressuremeter tests undertaken in Korea were used to develop their equation.
Galera et al. (2007)	$E_{rm} = E_i exp\left(\frac{RMR-100}{36}\right)$	Using 98 data of dilatometer tests from previously published works, they developed the equation.
Isik et al. (2008b)	$E_{rm} = (6.7RMR - 103.06) \times 10^{-3}$ $E_{rm} = 5.47 \times 10^{-3} GSI$	A total of 27 pressuremeter tests were undertaken on weak, heavily jointed, sheared and/or blocky greywacke rock masses and used in this study, and their developed equation is valid when $RMR \geq 27$.
Chun et al. (2009)	$E_{rm} = \frac{5.992Depth^2+1.883\sigma_c^4+4.851RQD^3+0.031JS^5+2399.530JC}{10000}$ JS: joint spacing and JC: joint condition.	There were 61 data sets collected from road and railway construction sites in Korea where the deformation modulus values were measured using pressuremeter tests in most cases.
Beiki et al. (2010)	$E_{rm} = \sqrt[3]{\sigma_c} tan\left(\sqrt{1.56+(lnGSI)^2}\right)$ $E_{rm} = \sqrt[3]{RQD} log(\sigma_c).tan(ln(GSI))$	They had 150 data of plate loading tests used to develop two equations based on the genetic programming approach.

Table 1 (continued)

Researcher	Equation	Description of the study
Mohammadi and Rahmannejad (2010)	$E_{rm} = 0.0003RMR^3 - 0.0193RMR^2 + 0.3157RMR + 3.4064$	They used data sets of Plate-loading test in a dam site to develop a new statistical equation.
Martins and Miranda (2012)	$E_{rm} = -8.1372 + 0.10005Depth + 0.6435UCS + 1.11458JC$	Using the database of Chun et al. (2009), they proposed a new formula based on the data mining algorithm of Support Vector Machines.
Shen et al. (2012)	$E_{rm} = 1.14E_i exp\left(-\left(\frac{RMR-116}{41}\right)^2\right)$	In situ data from Bieniawski (1978a), Serafim and Pereira (1983) and Stephens and Banks (1989) were used to derive a Gaussian equation.
Kang et al. (2013)	$E_{rm} = 10^{(0.32logQ + 0.585)}$ $E_{rm} = 10^{\left(\frac{10RMR-16}{50}\right)}$	A large data set of in-situ deformation modulus measurement tests undertaken in Korea were used to develop equations where the data set involved 314 values of Q in the range from 0.01 to 200 and 875 data of RMR in the range of 0 to 98.
Khabbazi et al. (2013)	$E_{rm} = 9 \times 10^{-7}RMR^{3.868}$	Eighty-two dilatometer test results gathered from two dam sites and a tunnel site were used to derive the equation where $39 \le RMR \le 87$.
Ajalloeian and Mohammadi (2014)	$E_{rm} = -0.016Q^2 + 1.581Q + 0.961$	Twenty-eight data from plate loading tests were used and the equation which is valid in the range of $0 < Q < 50$.
Sanei et al. (2013)	$E_{rm} = 0.0222GSI^2 - 2.1172GSI + 54.24$	Using the database of 47 plate loads, 86 dilatometers and 9 flat jack tests in a dam project, they developed three equations based on statistical analyses.
Nejati et al. (2014)	$E_{rm} = 7.192 + 0.06469UCS + 0.20481RQD + 0.30974JSR + 0.38384JCR + 0.01716GWR$ JCR: joint condition rating, JSR: joint spacing rating and GWR: ground water rating	Results of 8 plate bearing and 44 dilatometer tests were used and based on the statistical and neural network modelling, a new equation was developed.
Kavur et al. (2015)	$E_{rm} = 4^{\left(\frac{RMR-20}{20}\right)}$	Results of 69 large flat jack and plate jacking tests at three large hydroelectric projects with field data reported by Bieniawski (1978a), Serafim and Pereira (1983) and Stephens and Banks (1989) were used to derive their proposed equation.
Alemdag et al. (2015)	$E_{rm} = 0.058\, exp\,(0.0785RMR)$	A total of 50 pressuremeter tests were carried out in this study at four case sites in limestone where the RMR values ranging between 41 and 62 with a mean value of 53.
Kallu et al. (2015)	$E_{rm} = exp\,(-0.731 + 0.08465SF + 0.382\ln(BS) + 0.134RF + 0.157IF)$ SF: transformed strength factor, BS: average block size or joint spacing in cm, RF: transformed roughness factor and IF: transformed infilling hardness	Twenty-six data from plate loading tests in weak rock masses taken from one mine and two dam sites were used to develop an empirical equation where RMR_{76} was varying in the range of 15 to 55.

loading plates in the test (Bieniawski 1978a; Palmström and Singh 2001). Therefore, depending on the chosen correction factor, different results can be obtained. The pressuremeter test is based on the expansion of infinitely long cylindrical cavity theory. As the effected volume during borehole expansion tests may not be representative of the whole rock mass and also the presence of disturbed annulus around the borehole can result in underestimation of the deformation modulus (Isik et al. 2008b; Wittke 1990). Hoek and Diederichs (2006) stated that the in-situ down-hole jacks and borehole pressuremeter tests are the least reliable methods due to difficulty in interpretation of results particularly in hard and jointed rock masses where the stressed rock volume is too small.

It is believed that the flat jack test provides less reliable results due to the small volume of rock tested near the surface as well as difficulties in interpretation of results due to a wide scatter even in a very uniform rock mass (Bieniawski 1978a; Bieniawski 1979).

In the plate jacking test, the rock deformation is measured by extensometers placed in the drill holes. These extensometers are less sensitive to variation of pressure distribution at the surface of the loading area and also measurements of deformation at different depths provide a check against gross errors of the measurements. In this method, it is possible to assess the behaviour of rock at different depths. Therefore, the plate jacking test generally gives the best results (Benson et al. 1970; Palmström 2001; Ribacchi 1988).

