



An interval fuzzy comprehensive assessment method for rock burst in underground caverns and its engineering application

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Received: 28 June 2018 / Accepted: 21 December 2018 / Published online: 15 January 2019
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Abstract

Rock burst is a dynamic process involving the sudden release of elastic energy accumulated in overstressed rocks and coal masses during underground excavations, capable of causing a great number of fatalities, failure of supporting structures, and damage to equipment. To minimize the risk, a fuzzy comprehensive assessment model was established to predict and evaluate rock burst in underground caverns, and some indices were selected and analyzed. The result vectors and weight vectors were expressed by interval numbers and the membership degrees were determined by a sigmoid membership function. In addition, synthetic weight was obtained by combining subjective weight with objective weight, and final vectors were categorized into different levels through possibility ranking analysis. Finally, the proposed method was applied to a practical case, the Jiangbian hydropower station in China, for further verification. Considering that some indices were conceptually dependent on one another, the multicollinearity of these indices was evaluated according to results based on different indicator systems. The evaluation results showed a high consistency with the actual situation. Moreover, the indicator system, which made full use of the obtained data, had more reliable results than other indicator systems. The proposed fuzzy comprehensive assessment method provides valuable guidance for rock burst assessment.

Keywords Rock burst · Underground caverns · Assessment · Fuzzy set theory · Interval number

Introduction

Rock burst is an inherent and hazardous problem found in overstressed rocks and coal masses during underground excavations. Rock bursts are usually accompanied by a sudden and violent release of elastic energy as well as extreme rock deformation and splash (He et al. 2010; Cai 2013; Li et al. 2017; Hu et al. 2018). The high stress concentration caused by

excavation often triggers rock bursts around the openings. With the increasing depth and complexity of underground projects in the last few decades, rock burst disasters occur frequently and have become a serious problem. Severe accidents caused by rock burst events around the world have resulted in a large number of casualties, failure and deformation of supporting structures, and damage to equipment (Chen et al. 2015; Zhang et al. 2015). Therefore, the prediction and

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control of rock burst hazard is critical, and has become a key scientific and technical challenge during underground activity.

At present, research on rock burst hazard focuses on long-term and short-term prediction and assessment. At the project design stage, the possibility of rock burst is preliminarily determined through long-term prediction and assessment, which provides guidance for creating an excavation plan and determining a construction method. At the excavation stage, constructors predict the location and time of rock bursts based on the on-site data (Peng et al. 2010).

For short-term prediction and assessment, scholars have carried out extensive theoretical and experimental study on rock failure mechanisms, prediction methods and control techniques. The single-face dynamic unloading test under true-triaxial conditions was carried out for limestone blocks as a useful tool for better understanding the characteristics of rock burst (He et al. 2010). The relation between electromagnetic radiation generated and applied loads was studied, and the working face of a coal mine was monitored by monitoring instruments (Wang and Peng 2014). An experimental study on precursors of rock burst was carried out based on acoustic emission signal dominant-frequency and entropy (Zhang et al. 2015). The failure behaviors of granite, red sandstone, and cement mortar were investigated using a novel testing system of slabbing and rock burst, coupled with true-triaxial static loads and local dynamic disturbances (Du et al. 2016). Miao et al. (2016) predicted the probable location and intensity of rock burst through theoretical analysis and numerical simulation based on in situ stress and energy accumulation theory.

For long-term prediction and assessment, artificial intelligence and data mining methods have been extensively applied in this field. Zhang et al. (2012) studied the effect of reducing strain type and fault-slip type rock bursts by numerical analysis based on the failure approaching index, energy release rate, and excess shear stress. A support vector machine model was established to predict rock burst risk by Peng et al. (2014). Zhou et al. (2016) applied a cloud model with entropy weight based on the uncertainty in artificial intelligence for the assessment and classification of rock burst. A new rock burst evaluation method based on attribute reduction operation in rough set theory and weight analysis of extension theory was also established (Xue et al. 2017). Li et al. (2017) proposed a novel application of a Bayesian network (BN) to predict the occurrence of rock burst, as BNs have the advantage of naturally dealing with the conditional dependency relations between the observed and unobserved random variables of a statistical model. The present study deals with long-term assessment of rock burst.

With the development of nonlinear mathematics, relevant methods have been applied for the assessment and prediction of rock burst risk; fuzzy mathematics in

particular has been applied to many studies. To evaluate various influential factors of rock burst, the concepts and methods of variable fuzzy sets are defined and established based on fuzzy set theory. On the basis of indices standard intervals of rock burst with a series of underground rock projects, occurrence and intensity of rock burst with rank feature values were predicted (Chen and Guo 2005). A quality evaluation system framework of burst-proneness risk indices was established and a comprehensive fuzzy evaluation method was applied to evaluate and solve the unclear boundary of some burst-proneness risk indices and inaccurate fuzziness (Guo and Jiang 2010). Adoko et al. (2013) conducted research to predict rock burst intensity based on a fuzzy inference system, an adaptive neuro-fuzzy inference system and field measurement data. A fuzzy comprehensive evaluation model for rock burst assessment was developed and the membership function was built using Gaussian shape combined with the exponential distribution function (Cai et al. 2018). The deformation and failure characteristics of engineering rock masses are extremely complex and highly nonlinear. The above fuzzy comprehensive evaluation method fully considered that the influential factors of rock burst have ambiguity. In the past, only one qualitative indicator was used to determine the possibility of rock burst occurrence. The fuzzy evaluation method compensated for this defect and made the result closer to the real situation. Previous studies analyzed multiple factors and comprehensively evaluated the possibility and intensity of rock burst occurrence based on the rock burst rock mechanical test and fuzzy mathematics. These studies provide guidance and scientific basis for the assessment of rock burst.

In practical engineering, the occurrence of rock burst in underground caverns is affected by the combined action of multiple factors, because of the actual engineering complexity and differences in multifarious geological conditions. Generally, it is difficult to acquire sufficient data for rocks. Moreover, the measurement data collected at the engineering site, such as drilling bit information, microseismic monitoring, and acoustic emission, is often not constant. To fully reflect the uncertainty of data, a novel method based on a multi-index system and fuzzy set theory is proposed in this paper for predicting and evaluating rock burst in underground caverns. The factors, result vectors and weight vectors are expressed by interval numbers. Moreover, subjective weight and objective weight are combined to obtain ideal synthetic weight. The proposed method is applied to a practical case in China, whose evaluation results are in good agreement with the actual situation. The multicollinearity of these indices is discussed by results based on different indicator systems. The validity and feasibility of the proposed method are well demonstrated below.

Multi-index system of rock burst

Rock burst is a fracture instability phenomenon produced by brittle surrounding rock in underground excavation. Rock bursts are prone to occur when the energy stored in the rock mass is high, as high energy is the material basis for rock burst events (Shang et al. 2013; Huang et al. 2014). When the rock mass is excavated, the surrounding environment of the surrounding rock is altered, and the stress field of the surrounding rock is redistributed, which is the external factor affecting rock burst. The process of rock burst is very complex, and the mechanism leading to rock burst occurrence is affected by many factors. As to the main influential factors of rock burst, scholars around the world have put forward a series of corresponding criteria. Since the 1960s, several theories have been advanced to explain the mechanism of this hazard, including strength theory, rigidity theory and energy theory (Zhang et al. 2014; Chen et al. 2015; Hao et al. 2016).

In this study, combined with previous studies (Yu et al. 2009; Qiu et al. 2011; Jia et al. 2014; Li et al. 2017; Xue et al. 2017), 11 factors with a potential influence on rock burst will be analyzed. These factors can be generally classified into rock mechanical properties (b_1), stress conditions (b_2) and surrounding rock conditions (b_3) in this study. These factors are the coefficient of deformation brittleness (C1) and coefficient of strength brittleness (C2), the elastic strain energy index (C3), elastic strain energy (C4), energy storage index (C5), stress index (C6), stress coefficient (C7), Turchaninov criterion (C8), RQD (C9), rock integrity coefficient (C10) and buried depth of rock (C11). C1–C5 belong to lithological conditions. C6–C8 belong to stress conditions. C9–C11 belong to surrounding rock conditions. These evaluation indices are quantitatively classified as four grades: none (micro-risk or no risk), weak (low risk), moderate (medium risk) and strong (high risk) as shown in Table 1. A brief description of these parameters is presented below.

Rock mechanical properties

When the elastic strain stored in surrounding rock is greater than the energy consumed by rock fragmentation, rock fragments will collapse outward. The rock mass, which is prone to rock burst, is usually capable of storing more elastic strain energy. The hard and intact rock mass has a high ability to store strain energy and a tendency toward rock burst. Moreover, the uniaxial and triaxial compression tests of rock show that the large brittle rock will soon undergo macroscopic damage after loading peaks. Thus, brittle fracture of rock is one of the necessary conditions for rock burst. The brittleness of rock often reflects the proneness of rock burst. The coefficient of deformation brittleness K_u (C1) and the coefficient of strength brittleness R (C2) are common brittleness parameters, which can be expressed as follows.

$$K_u = u / u_l \tag{1}$$

$$R = \sigma_c / \sigma_t \tag{2}$$

In Eqs. (1) and (2), u and u_l are the total deformation and permanent deformation before peak load obtained by loading and unloading curves; σ_c and σ_t are the uniaxial compressive and tensile strength of rock, respectively. The coefficient of deformation brittleness K_u (C1) describes the characteristic of rock brittleness from the perspective of deformation. And the coefficient of strength brittleness R (C2) describes the characteristic of rock brittleness from the perspective of strength.

