ORIGINAL PAPER

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New methodology for estimating the shear strength of layering in slate by using the Brazilian test

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Abstract

A new method is proposed in order to estimate the shear strength of schistosity planes in slate in terms of Mohr–Coulomb cohesion and internal friction angle. The procedure consists in carrying out the Brazilian method under different loading-foliation angles, for which experimental tests were achieved in slates from the northwest of the Iberian Peninsula (Spain). The experimental fracture patterns were analytically studied and justified by simulating the stress field in the discontinuity planes contained in the whole sample, taking into account the first failure registered in the tests. By combining experimental and analytical studies and a procedure based on the representation of the threshold state of stresses—in the elastic regime—in the failure plane, it is possible to estimate the foliation's strength envelope through a lineal adjustment according to the Mohr–Coulomb criterion and, thus, to characterize the layering. Finally, the proposed procedure was validated by the direct shear test. The cohesion and the internal friction angle obtained with this convencional test were very close to that calculated by the proposed method, verifying the methodology developed by the authors. This procedure may be interesting in various engineering applications, either in the study of the properties of cleavage in slate, which is commonly used as an industrial rock, or in dam foundations, underground excavations and slope engineering, since one of the main failures in civil engineering is due to sliding along weak planes.

Keywords Brazilian test · Slate · Foliation · Internal friction angle · Cohesion · Strength parameters

Introduction

Slate is a foliated metamorphic rock which shows a texture characterized by a set of parallel well-defined discontinuities, due to the disposition of flat or long grain minerals. These discontinuities, called cleavage or foliation in slates, constitute planes of relative weakness in the rock. This fabric structure affects its mechanical behavior, tending to fail along the discontinuities according to the direction of applied stresses. This characteristic has been taken advantage of in its use as an industrial rock. Slate cleavage allows that, during the roofing slate production process, blocks are mainly hand–split one by one by qualified operators into large, thin, flat sheets of the finished thickness. For this reason, slate is commonly used as roofing or flooring, being an important economic sector in Spain, which is the world leader in roofing slate production. Besides its popular use in building, slate is frequently encountered in various engineering applications, such as dam foundations, underground excavations and slope engineering (Chen et al. 2016). Thus, to establish the mechanical properties of jointed rocks along weak planes is important for the exploitation, design and construction of civil engineering works.

Direct shear testing is the favorable way to characterize the discontinuities. The portable shear box described by Brown and Walton (1975) is one of the most used testing systems in order to calculate the cohesion and the internal friction angle by assuming the Mohr–Coulomb criterion. Once the specimen containing a discontinuity is selected, it is cast in two stages into special molds using mortar or plaster, ensuring that the discontinuity is correctly positioned and aligned. In the main, this device uses jacks to apply normal and shear loads through wire slings. Although the portable shear box method is widely used for testing, many workers prefer to use a shear testing apparatus based on the conventional soil shear box method.

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However, in this apparatus, the capacity loading system is lower and, in many cases, not applicable for jointed rocks (Priest 1993). Konietzky et al. (2012) and Meng et al. (2016) describe alternative devices for shear testing, but these are large-scale direct shear tests and robust devices developed in their own laboratories, which are not accessible for all laboratories testing rock materials.

Another way to study the properties involved in shear behavior is taking into account that the failure mode of anisotropic rocks depends on both the confining pressure and the orientation of the samples' discontinuities. In uniaxial compression tests, when the angle between the orientation of the planes of weakness and the load applied is around 20°–50°, sliding mode failure occurs. In triaxial tests, under higher confining pressures, the sliding mode band narrows and eventually disappears (Asadi and Bagheripour 2015). Jaeger (1960) first studied the variation of the failure behavior with the orientation angles under various confining pressures. He introduced an instructive analysis of the case in which rocks contain a well-defined parallel discontinuity. Many investigations (Duveau and Shao 1998; Tien and Kuo 2001) continued Jaeger's work to express the strength along discontinuities, as well as the proposing of new empirical criteria to determine the strength of transversely anisotropic rocks (Saeidi et al. 2013).

In the same manner, many researchers have studied the fracture pattern of layered slates by using the Brazilian test. Tan et al. (2015) differentiated five types of failure patterns in transversely isotropic rock according to the foliation-loading angle (θ). In their study, cracks are extended along schistosity planes at foliation-loading angles of 0°, 15°, 30° and 45°, and extended into the rock matrix at 90°. In addition, a mixed mode failure type characterized by combined tensile splitting and shear failure occurs at 60° and 75°, meaning that fracturing takes place along the foliation as well as in the rock matrix. Several scholars (Debecker and Vervoort 2009; Yun-Si et al. 2012; Gholami and Rasouli 2014; Xie et al. 2014) have also observed a variation in the failure mechanism in slate depending on the foliation orientation by using the indirect tensile tests.

In the current study, a new method is proposed for determining the shear strength of the weak planes in slate by using the simple Brazilian test. The Mohr–Coulomb criterion is adopted, since it is one of the most widely used in order to predict the shear strength of discontinuities, allowing the determination of both the cohesion and the internal friction angle of the weak planes. Experimental Brazilian tests were carried out in slate under different loading–foliation angles. The fracture pattern was analytically studied by simulating the field stress in the sample. Finally, a new method is proposed to calculate the cohesion and the internal friction angle of the layering, which was validated by direct shear testing.

Experimental procedure

Experimental method

The Soldon slate formation, from Galicia (Spain), has been chosen for this study, which is stratigraphically located in Ordovician levels from the northwest of the Iberian Peninsula. In general terms, this slate is characterized by a mechanical behavior and mineral properties beneficial to its use for roofing (according to the European Standard 2004). In fact, the samples were drilled out of a prismatic block obtained from a factory which commercializes the Soldon slate as building material. In order to obtain this block, during the manufacturing process of the slate tiles, the slaty cleavage and the sedimentation beds were previously identified in the quarry. The intersection of both forms a set of lineations which give the best bending strength for the slate and for this reason the material is sawn lengthwise along these lineations.

Figure 1a shows the primary block used in order to obtain all the specimens for the present study, so similar petrographic
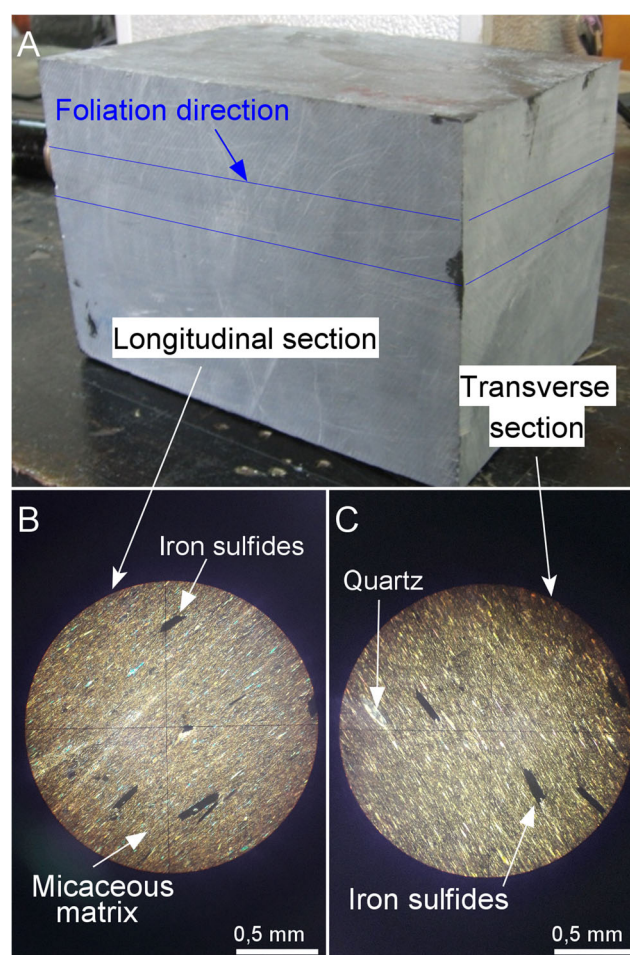

Fig. 1 Schematic of the section's orientations taking into account the slate fabric. **a** Primary block from which all samples tested were obtained. **b** Longitudinal thin section. **c** Transverse thin section

and mineralogical characteristics can be assumed for all the samples tested. Thin sections were prepared following the spatial directions of the block, which correspond with the longitudinal (Fig. 1b) and transverse (Fig. 1c) sections, since both allow the identification of the mineralogy and the fabric of the slate. Petrographical examination of the sections showed a porphyroblastic fabric, in which iron sulfide crystals (such as pyrites) are clearly distinguished from the matrix, which is constituted mainly from mica, quartz and feldspars with a fine grain size varying from c. 0.02 to 0.06 mm. Iron sulfide mineral is characterized by its disposition being elongated following the foliation direction defined by the micaceous matrix. In addition, on a larger scale, this type of slate shows well-defined fissility, displaying a penetrative cleavage, which generates more or less flat surfaces when samples fail along weak planes.