In spite of high costs and operational difficulties in undertaking in-situ tests, there are several sources of uncertainties in these tests caused by blast damage, testing method and test procedure, which may result in high variability of results even in well understood methods (Aksoy et al. 2012; Bertuzzi 2017; Bieniawski 1978a; Palmström and Singh 2001). For example, the deformation modulus in a drill-and-blast audit can typically be a third of the one carefully excavated (Palmström and Singh 2001). Due to high variability of results, a large number of in-situ tests are required to evaluate the rock mass deformability by a mean value. Therefore, a good characterisation of the rock mass may provide a comparable or even better estimation of the deformation modulus using rock mass classification systems. Empirical relationships have been proposed to estimate the rock mass deformation modulus using classification systems such as the rock mass rating (RMR), the rock quality designation (RQD), the tunnelling quality index (Q) and geological strength index (GSI). However, selection of an appropriate equation among the large number of empirical relationships has been made as a challenge for practical rock engineers.

This paper aims to investigate the reliability of empirical equations in estimation of the deformation modulus. The manuscript combines a review of empirical methods and statistical analyses for evaluating the predictive performance of these methods. A comprehensive review of empirical approaches in estimation of the deformation modulus is provided in the Section "Review of empirical approaches". Results of 99 plate jacking tests undertaken at three dam and hydropower project sites in Iran are then statistically analysed, and the predictive performance of empirical methods are evaluated.

Review of empirical approaches

As noted in a previous section, a large number of empirical equations are available for estimation of the deformation modulus. A review of empirical relationships is presented in Table 1. These empirical equations are classified based on their input rock mass classification system in Table 2. Although these equations are simple and cost-effective, some uncertainties exist in the reliability of relationships which depends on the number and quality of employed data. Input parameters of these equations should be determined in a quantitative approach and a large database of reliable in-situ measurements are required for comparison between the measured and predicted values.

Some effort has been undertaken in recent years to develop new empirical models for predicting a deformation modulus using neural networks, neuro-fuzzy modelling, Bayesian models and support vector regression (Alemdag et al. 2016; Fattahi 2016; Feng and Jimenez 2015; Gokceoglu et al. 2004; Nejati et al. 2014; Radovanović et al. 2017; Rezaei et al. 2015). However, these models suffer from the lack of physical logic in relating the modulus to input parameters in an analytical form and also do not use a parametric approach, unlike the statistical methods (Nejati et al. 2014).

As shown in Table 2, more than forty empirical equations have been developed by researchers for estimation of the deformation modulus in the last 50 years, and the number of these equations has been increased considerably in recent years. However, practical rock engineers have encountered the question which equation(s) should be used for estimation of the deformation modulus, and which one provides the most reliable result. In the following section, the reliability of empirical equations are evaluated based on the in-situ measurements in Iran. The relationships noted in Table 2 with input parameters that can be acquired easily and widely mentioned in the literature are used for the analysis.

Methodology

As noted in the Section "Introduction", previous studies have shown that the plate jacking test provides the most reliable approach for in-situ measurement of the deformation modulus. To investigate the ability of empirical methods in estimation of deformation modulus, 99 plate jacking tests data from three dam and hydroelectric project sites in Iran were provided.

The Bakhtiary dam and hydropower project are located downstream of the Bakhtiary river in Lorestan province, in the southwest of Iran in the Zagros Mountains. It includes a 325 m tall double-curvature concrete arch dam and a hydropower plant as an underground powerhouse complex with a total capacity of 1500 MW. The dam site is located in the marly and siliceous limestone beds of the Middle Cretaceous Sarvak formation in the north-western part of the folded Zagros. Laboratory experiments undertaken on intact core samples indicate that the saturated uniaxial compressive strength is 109 ± 29 MPa and the tangential elastic modulus is 69 ± 10 GPa.

The Karun III dam site is located on the Karun river in the western Zagros mountain range, in the north-east of the Khuzestan province, southwest of Iran. The Karun III dam is a 205 m high, double-curvature concrete arch dam and the underground power plant includes 8×285 Francis-type turbines. The dominant geological features in this region are Asmari and Pabdeh formations which consist of folded Oligocene and Miocene-age sedimentary rocks. The