In addition, the elastic strain energy index (C3), elastic strain energy (C4) and energy storage index (C5) are also commonly used indices for assessing the proneness of rock burst at present (Jiang et al. 2010; Liu et al. 2013a, b; Xue et al. 2017). The elastic strain energy accumulated in rocks is one of the dominant factors of rock burst. The elastic strain energy index W_{et} (C3) is also called the rock burst proneness index for assessing rock burst, first proposed by Kidybiński (1981). It can be determined when the rock specimen is loaded to 70–

Table 1 Evaluation indices and grading standards of evaluation indices of rock burst

Evaluation index	I	II	III	IV
Coefficient of deformation brittleness (C ₁)	<2.0	2.0–6.0	6.0–9.0	>9.0
Coefficient of strength brittleness (C ₂)	<10	10–14	14–18	>18
Elastic strain energy index (C ₃)	<2.0	2.0–3.5	3.5–5.0	>5.0
Strain energy for uniaxial test (C ₄) / (kJ/m ³)	<40	40–100	100–200	>200
Energy storage index (C ₅)	<20	20–100	100–130	>130
Stress index (C ₆)	>14.5	5.5–14.5	2.5–5.5	<2.5
Stress coefficient (C ₇)	<0.3	0.3–0.5	0.5–0.7	>0.7
Turchaninov criterion (C ₈)	<0.3	0.3–0.5	0.5–0.8	>0.8
RQD (C ₉)	<55%	55%–70%	70%–85%	>85%
Rock integrity coefficient (C ₁₀)	<0.55 /V VI	0.55–0.65 /IV	0.65–0.75 /III	>0.75 /II I
Depth of rock (C ₁₁) /m	<50	50–200	200–700	>700

80% of its UCS and then unloaded to 50% of its UCS. The elastic strain energy index can be calculated as follows.

$$W_{et} = \Phi_{sp}/\Phi_{st} \quad (3)$$

where Φ_{sp} is elastic strain energy (total area below the loading curve) and Φ_{st} is dissipative strain energy (area between the loading curve and unloading curve) (Shan and Yan 2010). According to the State Standard of Poland, if $W_{et} \geq 5.0$, strong rock burst would take place, and if $2.0 \leq W_{et} < 5.0$, weak or medium class rock burst would occur.

The stored elastic strain energy before the rock reaches its peak strength, plays an important role in rock burst. In uniaxial compression tests, the strain energy for the uniaxial test W_e can be calculated as follows:

$$W_e = \frac{\sigma_c^2}{2E_s} \quad (4)$$

where σ_c is the uniaxial compressive strength and E_s is the effective Young's modulus. The stored elastic strain energy is divided into four grades, as shown in Table 1.

Moreover, the storage capacity of elastic deformation energy for rock should be taken into account, to better assess the proneness of rock burst. The accumulation and dissipation of energy should also be considered. Then, the energy storage index k is defined (Tang and Wang 2002).

$$k = \frac{\sigma_c \cdot \varepsilon_f}{\sigma_t \cdot \varepsilon_b} \quad (5)$$

where σ_t and σ_c are the uniaxial tensile and compressive strength of rock, respectively. ε_f and ε_b are the total strain before and after the peak in the stress-strain curve for the uniaxial test, respectively. The elastic strain energy index W_{et} (C3) is determined by the uniaxial loading and unloading test and the strain energy for the uniaxial test W_e is determined by the uniaxial compression test data. The energy storage index k shall take the accumulation and dissipation of energy into account. These indices are to assess the proneness of rock burst from the perspective of strain energy. These evaluation indices can be divided into four grades based on previous studies (Liu et al. 2013b; Xue et al. 2017), as shown in Table 1.

Stress conditions

In situ stress can objectively reflect the magnitude of the elastic strain energy accumulated in the rock. Rock mass will gather higher strain energy under conditions of high in situ stress. The excavation and mining of underground engineering results in stress redistribution, which further results in stress concentration. Rock burst can easily occur once there is an engineering disturbance and the stress is concentrated to critical stress. The stress criterion for rock bursts should be

taken into account for both the stress state and the rock mechanical properties. Hence, factors associated with stresses are most often used as criteria for the assessment of rock burst intensity (Liu et al. 2013b).

Specifically, the main parameters reflecting the stress state include in situ stress, tangential stress, axial stress and uniaxial compressive strength (Kong and Shang 2018; Aliyu et al. 2019). At present, a number of rock burst stress criteria have been proposed worldwide, including the Russenes criterion and the Turchaninov criterion. Specifically, the Russenes criterion (Russenes 1974) sets up the classification criterion for rock burst strength by means of the ratio of the maximum tangential stress around the tunnel σ_θ to the uniaxial compressive strength σ_c . The Turchaninov criterion (Turchaninov et al. 1972) assesses the occurrence time and intensity of rock burst using the ratio of σ_c and the sum of σ_θ and σ_L , where σ_L is the axial stress around the opening. Based on the tangential stress criterion, the ratio of in situ stress σ_1 of the surrounding rock to the compressive strength σ_c is defined as the stress index S (C6) (Cook et al. 2016); the ratio of the tangential stress σ_θ of the surrounding rock to the compressive strength σ_c is defined as the stress coefficient P (C7) (Xue et al. 2017), with the consideration of stress state and rock mechanics of the rock mass. In this study, stress index S (C6), stress coefficient P (C7) and Turchaninov criterion T (C8) are selected as the main stress state parameters, as follows.

$$S = \frac{\sigma_c}{\sigma_1} \quad (6)$$

$$P = \frac{\sigma_\theta}{\sigma_c} \quad (7)$$

$$T = \frac{\sigma_\theta + \sigma_L}{\sigma_c} \quad (8)$$

where σ_1 is in situ stress; σ_θ is the tangential stress around the openings; σ_L is axial stress; σ_c is the uniaxial compressive strength of rock. These indices are quantitatively classified according to four risk levels (Table 1).

Surrounding rock conditions

Surrounding rock is the carrier of stress in the process of rock burst. The mechanical properties of surrounding rock, such as strength, integrity and depth, are influential factors of rock burst, which can determine the possibility of rock burst directly. In this study, we chose RQD (C9), rock integrity coefficient K_V (C10) and buried depth of rock (C11) as factors, as shown in Table 1.

Rock quality designation (RQD) is an important index that reflects the integrity and strength of rocks and fracture development. The integrity of the fractured rock mass is relatively poor. Elastic strain energy is not easily accumulated in a fractured rock mass, and the stress concentration phenomenon

does not easily occur. Thus, RQD is low in fractured rock mass, and the possibility for rock burst is small. On the contrary, the intact rock mass has relatively strong ability to store elastic strain energy. The more integrity of the rock mass, the more likely rock burst occurs. In other words, if the integrity of the surrounding rock is good, the strength is high and the depth is low, it will lead to great deformation resistance. Moreover, under the construction disturbance, large deformation will not occur. Thus it will reduce the possibility of rock burst and other geological disasters. Therefore, the integrity coefficient of rock mass K_V (C10) is also an important index for rock burst (Huang et al. 2011).

$$K_V = \frac{v_{pm}^2}{v_{pr}^2} \tag{9}$$

where v_{pm} is the average speed of sound in coal and rock mass; v_{pr} is the average speed of sound in rock. K_V is applied to indicate the integrity of coal and rock mass.

The observation of real cases shows that rock burst mainly occurs in deep rock engineering. Though the assessment of rock burst would probably be more accurate with in situ rock stress, the measurement of ground stress is expensive, and it is often difficult to accurately estimate in the early stages of projects. Therefore, tunnel depth is usually selected as an alternative (Li et al. 2017; Xue et al. 2017). The greater the buried depth, the larger the in situ stress will be. Under conditions of high in situ stress, soft rock will produce plastic flow deformation and brittle hard rock is prone to rock burst. In the present paper, the depth of rock is divided into four grades (Table 1).

In summary, 11 factors with a potential influence on rock burst are considered. These factors are categorized into three groups, i.e., rock mechanical properties, stress conditions and surrounding rock conditions. However, in fact, the sufficient data and detailed information of rock is generally difficult to capture in general. It is difficult to assess the rock burst risk based on all the 11 factors in a specific tunnel case. Thus, according to the three group factors, one or two of the dependent influencing factors could be selected to establish a new evaluation index system when applying the fuzzy comprehensive assessment method, with high feasibility and effectiveness. Therefore, the evaluation index system is not fixed. According to the specific engineering project, various combination of these indices can form different multi-index assessment systems for rock burst. The proposed interval assessment model can analyze different evaluation systems for the different specific projects. Moreover, conceptually some of the factors are dependent on each other, which have a similar physical concept. For instance, C1 and C2 are related to the rock brittleness. These dependent influencing factors may create bias in the results. Therefore, it is necessary to evaluate the multi-collinearity of parameters in this model. In this paper, it was analyzed and discussed in the section of case study.

Fuzzy comprehensive assessment method

Computational model

Fuzzy comprehensive evaluation theory provides a novel method to better deal with imprecision and ambiguity of fuzzy factors in human judgments. It has been widely used in the assessment of geological disasters (Guo and Jiang 2010; Cai et al. 2018; Wang et al. 2019). The evaluation theory is based on fuzzy set theory and can make it possible to represent imprecise concepts accurately. In this paper, the fuzzy comprehensive evaluation theory is used to assess rock burst disasters. In actual conditions, it is worth noting that we need to consider that evaluation indicators obtained from field exploration are usually interval numbers. In other words, it is hard to be represented by a definite value, for which is difficult to reflect the uncertainty of the fuzzy evaluation index in the quantification process. Thus, interval numbers are applied to the assessment of rock burst in this study to clearly reflect the uncertainty and fuzziness of fuzzy evaluation.