The recommendations of the International Society for Rock Mechanics (ISRM) (1978) and the Spanish Association for Standardisation (UNE) (1990) were strictly followed during sample preparation and testing. Cylindrical cores with a diameter of 50 mm were drilled out of the larger block shown in Fig. 1a, taking into account the slate fabric. In this sense, it was specifically ensured that the direction of coring was parallel to the strike of the weak planes. The cores were then cut into test specimens of c. 25 mm thickness with a diamond saw. The dry density of the slate was measured in each sample, obtaining a value around 2.65 ± 0.05 g/cm^3. With this procedure, 18 samples were obtained, and the orientation of the foliation planes (or layering) was identified in all of them by the alignment of the micaceous minerals and the iron sulfide.

Brazilian tests were carried out by using the uniaxial rock mechanical test system, CONTROLS (loading capacity of 250 kN), and a constant rate of loading (c. 200 N/s) until its failure. Load and vertical displacements were registered with a load cell and a LVDT sensor, respectively. Two steel loading jaws of predefined internal radius of curvature $R_j = 1.5R$ were used, where R is the radius of the specimen and R_j is the radius of the jaw. In general terms, in a rock material such as slate with high stiffness, this creates a contact over a finite arc (2α) of c. 10°. The experimental arrangement for the Brazilian test is shown in Fig. 2, in which the foliation angle, β (the angle between the perpendicular loading direction and the layering), is highlighted. Six different configuration angles of 90°, 85°, 80°, 75°, 60°, and 45° and three samples for each angle β were tested. With this selection of angles, the failure mode will be caused by sliding along the foliation planes, whereas, by using lower than 45° angles, a non-sliding failure mode governed by the rock matrix would have occurred (Tan et al. 2015). Thus, due to the fact that this paper only analyzes the layering, foliation angles, β, between 90° and 45° were chosen. In addition, by recording on video camera, it was possible to visualize the whole failure process.

Experimental results

The experimental research in this paper is focused on how the effect of the foliation orientation influences the failure load and the failure pattern, as well as its location. Fracture patterns observed in laboratory tests for each foliation angle, β, are shown in Fig. 3.

The above tested samples always fractured parallel to the weak planes. Most of the samples fractured along only one weak plane, and furthermore the repetitiveness on both the location of the first fracture plane and the failure load were observed by analyzing the failure pattern for each foliation angle, β. However, some samples also broke along more than just one layering, as happened, for instance, in samples with $\beta = 60°$. Nevertheless, by using the video camera, it was possible to identify the first failure plane, which is the main interest of this paper, since later failure planes are considered as post-breaking. Figure 4 shows the sequence of images recorded during the entire tests of one of the samples tested with $\beta = 60°$, which displays the first fracture plane located at $L = 0.47R$. In this work, the location of the fracture plane (L) is expressed as the radial distance from the center of the sample to the point where the failure is located, measured along the horizontal axis.

Moreover, the first failure was also identified by analyzing the experimental register in each test, since this initial failure is sometimes reflected as a peak in the register. This is shown in Fig. 5, in which the experimental register for a representative test of each foliation angle is shown, being the first failure highlighted. This initial failure corresponds to the threshold of the elastic behavior and the onset of a failure mechanism inelastic, which will not be studied in this paper. Thus, in the present work, the term failure always refers to this peak of initial rupture.

Therefore, once the repetitiveness of the experimental tests has been proven for each foliation angle, β, it is possible to average out the results in each one of them. Table 1 shows the experimental mean results obtained for the location of the first fracture plane ($L_{experimental}$) and the dimensions of the samples (where D is the diameter and t the thickness).

To understand and justify the location of the fracture planes, the effect of the foliation orientation has been studied in the following section from an analytical point of view.

Analytical procedure

Method

In this section, an analytical study has been carried out for determining the normal and shear stresses in the schistosity planes contained in a disc under a biaxial stress field, which is created by the Brazilian method.

Fig. 2 Brazilian test set-up with sample and curved loading jaws. Foliation direction is illustrated, which is identified by the disposition of the micaceous and iron sulfide minerals (not to be mistaken with *dipping lines* produced by the diamond saw during the sample preparation)
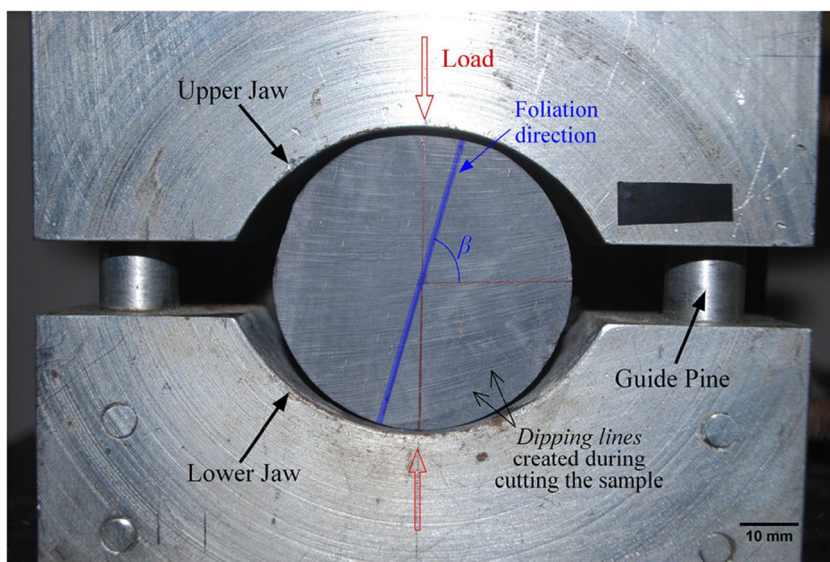
The Brazilian test is the most commonly used testing method to obtain the indirect tensile strength of brittle materials such as concrete, rock, and rock-like materials. In this test, a thin circular disc is diametrically compressed until failure, and the tensile strength is calculated by assuming that the failure is initiated at the center of the sample.

Nevertheless, several researchers have focused on establishing theoretical formulae to predict the elastic stress field in the disc until its initial failure. It was Hondros (1959) who first defined a complete stress solution for the case of a radial load distributed over a short finite circular arc of a disc, obtaining the full stress field formulated with series solutions by using the series expansion technique. Since then, several explicit expressions have been proposed, as for instance the formula developed by Ma and Hung (2008), who continued Hondros's work to obtain the analytical solution through an explicit form instead of using series solutions. Thus, by using the formulation suggested by Ma and Hung (2008), it is posible to calculate the state of stress at each point on the disc, the position of which is defined by r and θ (expressed in the polar coordinate system), with r the radial distance of a point from the origin, and θ the polar angle measured in an anticlockwise direction. This state of stress is given by the radial stress σ_r, the tangential stress σ_θ and the associated shear stress $\tau_{r\theta}$. Moreover, the above work also suggests the calculation of the stress field expressed in terms of principal stress, σ_1 and σ_3.
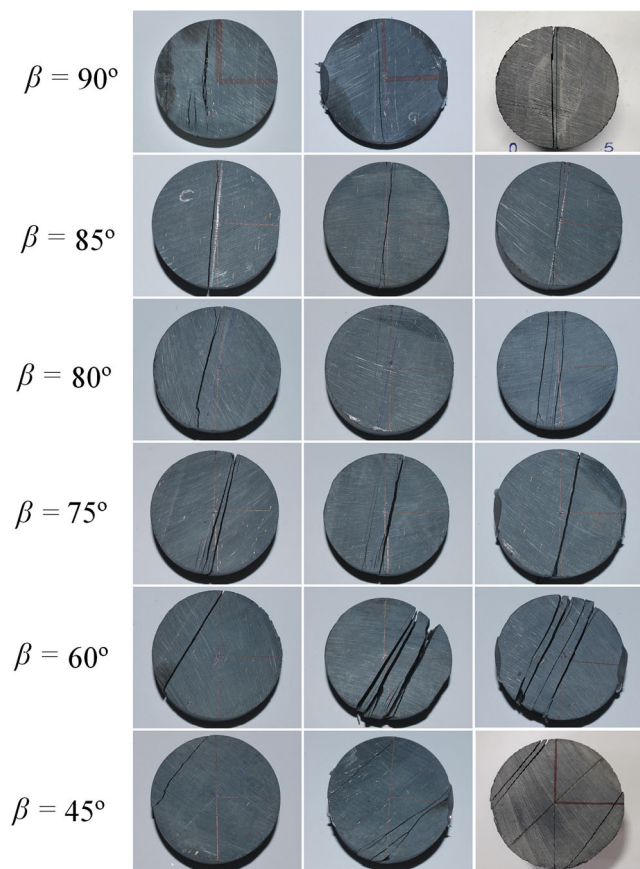
Consider now a planar discontinuity with an orientation, β, and located at a radial distance, L, with respect to the disc's center according to the geometric configuration shown in Fig. 6.