Table 2 Classification of empirical methods based on their input parameters

Classification System		Researchers	Equation
RMR	Only RMR	Bieniawski (1978a)	$E_{rm} = 2RMR_{76} - 100$
		Serafim and Pereira (1983)	$E_{rm} = 10^{\left(\frac{RMR_{76}-10}{40}\right)}$
		Mehrotra (1992)	$E_{rm} = 10^{\left(\frac{RMR-20}{38}\right)}$
		Aydan et al. (1997)	$E_{rm} = 9.7 \times 10^{-6} RMR^{3.54}$
		Read et al. (1999)	$E_{rm} = 0.1\left(\frac{RMR}{10}\right)^{3}$
		Diederichs and Kaiser (1999)	$E_{rm} = (7 \pm 3)\sqrt{Q'},\ Q' = 10^{\left(\frac{RMR-44}{21}\right)}$
		Ramamurthy (2004)	$E_{rm} = E_i exp\left(\frac{RMR-100}{17.4}\right)$
		Gokceoglu et al. (2003)	$E_{rm} = 0.0736\ exp\ (0.0755RMR)$
		Chun et al. (2006)	$E_{rm} = 0.3228\ exp\ (0.0485RMR)$
		Isik et al. (2008b)	$E_{rm} = (6.7RMR - 103.06) \times 10^{-3}$
		Mohammadi and Rahmannejad (2010)	$E_{rm} = 0.0003RMR^3 - 0.0193RMR^2 + 0.3157RMR + 3.4064$
		Kang et al. (2013)	$E_{rm} = 10^{\left(\frac{10RMR-16}{50}\right)}$
		Khabbazi et al. (2013)	$E_{rm} = 9 \times 10^{-7} RMR^{3.868}$
		Kavur et al. (2015)	$E_{rm} = 4^{\left(\frac{RMR-20}{20}\right)}$
		Alemdag et al. (2015)	$E_{rm} = 0.058\ exp\ (0.0785RMR)$
	RMR and E_i	Nicholson and Bieniawski (1990)	$E_{rm} = 0.01E_i\left(0.0028RMR^2 + 0.9exp\left(\frac{RMR}{22.82}\right)\right)$
		Mitri et al. (1994)	$E_{rm} = 0.5E_i\left(1-cos\left(\frac{\pi \times RMR}{100}\right)\right)$
		Sonmez et al. (2006)	$E_{rm} = E_i 10^{\left[\left(\frac{(RMR-100)(100-RMR)}{4000exp\left(\frac{-RMR}{100}\right)}\right)\right]}$
		Galera et al. (2007)	$E_{rm} = E_i exp\left(\frac{RMR-100}{36}\right)$
		Shen et al. (2012)	$E_{rm} = 1.14E_i exp\left(-\left(\frac{RMR-116}{41}\right)^2\right)$
GSI	Only GSI and Hoek-Brown parameters	Hoek and Brown (1997)	$E_{rm} = \sqrt{\frac{c}{100}}10^{\left(\frac{GSI-10}{40}\right)}$
		Hoek et al. (2002)	$E_{rm} = \left(1-\frac{D}{2}\right)\sqrt{\frac{c}{100}}10^{\left(\frac{GSI-10}{40}\right)}$
		Gokceoglu et al. (2003)	$E_{rm} = 0.1451\ exp\ (0.0654GSI)$
		Sonmez et al. (2004)	$E_{rm} = E_i(S^a)^{0.4}$
		Hoek and Diederichs (2006)	$E_{rm} = 100\left[\frac{1-\frac{D}{2}}{1+exp\left(\frac{75+25D-GSI}{11}\right)}\right]$
		Isik et al. (2008b)	$E_{rm} = 5.47 \times 10^{-3} GSI$
			$E_{rm} = \sqrt[3]{\sigma_c} tan\left(\sqrt{1.56 + (lnGSI)^2}\right)$
		Beiki et al. (2010)	$E_{rm} = 0.0222GSI^2 - 2.1172GSI + 54.24$
		Sanei et al. (2013)	
GSI, Ei or RQD		Hoek and Diederichs (2006)	$E_{rm} = E_i\left[0.02 + \frac{1-\frac{D}{2}}{1+exp\left(\frac{60+15D-GSI}{11}\right)}\right]$
		Beiki et al. (2010)	$E_{rm} = \sqrt[3]{RQD} log(\sigma_c).tan(ln(GSI))$
Q		Grimstad and Barton (1993)	$E_{rm} = 25logQ$
		Barton (1996)	$E_{rm} = 10Q_c^{1/3},\ Q_c = Q \times \frac{\sigma_c}{100}$
		Palmstrom and Singh (2001)	$E_{rm} = 8Q^{0.4}$
		Ajalloeian and Mohammadi (2014)	$E_{rm} = -0.016Q^2 + 1.581Q + 0.961$
		Kang et al. (2013)	$E_{rm} = 10^{(0.32logQ + 0.585)}$
RMi		Palmstrom (1995)	$E_{rm} = 5.6RMi^{0.375},\ RMi = 10^{\left(\frac{RMR-40}{15}\right)}$
		Palmstrom and Singh (2001)	$E_{rm} = 7RMi^{0.4}$
RQD		Conn and Merritt (1970)	$E_{rm} = E_i(0.0231RQD - 1.32)$
		Gardner (1987)	$E_{rm} = E_i(0.0231RQD - 1.32) \quad RQD > 64\%$
			$E_{rm} = 0.15E_i \quad RQD < 64\%$

Table 2 (continued)

Classification System	Researchers	Equation
	Kayabasi et al. (2003)	$E_{rm} = 0.135\left[\frac{E_i\left(1+\frac{RQD}{100}\right)}{WD}\right]^{1.1811}$
	Gokceoglu et al. (2003)	$E_{rm} = 0.001\left[\frac{\left(\frac{E_i}{c}\right)\left(1+\frac{RQD}{100}\right)}{WD}\right]^{1.5528}$
	Zhang and Einstein (2004)	$E_{rm} = E_i 10^{(0.0186RQD-1.91)}$
	Chun et al. (2009)	$E_{rm} = \frac{5.992Depth^2+1.883\sigma_c^4+4.851RQD^3+0.031JS^5+2399.530JC}{10000}$
	Nejati et al. (2014)	$E_{rm} = 7.192 + 0.06469UCS + 0.20481RQD + 0.30974JSR + 0.38384JCR + 0.01716GWR$
Other parameters	Ramamurthy (2001)	$E_{rm} = E_i \exp(-0.0115J_f)$
	Martins and Miranda (2012)	$E_{rm} = -8.1372 + 0.10005Depth + 0.6435UCS + 1.11458JC$
	Kallu et al. (2015)	$E_{rm} = exp\,(-0.731 + 0.08465SF + 0.382\ln(BS) + 0.134RF + 0.157IF)$

sediments are underlain by limestone and argillaceous sediments in this region and the project structure was founded on limestone and marly limestone.

Karun I dam and hydropower project is located 120 km downstream of the Karun III dam. The dam is a 200 high double-curvature concrete arch dam and the dam site houses two power stations with a combined generating capacity of 2000 MW. The underground power house No. 2 is located in the Asmari formation which is comprised of thick beds of limestone with the average uniaxial compressive strength of 100 MPa.

The histograms of in-situ measurements at project sites are illustrated in Fig. 1. The measured deformation modulus was in the range of 1 to 54 GPa and RMR values varied from 32 to 77. The RMR value for each plate jacking was measured, and the relationships between the RMR and the measured deformation modulus, as well as RMR and normalised deformation modulus (E_{rm}/E_i) are shown in Fig. 2, which show sigmoid shapes in both graphs. To investigate the ability of empirical methods in estimation of the deformation modulus, forty-one relationships were chosen. These equations were selected based on their input parameters, which are well-defined and can be acquired easily without any special difficulty for their estimation (Clerici 1993). For cases where Q, RMi or GSI values were not available in the filed measurements or there was a need for conversion between classification systems, these parameters were determined using the following well-known equations;

$$Q = 10^{\left(\frac{RMR-50}{15}\right)} \tag{1}$$

$$GSI = RMR_{76}\ or\ GSI = RMR_{89}-5 \tag{2}$$

$$RMi = 10^{\left(\frac{RMR-40}{15}\right)} \tag{3}$$

Five statistical approaches were employed to evaluate the predictive performance of these empirical methods. Root mean square error (RMSE) measures the departure of estimated values from the measured values and shows both bias and precision where low RMSE value indicates high predictive ability. RMSE is calculated as follows:

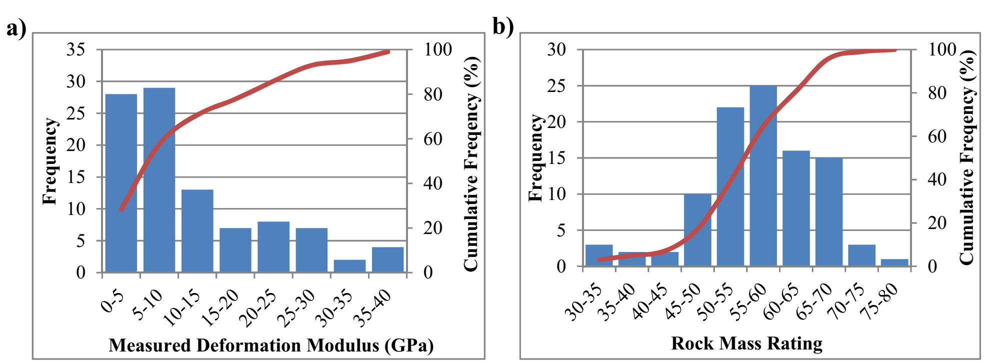

Fig. 1 Histograms of filed measurements: a) Measured deformation modulus in plate jacking tests and b) Rock mass rating (RMR)

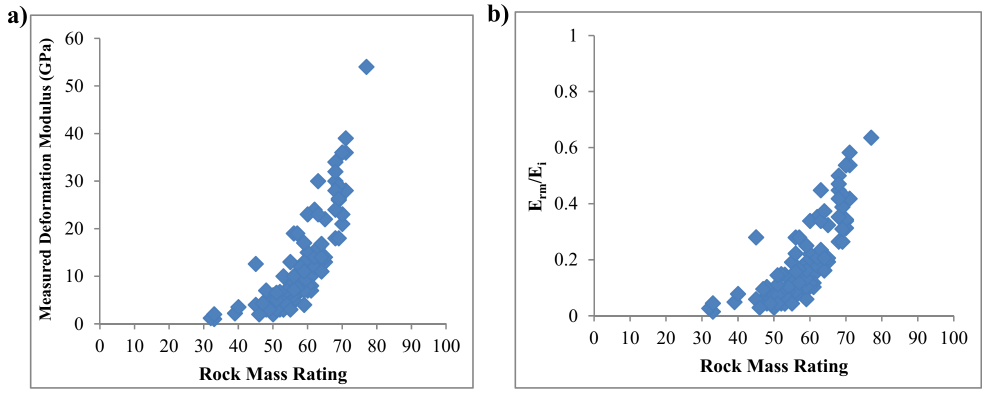

Fig. 2 The relationship between the measured deformation modulus and the rock mass rating (RMR): a) Measured deformation modulus versus RMR and b) Normalised deformation modulus (E_{rm}/E_i) versus RMR

$$RMSE = \sqrt{\frac{1}{N}\sum\left(A_{i\ meas} - A_{i\ pred}\right)^2} \quad (4)$$

where N is the number of data points, $A_{i\ meas}$ and $A_{i\ pred}$ are the measured and predicted deformation modulus, respectively.

Table 3 Evaluation of the predictive performance of empirical methods in estimation of the deformation modulus

Classification System		Researchers	RMSE	VAF	MAPE	R^2	F
RMR	Only RMR	Bieniawski (1978a)	6.40	**87.91**	**36.59**	69.25	124.03
		Serafim and Pereira (1983)	5.70	70.57	60.20	79.62	106.48
		Mehrotra (1992)	6.25	66.68	44.48	79.99	124.78
		Aydan et al. (1997)	7.42	77.34	100.10	77.50	149.29
		Read et al. (1999)	9.30	75.21	134.00	75.57	92.47
		Ramamurthy (2004)	9.74	47.61	42.83	77.12	131.92
		Gokceoglu et al. (2003)	8.93	57.31	**38.77**	**81.25**	**239.12**
		Chun et al. (2006)	10.97	34.02	44.46	78.25	63.02
		Isik et al. (2008b)	16.18	0.89	96.19	63.75	12.25
		Mohammadi and Rahmannejad (2010)	6.55	**79.94**	85.43	79.95	165.50
		Kang et al. (2013)	9.72	39.92	41.06	77.81	53.98
		Khabbazi et al. (2013)	9.79	47.17	41.08	78.47	190.00
		Kavur et al. (2015)	**5.67**	**80.77**	71.99	**80.84**	187.67
		Alemdag et al. (2015)	9.18	56.94	**40.74**	**81.40**	**264.84**
	RMR and E_i	Nicholson and Bieniawski (1990)	6.72	61.22	86.62	71.03	58.02
		Mitri et al. (1994)	28.89	45.41	431.70	55.28	26.05
		Sonmez et al. (2006)	**5.32**	74.43	47.64	78.15	192.14
		Galera et al. (2007)	10.54	60.57	176.40	64.95	26.11
		Shen et al. (2012)	5.76	71.49	43.91	77.55	206.67
GSI	Only GSI and Hoek-Brown parameters	Hoek and Brown (1997)	5.72	70.24	59.28	78.06	114.47
		Hoek et al. (2002)	**5.66**	71.88	65.17	77.11	119.51
		Gokceoglu et al. (2003)	10.93	41.20	47.18	**80.50**	157.70
		Sonmez et al. (2004)	11.38	59.31	183.81	61.66	32.48
		Hoek and Diederichs (2006)	**4.98**	77.65	**39.38**	**80.44**	**273.79**
		Isik et al. (2008b)	16.18	0.73	95.97	63.75	6.42
		Beiki et al. (2010)	9.10	42.03	44.62	73.63	58.54
		Sanei et al. (2013)	11.07	28.89	76.85	46.18	26.85
	GSI, Ei or RQD	Hoek and Diederichs (2006)	12.44	68.38	162.49	73.06	123.04
		Beiki et al. (2010)	7.94	47.82	46.25	71.29	51.23
Q		Grimstad and Barton (1993)	6.14	73.03	50.50	72.63	**222.08**
		Barton (1996)	6.88	73.87	101.36	76.53	84.24
		Palmstrom and Singh (2001)	5.77	73.41	62.21	79.51	59.38
		Ajalloeian and Mohammadi (2014)	**5.67**	**79.27**	**31.86**	79.45	**433.61**
		Kang et al. (2013)	10.64	36.20	43.08	78.35	65.49
RMi		Palmstrom (1995)	6.72	**78.48**	97.52	79.62	106.48
		Palmstrom and Singh (2001)	10.97	18.08	187.82	35.75	27.16
RQD		Conn and Merritt (1970)	18.59	3.02	123.36	49.20	60.30
		Gardner (1987)	14.21	9.49	128.58	53.20	81.66
		Kayabasi et al. (2003)	8.94	24.05	132.02	44.06	3.01
		Gokceoglu et al. (2003)	8.69	23.35	108.30	38.33	4.80
		Zhang and Einstein (2004)	9.57	17.11	81.93	55.72	116.57