In this study, an interval fuzzy comprehensive evaluation model is established with the aid of the multi-index assessment system and fuzzy set theory. The judgment matrix and weight vector have interval features and are expressed as an interval number. The result vectors and weight vectors can be expressed by interval numbers. The nonlinear fuzzy evaluation function is calculated by Eqs. (10) to (14) and the interval fuzzy comprehensive evaluation model is shown as follows.

$$A = W \cdot B_i \tag{10}$$

$$B_i = W_i \cdot C_i \tag{11}$$

$$A = [[a_{I}^-, a_{I}^+], [a_{II}^-, a_{II}^+], [a_{III}^-, a_{III}^+], [a_{IV}^-, a_{IV}^+]] \tag{12}$$

$$W = [[w_1^-, w_1^+], [w_2^-, w_2^+], [w_3^-, w_3^+]] \tag{13}$$

$$C_i = [c_{ijk}] = \begin{bmatrix} c_{i1I} & c_{i1II} & c_{i1III} & c_{i1IV} \\ c_{i2I} & c_{i2II} & c_{i2III} & c_{i2IV} \\ \vdots & \vdots & \vdots & \vdots \\ c_{ijI} & c_{ijII} & c_{ijIII} & c_{ijIV} \end{bmatrix} \tag{14}$$

In Eq. (14), A and B_i are result vectors of the destination layer (A) and the influence factor layer (B); W_i and W are the weight vectors of the evaluation index layer (C) and the influence factor layer (B); C_i is the judgment matrix consisting of membership degrees; c_{ijk} is the membership degree for the evaluation index c_i of influential factor b_j belonging to risk grade k , where $i = 1-11$ and $j = 1-3$. The weight vectors can be determined by the comprehensive weighting method.

Membership degree based on sigmoid function

There are several nonlinear membership functions, such as, Gaussian membership function, sigmoid membership

function and parabola membership function. In this paper, the membership degrees are determined by a sigmoid membership function. The curve of sigmoid membership function has good smoothness, clear physical meaning and zero degree, which is commonly used in research (Liu et al. 2013a). The quantitative index and qualitative index are two types of evaluation indices of rock burst hazard assessment in underground caverns. The coefficient of deformation brittleness (C1) and the coefficient of strength brittleness (C2), the elastic strain energy index (C3), the elastic strain energy (C4), the energy storage index (C5), stress index (C6), stress coefficient (C7), Turchaninov criterion (C8), and the buried depth of rock (C11) are quantitative indices. RQD (C9) and the rock integrity coefficient (C10) are qualitative indices. With the help of the proposed grade standard of indices system, the differential sigmoid membership functions can be constructed and the membership degree can be calculated as follows.

For index belonging to the boundary grades, the membership function should be calculated by the following formula.

$$c_{ijk} = \frac{1}{1 + e^{a(x-c)}} \tag{15}$$

$$c_{ijk} = \frac{1}{1 + e^{-a(x-c)}} \tag{16}$$

where the boundary grade is $[c, +\infty]$ or $[-\infty, c]$.

For index belonging to the intermediate grades, the membership function can be calculated as:

$$c_{ijk} = \frac{1}{1 + e^{-a_1(x-c_1)}} - \frac{1}{1 + e^{-a_2(x-c_2)}} \tag{17}$$

where the boundary grade is $[c_1, c_2]$.

In Eqs. (15), (16) and (17), the parameter a controls the curvature of the curve. In this paper, $a \in [\frac{20}{c_1}, \frac{80}{c_2}]$. The sigmoid membership function is inherently open to the right or to the left depending on the sign of the parameter a . If a is positive, the curve of it opens to right, vice versa. And $\forall x \in (-\infty, +\infty)$,

$$\sum_{k=1}^4 c_{ijk} = 1.$$

The membership function of quantitative index

Following the above method, the membership functions of the strain energy (C4) are established in the following formulas:

$$c_{41I} = \frac{1}{1 + e^{0.4(x-40)}} \tag{18}$$

$$c_{41II} = \frac{1}{1 + e^{-0.4(x-40)}} - \frac{1}{1 + e^{-0.4(x-100)}} \tag{19}$$

$$c_{41III} = \frac{1}{1 + e^{-0.4(x-100)}} - \frac{1}{1 + e^{-0.4(x-200)}} \tag{20}$$

$$c_{41IV} = \frac{1}{1 + e^{-0.4(x-200)}} \tag{21}$$

The membership function of other quantitative indices can be obtained as well. The upper and lower bounds of interval number for the quantitative evaluation index can be determined according to Table 1, respectively. Then according to the specific interval value, the membership degrees of quantitative evaluation indices can be obtained.

The membership function of qualitative index

The influential indices are mostly qualitative descriptions actually, which are obtained from preliminary project survey data. For convenience of calculation, the range of qualitative indices is represented by a number between 0 and 100. In accordance with the above method for RQD (C9), the sigmoid membership function can be established.

$$c_{93I} = \frac{1}{1 + e^{1.2(x-55)}} \tag{22}$$

$$c_{93II} = \frac{1}{1 + e^{-1.2(x-55)}} - \frac{1}{1 + e^{-1.2(x-70)}} \tag{23}$$

$$c_{93III} = \frac{1}{1 + e^{-1.2(x-70)}} - \frac{1}{1 + e^{-1.2(x-85)}} \tag{24}$$

$$c_{93IV} = \frac{1}{1 + e^{-1.2(x-85)}} \tag{25}$$

The index values obtained by the expert scoring method are also expressed in the form of interval numbers. According to the upper and lower bounds of these interval numbers, the membership degree can be calculated by the above sigmoid membership functions.

Weight determination analysis

In the evaluation and analysis of rock burst, the weights of each index directly affect the accuracy and validity of results. At the present stage, the commonly used weight determination methods are the subjective weighting method and the objective weighting method. Every weight determination method has its advantages and disadvantages. Objective weighting methods may cause deviation because of relying too much on data and the amount of information provided by each index. The proposed weight needs not only to reflect the subjective cognition of the mechanism of geological disasters, but also to reflect the law of index data objectively. In this paper, subjective weights and objective weights are combined to obtain ideal synthetic weights. Specifically, the subjective weights are calculated by an uncertain analytic hierarchy process, and the objective weights are calculated by the entropy method.

$$W = \alpha w_s + \beta w_o \tag{26}$$

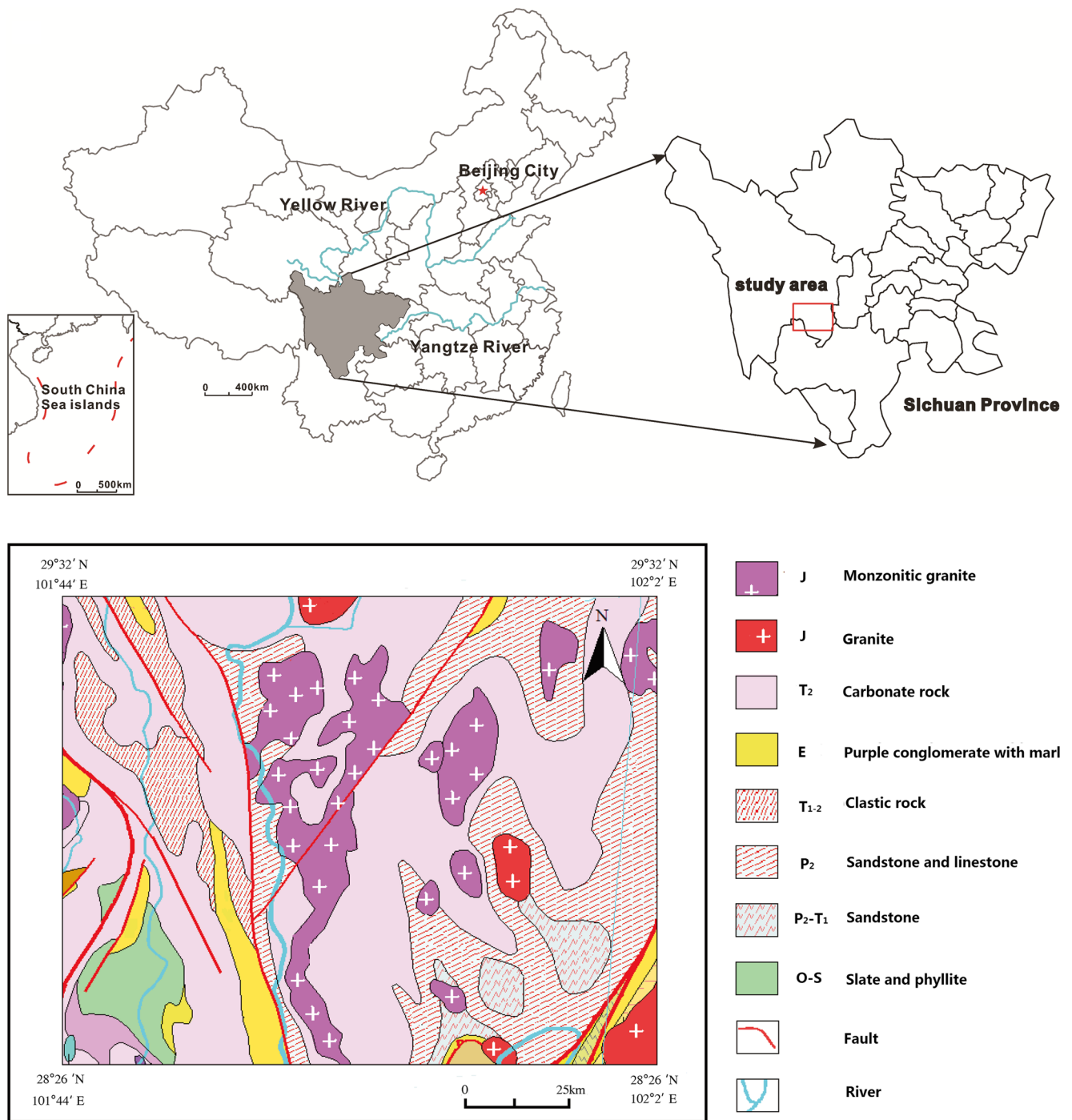


Fig. 1 Geological profile of Jiangbian hydropower station. Data from the China Geological Survey website (<http://www.cgs.gov.cn/>)

where α and β are weight coefficients of objective weights and subjective weights, and $\alpha + \beta = 1$. α and β will be taken as 0.5 and 0.5.