Normal stress (σ_{nn}) and shear stress (τ_{mn}) at any point included in the discontinuity can be obtained by using the formula expressed by Priest (1993):

Fig. 3 Fracture patterns of slate disc samples under different foliation angles

$$\sigma_{nn} = \sigma_r \, sin^2\gamma + \sigma_\theta \, cos^2\gamma + 2\tau_{r\theta} \, sin\gamma \, cos\gamma$$
$$\tau_{mn} = \sigma_r \, cos\gamma \, sin\gamma - \sigma_\theta \, sin\gamma \, cos\gamma + \tau_{r\theta}\left(cos^2\gamma - sin^2\gamma\right) \quad (1)$$
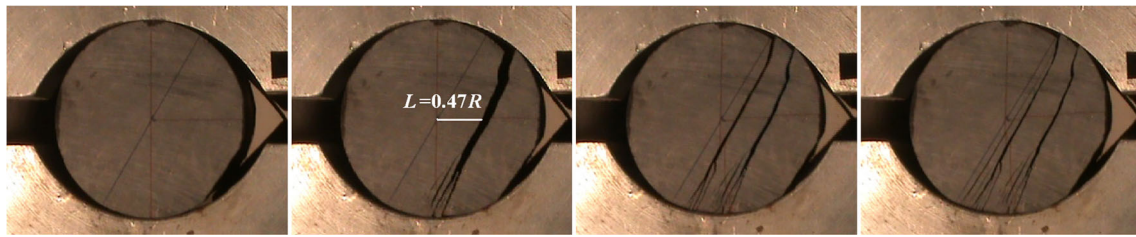
Fig. 4 Sequence of images recorded during the entire test in a sample with $\beta = 60°$

where γ is the angle between the discontinuity and the radial line drawn from the center to the point of interest, and is given by:

$$\gamma = \theta - \beta$$

By combining the expressions developed by Ma and Hung (2008) and Priest (1993), the material can be represented as an elastic and isotropic body subjected to conditions of simple strain, containing well-defined parallel planes of weakness. Furthermore, Eq. (1) implies that normal and shear stresses will vary at each point of the disc according to the foliation angle, β, and, thus, the anisotropic effect of this type of rock is taken into account in the model. The problem can also be confronted by using the constitutive equations of a transversely isotropic medium (Chen et al. 1998), being the layering plane considered as an isotropic plane. However, the theorem and mathematical computation procedures are complex (Chou and Chen 2008), and the problem is usually studied numerically with finite difference or finite element methods. Nevertheless, in order to verify if the approach considered above is adequate, the analytical study will be compared with the experimental one.

To summarize, an algorithm was written in MATLAB, which works similarly to a ubiquitous joint model, meaning that this model states that the failure may occur at any point in

the disc. One set of discontinuity planes was assumed in each foliation angle. First, the algorithm was implemented with the creation of a mesh of equispaced discontinuity planes; secondly, the state of stress in terms of normal and shear stresses in the whole of the disc at the initial failure—in other words, in the threshold of the elastic regime—is calculated according to the foliation angle. For this purpose, the experimental results shown in Table 1 were simulated. Moreover, classical assumptions in Brazilian tests have been considered, as regards the uniform distribution of pressure and 10° of contact angle (2α) created between the jaw and the disc.

Results

The analytical calculation carried out allows the determination of the weak plane in which, from a theoretical approach, failure is reached. A model composed by equispaceous discontinuity planes of 0.05 mm was elaborated for each angle β, taking into account the mean values of the initial failure experimental load, as well as the dimensions of the samples tested (Table 1).

Figure 7a shows the model used for this calculation considering the foliation angle $\beta = 45°$. Figure 7b shows the state of stress for each discontinuity plane. with $\beta = 45°$. in the sample, identifying each one according to the radial location,

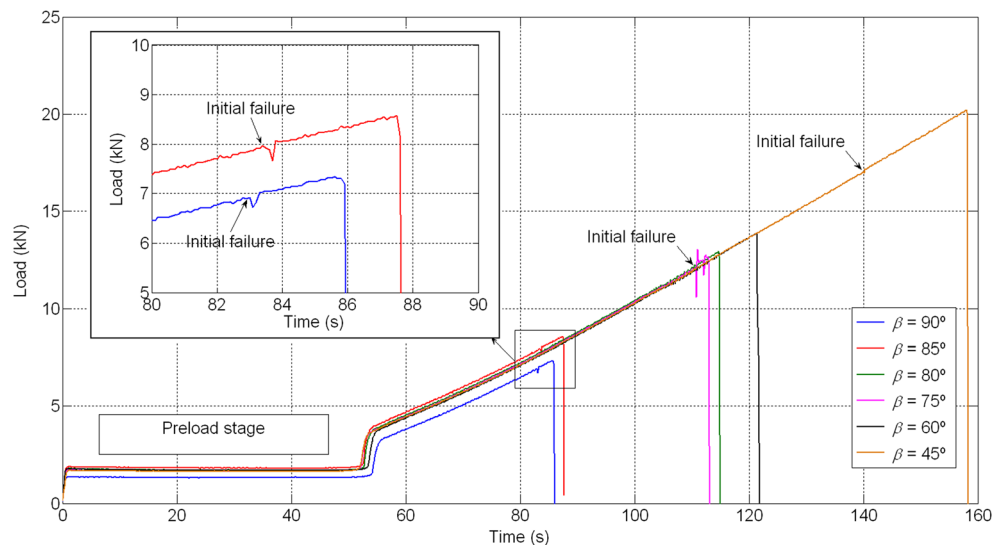**Fig. 5** Experimental register for a representative test of each foliation angle

Table 1 Experimental mean results

β (°)	D (mm)	t (mm)	$L_{experimental}$
90	50.75	19.30	0.11 R
85	50.70	24.63	0.05 R
80	50.73	26.26	0.14 R
75	50.66	24.63	0.17 R
60	50.73	25.23	0.48 R
45	50.70	26.35	0.84 R

L, with respect to the center of the disc. Stress curves are shown in $\sigma_{nn}-\tau_{mn}$, (for visualization purposes, weakness planes have been plotted every 2 mm, even though the calculation is made every 0.05 mm). It can be proven that the stress curve in $L = 0.91R$ (highlighted by a red dotted line) evolves the other stress curves and, also, reaches the maximum shear stress (τ_{mn}).

It is necessary to emphasize that, in the analytical model, elaborated antisymmetry was assumed according to the axis given by the plane of the foliation angle, β, that runs through the center of the sample (from now on, the mid-plane), meaning that, in the example shown in Fig. 7b, the state of stress will be the same at $L = 0.91R$ depending on whether it is measured from the horizontal axis to the left or the right from the center of the disc, while the location of the layers at the same distance above and below the mid-plane just have opposite signs.

This procedure was repeated for every foliation angle tested in the laboratory, identifying in which weakness plane the state of stress at failure evolves the rest of those in the disc. Table 2 shows a thorough detail of the location of the failure

planes coinciding with the cleavage, analytically calculated for each foliation angle.