• For each statistical analysis, five of the empirical relationships with the highest predictive performance are shown in bold

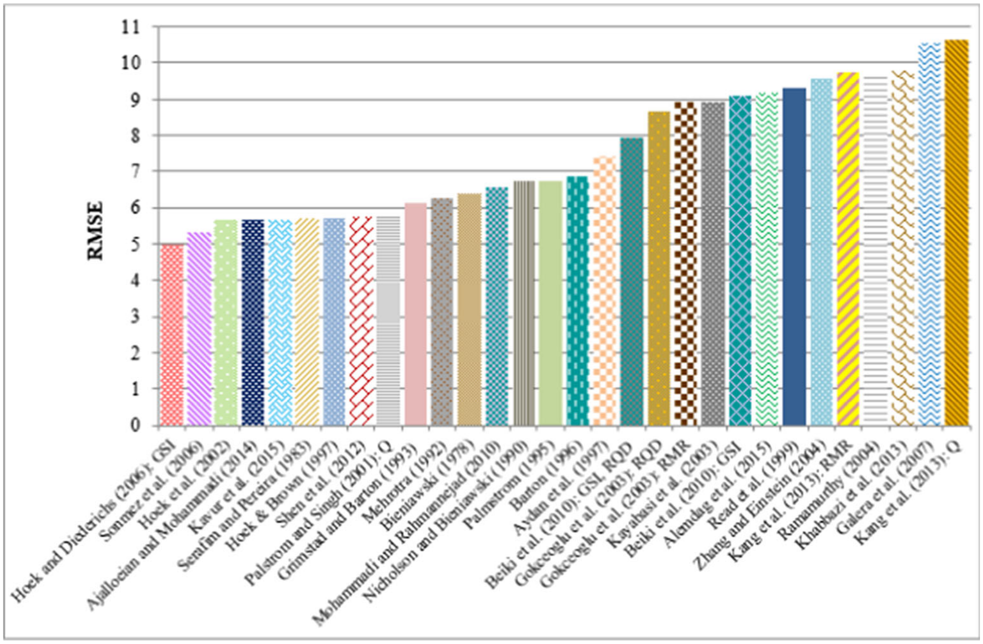

Fig. 3 Evaluation of empirical methods in estimation of the deformation modulus using RMSE

The variance accounted for (VAF) is the other statistical method which is used to measure preciseness of the prediction method, and the one with high VAF denotes high predictive performance for a given dataset. VAF is calculated as follows:

$$VAF = \left(1 - \frac{var\left(A_{meas} - A_{pred}\right)}{var\left(A_{meas}\right)}\right) \times 100 \quad (5)$$

The mean absolute percentage error (MAPE) is a measure of prediction accuracy of a forecasting method and usually expresses the accuracy as a percentage as:

$$MAPE = \frac{1}{N} \sum \left| \frac{A_{i\ meas} - A_{i\ pred}}{A_{i\ meas}} \right| \times 100 \quad (6)$$

Low MAPE value shows high predictive performance. The coefficient of determination (R^2) is a measure of how well the prediction regression equation approximates the measured data points where R^2 of 100% indicates that the prediction regression line perfectly fits the measured data.

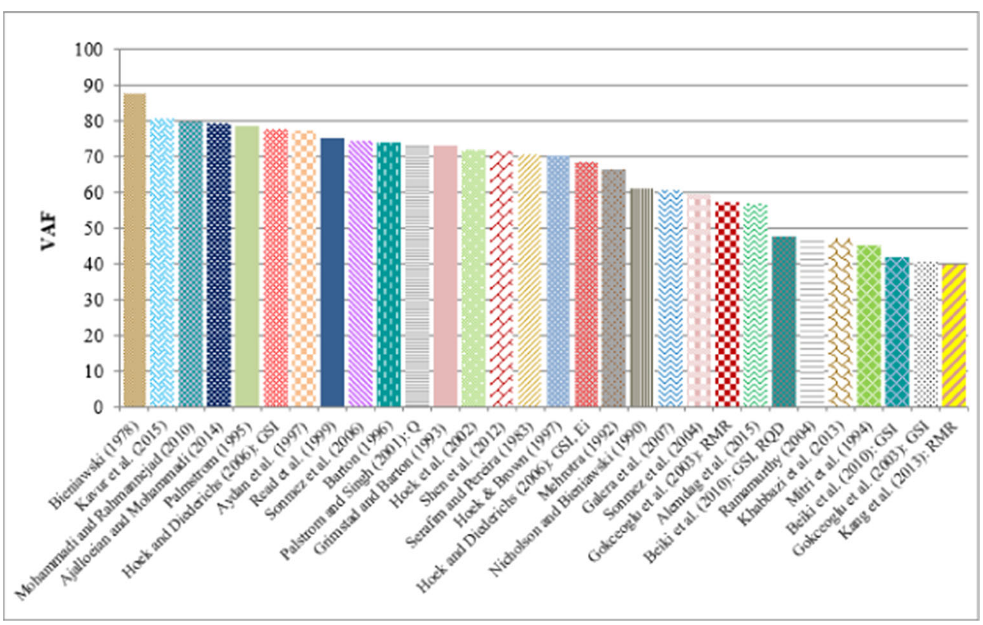

Fig. 4 Evaluation of empirical methods in estimation of the deformation modulus using VAF