Subjective weights

Based on the scale method (Saaty and Vargas 1987), paired comparisons are adopted to derive a scale of relative

importance for index. In this paper, the paired comparison is used as an interval scale according to an uncertain analytic hierarchy process (uncertain AHP). Suppose there are n evaluation indicators, the judgment comparison matrix $[a_{ij}]_{n \times n}$ ($j = 1-n$) of the interval number is acquired as an interval matrix by paired comparisons (Yang and Wu 2005; Gong et al. 2014), as well as the judgment matrixes of B grade indices and C. The consistency of the judgment matrixes has great

Table 2 The representative cavern sample data for the Jiangbian hydropower station

Sample	C2	C3	C6	C7	C10	C11
Construction branch tunnel	[19.35, 42.3]	[2.75, 4.6]	[5.3, 16.75]	[0.22, 0.66]	[0.36, 0.88]	[166, 318]
Traffic tunnel	[10.36, 27.3]	[3.2, 5.23]	[2.77, 4.85]	[0.42, 0.72]	[0.62, 0.92]	[277, 317]
Main powerhouse	[28.77, 32.8]	[2.97, 3.6]	[3.8, 12.25]	[0.42, 0.58]	[0.64, 0.71]	[148, 274]
Tailrace tunnel	[37.35, 42.7]	[2.17, 3.08]	[4.36, 15.2]	[0.28, 0.37]	[0.49, 0.66]	[155, 171]
High-pressure water diversion tunnel	[16.55, 29.7]	[3.82, 5.72]	[2.37, 5.35]	[0.69, 0.73]	[0.70, 0.90]	[265, 278]

influence on weight vectors. The comparison matrix with consistency has the following properties.

$$[a_{ij}]_{n \times n} = \left[[a_{ij}^-, a_{ij}^+] \right]_{n \times n} \quad (27)$$

$$a_{ij} = \frac{1}{a_{ji}} \quad (28)$$

$$a_{ij}a_{jk} = a_{ij}a_{ik} \quad (29)$$

The matrixes for $[a_{ij}^-]_{n \times n}$ and $[a_{ij}^+]_{n \times n}$ are constructed, which are the matrixes of the lower and upper bounds of the interval comparison judgment matrix. The weight vector of the comparison judgment matrix is established on the basis of the stochastic consistency and the characteristic root method. Assume that W is the interval weight vector and X is the

normalized eigenvector of the maximum eigenvalue, X is determined as follows (Wang et al. 2012):

$$X = \left[[x_{ij}^-, x_{ij}^+] \right]_{1 \times n} \quad (30)$$

$$x_{ij}^\pm = \sqrt[n]{\prod_{j=1}^n a_{ij}} \quad (31)$$

Then, the interval weight vector W can be constructed:

$$w_{ij}^\pm = x_{ij}^\pm / \sum_{i=1}^n x_{ij}^\pm \quad (32)$$

$$W = [W_{ij}]_{1 \times n} = \left[[\sqrt{m}w_{ij}^-, \sqrt{n}w_{ij}^+] \right]_{1 \times n} \quad (33)$$

In Eq. (33), $m = \sum_{j=1}^n \left[\left(\sum_{i=1}^n a_{ij}^+ \right)^{-1} \right]$ and $n = \sum_{j=1}^n \left[\left(\sum_{i=1}^n a_{ij}^- \right)^{-1} \right]$. The objective weights of indices can be determined by the above method.

Objective weights

Generally, entropy refers to disorder or uncertainty. In statistics, entropy is the average rate at which information is produced by a stochastic source of data and a measure of system disorder. The greater the entropy, the more chaotic the system and the greater the uncertainty will be, and the less information the system can provide. When the data source has a lower-probability value (i.e., the entropy is small), the event carries more information than when the source data has a higher-probability value. According to the characteristics of entropy, the randomness and degree of disorder of an event can be assessed by calculating the magnitude of the entropy. The greater the degree of dispersion of the indicator, the greater the impact of the indicator on the overall evaluation.

Table 3 Judgment matrix of B grade indices

Factors	B1	B2	B3	Weight of B grade indices
B1	[1, 1]	[2, 3]	[2, 4]	[0.52, 0.60]
B2	[1/3, 1/2]	[1, 1]	[1, 2]	[0.23, 0.27]
B3	[1/4, 1/2]	[1/2, 1]	[1, 1]	[0.17, 0.21]

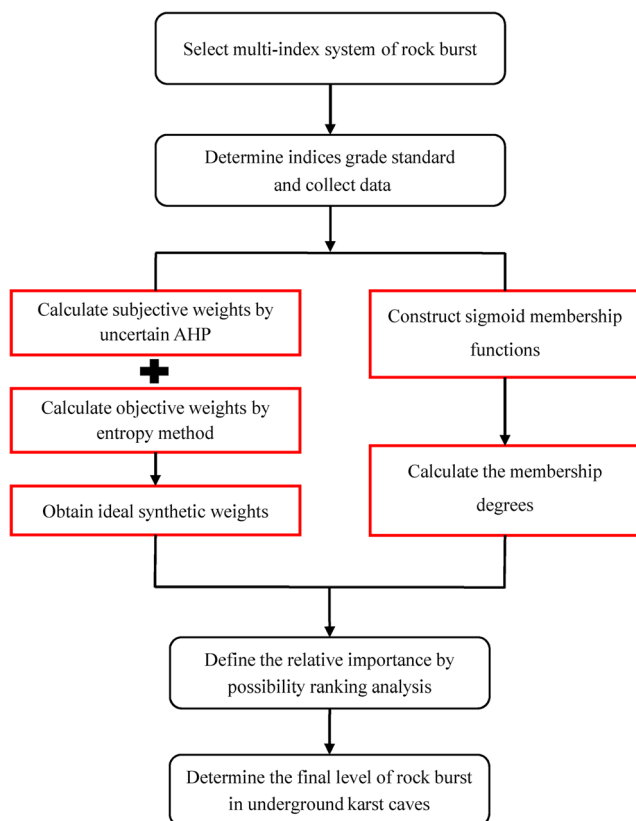
**Fig. 2** The predicted process of rock burst hazard in underground caverns

Table 4 Judgment matrix of C grade indices for the index systems composed of C₂, C₃, C₆, C₇, C₁₀ and C₁₁

Assessment index	C2	C3	C6	C7	C10	C11	Interval weight vector <i>W</i>
C2	[1, 1]	[1/4, 1/2]					[0.24, 0.28]
C3	[2, 4]	[1, 1]					[0.69, 0.78]
C6			[1, 1]	[2, 3]			[0.66, 0.76]
C7			[1/3, 1/2]	[1, 1]			[0.27, 0.31]
C10					[1, 1]	[1/3, 1/2]	[0.27, 0.31]
C11					[2, 3]	[1, 1]	[0.66, 0.76]

Therefore, the smaller the entropy value, the greater the weight should be given. The objective weight is calculated by the entropy method (Zhao and Chai 2015; Wang et al. 2018) in this paper.

Suppose that there are *m* feasible schemes (*A_m*) and *n* evaluation indices *G_n* for multiple attribute decision-making problems, and the interval index value is *b_{ij}*. A normalized judgment matrix *B* can be determined.

$$b_{ij} = [b_{ij}^-, b_{ij}^+] \tag{34}$$

$$B = [b_{ij}]_{m \times n} = \left[[b_{ij}^-, b_{ij}^+] \right]_{m \times n} \tag{35}$$

where *i* = 1, 2, ..., *m*, *j* = 1, 2, ..., *n*.

Based on the principle of objective weighting method, an evaluation index has greater effect on the optimization or ranking of the scheme when the deviation of the index value from the median value of the interval is larger. Thus, this evaluation index should be given a greater weight. On the other hand, the smaller the effect of the evaluation index on the optimization or ranking of the scheme, the smaller the weight of the evaluation index. Moreover, the deviation of the interval index value is reflected in the deviations of the upper bound *b_{ij}⁺* and the lower bound *b_{ij}⁻* when index values are expressed by interval numbers. The information entropy can accurately measure the deviation. The entropy of the evaluation index *G_j* can be expressed as:

$$E_j^- = -(\ln m)^{-1} \sum_{i=1}^m p_{ij}^- \ln p_{ij}^- \tag{36}$$

$$E_j^+ = -(\ln m)^{-1} \sum_{i=1}^m p_{ij}^+ \ln p_{ij}^+ \tag{37}$$

In Eqs. (36) and (37), $p_{ij}^- = b_{ij}^- / \sum_{i=1}^m b_{ij}^-$ and

$p_{ij}^+ = b_{ij}^+ / \sum_{i=1}^m b_{ij}^+$, *i* = 1, 2, ..., *m*, *j* = 1, 2, ..., *n*. Then the

importance *d_j* of evaluation index *G_j* and the entropy weight *w_j* are calculated.

$$d_j = 1 - E_j^\pm \tag{38}$$

$$w_j = \frac{d_j}{\sum_{j=1}^n d_j} \tag{39}$$

where $\sum_{j=1}^n w_j = 1$ and *j* = 1, 2, ..., *n*. According to the above method, the subjective weights can be determined.