These analytical results were compared with the experimental ones for verification pruposes. Figure 8 shows the comparison between the analytical results and the experimental fracture pattern according to the foliation angle. In this figure, normal stress (σ_{nn}) and shear stress (τ_{mn}) are represented in the whole of the disc at failure for the experimental failure load values. A black dotted line represents the analitycal failure planes shown in Table 2. The similarity of the location of the analytic failure plane with the failure pattern obtained experimentally is thus proven. Hence, when a slate sample is diametrically compressed, there is a point in the whole of the disc at which the maximum shear stress is reached for each of the foliation angles studied. Analytically, this point contained in each failure plane is located close to the loading area (at a radial distance r from the center of $r > 0.90R$). In general terms, the proximity of this point to the loaded arc is consistent with a recent work of the authors (Garcia-Fernandez et al. 2018) in which the initial failure point in the Brazilian test and the influence of the loading angle are analyzed in terms of principal stresses. Nevertheless, the present work studies the initial failure in terms of normal and shear stresses as well as the effect of the foliation angle, proving that, in the studied slate, the mentioned maximum shear plane is the one to fail, which is proven by the failure experimental patterns. Thus, both analytical and experimental results allow us to state that the failure is due to the fact that the maximum shear stress bearable is reached by the schistosity planes.

The good agreement between the experimental and the analytical results leads us to think that the solution adopted

Fig. 6 Geometric configuration for the stress components in a discontinuity plane included in a disc diametrically compressed
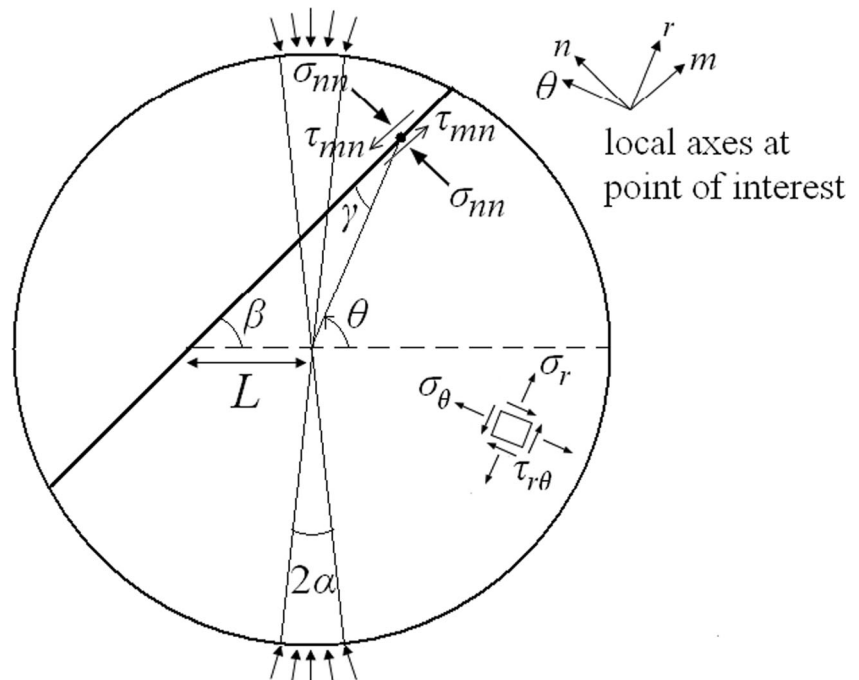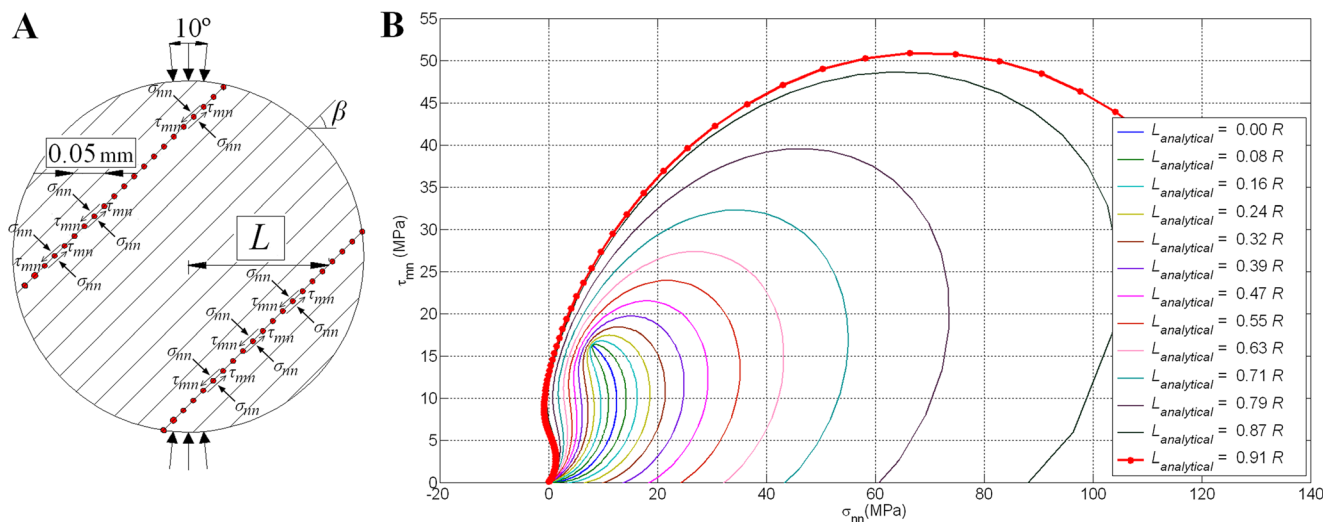
Fig. 7 Calculation of the state of stress in the weakness plane: **a** model used (not scaled); **b** stress curves at $\beta = 45°$

(regarding an elastic and isotropic body subjected to conditions of simple strain, containing well-defined parallel planes of weakness) is adequate, at least for the proposed objective of this work. This is due to the fact that this investigation refers to the initial or the final failure of the elastic regime under a sliding failure mode. The aim of the paper is not to give a full overview of the mechanical behavior of the slate, because, after the initial failure, an inelastic regime may take place. However, in order to establish a simple method to determine the shear strength properties of foliation in the elastic region—in terms of crack initiation strength—the adopted assumptions can be considered as valid according to the results achieved.

Moreover, other important aspects are also drawn out in the present study. Firstly, the justification of failure due to shear stress is specially striking for $\beta = 90°$. Traditionally, when the foliation occurs parallel or perpendicularly to the direction of the load in the Brazilian tests, tensile failure is assumed, either in the discontinuity planes or in the rock matrix, respectively (Dan et al. 2013), also an isotropic case is considered. By taking into account this assumption, the tensile strength of the layering would be 7.90 MPa, which corresponds to the value of σ_{nn} or σ_3 in the center of the disc. However, in order to verify this assumption, central vertical cracks should have been developed in the samples, which was not the case in the samples tested with $\beta = 90°$. In contrast, the fracture patterns

match up with the planes in which the maximum shear stress is reached, so, according to the results shown, when the cracks are not extended along the vertical central diameter in the Brazilian test, caution is required in the calculation of the tensile strength in jointed rocks. Nevertheless, the authors would like to assert that the aim of this paper is not to criticize or undermine the validity of the Brazilian test as a rough measure of the tensile strength (for which uniaxial tensile tests of the foliation should be carried out), but to use the biaxial stress created in the disc through the Brazilian method in order to determine the shear strength parameters of layering in the compression region.

Secondly, the coincidence of the failure plane location and the limit of the load rim in the jaw–disc interface have been proven. In other words, the planes located in the positions summarized in Table 2 (those in which the maximum shear stress is reached) correspond to the plane that limits the loaded area, and that without loading in the contour of the disc. This is schematically shown in Fig. 9, which represents the analytic failure planes for each β (black dotted line), expressing how its location is conditioned by the limit of the load rim (highlithed in red).

