ORIGINAL PAPER



# The characterization of rock slope stability using key blocks within the framework of GeoSMA-3D

Feili Wang<sup>1</sup> · Shuhong Wang<sup>1</sup> · Muhammad Zaffar Hashmi<sup>2</sup> · Zhanguo Xiu<sup>1</sup>

Received: 11 September 2017 /Accepted: 24 April 2018 /Published online: 4 July 2018 $\circled{c}$  Springer-Verlag GmbH Germany, part of Springer Nature 2018

#### Abstract

Considering the uncertainty and complexity of the influencing factors, the present study focused on the multi-level and multi-index evaluation system for analyzing rock slope stability. Quantitative analysis of the influence degree of the evaluation index on the rock slope stability was carried out by extension theory. The most significant factors affecting rock slope stability and the corresponding evaluation index were obtained. Further, the study presents a concept about the instability characterization coefficient of the key block, which is an important factor controlling slope stability. With this coefficient implemented into the search module of key blocks in the program Geotechnical Structure and Model Analysis-3D (GeoSMA-3D), developed by the corresponding author's team, a further determination and visualization of key blocks were achieved. However, in many previous studies, there was no good correlation between the theoretical key blocks and the actual rock slope engineering, which led to derailment between theoretical analysis and practical engineering. Hence, this paper proposed the characterization safety factor of rock slope stability that combined the instability characterization coefficient with the weight of key blocks. The influence degree of each key block on rock slope stability was determined by the size of the instability characterization coefficient of key blocks. The weight of each key block on the slope stability was determined by combining this coefficient with the analytic hierarchy process (AHP). The key block information was applied to characterize the rock slope stability. The present study proposed a convenient and feasible evaluation method regarding rock slope stability. For the specific rock slope engineering, the significance of each evaluation index was determined and the most significant index was obtained. The determination and visualization of key blocks and the judgment of the slope stability were investigated, which verified the applicability and feasibility of this evaluation method.

Keywords Rock slope . Evaluation indexes . Extension theory . Key blocks . GeoSMA-3D . AHP

# Introduction

The promotion of "the Belt and Road Initiatives" strategy has led to a large amount of exploitation and construction all over the world, especially in China. The stability of such largescale engineering activities on mountain or hilly slopes is directly related to the progress of the project and the safety of the workers. Hence, engineering construction, economics, and efficiency require the stability of a rock slope be judged quickly and accurately. The geological and environmental conditions are very complex, and the rock slope stability is controlled by multi-factor and multi-index. Establishing the multi-level and multi-index evaluation system and determining the significance of each evaluation index are an important part of rock slope stability analysis. Furthermore, the rock slope stability is controlled by the stability of the key blocks directly, and studies regarding the measurement of key blocks' stability are lacking. For example, how to apply the key block information to characterize the rock slope stability is a problem. So, there is dire need to understand these issues.

A lot of research has previously been carried out on rock slope stability. The analysis on influencing factors of rock slope stability is the basic and important one. Cao et al. [\(2013\)](#page-14-0) and Binod and Beena [\(2015\)](#page-14-0) studied the influence of shear strength on slope stability by the shear strength reduction approach, while Hisham et al. [\(2016](#page-14-0)) researched it by draining residual shear strength at effective normal stresses.

 $\boxtimes$  Shuhong Wang [wangshuhong@mail.neu.edu.cn](mailto:wangshuhong@mail.neu.edu.cn)

<sup>&</sup>lt;sup>1</sup> School of Resource and Civil Engineering, Northeastern University, Shenyang 110819, China

<sup>2</sup> Department of Meteorology COMSATS Institute of Information Technology, Islamabad, Pakistan

Also, Zhang et al. ([2016a,](#page-15-0) [b\)](#page-15-0) studied the shear behavior of rock joints in a laboratory method and gave a new peak shear strength criterion. Zhong et al. [\(2011\)](#page-15-0) and Chi et al. [\(2014\)](#page-14-0) analyzed the control effect of the structural plane on slope stability. Zheng et al. [\(2009\)](#page-15-0) and Qi et al. [\(2010\)](#page-14-0) discussed the failure mechanism and fracture surface of slope under earthquake. Deng et al. [\(2016\)](#page-14-0) explained the effect of water on slope stability. All the above research considered only a single factor. However, rock slopes are under complicated geological and environmental conditions in site engineering. Therefore, it is indispensable to establish a multifactor evaluation system, analyze the influence degree of each factor, and determine the most significant evaluation index for comprehensively evaluating the rock slope stability.

Key blocks originated from the block theory proposed by Goodman and Shi ([1985](#page-14-0)) that uses the theoretical method of judging key blocks by geometric representation, kinematic analysis, and mechanical calculation. Zhang and Wu ([2005\)](#page-15-0) further enriched the block theory in the morphological analysis of the complex concave block and multiple slidingplanes. Hao et al. ([2014](#page-14-0)) discussed block theory of limited trace lengths and its application to the probability analysis of block sliding of surrounding rock. For the analysis of wedge stability, Jiang et al. ([2013](#page-14-0)) established the criteria to identify failure modes of wedge failure and proposed a modified Hoek-Bray method, and Li et al. ([2016\)](#page-14-0) gave the layout and length optimization of anchor cables for reinforcing rock wedges. Jiang and Zhou ([2017\)](#page-14-0) also gave a rigorous solution for the stability of polyhedral rock blocks. Zheng et al. ([2013](#page-15-0)) and Guo et al. [\(2013\)](#page-14-0) gave the stability analysis method of block considering cracking of rock bridge. The visualization of key blocks also obtained fruitful achievements. Chen et al. ([2017](#page-14-0)) studied the automatic extraction of blocks from 3D point clouds of fractured rock. Wu et al. [\(2018\)](#page-15-0) simulated the block stability by DDA-3D. Yu et al. ([2005\)](#page-15-0) and Zhang and Wu ([2007](#page-15-0)) explained topological identification of spatial key blocks and developed the corresponding program. However, the existing investigations mainly improved the judgment of key block from geometrical analysis. Key blocks were determined by the traditional safety factor method and still not established very well. However, in site engineering practice, the key block that controlled the slope stability should satisfy the small safety factor while in large volume. In addition, in previous studies, there was no good correlation between the theoretical key blocks and the actual rock slope engineering, which led to derivation between theoretical analysis and practical engineering. For the slope engineering (Griffiths and Marquez [2007](#page-14-0); Kumsar et al. [2000](#page-14-0)), on the one hand, the evaluation indicators and key blocks that controlled the slope stability should be determined; on the other hand, key block information should try to be applied to characterize slope stability.