Possibility ranking of interval vectors

Based on the above analysis, the result vectors can be constructed by the nonlinear fuzzy evaluation function with the help of the determined weights and membership degrees. In this study, the possibility ranking analysis is used to sort the result vectors as suggested by Li et al. (2016). The information of each membership degree shall be considered fully during possibility ranking analysis. Therefore, the evaluation will be comprehensive and detailed. First, the relative dominance between two series of interval numbers was calculated and a judgment matrix was constructed. Then, the weight of the judgment matrix was determined and the rock burst risk level was determined by the principle of maximum weight. Specifically, the relative importance between different interval number vectors *A* = [*a*, *ā*] and *B* = [*b*, *b̄*] are as follows. If the boundary satisfies *a*⁻ ≤ *b*⁻ ≤ *b*⁺ ≤ *a*⁺, the relative importance between *A* and *B* can be defined as:

$$P(A > B) = \frac{a^+ - b^+}{l(a)} + \frac{b^+ - b^-}{2l(a)} \tag{40}$$

Table 5 Indices weight determined by comprehensive weighting method for the index systems composed of C₂, C₃, C₆, C₇, C₁₀ and C₁₁

Factors	C2	C3	C6	C7	C10	C11
Objective weights	[0.12, 0.17]	[0.36, 0.47]	[0.15, 0.21]	[0.06, 0.08]	[0.05, 0.07]	[0.11, 0.16]
Subjective weights	[0.08, 0.31]	[0.06, 0.10]	[0.14, 0.54]	[0.12, 0.27]	[0.04, 0.09]	[0.10, 0.13]
Synthetic weight	[0.10, 0.24]	[0.21, 0.29]	[0.15, 0.38]	[0.09, 0.17]	[0.05, 0.08]	[0.11, 0.15]

Table 6 Index weights for the index systems composed of C₂, C₃, C₆, C₁₀ and C₁₁

Factors	C2	C3	C6	C10	C11
Objective weights	[0.12, 0.17]	[0.36, 0.47]	[0.23, 0.27]	[0.05, 0.07]	[0.11, 0.16]
Subjective weights	[0.18, 0.42]	[0.09, 0.19]	[0.19, 0.27]	[0.12, 0.18]	[0.18, 0.19]
Synthetic weight	[0.15, 0.30]	[0.22, 0.33]	[0.21, 0.27]	[0.08, 0.12]	[0.15, 0.17]

If the boundary satisfies $b^- \leq a^- \leq b^+ \leq a^+$, the relative importance between A and B can be defined as:

$$P(A > B) = \frac{a^+ - b^+}{l(a)} + \frac{(b^+ - a^-)(a^- - b^-)}{l(a)l(b)} + \frac{(b^+ - a^-)^2}{2l(a)l(b)} \times \frac{(b^+ - a^-)}{l(b)} \tag{41}$$

In Eqs. (40) and (41), $l(a) = a^+ - a^-$ and $l(b) = b^+ - b^-$. Besides, the relative importance is 1 when the upper or lower boundary satisfies $b^- \leq b^+ \leq a^- \leq a^+$.

Let P_{ij} ($i, j = 1-4$) represent the relative importance $P(A > B)$; A and B are the interval numbers in the result vectors $[A, B, \dots]$, respectively. Then the relative dominance between the two pairs of intervals can be calculated by Eqs. (40) and (41). And the relative dominance matrix $[P] = [P_{ij}]_{4 \times 4}$ can be obtained, which is a fuzzy complementary matrix. By means of the ranking method of a fuzzy complementary matrix, the weight of a relative importance matrix $[P]$ is obtained as follows.

$$R_i = \frac{1}{n(n-1)} \left(\sum_{j=1}^n P_{ij} + \frac{n}{2} - 1 \right) \tag{42}$$

Then, the final risk level of rock burst hazard in underground caverns can be determined by the maximum weight principle.

Engineering application

Engineering background

Rock bursts often occur in underground caverns in mines and civil engineering sites. The Jiangbian hydropower station was

selected as a practical case in this study (Xue et al. 2017). It is located on the Jiulong River, a tributary of the Yalong River, in the southeast of Ganzi Tibetan Autonomous Prefecture, Sichuan Province, China (Fig. 1). This hydropower station includes several buildings and underground caverns, mainly consisting of a water diversion system and underground powerhouses. The bedrock of the underground powerhouse caverns is Yanshanian biotite granite, with moderate weathering. The rock is hard and the lithology is single. The rock mass structure is relatively complete, and the rock mass quality is considered to be good. The joint is closed and there are few faults. The study area is in a high ground stress zone, with maximum principal stress of 14 MPa at a depth of 302 m. The groundwater development is poor. At the initial stage of the high-pressure diversion tunnel construction, slight rock burst disasters occurred frequently, and strong rocks burst occurred several times (Xue et al. 2017).

Results

In this manuscript, 11 factors were considered which are categorized in three groups of rock mechanical properties, stress conditions and surrounding rock conditions. Some factors are dependent on each other and have a similar physical concept. Thus, one of the dependent influencing factors could be selected from each group to form a new evaluation index system based on the acquired data. In this case, the collection of data is a massive challenge for their applicability, and the data for all 11 influence factors are not fully available for this case study. Based on the available geological and hydrogeological conditions of underground caverns, the coefficients of strength brittleness (C₂), elastic strain energy index (C₃), stress

Table 7 The membership functions of the evaluation indices

Index	Membership function			
	I	II	III	IV
C2	$C_{21I} = \frac{1}{1+e^{4(x-10)}}$	$C_{21II} = \frac{1}{1+e^{4(x-10)}} - \frac{1}{1+e^{4(x-14)}}$	$C_{21III} = \frac{1}{1+e^{4(x-14)}} - \frac{1}{1+e^{4(x-18)}}$	$C_{21IV} = \frac{1}{1+e^{4(x-18)}}$
C3	$C_{31I} = \frac{1}{1+e^{16(x-2)}}$	$C_{31II} = \frac{1}{1+e^{-16(x-2)}} - \frac{1}{1+e^{-16(x-3.5)}}$	$C_{31III} = \frac{1}{1+e^{-16(x-3.5)}} - \frac{1}{1+e^{-16(x-5)}}$	$C_{31IV} = \frac{1}{1+e^{-16(x-5)}}$
C6	$C_{61I} = \frac{1}{1+e^{-16(x-14.5)}}$	$C_{61II} = \frac{1}{1+e^{-16(x-5.5)}} - \frac{1}{1+e^{-16(x-14.5)}}$	$C_{61III} = \frac{1}{1+e^{-16(x-2.5)}} - \frac{1}{1+e^{-16(x-5.5)}}$	$C_{61IV} = \frac{1}{1+e^{16(x-2.5)}}$
C7	$C_{72I} = \frac{1}{1+e^{120(x-0.3)}}$	$C_{72II} = \frac{1}{1+e^{-120(x-0.3)}} - \frac{1}{1+e^{-120(x-0.5)}}$	$C_{72III} = \frac{1}{1+e^{-120(x-0.5)}} - \frac{1}{1+e^{-120(x-0.7)}}$	$C_{72IV} = \frac{1}{1+e^{-120(x-0.7)}}$
C10	$C_{103I} = \frac{1}{1+e^{160(x-0.55)}}$	$C_{103II} = \frac{1}{1+e^{-160(x-0.55)}} - \frac{1}{1+e^{-160(x-0.65)}}$	$C_{103III} = \frac{1}{1+e^{-160(x-0.65)}} - \frac{1}{1+e^{-160(x-0.75)}}$	$C_{103IV} = \frac{1}{1+e^{-160(x-0.75)}}$
C11	$C_{113I} = \frac{1}{1+e^{0.4(x-50)}}$	$C_{113II} = \frac{1}{1+e^{-0.4(x-50)}} - \frac{1}{1+e^{-0.4(x-200)}}$	$C_{113III} = \frac{1}{1+e^{-0.4(x-200)}} - \frac{1}{1+e^{-0.4(x-700)}}$	$C_{113IV} = \frac{1}{1+e^{-0.4(x-700)}}$

