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Calibration of V_s -based empirical models for assessing soil liquefaction potential using expanded database

Chen Guoxing^{1,2} \cdot Kong Mengyun^{1,2} \cdot Sara Khoshnevisan³ \cdot Chen Weiyun^{1,2} \cdot Li Xiaojun 1,4

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Abstract The shear wave velocity (V_s) -based method of assessing the liquefaction susceptibility of soils has given geotechnical engineers a promising alternative for determining soil resistance to seismic liquefaction. Through the use of an expanded global case histories of soil liquefaction, the authors obtained a new calibrated correlation equation for liquefaction triggering analysis. This new liquefaction triggering correlation was found to be reasonably conservative, and yet insensitive to the variations in the components of the adopted simplified analysis framework. A probabilistic version of the liquefaction triggering correlation, obtained with the weighted maximum-likelihood technique, was validated using a

4. A probabilistic version of the new calibrated liquefaction triggering correlation is presented

 \boxtimes Chen Guoxing gxc6307@163.com

> Sara Khoshnevisan khoshnevisan.sara@gmail.com

- ¹ Institute of Geotechnical Engineering, Nanjing Tech University, Nanjing 210009, China
- ² Civil Engineering and Earthquake Disaster Prevention Center of Jiangsu Province, Nanjing 210009, China
- ³ Civil and Environmental Engineering Department, Clarkson University, Potsdam, NY 13699, USA
- ⁴ Institute of Geophysics, China Earthquake Administration, Beijing 100081, China

weighted empirical probability approach. Three zones of liquefaction potential, transitioning from non-liquefaction to liquefaction, were then defined based on liquefaction probability contours.

Keywords Soil liquefaction . Probability . Shear wave velocity . Case history

Introduction

The widespread liquefaction phenomena in loose saturated sandy sediments is an inevitable occurrence in coastal plains subjected to great earthquakes. For example, severe liquefaction occurred in the Tianjing coastal plain during the 1976 Tangshan $M_{\rm w}$ 7.6 earthquake (Fu and Tatsuoka [1984](#page-11-0); Liu [1986\)](#page-12-0), the Hanshin area during the 1995 Hyogo-ken Nanbu $M_{\rm w}$ 6.9 earthquake (Elgamal et al. [1996](#page-11-0)), the eastern suburbs and satellite townships of Christchurch, New Zealand during the 2010–2011 Canterbury $M_{\rm w}$ 5.2–7.1 earthquake sequence (Quigley et al. [2013\)](#page-12-0), and the reclaimed shore areas of Tokyo Bay during the 2011 M_w 9.0 earthquake off the Pacific coast of Tohoku (Bhattacharya et al. [2011](#page-11-0); Yamaguchi et al. [2012\)](#page-12-0).

The effect of soil liquefaction in an earthquake can be devastating. For example, the 2011 Tohoku Earthquake off the Pacific Coast also indirectly resulted in the radioactive contamination of the surface from the accident at the Fukushima Daiichi nuclear power plant. Similarly, although the reactors of the Kashiwazaki-Kariwa nuclear power plant were safely shut down during the 2007 Niigata Chuetsu-offshore M_w 6.8 earthquake off the coast of Japan, damage nonetheless occurred, and the subsequent soil liquefaction severely affected the structures, triggered a transformer fire, and simultaneously damaged the fire-suppression system and closed the wastewater treatment facility (RGHNCE [2008](#page-12-0)). These events clearly

Highlights

^{1.} A new V_s -based deterministic liquefaction triggering correlation was determined

^{2.} The new correlation is insensitive to variations in the components of liquefaction analysis framework

^{3.} An empirical equation for determining the probability of liquefaction was determined

indicate the vulnerability of nuclear power plants located in coastal plains from tremendous original and secondary disasters induced by seismic soil liquefaction. Thus, designing facilities against such devastating geotechnical and geoenvironmental issues from great earthquakes in coastal areas has presented a huge challenge to geotechnical engineers and designers. In practice, the engineer generally adopts simplified methods for assessing liquefaction potential and risk at a given project site.

The simplified method, pioneered by Seed and Idriss [\(1971](#page-12-0)), is the most widely adopted method for evaluating the seismic soil liquefaction potential in North America and throughout much of the world. This method is used to determine soil liquefaction resistance at a given depth of a soil profile subjected to the earthquake loading expressed as cyclic stress ratio (CSR) (e.g., Seed [1979](#page-12-0), [1987;](#page-12-0) Seed et al. [1985](#page-12-0); Idriss [1999;](#page-11-0) Cetin et al. [2002](#page-11-0), [2004](#page-11-0); Youd et al. [2001](#page-12-0); Idriss and Boulanger [2006](#page-11-0), [2008](#page-11-0), [2010](#page-11-0); Boulanger and Idriss [2012\)](#page-11-0). The main task of the liquefaction evaluation involves determination of the cyclic resistance ratio (CRR), i.e., the capacity of the soil to resist seismic liquefaction, using in situ tests such as the standard penetration test (SPT), the cone penetration test (CPT), and the shear-wave velocity (V_s) measurement. Although the SPT, the CPT, and the V_s measurement can all be used to assess the CRR and the liquefaction potential, only the latter is focused on in this paper.