Obviously, for this topic, there are three problems that need further discussion. Firstly, how to establish a multifactor evaluation system of slope stability and analyze the evaluation index quantitatively. Secondly, how to determine the key blocks that are more in line with engineering practice. Thirdly, how to apply the key block information to characterize the slope stability. Therefore, the objectives of the study are to establish a multi-level and multi-index evaluation system for analyzing the rock slope stability. The quantitative analysis on the influence degree of the evaluation index on the rock slope stability is carried out by establishing the matter element and calculating the correlation function, which are based on the extension theory. The most significant factors affecting the rock slope stability and the corresponding evaluation index are obtained. In addition, this paper also defines the concept of the instability characterization coefficient of key block and applies it to the evaluation of slope stability. This coefficient takes the safety factor and volume of the key block into account, which establishes a key block judgment method that is more in line with engineering practice. With this factor linked to the search module of key blocks in the program Geotechnical Structure and Model Analysis-3D (GeoSMA-3D), developed by our own team (Wang et al. [2009](#page-14-0)), a further determination and visualization of key blocks are achieved. The influence degree of each key block on rock slope stability is determined by the size of the instability characterization coefficient. Moreover, the weight of each key block on the slope stability is determined by combining this factor with AHP. The safety factor of the slope can be defined by the product of each key block's safety factor and its weight. The key block information is applied to characterize the stability of the slope. A convenient and feasible evaluation method about rock slope stability is given.

# Judge the significance of each evaluation index on slope stability

With the uncertainty and complexity of influencing factors, the geological and environmental conditions are so complex that the rock slope stability is controlled by multi-factors. It is indispensable to establish the multi-level and multi-index evaluation system, analyze the influence degree of each factor, and determine the most significant evaluation index for comprehensively evaluating the rock slope stability. This process is realized by extension theory (Cai [1999;](#page-14-0) Liu et al. [2013](#page-14-0)).

The theoretical pillars of extension theory are matter element theory (Cai [1994](#page-14-0)) and extension set theory (Cai [1990](#page-14-0)). The matter element theory studies matter-element and its transformations. The concept of matter-element is the logic cell of extenics and takes the matter, the <span id="page-2-0"></span>characteristic, and their measure into consideration. It can describe the intricacies of things in formalized languages. The extension set theory, which includes extension set and dependent function, is the quantitative tool of extension theory. Extension set has two definitions: one can be regarded as the quantitative tool to describe qualitative change, and another can describe qualitative change under certain conditions. Dependent functions express the dependent degree that a given matter possesses property P.

The extension theory is used to evaluate the significance of each index, which can analyze its influence on rock slope stability quantitatively. The concrete realization steps are shown in Fig. 1.

#### Establish the standard matter-element model

Considering the geological and environmental conditions, the factors which have important influence on the slope stability are selected as the evaluation indexes. The system of evaluation index is shown in Fig. [2](#page-3-0). Not all factors are included because different engineering works are controlled by different influencing factors. In this paper, the factors such as underground water level, faults, and fold are not considered because they do not have important influence on the engineering work of the  $K3 + 260 - K3 +$ 420 rock slope.

According to the specific experimental analysis (Zhao et al. [2015\)](#page-15-0), the relevant research (Liu [2016](#page-14-0)), and the current regulatory requirements (The Ministry of Water Resources, PRC [2015](#page-14-0)), the standard matter-element model was established. The standard matter-element models of geological and environmental evaluation indexes are shown in Tables [1](#page-3-0) and [2,](#page-3-0) respectively. Since the units of the evaluation indexes are not uniform, they are subjected to dimensionless processing. The standard matter-element model of evaluation indexes with dimensionless indices are given in Tables [1](#page-3-0) and [2](#page-3-0) with the values in brackets.

## Determination of the classical field and the limited field

According to extension theory and the study subject, the classical field represents the range of evaluation indexes about slope stability, as shown in Eq. (1).

$$
R_{j} = (P_{j}, c_{i}, v_{ji}) = \begin{bmatrix} P_{j} & c_{1} & v_{j1} \\ c_{2} & v_{j2} \\ \vdots & \vdots \\ c_{n} & v_{jn} \end{bmatrix}
$$

$$
= \begin{bmatrix} P_{j} & c_{1} & \langle a_{j1}, b_{j1} \rangle \\ c_{2} & \langle a_{j2}, b_{j2} \rangle \\ \vdots & \vdots \\ c_{n} & \langle a_{jn}, b_{jn} \rangle \end{bmatrix}
$$
(1)

Where  $P_i$  is the classified level of slope stability;  $c_i$  is the evaluation index of the slope stability;  $v_{ii} = \langle a_{ii}, b_{ii} \rangle$  is the range of the evaluation index  $c_i$  with respect to the corresponding level of slope stability.

According to the range of evaluation index in the evaluation system, the limited field is established as shown in Eq. (2).

$$
R_{p} = (P, c_{i}, v_{pi}) = \begin{bmatrix} P & c_{1} & v_{p1} \\ c_{2} & v_{p2} \\ \vdots & \vdots \\ c_{n} & v_{pn} \end{bmatrix}
$$

$$
= \begin{bmatrix} P & c_{1} & \langle a_{p1}, b_{p1} \rangle \\ c_{2} & \langle a_{p2}, b_{p2} \rangle \\ \vdots & \vdots \\ c_{n} & \langle a_{pn}, b_{pn} \rangle \end{bmatrix}
$$
(2)

Where  $P$  is the whole level of slope stability;  $c_i$  is the evaluation index of the slope stability;  $v_{pi} = \langle a_{pi}, b_{pi} \rangle$  is the range of the evaluation index with respect to P.







<span id="page-3-0"></span>Fig. 2 The multi-level and multiindicator evaluation system



# Establishment of the evaluated matter element model

According to the value of each evaluation index for the slope to be evaluated, the matter-element to be evaluated is established in Eq. (3).

$$
R_0 = (P_0, c_i, v_{0i}) = \begin{bmatrix} P_0 & c_1 & v_{01} \\ & c_2 & v_{02} \\ & \vdots & \vdots \\ & & c_n & v_{0n} \end{bmatrix}
$$
 (3)

Table 2 Standard matter-element model of environmental evaluation index (with dimensionless indices)



<span id="page-4-0"></span>Where  $P_0$  is the slope to be evaluated,  $v_{0i}$  is the value of  $c_i$  with respect to  $P_0$ .