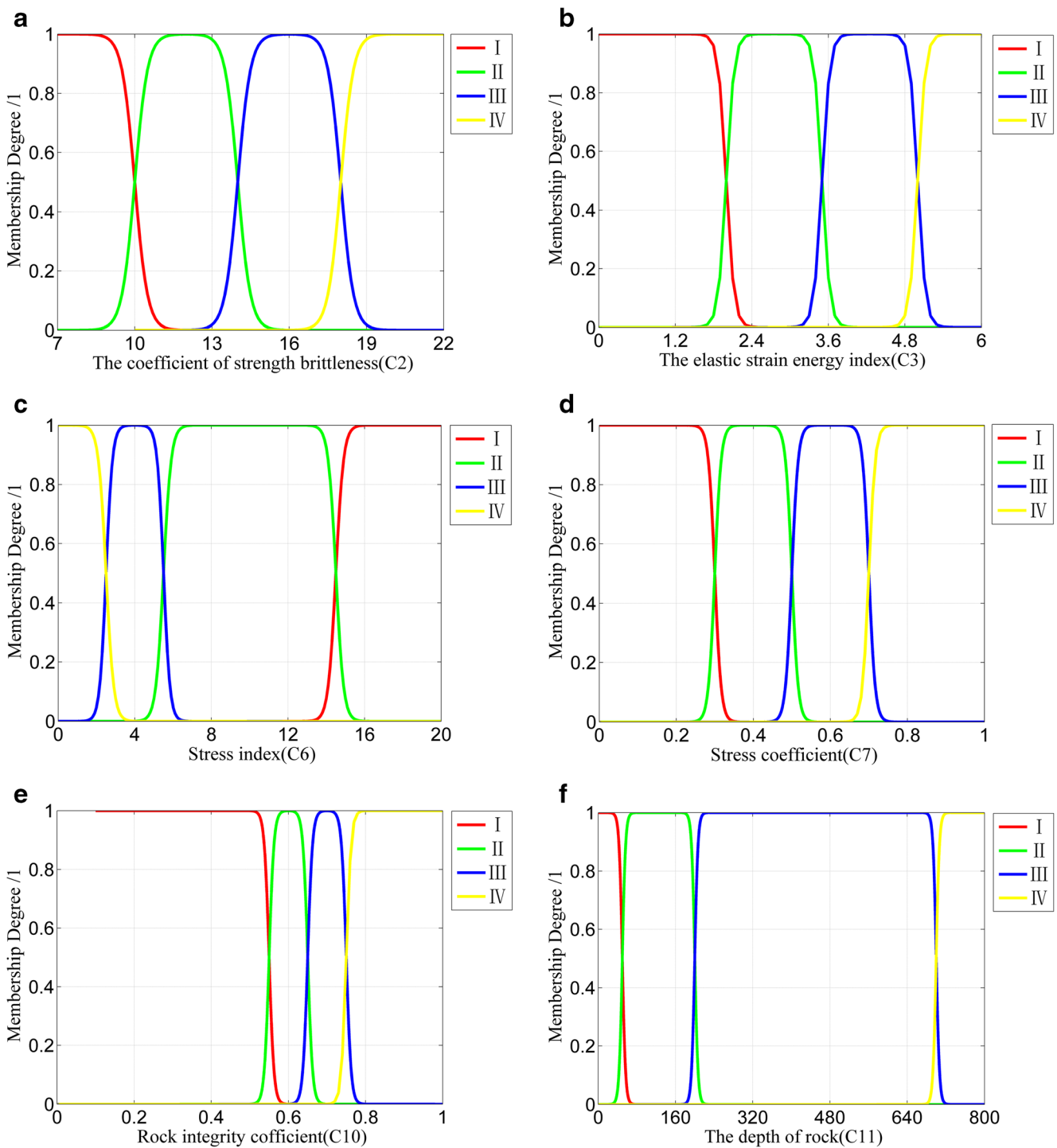


Fig. 3 Membership degree curves of the sigmoid membership function

index (C_6), stress coefficient (C_7), rock integrity coefficient (C_{10}) and the depth of rock (C_{11}) were selected as influence factors. The index value for rock burst risk was derived as shown in Table 2.

The proposed interval fuzzy comprehensive evaluation method was used to predict the occurrence of rock burst, and the predicted process is depicted in Fig. 2. The weight and

judgment matrixes are represented by interval numbers. The judgment matrixes are shown in Tables 3 and 4. The interval weight vector of subjective weights was then able to be constructed by Eqs. (27)–(33). For objective weights, the entropy of the evaluation index is expressed and the entropy weights are determined by Eqs. (35)–(38). The weight vectors are determined and ranked by Eqs. (39)–(41), as shown in

Table 8 Results of case study and comparison for the index systems composed of C_2 , C_3 , C_6 , C_{10} and C_{11}

Sample	Weight vector	Proposed level	Actual level
Construction branch tunnel	[[0, 0.106], [0.019, 0.252], [0, 0.250], [0.051, 0.138]]	II	II
Traffic tunnel	[[0, 0.027], [0.042, 0.331], [0.067, 0.439], [0, 0.334]]	III	III
Main powerhouse	[[0, 0.004], [0.018, 0.250], [0.001, 0.227], [0.052, 0.125]]	II	II
Tailrace tunnel	[[0, 0.085], [0.121, 0.241], [0, 0.074], [0.051, 0.124]]	II	II
High-pressure water diversion tunnel	[[0, 0], [0, 0.006], [0.024, 0.370], [0.001, 0.344]]	III	IV

Table 5. It is known from the table that the weight of stress index (C_6) is greater than stress coefficient (C_7). Two indicator systems were established to discuss the multicollinearity of the parameters evaluated in the model. The indicator systems (A) were composed of C_2 , C_3 , C_6 , C_{10} and C_{11} , and another indicator systems (B) were composed of C_2 , C_3 , C_6 , C_7 , C_{10} and C_{11} . In this paper, rock burst risk assessment was conducted for these indicator systems. Then, the weight vectors for the indicator systems (B) were determined by Eqs. (39)–(41), as shown in Table 6. With the aid of the proposed indices grade standard, the differential sigmoid membership functions could be constructed as shown in Table 7 and Fig. 3.

The result vectors can be constructed by the nonlinear fuzzy evaluation function (Tables 8 and 9). Then, the relative importance between different interval number vectors could be sorted by the possibility ranking analysis and determined by the maximum weight principle. The relative importance matrix weights P_i were obtained for the index systems (A) composed of C_2 , C_3 , C_6 , C_{10} and C_{11} , and the relative importance matrix weights P_i' were obtained for the index systems (B) composed of C_2 , C_3 , C_6 , C_7 , C_{10} and C_{11} .

$$P_1 = [0.158, 0.292, 0.283, 0.267] \quad (43)$$

$$P_2 = [0.125, 0.277, 0.320, 0.278] \quad (44)$$

$$P_3 = [0.125, 0.315, 0.314, 0.245] \quad (45)$$

$$P_4 = [0.216, 0.375, 0.129, 0.280] \quad (46)$$

$$P_5 = [0.125, 0.214, 0.333, 0.328] \quad (47)$$

$$P_1' = [0.161, 0.294, 0.289, 0.256] \quad (48)$$

$$P_2' = [0.125, 0.284, 0.322, 0.269] \quad (49)$$

$$P_3' = [0.125, 0.317, 0.305, 0.253] \quad (50)$$

$$P_4' = [0.244, 0.375, 0.138, 0.243] \quad (51)$$

$$P_5' = [0.125, 0.211, 0.334, 0.330] \quad (52)$$

Discussion

In the Jiangbian hydropower station, the actual level of rock burst intensity was determined by the drill cutting method. The advanced geological prediction was conducted to obtain the cuttings and cores of the surrounding rock. Then, the risk grade of rock burst was determined according to the ratio of the drilled rock volume to the theoretical borehole volume for the rock with lower strength. Moreover, the crack and the stuck phenomenon during the drilling process can also assist in determining the actual level of rock burst intensity. In this paper, the final risk level of rock burst for the two indicator systems are shown in Tables 8 and 9. It is known from the tables that the results calculated based on different indicator systems are consistent. The final levels for construction branch tunnel, main powerhouse, and tailrace tunnel were II, and traffic tunnel and high-pressure water diversion tunnel fall into level III. The assessment levels are in high agreement with the actual detection results (Xue et al. 2017), seen in Fig. 4. It could be concluded that the proposed fuzzy comprehensive evaluation model generated satisfactory results. Hence, the proposed fuzzy comprehensive assessment model is feasible and applicable in this research for the assessment of rock burst.

The results of the proposed method provided us with a lot of useful information. As an example, the results obtained based on the indicator systems (A) were analyzed. With the weights being $R_{III} = 0.320$, $R_{II} = 0.277$ and $R_{IV} = 0.277$, the risk level of

Table 9 Results for the index systems composed of C_2 , C_3 , C_6 , C_7 , C_{10} and C_{11}

Sample	Weight vector	Proposed level
Construction branch tunnel	[[0, 0.140], [0.014, 0.316], [0, 0.313], [0.052, 0.139]]	II
Traffic tunnel	[[0, 0.027], [0.042, 0.377], [0.053, 0.514], [0, 0.335]]	III
Main powerhouse	[[0, 0.003], [0.018, 0.314], [0.001, 0.291], [0.052, 0.125]]	II
Tailrace tunnel	[[0, 0.146], [0.123, 0.305], [0, 0.099], [0.051, 0.124]]	II
High-pressure water diversion tunnel	[[0, 0], [0, 0.008], [0.023, 0.424], [0.005, 0.406]]	III