This result implies that the location of the failure plane will depend on the contact angle created in the loaded area. In this paper, it has been shown that, by considering a contact angle (2α) of 10° and a uniform distribution of pressure, which are the classical hyphotesis in the Brazilian tests for brittle materials, there is a good correspondance between the analytical and experimental results. Analytically, if the contact angle is modified, the plane in which the maximum shear stress is reached will vary, but will remain located at the limit of the loaded rim. So, this plane will modify its location according to the contact angle, 2α. The variation of this angle 2α may be reached either by modifying the relationship of the curvatures R_j/R, or with another type of jointed rock with different elastic

Table 2 Location of the failure planes analitically obtained ($L_{analytical}$)

β (°)	$L_{analytical}$
90	0.10R
85	0.00R
80	0.10R
75	0.18R
60	0.49R
45	0.91R

Fig. 8 Comparison between
analytical and experimental
results: *left* shear stress; *middle*
normal stress and *right* fracture
patterns obtained in laboratory
tests (*black dotted lines* represent
the location of the failure planes
obtained analytically)
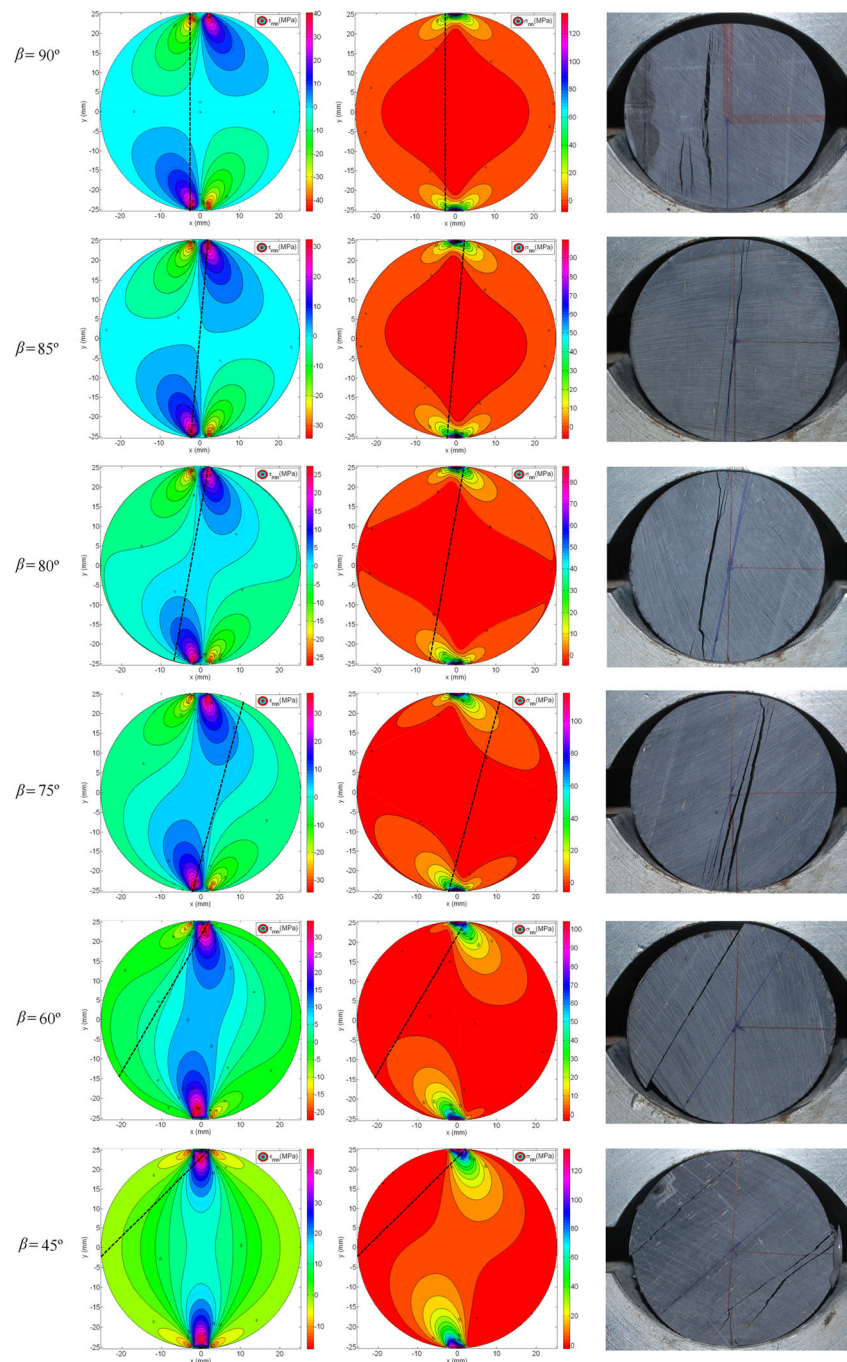
parameters, since the contact angle is conditioned by the relative stiffness between jaw and disc. This could be interesting in order to determine accurately the contact angle created in the jaw–disc interface in the Brazilian test for foliated rocks. Nevertheless, the influence of this parameter 2α will not be quantified because it is out of the scope of this paper.

The above is related to the fact that the failure plane occurs nearly through the center of the sample at $\beta = 85°$. Without a detailed analysis, it could be thought a priori that the failure is in accordance with the classical hypothesis of the Brazilian

test, because it goes through the center of the sample. However, failure is analytically justified due to the combination of the foliation angle and the contact angle applied in the sample. When the condition $90 - \beta = \alpha$ is verified, the plane with the maximum τ_{mn} will go through the center, but the failure will be controlled by shear stress. Analytically, by modifying the contact angle, there is one plane with the maximum shear stress that still runs through the center of the disc: for example, with $2\alpha = 20°$, the plane with the maximum τ_{mn} will go through the center when $\beta = 80°$; with $2\alpha = 30°$, it will
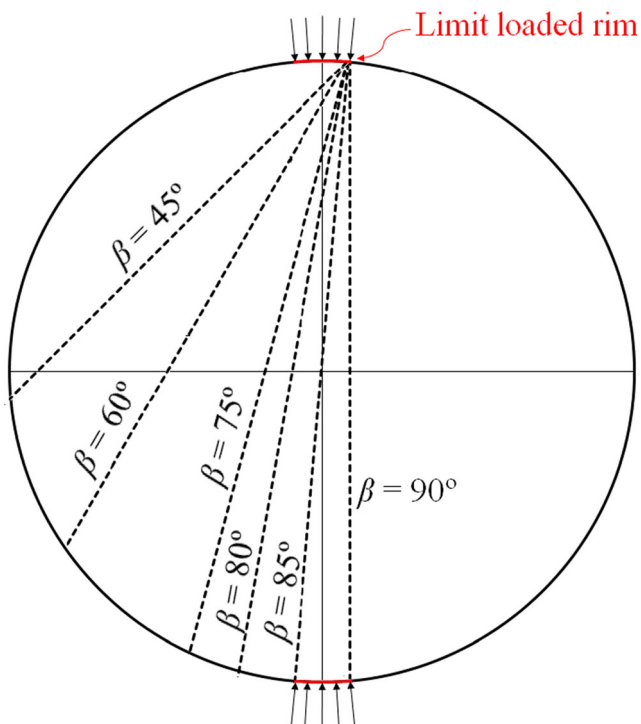
Fig. 9 Location of thefailure planes (*black dotted lines*) according to the foliation angle β

be for $\beta = 75°$; and successively. In further investigations, it will be tested if this tendency is experimentally repeated.

Moreover, it may be taken into account that, although antisymmetry was considered in the analytical model, the experimental reality shows that this condition is not always accomplished, because two antisymmetric failure planes (located at the same distance from the mid-plane) should take place in each foliation angle emerging from the loaded rim limit. However, in laboratory tests, failure is dominated by only one plane. This could be due to several causes: no homogeneity of the specimens, slight variations in the cylindrical geometry of the sample, or the asymmetry both in the contact angle and the distribution of pressure due to the operating mechanism of the test press. This last could be the most likely cause, because these types of machines use a hydraulic cylinder to generate a compressive force. This implies that, when the sample and jaws are in contact, a state of strain is generated which develops a stress field in the disc. For this reason, strictly speaking, forces diametrically opposed are not applied as in the theoretical models, but displacements instead. This generates slight deviations that preclude ideal boundary conditions to those assumed by the classical theory, but acceptable in experimental practice.

In addition, it is important to emphasize that, although only one type of slate was studied in this work, the failure patterns obtained in the laboratory along the weak planes are similar to those observed by other researchers (Tan et al. 2015), who carried out Brazilian tests in slate while modifying the foliation angle. Thus, the results (as regards the initial location failure) shown in this paper are not only exclusive for the type of slate tested but can be extrapolated to other foliated rocks in which failure was induced along the weak planes under the conditions of the Brazilian method tested in this paper (because other conditions as, for instance the use of flat loading platens, could produce different results regarding the crack location).

On the oher hand, a uniform distribution of pressure was considered in the present study, because when the contact angle (2α) created in the jaw–disc is approximately 10°, this type of distribution can be assumed (Hondros 1959). Nevertheless, the consideration of another type of distribution (such as sinusoidal or parabolic) will hardly modify the results presented above. The aim of this paper is not to state the distribution of pressure or the most reliable boundary conditions in the indirect tensile test but to establish a simple procedure in which the mechanic parameters of the foliation planes may be determined by the Brazilian test. So, with this classic assumption, a satisfying analitycal and experimental correlation has been proven by this method.