## Determination of dependent functions and obtaining the significance of each index

According to the analysis of the classical field and the matter-element model, the simple dependent function is calculated by Eq. (4).

$$
k_j(v_{0i}) = \begin{cases} \frac{2(v_{0i} - a_{ji})}{b_{ji} - a_{ji}}, & v_{0i} < \frac{b_{ji} + a_{ji}}{2} \\ \frac{2(b_{ji} - v_{0i})}{b_{ji} - a_{ji}}, & v_{0i} \ge \frac{b_{ji} + a_{ji}}{2} \end{cases}
$$
(4)

The primary dependent function of each evaluation index is calculated according to Eq. (5). At the same time, the influence degree of the evaluation indicators on slope stability is determined.

$$
K_j(\nu_{0i}) = \begin{cases} \frac{\rho(v_{0i}, v_{ji})}{\rho(v_{0i}, v_{pi}) - \rho(v_{0i}, v_{ji})}, \rho(v_{0i}, v_{pi}) - \rho(v_{0i}, v_{ji}) \neq 0\\ -\rho(v_{0i}, v_{ji}) - 1, \rho(v_{0i}, v_{pi}) - \rho(v_{0i}, v_{ji}) = 0 \end{cases}
$$
(5)

Where  $\rho(v_{0i}, v_{ii})$  is the distance on real axis between point  $v_{0i}$  and a given real interval  $v_{ji} = \langle a_{ji}, b_{ji} \rangle$  and can be defined as Eq. (6).

$$
\rho(v_{0i}, v_{ji}) = \left| v_{0i} - \frac{b_{ji} + a_{ji}}{2} \right| - \frac{b_{ji} - a_{ji}}{2}
$$

$$
= \begin{cases} a_{ji} - v_{0i}, & v_{0i} \le \frac{b_{ji} + a_{ji}}{2} \\ v_{0i} - b_{ji}, & v_{0i} > \frac{b_{ji} + a_{ji}}{2} \end{cases}
$$
(6)

Similarly,  $\rho(v_{0i}, v_{pi})$  is the distance on real axis between point  $v_{0i}$  and a given real interval  $v_{pi} = \langle a_{pi}, b_{pi} \rangle$  and can be defined as Eq. (7).

$$
\rho(v_{0i}, v_{pi}) = \left| v_{0i} - \frac{b_{pi} + a_{pi}}{2} \right| - \frac{b_{pi} - a_{pi}}{2}
$$
\n
$$
= \begin{cases}\n a_{pi} - v_{0i}, & v_{0i} \le \frac{b_{pi} + a_{pi}}{2} \\
v_{0i} - b_{pi}, & v_{0i} > \frac{b_{pi} + a_{pi}}{2}\n\end{cases} (7)
$$

According to the calculation of the dependent degree for each evaluation index, the significance of each index can be obtained. The quantitative analysis of the influence degree of the evaluation index on the rock slope stability was carried out by establishing the matter element and calculating the dependent function based on the extension theory. The most significant factors affecting the rock slope stability and the corresponding evaluation index were obtained, which provided a theoretical basis for establishing effective reinforcement measures.

### Determination and visualization of key blocks

According to block theory and modern computer techniques, our team developed a three-dimensional numerical analysis software GeoSMA-3D (Yang et al. [2009](#page-15-0)) for identifying the key blocks (Wang et al. [2009](#page-14-0)). The main analysis procedures include three dimension discontinuities network simulation, analysis of intersecting lines between discontinuities and surfaces, analysis of primary loops, loops location analysis, isolated loops deleting, relative loop analysis, and closed block identification (Wang et al. [2013a,](#page-15-0) [b;](#page-15-0) Zhang et al. [2016a,](#page-15-0) [b\)](#page-15-0).

In the original procedure, the block search module established to determine the block stability and the key block was based on the typical block theory. The key blocks are determined by the safety factor in the original search module. In order to establish a key block judgment method that is more in line with engineering practice, this paper presents a concept about the instability characterization coefficient of the key blocks, which is defined by the ratio of key block's safety factor to its relative volume. With the factor linked to the search module of the key blocks in the GeoSMA-3D program, a further determination and visualization of key blocks were achieved.

## The theoretical derivation of the instability characterization coefficient of key blocks

The instability characterization coefficient of key blocks took the safety factor and volume into account. It was defined by the ratio of key block's safety factor to its relative volume, as shown in Eq.  $(8)$ .

$$
\xi = \frac{f_{si}}{\Delta v_i} \tag{8}
$$

Where  $\xi$  is the instability characterization coefficient of key block;  $f_{si}$  is the key block's safety factor;  $\Delta v_i = \frac{v_i}{v}$  is the relative volume of each key block,  $v_i$  is the volume of each key block, and  $\nu$  is the whole volume of all key blocks.

The calculation of  $\xi$  is related to the key block's safety factor and volume. The calculation of safety factor is carried out by block theory, which includes three conditions, as shown in Table [3](#page-5-0).

Another amount that needs to be calculated is the key block's volume. As tetrahedron is the common form of block, this paper uses the tetrahedron for the calculation of block volume. In order to make the established calculation formula of volume more in line with engineering practice, the volume

Failure mode	Schematic figure	Calculation formula	Remarks
Lifting		$f_{si} = 0$	$G$ is the gravity of block, $N_1, N_2$ is the normal force of the
Single-plane sliding		$f_{si} = \frac{G\cos\theta\tan\varphi + cS}{G\sin\theta}$	slipping plane, $S$ is the area of the sliding plane, $\varphi(\varphi_1, \varphi_2)$ and $c(c_1, c_2)$ are friction
Double-plane sliding		$f_{si} = \frac{N_1 \tan \varphi_1 + N_2 \tan \varphi_2 + c_1 S_1 + c_2 S_2}{G \sin \theta}$	angle and cohesion of plane structural respectively, $\theta$ is the dip angel of the slipping plane.

<span id="page-5-0"></span>Table 3 The calculation of the key block's safety factor

of the tetrahedron is divided into a planar tetrahedron and curved tetrahedron for calculation (Xie et al. [2006](#page-15-0)).