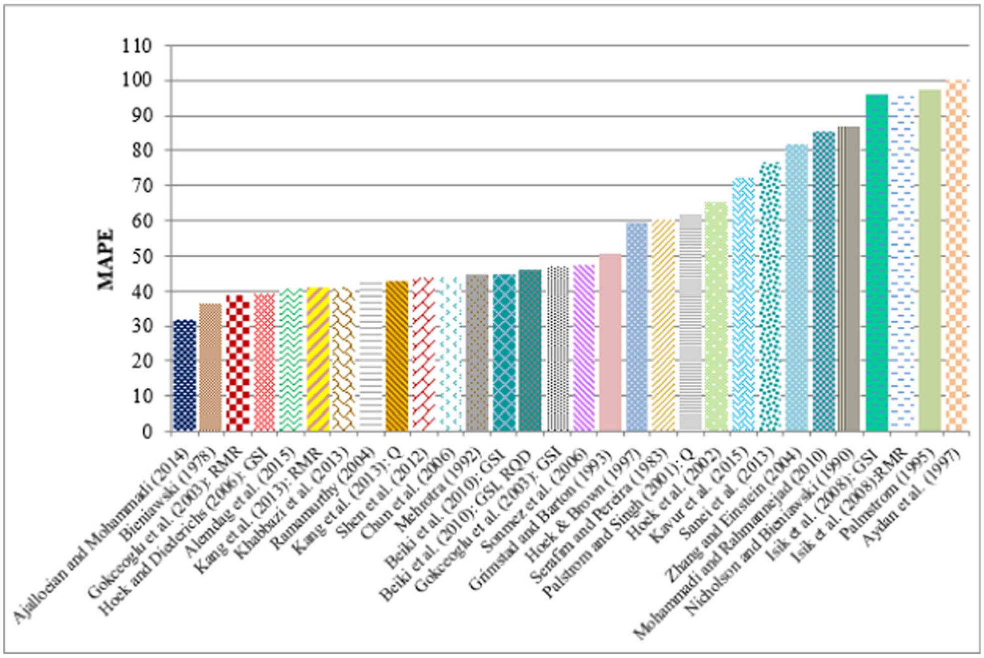

Fig. 5 Evaluation of empirical methods in estimation of the deformation modulus using MAPE

$$R^2 = 100 \times \left\{ \frac{\sum\left(A_{i\ pred} - \overline{A}_{pred}\right)\left(A_{i\ meas} - \overline{A}_{meas}\right)}{\sqrt{\sum\left(A_{i\ pred} - \overline{A}_{pred}\right)^2 \sum\left(A_{i\ meas} - \overline{A}_{meas}\right)^2}} \right\}^2 \tag{7}$$

where $\overline{A}_{pred}$ and $\overline{A}_{meas}$ are the mean of predicted and measured deformation modulus values, respectively.

The F-test is a statistical analysis, which is used to compare the variances of two data sets, and the F-statistic can be used to evaluate the quality of regressions. The high F-statistic indicates that the error is low relative to the predicted value, and the equation with higher F-statistic has better predictive performance for a given dataset. The F-statistic is calculated as follows:

$$F = \frac{var\left(A_{pred}\right)}{\frac{1}{N-2} var\left(A_{meas} - A_{pred}\right)} \tag{8}$$

As the measured rock mass modulus values varies considerably in this study and the F-statistic is strongly influenced by the error of largest values, logarithmic transformation was applied on the input data to address this variation (Kallu et al. 2015).

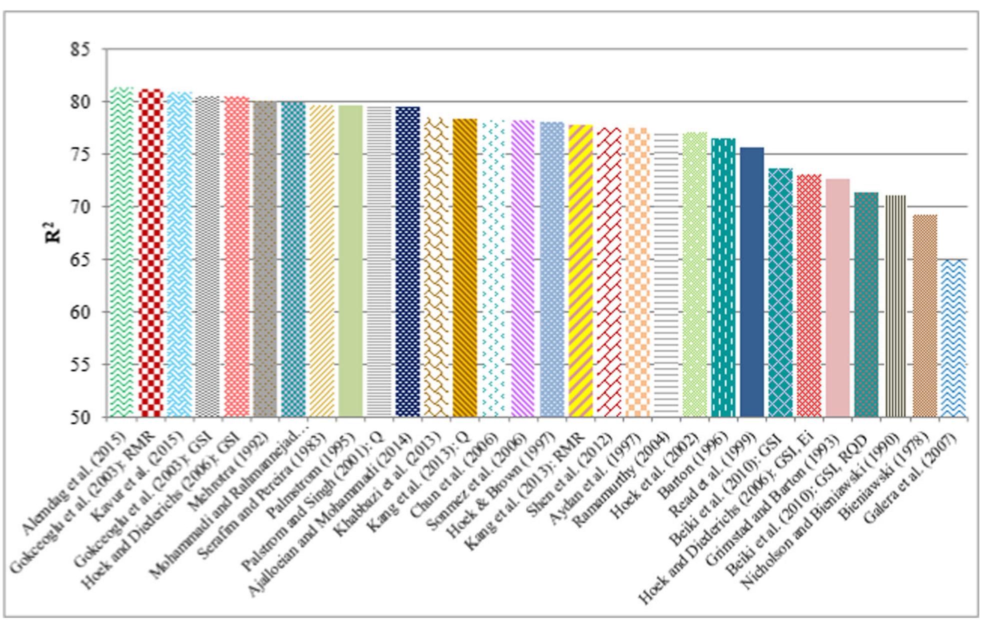

Fig. 6 Evaluation of empirical methods in estimation of the deformation modulus using R^2