Determination of shear strength of layering

Method suggested by using the Brazilian test

In order to calculate the shear strength of layering in the studied slate (regarding cohesion and internal friction angle), the Mohr–Coulomb criterion (Mohr 1900) has been adopted, which is one of the most widely used linear failure criterion for planes of discontinuity. This criterion is based on the hypothesis that failure happens when shear stress in any plane reaches the failure shear stress τ_{mn} defined as:

$$\tau_{mn} = C_j + \sigma_{nn} \tan \Phi_j \tag{2}$$

where σ_{nn} is the effective normal stress acting on the failure plane, and C_j and Φ_j are the cohesion and the internal friction angle of the weak planes, respectively. If the criterion is extrapolated to the tension region, several studies recommend interrupting the criterion when a normal stress similar to the tensile strength is reached, making the Mohr–Coulomb a bilineal criterion. For this reason, in order to model the behavior of planes of discontinuity in the tensile stress region, the concept of tension cut-off was introduced by Paul (1961), which constitutes a more realistic approximation as regards the behavior of the rocks in the tension region. Due to the fact that the Mohr-Coulomb is one of the most used simple failure criterion to predict the mechanical behavior of joints at failure, and allows the calculation of these mechanical parameters of discontinuities, an approximation based on this lineal adjustement will be carried out in this work. Nevertheless,

the procedure exposed in this paper is not exclusive for lineal adjustement, since it could be used to establish failure criteria by other approximations such as hyperbolic or parabolic.

According to the analytical study achieved in the previous section, it was possible to obtain the stress curves (σ_{nn}, τ_{mn}) in all the failure planes (by the procedure shown in Fig. 7). Thus, the failure criterion will be the maximum state of stress allowed in the material. A simple calculation of the criterion could be established by averaging the stress curve for each foliation angle, expressing it as a single value, and then determining the lineal regression that best adjusts when considering the results for each β.

By considering a discontinuity plane with an angle β inclined with respect to the horizontal axis, this calculation is similar to decomposing the stresses in the weak plane at failure, and considering both the angle β and the area of the failure plane (A):

$$\sigma_{nn} = \frac{P}{A}\cos\beta$$
$$\tau_{mn} = \frac{P}{A}\sin\beta \qquad (3)$$

Equation (3) shows simple expressions based on the hypothesis that the stress distribution in the inclined failure plane is uniform. By using this method, various researchers have theoretically calculated the triaxial strength of anisotropic jointed rocks, in order to prove that it depends on the angle between the load direction and the discontinuity planes. However, this simple method is not applicable for the purpose of this paper.

This assertion can be deduced by considering the failure at $\beta = 45°$. In this condition, it was experimentally proven that failure occurred at 17.09 kN of load, and analytically the fracture plane was located at $L = 0.91R$. Taking into account that the failure area of this plane is 10.17 cm^2, by applying Eq. (3), values of $\sigma_{nn} = 11.88$ MPa and $\tau_{mn} = 11.88$ MPa are reached. These results are similar to those obtained by averaging the stress curve in the failure plane according to the analytical study ($\sigma_{nn} = 11.92$ MPa and $\tau_{mn} = 11.88$ MPa).

Figure 10 shows this calculation repeated for the state of stress in the failure planes with $\beta = 60°$, 75°, 80° and 85°, in terms of normal and shear stresses. Both the stress curves (continueouslines) for each fracture plane are plotted in this figure according to the foliation angle at failure, and its corresponding averaged values for each one (highlighted as points). Figure 10 shows that the averaged values are considerably below their respective stress curves.

It is important to emphasize that the average values could be used to obtain a conservative failure criterion, which is favorable from the point of view of, for instance, civil engineering support. However, the aim of the present paper is to polish the calculation of the lineal Mohr–Coulomb failure criterion in order to estimate the mechanical parameters both of cohesion and internal friction angle of foliation, so this calculation will not be carried out with the average values, but with the procedure exposed below.

In the previous section, the field stress in the disc diametrically compressed was analyzed for each foliation angle. It was analytically proven that, according to the orientation of schistosity planes, there is a weak plane in which state of stress (in terms of σ_{nn} and τ_{mn}) evolves the others' stress curves corresponding to each discontinuity plane. By analyzing the fracture experimental patterns, it was proven that the failure matches up with the foliation planes with stress curve envelopes.

Figure 11a shows the stress curve in the failure plane at $\beta = 45°$ (orange line) in normal stress versus shear stress. As was mentioned above, each point contained in this stress curve (red circles) represents the state of stress in each point contained in the failure plane, which can also be expressed as Mohr's circles (black lines). From a theoretical point of view, the intersection from each point to its respective center of the Mohr circle defines a radius (highlighted in the dotted line) with an angle 2θ (clockwise) from the abscissa, with θ the angle between the failure plane and the maximum principal stress in the considered point (detailed in Fig. 11b).

As was previously exposed, a common tendency was observed in all the foliation angles tested: samples experimentally failed along the plane that contained the point with the peak of shear stress in the whole of the disc, as was analytically proven. Thus, in order to simplify the procedure, the point at which the maximum τ_{mn} is reached for each orientation β was selected for the calculation of the failure envelope.

By knowing the limit state of stresses for each foliation angle, the calculation of the failure envelope for the schistosity planes in the studied slate was carried out, according to the experimental and analytical studies. The state of stress in the failure in each foliation angle is represented by a Mohr's circle taking into account the above. Figure 12a shows the results regarding the failure of the foliation planes, which is expressed by considering both the peak of shear stress and its respective Mohr's circles. The best-fit line that connects the failure values of normal and shear stress in each angle of foliation studied is termed the Mohr failure envelope. The failure envelope obtained in the compression region (continuous red line) is characterized by a cohesion (C_j) of 11.90 MPa and internal friction angle (Φ_j) of 30.37°. Both values are in typical magnitude orders in slate (Tan et al. 2015), which tends to be defined by values of cohesion between 5.00 MPa and 13.20 MPa, and internal friction angles from 22.60° to 44.40°.