The vertex coordinates of the planar tetrahedron are  $P_i(x_i)$ ,  $y_i, z_i$ ,  $i = 0, 1, 2, 3$ , as shown in Fig. 3a, and the volume V of planar tetrahedron can be calculated by Eq. (9).

$$
V = \frac{1}{6} \begin{vmatrix} x_0 & y_0 & z_0 & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix}
$$
 (9)

When the tetrahedron is a curved tetrahedron, as shown in Fig. 3b, the calculation formula of volume is changed, as shown in Eq.  $(10)$ .

$$
V = V_0 - V_1 - V_2 - V_3 \tag{10}
$$



Fig. 3 Model of the tetrahedron. (a) planar tetrahedron (b) curved tetrahedron

Where  $V_0$  comprises vertexes of  $P_0$  and  $P_1$ ,  $P_2$ ,  $P_3$ ;  $V_1$ comprises vertexes of  $P_4$  and  $P_1$ ,  $P_2$ ,  $P_3$ ;  $P_4$  is the node of plane  $y = y_i$  and  $z = z_j$  on structure J;  $V_2$  comprises vertexes of  $P_4$  and  $P_2$ ,  $P_3$  and the curved plane  $P_R$ ;  $V_3$  comprises vertexes of  $P_4$  and  $P_1$ ,  $P_3$  and the curved plane  $Q_R$ .

Volumes  $V_0$  and  $V_1$  are calculated by Eq. (9), volume  $V_2$  is calculated by Eq. (11), and volume  $V_3$  is calculated by Eq. (12).

$$
V_2 = \frac{2}{3} \left(\frac{a+b}{2}\right)^3 \left(\frac{B_2'}{A_2} - \frac{B_3'}{A_3}\right) \sin^3 \alpha_0 \tag{11}
$$

$$
V_3 = \frac{2}{3} \left(\frac{a+b}{2}\right)^3 \left(\frac{B_1'}{A_1} - \frac{B_3'}{A_3}\right) \sin^3 \alpha_0 \tag{12}
$$

Where a, b is the radius of arc;  $\alpha_0$  is the angle between the X coordinate axis and the north direction; and  $B'_1, B'_2, B'_3$  satisfy Eq. (13).

$$
B'_{1} = B_{1} \cos \alpha_{1} + C_{1} \sin \alpha_{1}
$$
  
\n
$$
B'_{2} = B_{2} \cos \alpha_{1} + C_{2} \sin \alpha_{1}
$$
  
\n
$$
B'_{3} = B_{3} \cos \alpha_{1} + C_{3} \sin \alpha_{1}
$$
  
\n
$$
\alpha_{1} = arccot \left(\frac{z_{j} + z_{i}}{y_{j} + y_{i}}\right)
$$
\n(13)

Therefore, the volume of the complex block can be determined by calculating the volume of tetrahedron. That is to say,

<span id="page-6-0"></span>when the block is divided into  $n$  tetrahedrons, its volume can be calculated by Eq. (14).

$$
V = \sum_{i=1}^{n} V_i \tag{14}
$$

In the traditional method, the stability of the block was determined only by the safety factor. When its safety factor was less than 1, the block was defined as a key block. Considering the engineering practice, the volume of the block was introduced into the evaluation of block stability. Through the calculation of the safety factor and the block's volume, combined with the instability characterization coefficient of key blocks, the judgment method of the key block in different sliding models was given.

## Program implementation of the instability characterization coefficient of key blocks

By combining the block search module in GeoSMA-3D with the collection system of structural plane information in ShapeMetriX3D, the determination and visualization of key blocks can be realized (Wang et al. [2011](#page-14-0); Wang and Ni [2014\)](#page-14-0). However, the key blocks are determined only by the safety factor in GeoSMA-3D. In order to make the key blocks determined more in line with engineering practice, the judgment method should be improved.

The block search module in GeoSMA-3D (Wang et al. [2017\)](#page-15-0) cannot only realize the visualization of the key blocks but also can obtain information, such as the safety factor, volume, sliding surface and so on, of key blocks. The numerical solution for calculating the instability characterization coefficient of key blocks would be obtained. Using the safety factor and volume of key blocks derived from GeoSMA-3D with the instability characterization coefficient, further determination of key blocks can be realized. The concrete implementation steps and numerical verification of engineering application will be given in a later section.

## Characterization of slope stability

Accurately judging the key block is the basic function of the rock slope stability analysis, and how to apply the key block information to characterize the rock slope stability is more important. However, in previous studies, there was no good correlation between the theoretical blocks and the actual rock slope engineering, which led to derivation between theoretical analysis and practical engineering. Due to this, the present paper proposed the characterization safety factor of rock slope stability, which combined the instability characterization coefficient with the weight of key blocks. The influence degree of each key

Table 4 The Satty 1–9 scale system

Quantization scale	Comparison of two-factor importance		
1	<i>i</i> is as important as j		
3	i is slightly more important than j		
5	i is significantly more important than j		
7	i is strongly more important than i		
<sup>Q</sup>	i is extremly more important than j		
2, 4, 6, 8	The median of the above adjacency judgments		
$1/1, 1/2 \cdots 1/9$	Represents the comparison of j with i, as opposed to the comparison of i with j		

block on slope stability was determined by the size of the instability characterization coefficient of key blocks. The weight of each key block on the slope stability was determined by combining this factor with AHP (Wen et al. [2017](#page-15-0)). The safety factor of the slope can be defined by the product of each key block's safety factor and its weight. The key block information was applied to characterize the slope stability. The specific implementation steps are as follows:

① According to the instability characterization coefficient of key blocks, the influence degree of each key block on rock slope stability is determined. Then the judgment matrix D is established by AHP (Huang et al. [2007\)](#page-14-0). Judgment matrix D is represented by Eq. (15).

$$
D = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1n} \\ w_{21} & w_{22} & \cdots & w_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n1} & w_{n2} & \cdots & w_{nn} \end{bmatrix} = \begin{bmatrix} 1 & \frac{x_1}{x_2} & \cdots & \frac{x_1}{x_n} \\ \frac{x_2}{x_1} & 1 & \cdots & \frac{x_2}{x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{x_n}{x_1} & \frac{x_n}{x_2} & \cdots & 1 \end{bmatrix} (15)
$$

The Satty 1–9 scale system (Zhou et al. [2012\)](#page-15-0), as shown in Table 4.