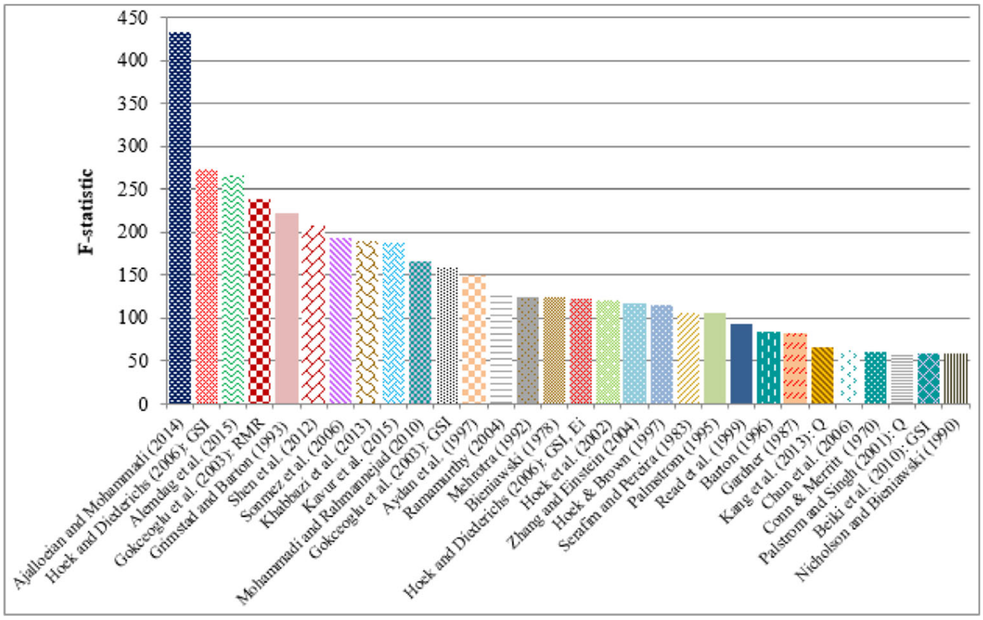

Fig. 7 Evaluation of empirical methods in estimation of the deformation modulus using F-statistics

Results and discussion

The predictive performance of empirical methods was evaluated using five statistical analyses of RMSE, VAF, MAPE, R^2 and F-statistic, and the results are presented in Table 3. Based on each analysis, the calculated statistical values were sorted, and the results are presented in Figs. 3, 4, 5, 6 and 7 (to provide better comparisons in these figures, thirty empirical equations with higher predictive performance are shown). The number of data in statistical analyses for equations of Coon and Merritt (1970), Bieniawski (1978a, b), Grimstad and Barton (1993) and Palmström (2001) were lower than other equations due to limitation of the range of applicability of these equations, as noted in Table 1. In these equations, Mitri et al. (1994) equation tends to overestimate the deformation modulus while Isik et al. (2008a, b) equations underestimate the deformation modulus in most cases.

Comparison of the results show that different empirical equations show the highest predictive performance based on each statistical analysis. However, empirical approaches of Hoek and Diederichs (2006), Ajalloeian and Mohammadi (2014), Alemdag et al. (2015), Kavur et al. (2015) and Gokceoglu et al. (2003) in most of the analyses show high estimation accuracy compared to the