All the failures shown in this paper were justified by shear stress, so the results do not allow the limiting of the tension region (dotted red line in Fig. 12a). Nevertheless, as was previously mentioned, several investigations state that, with $\beta = 90°$, the failure is initiated at the center of the sample, being controlled by the tensile strength along the weak planes. In
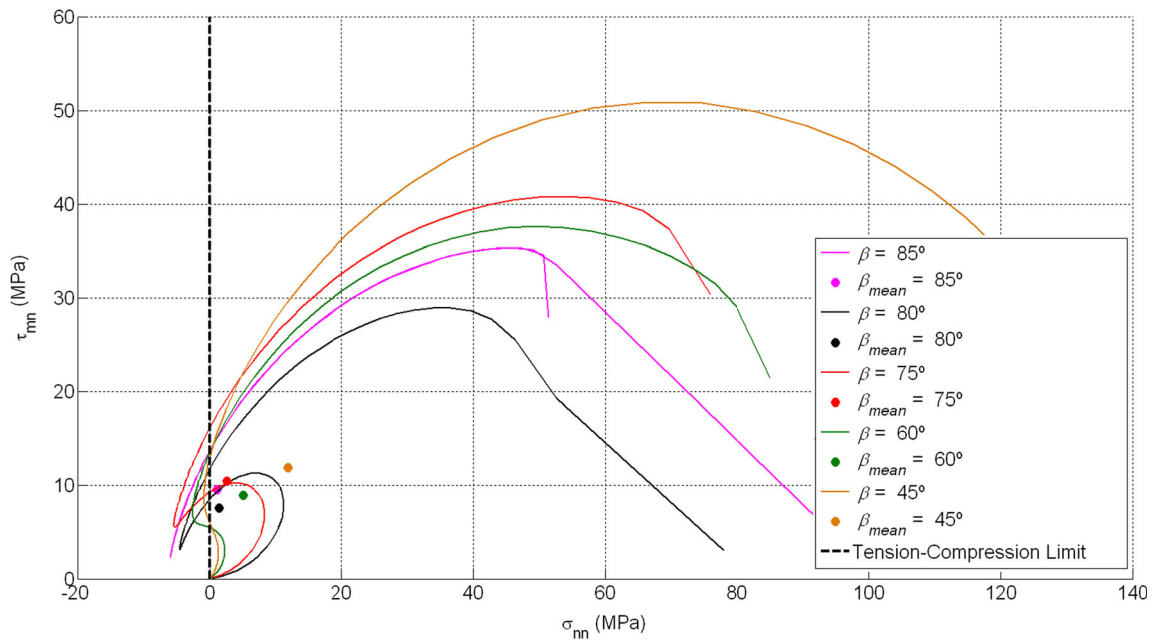
Fig. 10 Comparative between stress curves (*continuous line*) obtained for each failure plane according to β and its correspondant average values (*points*)
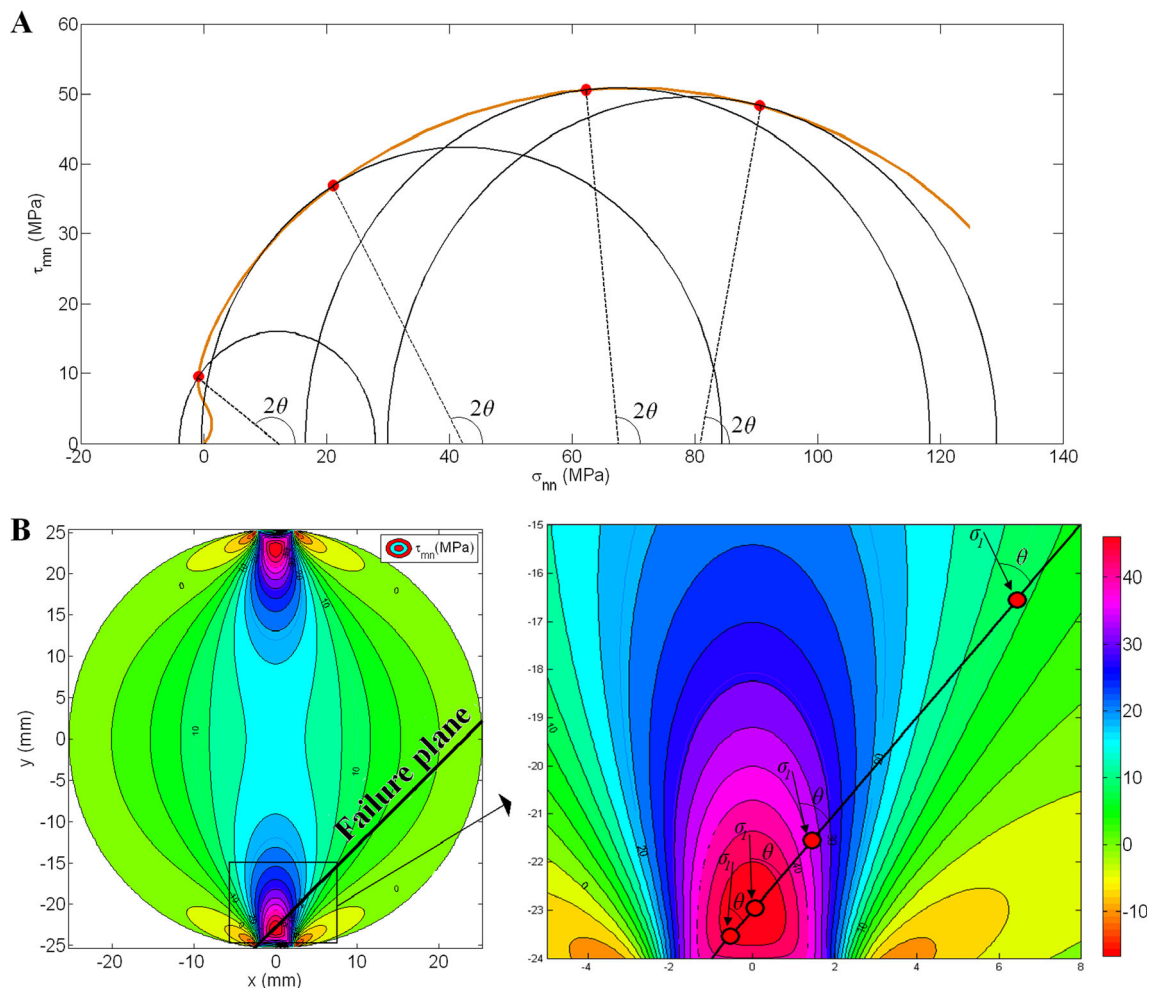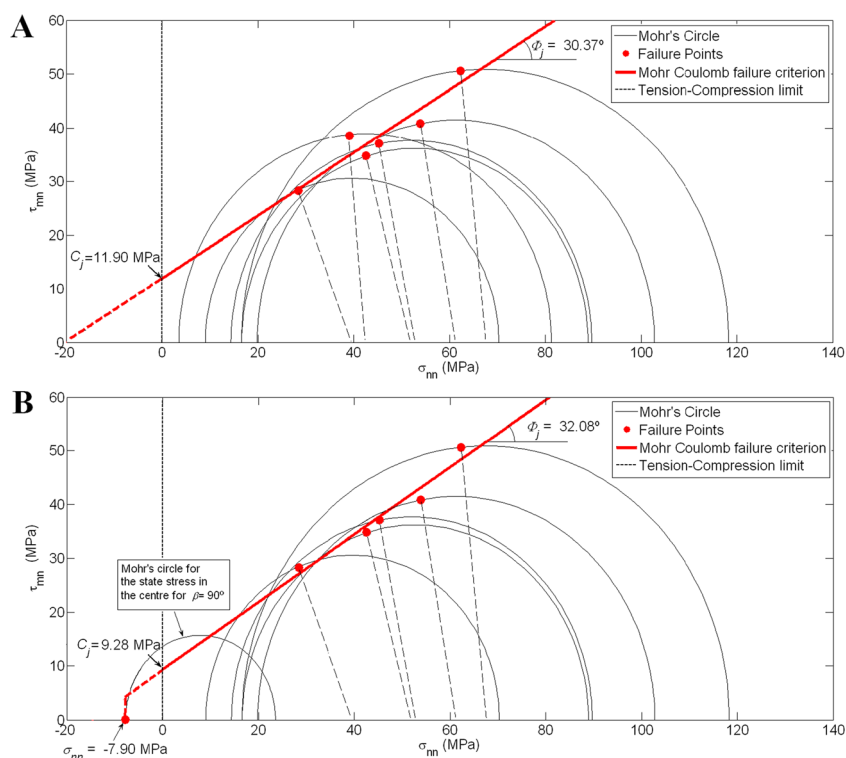

Fig. 11 **a** Stress curve in the failure plane when $\beta = 45°$ and corresponding Mohr circle in four points; **b** angle θ represented in the disc

Fig. 12 Mohr–Coulomb failure criterion obtained with the proposed method. **a** Failure by shear stress is assumed at $\beta = 90°$; **b** Failure by tensile stress in the center of the disc is assumed at $\beta = 90°$

this case, normal stress at the center (which is equal to the minor principal stress σ_3) is assumed as equivalent to the tensile strength of the layering, and is commonly assumed to establish the tension cut-off. For this reason, an additional calculation was carried out by assuming that the failure with $\beta = 90°$ is due to the tensile stress at the center of the specimen. Subsequently, the corresponding Mohr's circle (highlighted in Fig. 12b) was calculated in order to represent the states of stress σ_1 and σ_3 in the center. By considering this hypothesis, the failure envelope obtained in the compression region (continuous red line in Fig. 12b) is characterized by a cohesion of 9.28 MPa, and internal friction angle of 32.08°. Also, in this case, the failure criterion would be limited in the tension region for a value of $\sigma_{nn} = -7.90$ MPa (the negative sign denotes the tensile stress). Thus, the failure envelope in the tension region would be defined by a bilineal criterion (dotted red line in Fig. 12b). Nevertheless, the last calculation was carried out just for comparison purposes, because the called hybrid failure -the transition between extension (tensile) failure and shear failure- remained controversial (Ferrill et al. 2017) and other adjustements would be necessary to carefully develop this region.

Therefore, although there are slight uncertainties regarding the failure mode being $\beta = 90°$, for which further analysis would be needed, the results obtained do not reduce the validity of the procedure exposed in this paper, which could constitute a new simple method in order to determine the Mohr–Coulomb failure criterion and to calculate the mechanical parameters C and Φ in jointed rocks.