Fig. 4 The determination process of key blocks' weight

<span id="page-7-0"></span>

② The maximum eigenvalue of the judgment matrix and the corresponding eigenvector are calculated by square root method (Xia et al. [2011\)](#page-15-0). The eigenvector represents the weight of each key block's influence on the rock slope stability (Li et al. [2009\)](#page-14-0). The implementation is shown in Fig. [4.](#page-6-0)

Where CR is consistency ratio and when  $CR < 0.1$ , the consistency of judgment matrix satisfies the requirement;  $CI = (\lambda_{\text{max}} - n)/(n - 1)$  is consistency index; RI is random consistency index and can be obtained from Table 5 (Guo et al. [2017](#page-14-0)).

#### ③ Measurement of the safety factor of rock slope stability.

By calculating the instability characterization coefficient of each key block, the weight of each key block has been determined. The key block information is applied to characterize the slope stability by combining this coefficient with AHP. Based on this, the safety factor of slope can be calculated by the product of each key block's safety factor and its weight, which can be calculated by Eq.  $(16)$ .



**Fig. 6** The rock slope of  $K3 + 260 - K3 + 420$ 

$$
F_s = \sum_{i=1}^n f_{si} \cdot w_i \tag{16}
$$

Where  $F_s$  is the safety factor of slope;  $f_{si}$  is the safety factor of each key block;  $w_i$  is the weight of each key block; *i* is the number of key block.

The slope stability can be determined with the safety factor  $F_s$ , which was calculated by the key block information. The key block information was applied to characterize the stability of the slope. A convenient and feasible evaluation method about rock slope stability was given. In addition, this evaluation methodology can not only ensure the safety of the slope but also provide a theoretical basis for reasonably reducing the support cost.



Evaluation index	Uniaxial compressive strength/ Integrity MPa	coefficient	Deformation modulus/ GPa		Internal friction $RQD$ angle/ $\circ$	Cohesion /MPa
Value	74	0.36	5.10	57	Q	0.03
Value with dimensionless indices	0.74	0.36	0.82	0.57	0.20	0.19

<span id="page-8-0"></span>**Table 6** The geological indexes of the  $K3 + 260 - K3 + 420$  rock slope

**Table 7** The environmental indexes of the rock slope  $K3 + 260 - K3 + 420$  area

Evaluation index	The height of slope/m	The angle of slope $\sqrt{\ }$	Cumulative rainfall monthly/mm	Softening coefficient	Weathering coefficient	Basic seismic acceleration/g
Value	52	18	87	0.70	0.50	0.10
Value with dimensionless 0.48 indices		0.80	0.71	0.73	0.52	0.70

## The application in engineering work

For engineering works, the significance of each index should be evaluated, which can analyze the influence of each index on slope stability quantitatively. The most significant factor affecting rock slope stability and the corresponding evaluation index were obtained. The key blocks that controlled the rock slope stability should be further determined and visualized. In addition, the judgment method of the slope stability should be carried out in the specific rock slope engineering and analyze the similarity between the theoretical analysis and engineering practice. The research steps are shown in Fig. [5](#page-7-0).

## Introduction of the engineering work

In this paper, the rock slope (Fig. [6\)](#page-7-0) at  $K3 + 260 - K3 +$ 420 of Road Construction Project in Yiyang, Hunan Province, P. R. China is selected as the research subject. This Road Construction Project is an urban thoroughfare connecting the east-west traffic in Yiyang, which is ecoefficient and green economy with an international advanced level. The road starting and terminal point is  $K0 + 000$  and  $K13 + 541.436$  respectively. The total length of this road is about 13.541 km and the designed speed is 60 km/h.

#### The geological and environmental conditions

According to the geological data of the slope  $K3 + 260$ - $K3 + 420$  area, the stratigraphic lithology belongs to complete soft rock, whose rock uniaxial compressive strength is 74 MPa and rock integrity coefficient is 0.36. The properties of rock structure are not very good, its deformation modulus is 5.10 GPa and RQD is 57. The structural plane has weak interlayer so its nature is poor, the internal friction angle is 9°, and the cohesion is 0.03 MPa. Its geological condition is shown in Table 6. When the geological evaluation indexes are subjected to dimensionless processing, the characteristic parameters are given in Table 6.

By combination of the location and climate of the slope  $K3 + 260 - K3 + 420$  area, the environmental evaluation

**Table 8** Joint formation of the  $K3 + 260-K3 + 420$  rock slope

Group		First group	Second group	Third group
Maximum	dip direction $\varphi$ <sup>(<math>\circ</math></sup> )	295.346	242.075	230.738
	dip angle $\theta$ <sup>(°)</sup>	55.033	70.788	89.461
Minimum	dip direction $\varphi$ <sup>(<math>\circ</math></sup> )	216.708	209.835	27.143
	dip angle $\theta$ <sup>(°)</sup>	20.685	48.177	44.681
Average value	dip direction $\varphi$ <sup>(<math>\circ</math></sup> )	252.307	61.844	225.032
	dip angle $\theta$ <sup>(°)</sup>	42.064	64.389	70.470
The number of structure		27	12	16



Fig. 7 Stereographic projection of the deterministic structural planes and its distribution

indexes can be determined. The height of the slope is 52 m and the angle is 18°. As the slope is in a subtropical monsoon humid climate, the rainfall is abundant and the wind is strong. As a result, the cumulative rainfall monthly is 87 mm, the softening coefficient is 0.70, and the weathering coefficient is 0.50. The slope  $K3 + 260 - K3 + 420$  area in Hunan province has the basic seismic acceleration 0.10 g. The environmental condition of rock slope is shown in Table [7](#page-8-0). When the environmental evaluation indexes are subjected to dimensionless processing, the characteristic parameters are given in Table [7.](#page-8-0)

### The structural plane information

The structural plane information in the  $K3 + 260-K3 + 420$  rock slope is obtained by ShapeMetriX3D software, which is shown in Table [8](#page-8-0). The stereographic projection is shown in Fig. 7.

<span id="page-10-0"></span>Fig. 8 Dependent degree of geological evaluation index



# The significance analysis of evaluation indexes

specific value of each evaluation index for the Yiyang slope, the matter element  $R_0$  to be evaluated is established.