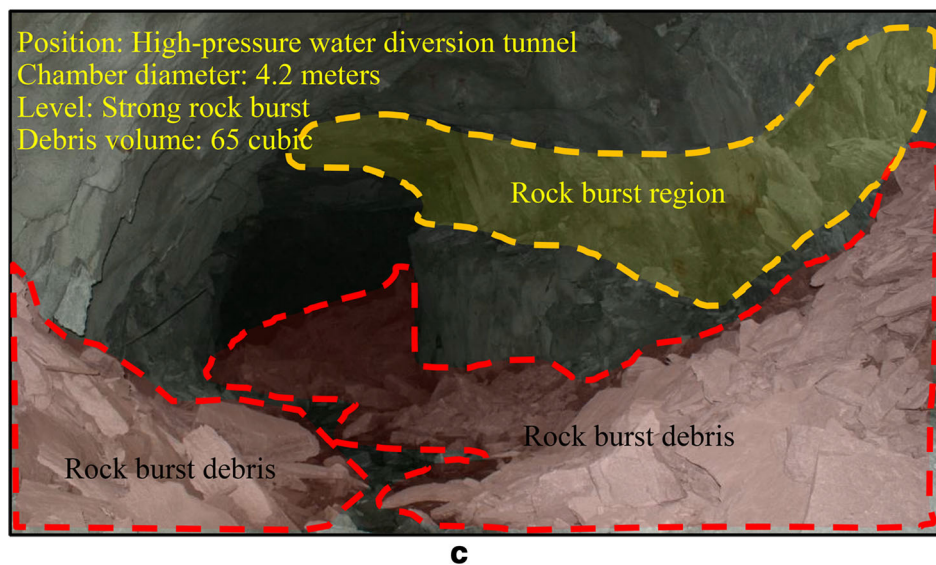
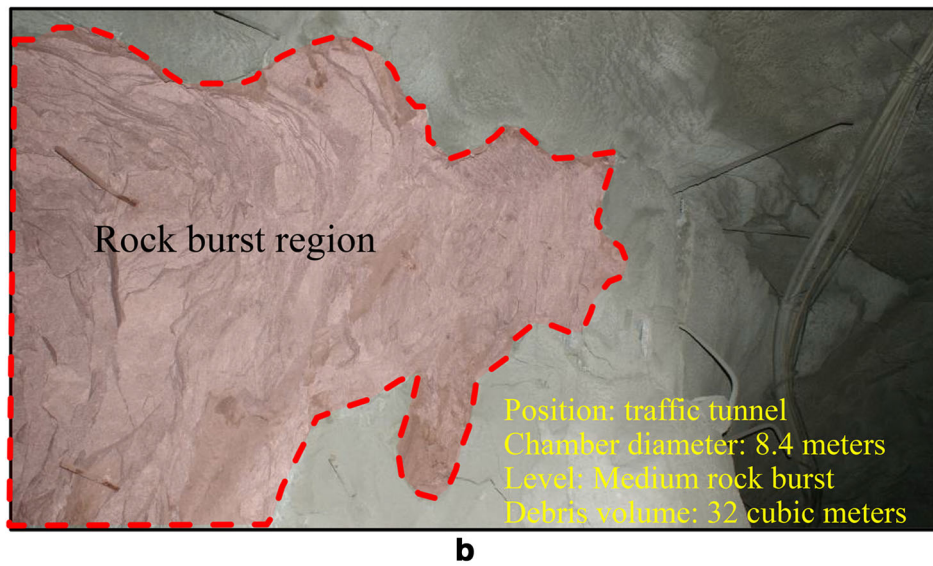
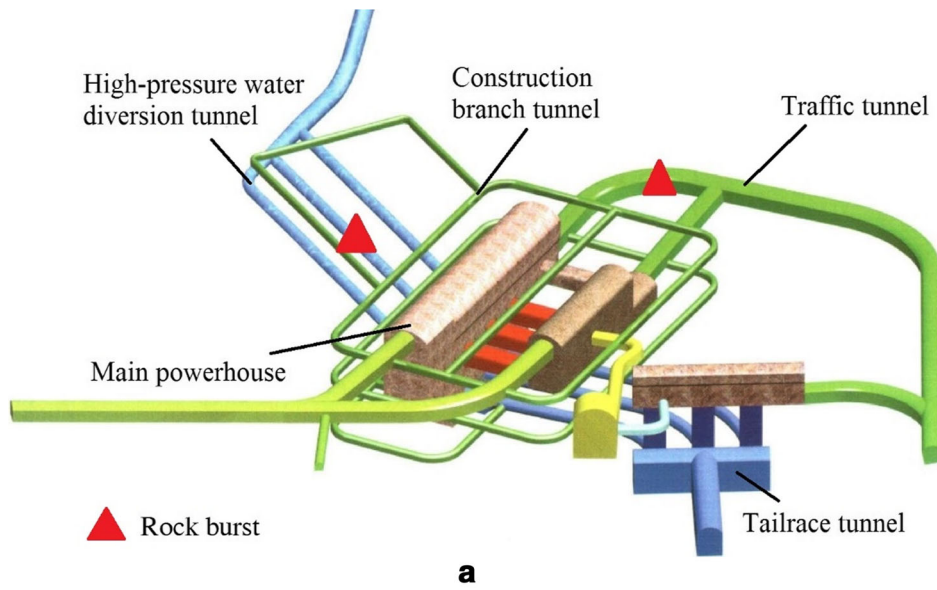


Fig. 4 Verification by practical situation (Xue et al. 2017)

the traffic tunnel is likely to fall into level III, or level II and level IV. In Table 8, the assessment level of the high-pressure water diversion tunnel is III, but the actual level is IV. The weights of the relative importance of high-pressure water diversion tunnel are $R_{III}=0.333$ and $R_{IV}=0.328$, and they are almost the same. Consequently, the high-pressure water diversion tunnel has almost the same possibility of being level III and IV. The final risk levels for construction branch tunnel, main powerhouse, and tailrace tunnel are of level II. The relative importance matrix weights $R_{II}=0.375$ for the tailrace tunnel is greater than $R_{II}=0.277$ for the construction branch tunnel and $R_{II}=0.315$ for the main powerhouse. Thus, the proposed level of the tailrace tunnel is more inclined to belong to II in the light of the obtained weight vectors and relative importance matrix weights. The risk level for the high-pressure water diversion tunnel can be opposed to the main powerhouse in a like manner and the main powerhouse shares almost the same possibility of being level II and III.

In this case, stress index (C6) and stress coefficient (C7) are dependent on each other conceptually, which have a similar physical concept. Therefore, the multicollinearity of the parameters evaluated of the indicator systems (A) and (B) was discussed by results. From Tables 8 and 9, the results based on the two indicator systems were consistent, and these index systems both had credibility. It was worth noting that the result based on the indicator systems (B), which was composed of $C_2, C_3, C_6, C_7, C_{10}$ and C_{11} , was more consistent with the actual rock burst risk level. For example, the rock burst final risk level of the traffic tunnel was III. Specifically, the relative importance matrix weights for risk level III were $R_{III}=0.320$ based on the indicator systems (A) and $R_{III}=0.322$ based on the indicator systems (B). The other cases could be similarly analyzed. In addition, the relative importance matrix weights $R_{IV}=0.330$ of the indicator systems (B) is greater than $R_{IV}=0.328$ of the indicator systems (A) for the high-pressure water diversion tunnel. Considering that the actual risk level for high-pressure water diversion tunnel was IV, the result based on the indicator systems (B) was more consistent with the actual rock burst risk level. Thus, it could be concluded that the indicator systems (B) made full use of all the data obtained and comprehensively reflect the rock burst risk. Reliable results may be obtained by evaluating as many indicators as possible instead of limiting the usage of dependent factors to one of them when data are available.

Six factors of rock burst were selected and the joint effect had a certain effect on the final assessment results. Table 5 shows the importance of these proposed factors. Specifically, the synthetic weight of the coefficient of the elastic strain energy index (C3) and stress index (C6) were larger than the other indices. Moreover, these factors are not only of great

significance to predict rock burst, but also a promising orientation in the selection of prevention measures to avoid and minimize rock burst. Rock burst risk could be reduced by controlling these factors. It is an effective method to change the shape and diameter of the openings, which are considerably easier to be controlled with proper design. Therefore, for the traffic tunnel and the high-pressure water diversion tunnel, the main parameters of the project layout scheme should be modified as the primary measure for rock burst hazard reduction, i.e., the location, sequence and method of excavation and the shape and diameter of the openings.

At present, rock burst is still often occurring and far from being under control. The theoretical study of rock burst needs to be supplemented continuously. This research provides a new idea for assessment of rock burst. Still, large data sets are needed to better study this event more comprehensively, then feasible measures can be taken to prevent rock burst hazards.

Conclusion

The present study provided a novel method for predicting and evaluating rock burst in underground caverns based on a multi-index system and fuzzy set theory. Eleven factors were analyzed that had potential influence on rock burst. This multi-index system was a good generalization of influence factors, which were generally classified into lithological conditions, stress conditions and surrounding rock conditions, and the quantitative standards of the indices were determined. Some factors were dependent on each other and have a similar physical concept. The proposed interval assessment model analyzed and predicted a rock burst evaluation index system established by a different combination of mentioned indices.

A fuzzy comprehensive assessment model was established in this paper and the result vectors and weight vectors were expressed by interval numbers. The membership degrees could be determined by a sigmoid membership function. Moreover, the synthetic weights were obtained by combining subjective weights and objective weights. Specifically, these weights were calculated by uncertain an analytic hierarchy process and entropy method, respectively. The proposed weight reflected the subjective cognition of a geological disasters mechanism and the law of index data objectively. Finally, the possibility ranking analysis was adopted to sort the final vectors.

In addition, this model had been applied to the Jiangbian hydropower station, Sichuan, China. Results suggested that the elastic strain energy index was the most influential parameter. The weights of strength brittleness coefficient and stress index were roughly equal, while the weights of stress coefficient and rock integrity coefficient were relatively small. Traffic tunnel and the high-pressure water diversion tunnel

fall into level III. The results were in good agreement with the actual situation. Moreover, the multicollinearity of the parameters evaluated in the proposed model was studied by results based on the different indicator systems. It could be concluded that the indicator system, which made full use of all the data obtained, comprehensively reflects the rock burst risk. Reliable results could be obtained by evaluating as many indicators as possible when data are available. The validity and feasibility of the proposed evaluation method for rock burst in underground caverns was verified.

The interval fuzzy comprehensive assessment model provides a reference for the rock burst assessment and guidance for related studies in the future. And it is worth noting that numerical simulation, field testing, monitoring and other methods are also commonly used in current practice. The proposed method should be combined with other methods to predict and evaluate rock burst hazard and to guide subsequent control measures.

Acknowledgments Much of the work presented in this paper was supported by the National Natural Science Foundation of China (Grant Nos. 51879153, 51422904, 51379112, 41877239) and the Fundamental Research Funds of Shandong University (Grant Nos. 2017JC001, 2017JC002).