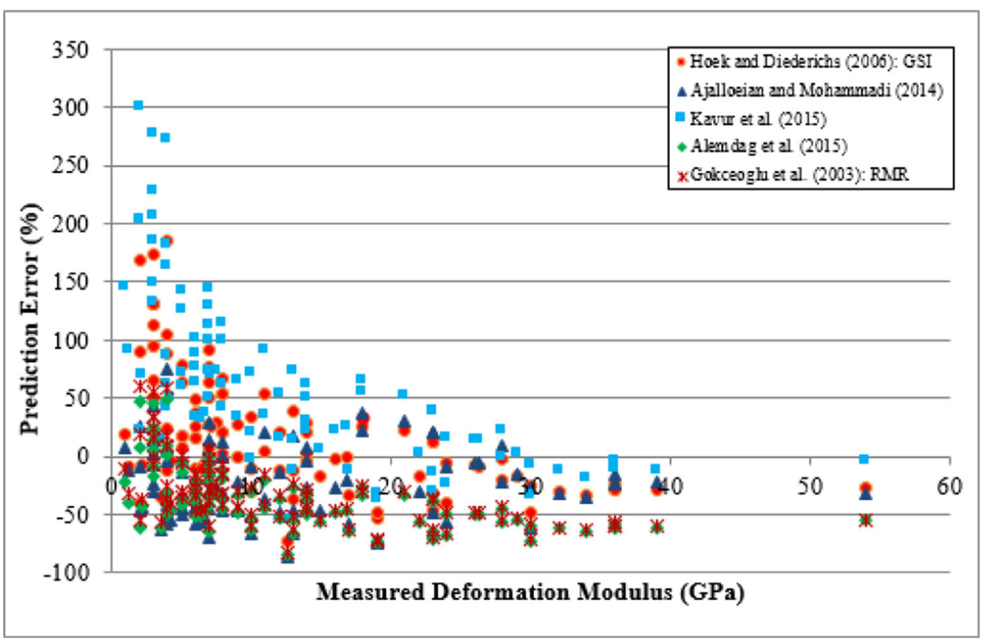

Fig. 8 Prediction error of empirical approaches for each measured data point

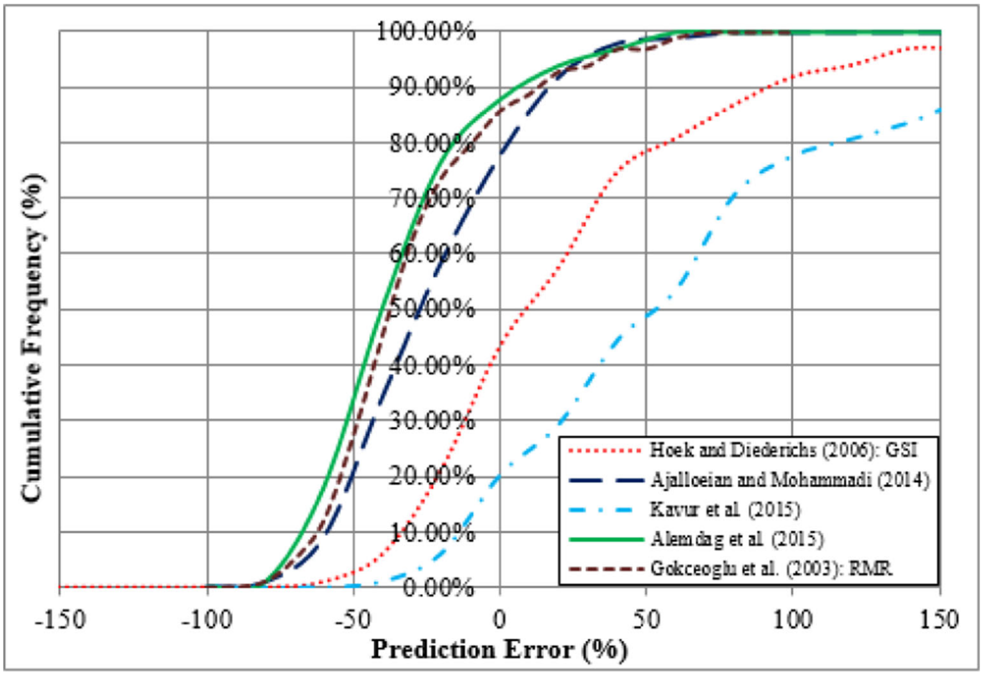

Fig. 9 Cumulative frequency of the prediction error of the selected empirical approaches

others. Therefore, these equations were chosen for detailed analyses.

The prediction error (PE) is a measure of prediction accuracy for each data point and is usually expressed as a percentage as:

$$PE(\%) = \frac{A_{pred} - A_{meas}}{A_{meas}} \times 100 \tag{8}$$

The negative value of PE shows underestimation while its positive value shows overestimation of the deformation modulus. The graph of prediction error for measured data points is depicted in Fig. 8. Comparison of the PE of the selected empirical approaches show that the Ajalloeian and Mohammadi (2014) and Hoek and Diederichs (2006) equations provide the lowest error in estimation of the deformation modulus while the Kavur et al. (2015) equation tends to overestimate and Gokceoglu et al. (2003) and Alemdag et al. (2015) equations tend to underestimate the deformation modulus.

The PE cumulative frequency graph of the selected empirical approaches is shown in Fig. 9. The steep curve of PE cumulative frequency, which is located close to the vertical axis, can be a good representative of prediction precision and accuracy. The Ajalloeian and Mohammadi (2014) and Hoek and Diederichs (2006) equations are the closest curves to the vertical axis. The Ajalloeian and Mohammadi (2014) equation provides the most precise estimation of the deformation modulus (due to steepness of the curve compared to the Hoek and Diederichs (2006)

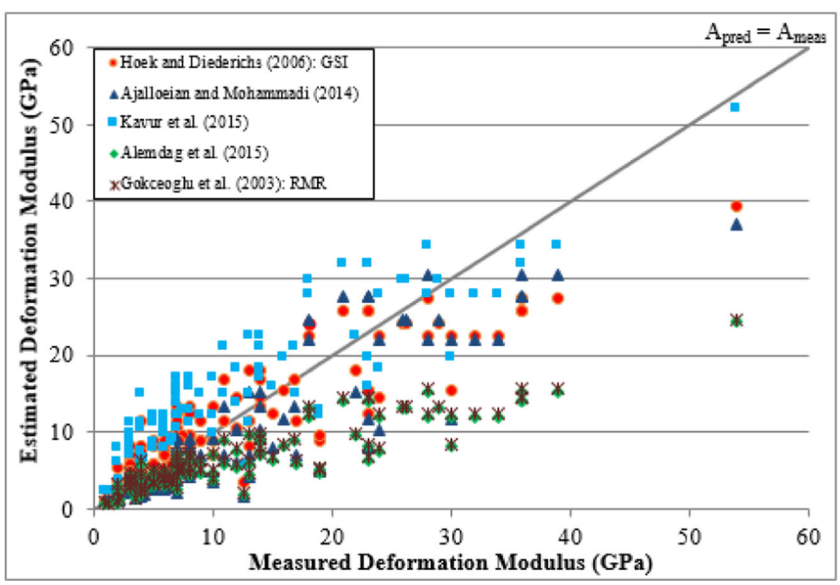

Fig. 10 Predicted deformation modulus versus the measured one for the selected empirical approaches

equation). The vertical axis intercept of the PE cumulative frequency curve can be a measure of the empirical equation trueness. The Ajalloeian and Mohammadi (2014) equation underestimates the deformation modulus in 78% of the data points while the Hoek and Diederichs (2006) equation shows underestimation in 44% of the data points.

The graph of estimated deformation modulus versus the measured one for the selected empirical equations is illustrated in Fig. 10. This graph shows that equation of Kavur et al. (2015) overestimates the deformation modulus while Alemdag et al. (2015) and Gokceoglu et al. (2003) equations underestimate the deformation modulus. Empirical relationships of Hoek and Diederichs (2006), Ajalloeian and Mohammadi (2014) slightly underestimate the deformation modulus based on this graph, but they are on the safe side from practical design considerations.

Conclusions

In this study, the predictive performance of empirical methods in estimation of the deformation modulus was evaluated. There were 99 data of plate jacking tests from three dam and hydropower project sites in Iran provided, and the reliability of empirical equations was evaluated using different statistical analyses. Results of this study clearly show that empirical equations of Hoek and Diederichs (2006) and Ajalloeian and Mohammadi (2014) provide the most precise and accurate estimation of the deformation modulus.

One of the main sources of inaccuracy in empirical estimation of the deformation modulus is related to employed data for developing the empirical equations. Several approaches have been proposed for measurement of the deformation modulus where in some of them there is a need for a correction factor due to small volume of the stressed rock mass and disturbance of the rock near the loading place. The difficulty in interpretation of the experiment results especially in anisotropic rock masses can lead to misleading estimation of the deformation modulus. The other source of inaccuracy in empirical equations is related to the limited number of measurement data in a narrow range of rock mass classification systems which results in uncertainty beyond the range for which these equations have been derived. The Hoek and Diederichs (2006) equation, which is developed based on around 500 data in a wide range of rock characteristics, provides the most reliable estimation of the rock mass modulus based on the results of this study. However, development of new equations based on a wide range of data set in well-characterised rock masses are recommended for future studies to increase the accuracy and precision of the deformation modulus estimation for practical purposes. New improvements in numerical modelling, especially in discrete element modelling, can also be employed to enhance our understanding regarding the effect of rock mass characteristics on the deformation modulus.

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