The classical field  $R_i(i = 1, 2, \dots, 5)$  and the limited field  $R_p$  can be determined by Eqs. ([1](#page-2-0)) and ([2\)](#page-2-0). Also, according to the

$$
R_1 = \begin{bmatrix} P_1 & c_1 & \langle 0.00, 0.30 \rangle \\ c_2 & \langle 0.00, 0.15 \rangle \\ c_3 & \langle 0.00, 0.30 \rangle \\ c_4 & \langle 0.00, 0.30 \rangle \\ c_5 & \langle 0.00, 0.20 \rangle \\ c_6 & \langle 0.00, 0.20 \rangle \\ c_7 & \langle 0.00, 0.20 \rangle \\ c_8 & \langle 0.00, 0.20 \rangle \\ c_{10} & \langle 0.00, 0.56 \rangle \end{bmatrix} \quad R_2 = \begin{bmatrix} P_2 & c_1 & \langle 0.30, 0.50 \rangle \\ c_2 & \langle 0.15, 0.35 \rangle \\ c_3 & \langle 0.35, 0.67 \rangle \\ c_4 & \langle 0.30, 0.50 \rangle \\ c_5 & \langle 0.29, 0.42 \rangle \\ c_7 & \langle 0.20, 0.40 \rangle \\ c_8 & \langle 0.00, 0.20 \rangle \\ c_{10} & \langle 0.00, 0.20 \rangle \\ c_{11} & \langle 0.00, 0.20 \rangle \\ c_{12} & \langle 0.00, 0.56 \rangle \\ c_{13} & \langle 0.00, 0.50 \rangle \\ c_{11} & \langle 0.00, 0.50 \rangle \\ c_{12} & \langle 0.00, 0.50 \rangle \\ c_{13} & \langle 0.46, 1.00 \rangle \\ c_{14} & \langle 0.20, 0.40 \rangle \\ c_{15} & \langle 0.45, 0.70 \rangle \\ c_{16} & \langle 0.45, 0.70 \rangle \\ c_{17} & \langle 0.46, 1.00 \rangle \\ c_{18} & \langle 0.46, 1.00 \rangle \\ c_{19} & \langle 0.46, 1.00 \rangle \\ c_{10} & \langle 0.82, 1.00 \rangle \\ c_{11} & \langle 0.80, 1.00 \rangle \\ c_{12} &
$$





Based on the determination of the classical field  $R_i(i = 1, 2, ...)$  $\cdots$ , 12) and the limited field  $R_p$  and the establishment of the matter-element to be evaluated, the dependent degree of each evaluation index can be calculated by Eqs. [\(4\)](#page-4-0)-[\(7](#page-4-0)). The calculation results are shown in Figs. [8](#page-10-0) and 9.

dependent degree

The maximum dependent degree of each evaluation index and its associated slope level can be determined by calculating the dependent degree between each index and the level of slope stability. As a result, the significance of each evaluation index can be obtained. The maximum dependent degree of each evaluation index and the associated level of slope stability are shown in Fig. 10.

The level of slope stability follows the order:  $V > IV > III >$  $II > I$ , the grade I level of slope stability is the worst and the grade V level of slope stability is the best. The evaluation index associated with the slope of grade I has the greatest influence on the rock slope stability, followed by grade II, and so on. The results in Fig. 10 indicated that the influence degree of each evaluation index on rock slope stability is: internal friction angle > cohesion > RQD > weathering coefficient > basic seismic



<span id="page-12-0"></span>

Fig. 11 The key blocks in the  $K3 + 260 - K3 + 420$  rock slope

acceleration > rock integrity coefficient > softening coefficient > rock uniaxial compressive strength > cumulative rainfall monthly  $>$  the angle of slope  $>$  the height of slope. Therefore, the influence factor that controlled the rock slope stability is structural plane, and the most significant indexes are the internal friction angle and cohesion.

#### Determination and visualization of key blocks

By combining the structural plane information derived from ShapeMetriX3D with the block search module in GeoSMA-3D software, the judgment of block stability and the visualization of key blocks can be achieved. The key blocks shown in Fig. 11 are based on the safety factor method, which is the traditional way.

The volume, the number of sliding surface, the safety factor, and other key block information analyzed by GeoSMA-3D software are shown in Table 9.

The block search module in GeoSMA-3D was based on block theory, and the key blocks searched in Fig. 11 were defined by the safety factor. In the traditional view, the block whose safety factor was small had a negative effect on the slope stability. The smaller the safety factor of the block, the worse the stability of the block. From the key block information shown in Table 9, we can see that the degree of influence on

Table 9 Key block data and information

The number of key blocks 9						
N <sub>0</sub>	$v_i$	Number of block surfaces	Number of sliding surfaces	$f_{si}$		
01	1.615	5	2	0.141		
02	0.296	4	$\overline{2}$	0.967		
03	8.879	4	$\overline{2}$	0.934		
04	2.903	5	3	0.200		
05	11.341		4	0.569		
06	0.001	4	$\overline{2}$	0.424		
07	0.705	5	$\overline{2}$	0.578		
08	12.262	6	3	0.858		
09	0.116	6	3	0.662		



Table 10 Instability characterization coefficient of key blocks

rock slope stability was: key block  $1 > \text{key}$  block  $4 >$ key block  $6 > \text{key}$  block  $5 > \text{key}$  block  $7 > \text{key}$  block  $9 > \text{key block } 8 > \text{key block } 3 > \text{key block } 2$ .

Using the key block determination method proposed earlier, the key blocks were further determined by considering the influence of volume and safety factor. According to the safety factor and volume of key blocks obtained in Table 9, the instability characterization coefficients of key blocks were calculated by Eq. [\(8](#page-4-0)). The results are shown in Table 10.

According to the definition of the instability characterization coefficient, the rock slope stability is more dangerous when the safety factor of the key block is smaller and the volume of the key block is larger, that is, the instability characterization coefficient becomes smaller. From the size of the instability characterization coefficient in Table 10, we can see that the degree of influence on rock slope stability is: key block 5 > key block 4 > key block 8 > key block 1 > key block  $3 >$  key block  $7 >$  key block  $2 >$  key block  $6 >$  key block 9.

Obviously, the key blocks determined by these two methods are different. The most unfavorable key block determined by safety factor is block 1, while block 5 is determined by the instability characterization coefficient.

Through the actual situation and further analysis of the slope, it can be found that the first slippery block is key block 1, whose safety factor is the smallest. The slip of



Fig. 12 The slide of the first key block



Fig. 13 The slide of the fifth key block

block 1 does not pose a threat to the rock slope stability, as shown in Fig. [12.](#page-12-0) However, when key block 5, whose instability characterization coefficient is the smallest, slips it causes a chain slip of the block and results in slope failure, as shown in Fig. 13.