Unlike the SPT and CPT penetration methods, V_s actually measures the fundamental property of the soil and is considerably less sensitive to the problems of soil compression and penetration resistance reduction in the presence of fine soils (Kayen et al. [2013](#page-12-0)). It is also more suitable for sites underlain by soils that are difficult to penetrate or sample (Andrus and Stokoe [2000](#page-11-0); Kayen et al. [2013\)](#page-12-0). Although not as widely adopted as the SPT-based and CPT-based methods, the V_s based simplified method for the liquefaction potential evaluation has gained significant attention in the geotechnical earthquake engineering communities (e.g., Dobry et al. [1982](#page-11-0); Alba et al. [1984;](#page-11-0) Tokimatsu and Uchida [1990](#page-12-0); Kayen et al. [1992](#page-12-0); Robertson et al. [1992;](#page-12-0) Lodge [1994](#page-12-0); Andrus and Stokoe [2000](#page-11-0); Juang et al. [2001](#page-11-0); Youd et al. [2001;](#page-12-0) Juang et al. [2002a;](#page-11-0) Andrus et al. [2004;](#page-11-0) Chen et al. [2005;](#page-11-0) Juang et al. [2005](#page-11-0); Liu and Mitchell [2006;](#page-12-0) Zhou and Chen [2007;](#page-12-0) Uyanik and Taktak [2009](#page-12-0); Zhou et al. [2010;](#page-12-0) Kayen et al. [2013](#page-12-0); Ahmadi and Paydar [2014](#page-11-0); Dobry et al. [2015](#page-11-0)).

Juang et al. ([2002a\)](#page-11-0) presented a probability-based method for their evaluation of liquefaction potential using V_s . Based on the Bayesian mapping functions developed by Juang et al. [\(2002a](#page-11-0)), a factor of safety against liquefaction of $F_s = 1.2$ obtained with the V_s -based method of Andrus and Stokoe [\(2000](#page-11-0)) corresponds to an expected liquefaction probability of 16%. Similarly, Kayen et al. ([2013](#page-12-0)) recommended the adoption of a probability of liquefaction of $P_L = 15\%$, which corresponds to $F_s = 1.17$ in their calibration study, for use as

the single deterministic boundary curve for evaluating the liquefaction potential.

Kayen et al. [\(2013\)](#page-12-0) compiled a global database of 415 V_s based case histories of field liquefaction performance in an effort to better define the boundary curve at high CSR values. They concluded that there was no need to artificially set a limiting upper bound of the effective overburden stresscorrected shear wave velocity of clean soils (V_{s1}) . This limiting upper bound of the V_{s1} had previously been assumed to vary from 200 m/s for soils with a fine content of 35% to 215 m/s for soils with a fine content of 5% or less at liquefaction sites (Andrus and Stokoe [2000;](#page-11-0) Youd et al. [2001\)](#page-12-0).

Figure [1](#page-2-0) compared the liquefaction triggering correlations of clean sands from the 15% probability contour developed by Kayen et al. [\(2013\)](#page-12-0), and the 10% and 50% probability contour developed by Juang et al. [\(2002a](#page-11-0)), against the deterministic boundaries developed by Andrus and Stokoe ([2000](#page-11-0)), and the lower boundaries developed by Zhou and Chen ([2007](#page-12-0)). Although the liquefaction triggering correlation developed by Kayen et al. ([2013\)](#page-12-0) may be useful in civil engineering projects with low-to-medium potential risk, it appears less suitable for critical infrastructures such as nuclear power plants. Therefore, it is necessary to re-calibrate the liquefaction triggering correlations using the expanded V_s -based case history database to reduce the probability of mistaking liquefiable soil for non-liquefiable soil when evaluating the liquefaction potential for projects with different risk levels.

In this paper, a new empirical liquefaction triggering correlation is obtained through an updated calibration using an expanded database, and a conservative assessment procedure is established for evaluation of the soil liquefaction potential at nuclear power plant project sites with extremely high-risk potential. Then, a simplified probabilistic version of the proposed correlation, expressed in terms of the P_L and the nominal safety factor (F_s) , is presented. It should be noted that the nominal value of the safety factor was defined as the ratio of the nominal cyclic resistance ratio (CRR) over the nominal cyclic stress ratio (\widetilde{CSR}), following the probability-based analysis framework developed by Juang et al. [\(2012](#page-11-0), [2013](#page-11-0)).