Validation through direct shear test

Experimental shear tests were carried out in the studied slate in order to verify the results obtained with the proposed method. The equipment used for the direct shear tests was a portable shear box, which allows the testing of samples with both regular and irregular forms. It is composed of an upper and a lower shear box and a normal and shear load hydraulic systems with a load capacity of 50 kN. The recommendations of the ASTM D5607-16. (2016) and ISRM (2014) were strictly followed during sample preparation and testing. Cylindrical cores with a diameter of 50 mm and height of 110 mm were drilled out of the same block from which the samples for the Brazilian tests were obtained. Before the direct shear tests, the samples were fixed in the two parts of the shear boxes by high-strength cement.

The procedure of this test consists of, first of all, placing the specimen in the shear box; secondly, applying a normal force on its top; and, finally, the shear force is exerted on the sample until failure. By modifying the normal stress applied to the specimen, the shear stress needed in order to break the sample varies. In this work, samples were tested with the direction of foliation parallel to the shear force, thus the mechanical parameters of the weak planes in the slate, such as cohesion and internal friction angle, can be determined.

Figure 13 shows the direct shear test carried out in the laboratory. In Fig. 13a, one of the samples tested is shown before the device assembly, in which the foliation direction
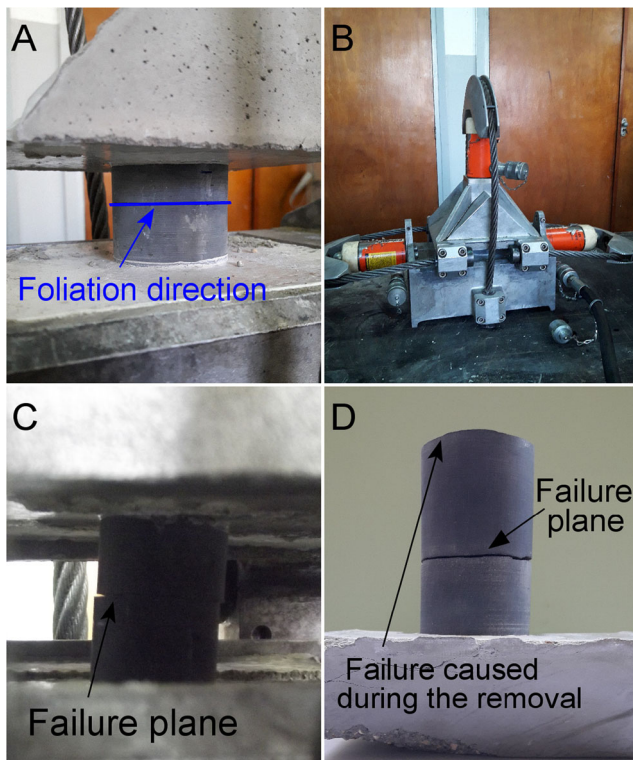
Fig. 13 Direct shear test: **a** slate sample in cement mortar; **b** portable shear box; **c** first failure plane; **d** failure pattern in slate

is marked. Figure 13b displays the portable shear box used. Figure 13c and d represents two of the samples tested in the laboratory. Figure 13c illustrates the moment at which the failure is reached by shear stress in one of the samples, while Fig. 13d shows the failure patterns after the shear test in a second sample. It is important to emphasize that only the normal and shear loads for the first failure were taken into account for the stress calculations, meaning that, in spite of the fact that two failure planes are visualized in the sample (Fig. 13d), only the central failure was considered, as the other took place during the removal of the device.

Thus, according to the procedure exposed in ISRM (2014), cohesion of 7.20 MPa and internal friction angle of 29.50° were obtained in the slate. In general terms, the values obtained are very close to the parameters achieved with the procedure developed by the authors in this paper (Fig. 12).

Although the portable shear box is probably the most widely used method for determining the shear properties of discontinuities, several researchers have also developed their own devices in order to carry out shear tests, which are not approachable for all rock mechanic laboratories. However, the method proposed in this paper could be a simpler procedure achievable by any laboratory, by using the Brazilian test and taking into account the failure pattern along the weak planes according to the foliation angle.

Conclusions

An approach to estimate the Mohr–Coulomb shear strength parameters of the foliation in slate has been presented. The procedure is based on the use of the Brazilian method and the anisotropic behavior of jointed rocks when the orientation of foliation is modified according to the direction of the stresses. Experimental tests in slates from the northwest of the Iberian Peninsula were carried out under different loading–foliation angles, β, in order to determine the initial failure load and the location of the first failure plane. Then, an analytical study of the stress field in the discontinuity planes in the whole of the disc at failure was implemented by considering the experimental results, regarding the failure loads for each angle β, the dimensions of the samples tested, and the conditions for the Brazilian test (such as uniform pressure and loading angle, $2\alpha = 10°$). For this purpose, an algorithm was created in MATLAB, in which the material is represented as an elastic and isotropic body subjected to conditions of simple strain, containing well-defined parallel planes of weakness. By combining the experimental and the analytical study, a procedure to calculate the Mohr–Coulomb failure criterion was established based on determining the maximum shear stress reached in the disc for each angle, β, and its corresponding normal stress (although the procedure could be used to establish failure criteria by other approximations such as hyperbolic or parabolic).

The main conclusions can be summarized as follows:

– Brazilian tests were carried out with six foliation angles, β (90°, 85°, 80°, 75°, 60° and 45°), by using curved loading jaws. All the samples failed along weak planes for the angles tested. By analyzing the failure pattern for each foliation angle, the repetitiveness on both the location of the first fracture plane and the failure load was observed.
– By simulating laboratory results, the analytical study allowed us to determine the weak planes in which the maximum shear stress is reached for each β. The locations of these planes are highly matched with those experimental failure planes, so the solution adopted was adequate at least for the proposed objective of this work. All the failure patterns can be justified due to peak shear stress, which is striking at $\beta = 90°$, since with this foliation angle the failure is traditionally assumed to be caused by the tensile splitting along the foliation. Moreover, the influence of the load angle in the jaw–disc interface was shown, since failure is located in the foliation plane which matchs the limit of the load rim.
– Failure patterns along the weak planes obtained in the slate studied in the laboratory are similar to those observed by other researchers, so the results (regarding the initial location frailure) shown in this paper are not exclusive to the type of slate tested but can be extrapolated to

other foliated rocks in which failure along the weak planes was induced under the conditions of the Brazilian method tested in this paper.

– A methodology was established to determine the Mohr–Coulomb failure criterion by knowing the limit state of stress in the failure plane according to the foliation angle. This methodology is based on determining the peak of shear stress (and its corresponding normal stress) for each β and estimating the strength envelope of foliation through a lineal adjustement. Each peak of shear stress establishes the limit state of stress bearable by the schistosity planes. This procedure was selected after proving that the assumption of the uniform distribution of stresses in the failure plane was too conservative in order to determine the failure criterion, and the common tendency in all the orientations tested to fail along the plane which contains the maximum shear stress. Thus, both a cohesion of 11.90 MPa and an internal friction angle of 30.37° in the studied slate were obtained in the compression region. Nevertheless, an additional calculation was carried out assuming that failure with $\beta = 90°$ is due to tensile splitting in the center of the disc. With this assumption, the strength envelope of foliation in the compression region is characterized by $C_j = 9.28$ MPa and $\Phi_j = 32.08°$. In this case, the failure criterion can be limited in the tension region to $\sigma_{nn} = -7.90$ MPa, taking into account the concept of tension cut-off. Nevertheless, the last calculation was just carried out for comparison purposes, because the study of the hybrid failure is out of the scope of this paper.

The reliability of the procedure proposed was verified by direct shear tests, in which a cohesion of 7.20 MPa and an internal friction angle of 29.50° were obtained in the slate studied. The values obtained are very close to the parameters achieved with the procedure developed by the authors in this paper.

It is important to emphasize that this work assumes an elastic solution for calculating the stress field in the disc. This is due to the fact that this investigation refers to the initial failure or the threshold of the elastic regime, so the term failure always refers to the peak of initial rupture, which can be observed in the experimental register. The aim of the paper is not to give a full overview of mechanical behavior in the failure of the slate, because, after the initial failure, an inelastic regime may take place. However, for small civil engineering works, the method proposed in this paper allows a reasonable estimation of (Mohr–Coulomb) internal friction angle and cohesion values of layering in slate, with the main advantage of being a simple procedure achievable to any laboratory, and low cost in obtaining these mechanical parameters.

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