From the analysis above, we can see that the key blocks determined by the instability characterization coefficient are more in line with the engineering practice, and it is theoretically significant in providing effective supporting measures for the unstable slope.

#### The characterization of the slope stability

According to the instability characterization coefficient of key blocks, the influence degree of each key block on rock slope stability is determined. Then the judgment matrix D is established by 1~9 scale system and AHP.

$$
D = \begin{bmatrix} 1 & 4 & 2 & 1/3 & 1/4 & 5 & 3 & 1/2 & 6 \\ 1/4 & 1 & 1/3 & 1/6 & 1/7 & 2 & 1/2 & 1/5 & 3 \\ 1/2 & 3 & 1 & 1/4 & 1/5 & 4 & 2 & 1/3 & 5 \\ 3 & 6 & 4 & 1 & 1/2 & 7 & 5 & 2 & 8 \\ 4 & 5 & 7 & 2 & 1 & 8 & 6 & 3 & 9 \\ 1/5 & 1/2 & 1/4 & 1/7 & 1/8 & 1 & 1/3 & 1/6 & 2 \\ 1/3 & 2 & 1/2 & 1/5 & 1/6 & 3 & 1 & 1/4 & 4 \\ 2 & 5 & 3 & 1/2 & 1/3 & 6 & 4 & 1 & 7 \\ 1/6 & 1/3 & 1/5 & 1/8 & 1/9 & 1/2 & 1/4 & 1/7 & 1 \end{bmatrix}
$$

Table 11 Calculation date of each value

The maximum eigenvalue of the judgment matrix and the corresponding eigenvector are calculated by square root method, and the eigenvector represents the weight of each key block's influence on the rock slope stability. The calculation steps in Fig. [4](#page-6-0) are used, and the calculation dates are shown in Table 11.

For this slope,  $CI = \frac{\lambda_{\text{max}} - n}{n-1} = \frac{\lambda_{\text{max}} - n}{n-1} = 0.0496; RI =$ 1.46.And  $CR = \frac{CI}{RI} = \frac{CI}{RI} = 0.034 < 0.1$ , that is to say, the consistency of the judgment matrix satisfies the requirement, and the weight of key blocks are reasonable.

The stability of the slope can be characterized by the product of each key block's safety factor and its weight calculated above. For this slope, the safety factor is as follows:

$$
F_s = \sum_{i=1}^{9} f_{si} \cdot w_i
$$
  
= 0.108 × 0.141 + 0.035 × 0.967 + 0.074 × 0.934  
+0.224 × 0.200 + 0.308 × 0.569 + 0.025 × 0.424  
+0.051 × 0.578 + 0.157 × 0.858 + 0.018 × 0.662  
= 0.525

From the analysis, it can be seen that this rock slope is unstable. The most significant indexes are the internal friction angle and cohesion, and key block 5 is the biggest threat to the slope stability. Therefore, there is a need for the slope to take reinforcement measures. Targeted measures should be taken for key block 5. Further, the influence of the internal friction angle and cohesion on this slope should be considered primarily.

## Conclusion

In this paper, the significance of evaluation indexes was analyzed by extension theory and the weight of key blocks was calculated by AHP. The most significant evaluation index and key block were determined, which provided a theoretical basis for establishing effective reinforcement measures for an unstable slope. The key block information was applied to characterize the slope stability and a convenient and feasible evaluation method about rock slope stability was given. This evaluation method was applied to engineering work, which verified its applicability and feasibility. Through analysis, the following conclusions were obtained:



- <span id="page-14-0"></span>1. The multi-level and multi-index evaluation system for analyzing rock slope stability was established from geological and environmental conditions. Quantitative analysis about the influence degree of each evaluation index on rock slope stability was carried out by extension theory. Further, the most significant factors affecting rock slope stability and the corresponding evaluation index were obtained, which provided a theoretical basis for establishing effective reinforcement measures.
- 2. The instability characterization coefficient of key block was proposed, which established a key block judgment method that was more in line with engineering practice. With this coefficient implemented into the key blocks' search module in GeoSMA-3D, further determination and visualization of key blocks were realized.
- 3. In order to apply the theoretical analysis to practical engineering, this paper proposed the characterization safety factor of rock slope stability, which combined the instability characterization coefficient with the weight of key blocks by AHP. The key block information was applied to characterize the slope stability. A convenient and feasible evaluation method about rock slope stability was given.
- 4. The research methods and analytical steps proposed in this paper are used to analyze the stability of Yiyang slope. The results indicated that this rock slope is unstable. The most significant indexes were the internal friction angle and cohesion, and key block 5 was the biggest threat to slope stability. Therefore, there was a need for this slope to take reinforcement measures. Targeted measures should be taken for key block 5. The influence of the internal friction angle and cohesion on this slope should be considered primarily.

Acknowledgements This work was conducted with supports from the National Natural Science Foundation of China (Grant Nos. 51474050 and U1602232), the Program for Liaoning Excellent Talents in University (Grant No. LN2014006) to Dr. Shuhong Wang.