Calibration of the new liquefaction triggering correlation based on expanded database

Case history database

The V_s -based liquefaction and non-liquefaction case history database in this paper has been expanded from the databases of Andrus et al. ([1999](#page-11-0)), Chu et al. [\(2004\)](#page-11-0), Saygili ([2005](#page-12-0)), and Kayen et al. [\(2013\)](#page-12-0). These data were combined into a global catalog of 618 case histories from 49 earthquake events with V_s -based liquefaction performance observations, of which 380

Fig. 1 Comparison of the liquefaction triggering correlation ($M_w = 7.5$) for the deterministic V_s -based assessment of liquefaction for clean sand

cases exhibited surface evidence of liquefaction, 234 cases exhibited no surface evidence of liquefaction, and four cases from 2003 Tokachi Oki $M_{\rm w}$ 7.8 earthquake were in the margin between liquefaction and non-liquefaction. In general, the occurrence of ground failure at the surface (e.g., hydraulic fracturing, sand boiling, ground cracking, lateral spreading or sliding) and its resulting effects (e.g., the uplifting of the manholes and sewage drainage pipes, the settling and tilting of the buildings and houses) were observed at the liquefaction sites. The contents of these expanded databases are detailed below.

Of the 12 case history data developed by Chu et al. [\(2004\)](#page-11-0) from the 1999 Chi-Chi, Taiwan $M_{\rm w}$ 7.6 earthquake, nine cases indicated surface evidence of liquefaction, three cases indicated no evidence of surface liquefaction, and one case was in the realm between liquefaction and non-liquefaction (called marginal cases). Of the 41 case histories developed by Saygili [\(2005](#page-12-0)) from the 1999 Chi-Chi earthquake and the 1999 Kocaeli, Turkey M_w 7.4 earthquake, 24 cases indicated surface evidence of liquefaction, and 17 cases indicated no surface evidence of liquefaction. Of the 415 case histories developed by Kayen et al. ([2013](#page-12-0)) from the earthquakes in Mainland China, Greece, Japan, Taiwan (China), and the United States, 287 cases indicated a surface evidence of liquefaction, 124 cases indicated no surface evidence of liquefaction, and four cases were marginal cases. Of the 225 case histories developed by Andrus et al. [\(1999\)](#page-11-0) from earthquakes in Mainland China, Japan, Taiwan (China), and the United States, 75 cases were the same as those developed by Kayen et al. ([2013](#page-12-0)) and were removed from the final database. Thus, only 150 cases from the database by Andrus et al. [\(1999\)](#page-11-0) were included in the final database. Of these 150 cases, 60 indicated a surface evidence of liquefaction and 90 indicated no surface evidence of liquefaction.

In summary, the final database included 380 liquefaction cases and 234 no-liquefaction cases for a total of 614 case histories from 49 earthquakes. The details of the final database are presented in Table [1](#page-3-0).

Normalized shear-wave velocity and effect of fines content

The shear wave velocity (V_s) is a small-strain property related to the undisturbed shear modulus and the mass density of the soil. The V_s of a soil is primarily influenced by the principal stresses in the direction of wave propagation and particle motion, and soil conditions, such as void ratio, stress history, grain characteristics, cementation, and geologic age (Andrus and Stokoe [2000;](#page-11-0) Ahmadi and Paydar [2014\)](#page-11-0). Of course, the measurement accuracy and precision of V_s depends on geophysical test methods, procedural details, soil conditions, and interpretation technique. The uncertainties associated with the calculation of the dispersion curve and the inversion of individual shear-wave velocity profiles were beyond the scope of this study, and will be elucidated in future work. A study by Moss [\(2008](#page-12-0)) indicates that the average combined coefficient of variation for the dispersion and inversion calculations of the test data of spectral-analysis-of-surface-waves is approximately 0.15. Typically, the field measurement of V_s is corrected to a normalized V_{s1} at a reference stress of 100 kPa as follows (Kayen et al. [1992;](#page-12-0) Robertson et al. [1992](#page-12-0); Andrus et al. [2004](#page-11-0)):