Appendix

For the interval number $X=[a, b]$ and $Y=[c, d]$, the binary arithmetic rule to which the interval number should be followed (Sun and Wu 2014) are shown in the following formula:

$$\begin{aligned} [a, b] + [c, d] &= [a + c, b + d] \\ [a, b] - [c, d] &= [a - d, b - c] \\ [a, b] \times [c, d] &= [\min\{ac, ad, bc, bd\}, \max\{ac, ad, bc, bd\}] \\ [a, b] \div [c, d] &= [a, b] \times \left[\frac{1}{c}, \frac{1}{d}\right] \\ e \times [a, b] &= \begin{cases} [ea, eb] & a > 0 \\ [eb, ea] & a < 0 \end{cases} c^X = [c^a, c^b] X^n = [(a)^n, (b)^n] \frac{1}{[a, b]} = \left[\frac{1}{a}, \frac{1}{b}\right], 0 \notin [a, b] \end{aligned}$$

References

- Adoko AC, Gokceoglu C, Wu L, Zuo QJ (2013) Knowledge-based and data-driven fuzzy modeling for rockburst prediction. *Int J Rock Mech Min* 61(4):86–95
- Aliyu MM, Shang J, Murphy W, Lawrence JA, Collier R, Kong F, Zhao Z (2019) Assessing the uniaxial compressive strength of extremely hard cryptocrystalline flint. *Int J Rock Mech Min Sci* 113:310–321
- Cai M (2013) Principles of rock support in burst-prone ground. *Tunn Undergr Space Technol* 36:46–56
- Cai W, Dou LM, Zhang M, Cao WZ, Shi JQ, Feng LF (2018) A fuzzy comprehensive evaluation methodology for rock burst forecasting using microseismic monitoring. *Tunn Undergr Space Technol* 80: 232–245. <https://doi.org/10.1016/j.tust.2018.06.029>
- Chen SY, Guo Y (2005) Model of variable fuzzy sets and its application to classification prediction of rockburst. *Chin J Rock Mech Eng* 24: 4603–4609
- Chen BR, Feng XT, Li QP, Luo RZ, Li S (2015) Rock burst intensity classification based on the radiated energy with damage intensity at Jinping II hydropower station, China. *Rock Mech Rock Eng* 48(1): 289–303
- Cook NGW, Hoek E, Pretorius JP, Ortlepp WD, Salamon MDG (2016) Rock mechanics applied to study of rockburst. *J South Afr Inst Min Metall* 66(10):435–528
- Du K, Tao M, Li XB, Zhou J (2016) Experimental study of slabbing and rockburst induced by true-triaxial unloading and local dynamic disturbance. *Rock Mech Rock Eng* 49(9):3437–3453
- Gong J, Hu NL, Cui X, Wang XX (2014) Rock burst tendency prediction based on AHP-TOPSIS evaluation model. *Chin J Rock Mech Eng* 33(7):1442–1448
- Guo YH, Jiang F (2010) Application of comprehensive fuzzy evaluation in burst-proneness risk of coal seam. *Int Conf Future Inf Technol Manag Eng IEEE* 38(6):404–407
- Hao J, Shi KB, Wang XL et al (2016) Application of cloud model to rating of rockburst based on rough set of FCM algorithm. *Rock Soil Mech* 37(3):859–866
- He MC, Miao JL, Feng JL (2010) Rock burst process of limestone and its acoustic emission characteristics under true-triaxial unloading conditions. *Int J Rock Mech Min* 47(2):286–298
- Hu J, Li S, Li L, Shi S, Zhou Z, Liu H, He P (2018) Field, experimental, and numerical investigation of a rockfall above a tunnel portal in southwestern China. *Bull Eng Geol Environ* 77(4):1365–1382
- Huang BX, Liu CY, Fu JH, Guan H (2011) Hydraulic fracturing after water pressure control blasting for increased fracturing. *Int J Rock Mech Min* 48(6):976–983
- Huang YR, Mao JX, Lin CY, Li SL, Hu JY (2014) The multi-criteria evaluation of rockburst proneness on deep buried large tunnel. *J Railw Eng Soc* 31(7):89–94
- Jia YP, Lv Q, Shang YQ, Du LL, Zhi MM (2014) Rockburst prediction based on rough set and ideal point method. *J Zhejiang Univ* 48(3): 498–503
- Jiang Q, Feng XT, Xiang TB, Su GS (2010) Rockburst characteristics and numerical simulation based on a new energy index: a case study of a tunnel at 2500m depth. *Bull Eng Geol Environ* 69(3):381–388
- Kidybiński A (1981) Bursting liability indices of coal. *Int J Rock Mech Min* 18(4):295–304
- Kong F, Shang J (2018) A validation study for the estimation of uniaxial compressive strength based on index tests. *Rock Mech Rock Eng* 51(7):2289–2297
- Li QD, Han GZ, Zeng WY, Yu XC (2016) Ranking method of interval numbers based on Boolean matrix. *Control and Decision* 31(4):629–634
- Li N, Feng X, Jimenez R (2017) Predicting rock burst hazard with incomplete data using Bayesian networks. *Tunn Undergr Space Technol* 61:61–70
- Liu Q, Chen X, Wu Z, Liu X (2013a) Determination method of nonlinear membership function based on the fuzzy density means cluster. *Res J Appl Sci Eng Technol* 5(8):2527–2530
- Liu ZB, Shao JF, Xu WY, Meng YD (2013b) Prediction of rock burst classification using the technique of cloud models with attribution weight. *Nat Hazards* 68(2):549–568
- Miao SJ, Cai MF, Guo QF, Huang ZJ (2016) Rock burst prediction based on in-situ stress and energy accumulation theory. *Int J Rock Mech Min* 83:86–94
- Peng Q, Qian A, Xiao Y (2010) Research on prediction system for rockburst based on artificial intelligence application methods. *J Sichuan Univ Eng Sci Ed* 42(2):18–24
- Peng YH, Peng K, Zhou J, Liu ZX (2014) Prediction of classification of rock burst risk based on genetic algorithms with SVM. *Appl Mech Mater* 628:383–389

- Qiu D, Zhang L, Li S, Zhang D (2011) Application of ideal point method in Rockburst prediction based on weight back analysis method. *GeoHunan Int Conf*. [https://doi.org/10.1061/47631\(410\)31](https://doi.org/10.1061/47631(410)31)
- Russenes BF (1974) Analysis of rock spalling for tunnels in steep valley sides (in Norwegian) Master Thesis of Science, Norwegian Institute of Technology, Trondheim
- Saaty TL, Vargas LG (1987) Uncertainty and rank order in the analytic hierarchy process. *Eur J Oper Res* 32(1):107–117
- Shan ZG, Yan P (2010) Management of rock burst during excavation of the deep tunnels in Jinping II hydropower station. *Bull Eng Geol Environ* 69(3):353–363
- Shang YJ, Zhang JJ, Fu BJ (2013) Analyses of three parameters for strain mode rockburst and expression of rockburst potential. *Chin J Rock Mech Eng* 32(8):1520–1527
- Sun J, Wu J, (2014) Optimization study on interval number judgment matrix weight vector based on immune evolution algorithm. *International Journal of Security and Its Applications* 8(3):355–362
- Tang LZ, Wang WX (2002) A new index of rock burst proneness. *Chin J Rock Mech Eng* 21(6):874–878
- Turchaninov A, Markov GA, Gzovsky MV et al (1972) State of stress in the upper part of the Earth's crust based on direct measurements in mines and on tectonophysical and seismological studies. *Phys Earth Planet Inter* 6:229–234
- Wang C, Peng W (2014) Rockburst prediction based on electromagnetic radiation technology and its application. *Adv Mater Res* 1010: 1564–1567
- Wang C, Lin L, Liu J (2012) Uncertainty weight generation approach based on uncertainty comparison matrices. *Appl Math* 3(5):499–507
- Wang XT, Li SC, Xu ZH, Hu J, Pan DD, Xue YG (2018) Risk assessment of water inrush in karst tunnels excavation based on normal cloud model. *Bull Eng Geol Environ*. <https://doi.org/10.1007/s10064-018-1294-6>
- Wang XT, Li SC, Xu ZH, Lin P, Hu J, Wang WY (2019) Analysis of factors influencing floor water inrush in coal mines: a nonlinear fuzzy interval assessment method. *Mine Water Environ*. <https://doi.org/10.1007/s10230-018-00578-x>
- Xue YG, Li ZQ, Li SC, Qiu DH, Tao YF et al (2017) Prediction of rock burst in underground caverns based on rough set and extensible comprehensive evaluation. *Bull Eng Geol Environ*. <https://doi.org/10.1007/s10064-017-1117-1>
- Yang J, Wu X (2005) Comprehensive forecasting method for estimating rock burst. *Chin J Rock Mech Eng* 24(3):411–416
- Yu HC, Liu HN, Lu XS, Liu HD (2009) Prediction method of rock burst proneness based on rough set and genetic algorithm. *J Coal Sci Eng* 15(4):367–373
- Zhang C, Zhou H, Qiu S, Wu W (2012) A top pilot tunnel preconditioning method for the prevention of extremely intense rockburst in deep tunnels excavated by TBMS. *Rock Mech Rock Eng* 45(3):289–309
- Zhang Y, Xiao PX, Chen LJ (2014) Method of layout design based on ratio of rock strength to in-situ stress for large underground caverns. *Chin J Rock Mech Eng* 33(11):2314–2331
- Zhang Y, Liang P, Liu X, Liu S, Tian B (2015) Experimental study on precursor of rock burst based on acoustic emission signal dominant-frequency and entropy. *Chin J Rock Mech Eng* 34:2959–2967
- Zhao S, Chai LH (2015) A new assessment approach for urban ecosystem health basing on maximum information entropy method. *Stoch Env Res Risk A* 29(6):1601–1613
- Zhou KP, Lin Y, Deng HW, Li JL, Liu CJ (2016) Prediction of rock burst classification using cloud model with entropy weight. *Trans Nonferrous Metal Soc China* 26(7):1995–2002