# References

- Binod T, Beena A (2015) Reduction in fully softened shear strength of natural clays with NaCl leaching and its effect on slope stability. J Geotech Geoenviron 141(1):1197–1209
- Cai W (1990) The extension set and non-compatible problem. Adv Math Mech China 2:1–21
- Cai W (1994) Matter-element models and their application. Science and Technology, Beijing
- Cai W (1999) Extension theory and its application. Chin Sci Bull 17: 1538–1548
- Cao LJ, Liu YN, Shang guan ZC (2013) Stability analysis of jointed rock slope using shear strength reduction approach. Appl Mech Mater 2746(423):1321–1324
- Chen N, Kemeny J, Jiang QH, Pan ZW (2017) Automatic extraction of blocks from 3D point clouds of fractured rock. Comput Geosci 109: 149–161
- Chi EA, Tao TJ, Zhao MS, Kang Q (2014) Failure mode analysis of bedding rock slope affected by rock mass structural plane. Appl Mech Mater 3365(602):594–597
- Deng HF, Zhou ML, Li JL, Sun XS, Huang YL (2016) Creep degradation mechanism by water-rock interaction in the red-layer soft rock. Arab J Geosci 9:42–57
- Goodman RE, Shi GH (1985) Block theory and its application to rock engineering. Prentice-Hall, Englewood
- Griffiths DV, Marquez RM (2007) Three-dimensional slope stability analysis by elasto-plastic finite elements. Geotechnique 57(6): 537–546
- Guo MD, Zhu FS, Wang SH, Zhang SC, Zhang J (2013) Research on rock bridge coalescence law of rock mass containing coplanar structural planes. Rock Soil Mech 06:1598–1604
- Guo ZQ, Chen WW, Zhang JK, Ye F, Liang XZ, He FG, Guo QG (2017) Hazard assessment of potentially dangerous bodies within a cliff based on the fuzzy-AHP method: a case study of the Mogao grottoes, China. Bull Eng Geol Environ 76:1009–1020
- Hao J, Shi KB, Chen GM, Bai XJ (2014) Block theory of limited trace lengths and its application to probability analysis of block sliding of surrounding rock. Chin J Rock Mech Eng 07:1471–1478
- Hisham TE, Khaled HR, Dharma W (2016) Drained residual shear strength at effective normal stresses relevant to soil slope stability analyses. Eng Geol 204(8):94–107
- Huang JW, Li JL, Zhou YH (2007) Application of fuzzy analysis based on AHP to slope stability evaluation. Chin J Rock Mech Eng 26(1): 2627–2632
- Jiang QH, Zhou CB (2017) A rigorous solution for the stability of polyhedral rock blocks. Comput Geotech 90:190–201
- Jiang QH, Liu XH, Wei W, Zhou CB (2013) A new method for analyzing the stability of rock wedges. Int J Rock Mech Min Sci 60(6):413–422
- Kumsar H, Aydan O, Ulusay R (2000) Dynamic and static stability assessment of rock slopes against wedge failures. Rock Mech Rock Eng 33(1):31–51
- Li KG, Hou KP, Li W (2009) Research on influences of factors dynamic weight on slope stability. Rock Soil Mech 30(2):492–496
- Li CD, Wu JJ, Wang J, Li XK (2016) Layout and length optimization of anchor cables for reinforcing rock wedges. Bull Eng Geol Environ 75(4):1399–1412
- Liu CH (2016) On improving the classification accuracy of extension theory. Intell Decis Technol 10(1):27–36
- Liu GS, Qi CM, Nie CL, Hu J (2013) Slope stability evaluation by the improved AHP and extension theory. Appl Mech Mater 2685(405): 106–110
- Qi SW, Xu Q, Lan HX, Zhang B, Liu JY (2010) Spatial distribution analysis of landslides triggered by 2008.5.12 Wenchuan earthquake, China. Eng Geol 116(1):95–108
- The Ministry of Water Resources, PRC (2015) Standard for engineering classification of rock mass, GB50218-2014. China Planning Press, Beijing
- Wang SH, Ni PP (2014) Application of block theory modeling on spatial block topological identification to rock slope stability analysis. Int J Computat Methods 11(1):1–24
- Wang SH, Guo MD, Yang Y (2009) Enhancing block rock failure understanding through GeoSMA-3D numerical analysis. ISRM international symposium on rock mechanics: rock characterization, modelling and engineering design method, SINOROCK 2009, Hong Kong, pp 158–164
- Wang SH, Ni PP, Yang H, Xu Y (2011) Modeling on spatial block topological identification and their progressive failure analysis of slope and cavern rock mass. Procedia Eng 10:1509–1514
- <span id="page-15-0"></span>Wang SH, Huang RQ, Ni PP (2013a) Fracture behavior of intact rock using acoustic emission: experimental observation and realistic modeling. Geotech Test J 36(6):903–914
- Wang SH, Ni PP, Guo MD (2013b) Spatial characterization of joint planes and stability analysis of tunnel blocks. Tunn Undergr Space Technol 38(1):357–367
- Wang SH, Wang FL, Zhang ZS (2017) Blocks searching and program development based on overlapping technology. J Northeast Univ (Nat Sci) 38(02):265–269
- Wen HJ, Xie P, Xiao P, Hu DP (2017) Rapid susceptibility mapping of earthquake-triggered slope geohazards in Lushan County by combining remote sensing with the AHP model developed for the Wenchuan earthquake. Bull Eng Geol Environ 76:909–921
- Wu W, Zhu HH, Lin JS, Zhuang XY, Ma GW (2018) Tunnel stability assessment by 3D DDA-key block analysis. Tunn Undergr Space Technol 71(1):210–214
- Xia CY, Liu YJ, Liu DM (2011) Method of weight assignment for multicriterion based on Grey interval AHP. Adv Mater Res 1269(243): 5285–5288
- Xie Y, Liu J, Li ZK, Zhang ZY (2006) Stability analysis of block in surrounding rock mass of large underground excavation. Chin J Rock Mech Eng 02:306–311
- Yang Y, Wang SH et al (2009) Visual program based on general block theory. New development in rock mechanics and engineering & Sanya forum for the plan of city and city construction, Sanya, pp 351–358
- Yu QC, Chen DJ, Xue GF, Daxi YS (2005) Preliminary study on the general block theory of fractured rock mass. Hydrogeoloogy Eng Geol 06:42–48
- Zhang QH, Wu AQ (2005) Morphological analysis method of block based on classification of concave zone. Chin J Geotech Eng 03: 299–303
- Zhang QH, Wu AQ (2007) General methodology of spatial block topological identification with stochastic discontinuities cutting. Chin J Rock Mech Eng 10:2043–2048
- Zhang MS, Huang RQ, Wang SH (2016a) Spatial block identification method based on meshing and its engineering application. Chinese Journal of Geotechnical Engineering 38(3):477–485
- Zhang X, Jiang Q, Chen N, Wei W, Feng X (2016b) Laboratory investigation on shear behavior of rock joints and a new peak shear strength criterion. Rock Mech Rock Eng 49:3495–3512
- Zhao B, Xu WY, Liang GL, Meng YD (2015) Stability evaluation model for high rock slope based on element extension theory. Bull Eng Geol Environ 74(2):301–314
- Zheng YR, Ye HL, Huang RQ (2009) Analysis and discussion of failure mechanism and fracture surface of slope under earthquake. Chin J Rock Mech Eng 28(8):1714–1723
- Zheng YH, Xia L, Yu QC (2013) Stability analysis method of block considering cracking of rock bridge. Rock Soil Mech S1:197–203
- Zhong W, Tan ZY, Qiao L (2011) Stability analysis of rock slope based on preferred structural plane. Adv Mater Res 1269(243):243–249
- Zhou ZJ, Liang H, Wang XD (2012) Study on stability of rock at reservoir banks slop based on AHP. Appl Mech Mater 1975(204):2309–2317