$$
V_{s1} = V_s \left(\frac{P_a}{\sigma_v'}\right)^{0.25} \tag{1}
$$

Three methods are available to assess the effect of the fines content (FC) : (1) the use of inter-granular soil mechanics, (2) the undertaking of field SPT and CPT penetration measurements, and (3) the use of V_s for low-strain measurements (Kayen et al. [2013](#page-12-0)). For penetration measurements, the increase of the non-plastic fines causes the apparent decrease of the frictional resistance and increases excess pore pressures. As such, the effect of FC as applied to the penetration-based liquefaction correlation boundaries is quite strong (Polito [1999](#page-12-0)). On the other hand, the V_s measurement is noninvasive and it measures the small strain shear modulus G_0 through V_s [$G_0 = \rho V 2$ S, where ρ is the mass density of soil]. Given the very small difference between the G_0 of sand and silty or clayey sand, all these soils exhibit the same initial stiffness when undergoing strain (Iwasaki and Tatsuoka [1977\)](#page-11-0), which renders the V_s measurements relatively insensitive to any small discrepancies in the FC. The work of Kayen et al. ([2013](#page-12-0)) on the liquefaction correlation boundary curve shows that the required adjustment of V_s to account for the effect of FC (from 5% to 35%) is generally less than 5 m/s. This is quite consistent with the findings of previous studies (Andrus and Stokoe [2000;](#page-11-0) Huang et al. [2004;](#page-11-0) Liu and Mitchell [2006;](#page-12-0) Zhou and Chen [2007;](#page-12-0) Dabiri et al. [2011](#page-11-0)).

Table 1 (continued)

(continued)

Compared with other aspects of the liquefaction potential evaluation, the uncertainty associated with FC is relatively minor and thus the influence of FC on the normalized V_{s1} was not considered in this study.

Calibration of V_s -based liquefaction triggering correlation

The V_s -based liquefaction triggering correlation for soil sites was studied herein using the liquefaction analysis framework by Boulanger and Idriss [\(2012\)](#page-11-0). Specifically, the following liquefaction analysis framework was used in this study: (1) the earthquake-induced cyclic stress ratio (CSR) at a given depth of the soil profile was first calculated, and (2) then the cyclic resistance ratio (CRR) was computed based on the V_{s1} of the soil. The liquefaction of soil is said to occur when $CSR > CRR$; otherwise, no soil liquefaction occurs. The CSR and CRR are computed at a given depth, which is generally taken at a representative depth of a liquefiable layer at a given test location. The occurrence of liquefaction is based on surface manifestation at the site. Strictly speaking, the exactly one (a set of input data) to one (an observation) correspondence cannot be assured of, as the simplified approach is not about the controlled experimentation. Here, CSR is expressed as (Idriss and Boulanger [2006](#page-11-0); Boulanger and Idriss [2012\)](#page-11-0):

$$
CSR = 0.65\frac{\sigma_v}{\sigma_v'}\frac{a_{\text{max}}}{g}r_d\frac{l}{MSFK_{\sigma}}
$$
\n(2)

where, σ_{ν} and σ_{ν} are the vertical total and effective overburden stresses, respectively; g is the acceleration of gravity; a_{max} is the maximum horizontal peak ground surface acceleration; r_d is the shear stress reduction factor; *MSF* is the magnitude scaling factor (to adjust all data to $M_w = 7.5$); and K_{σ} is the overburden correction factor for the cyclic stress ratio $(K_{\sigma} = 1.0$ according to Kayen et al. [2013](#page-12-0)).

The proposed liquefaction triggering correlation between the CRR and V_{s1} takes the form of Eq. (3) with five parameters (a_1-a_5) in this paper:

$$
CRR = \exp\left[\frac{V_{s1}}{a_1} + \left(\frac{V_{s1}}{a_2}\right)^2 - \left(\frac{V_{s1}}{a_3}\right)^3 + \left(\frac{V_{s1}}{a_4}\right)^4 - a_5\right] \quad (3)
$$

These five parameters could be determined by trial and error using the following two principles: (1) the liquefaction triggering correlation for $V_{s1} \leq 200$ m/s is generally controlled by the lower bound of the liquefaction case history data and (2) the liquefaction triggering correlation for V_{s1} > 200 m/s is generally controlled by the upper bound of the nonliquefaction case history data. To further elaborate this point, the parameter a_5 is given a trial value in priori. Four points at the boundary of the liquefaction triggering correlation (marked with larger open circles, as shown in Fig. [2\)](#page-5-0) are then chosen to back-fit the parameters a_1-a_4 . The procedure is

repeated in this trial-and-error procedure to determine the values of parameters a_1-a_5 with an objective of best fitting the boundary that distinguishes regions of high- and lowlikelihood of liquefaction occurrence. Therefore, the chance of mistaking a liquefaction case history for a non-liquefaction case history is reduced, thus ensuring that the liquefaction triggering correlation is reasonably conservative. Following these principles, the following equation is obtained:

$$
CRR = \exp\left[\frac{V_{\rm sl}}{86.4} + \left(\frac{V_{\rm sl}}{134.0}\right)^2 - \left(\frac{V_{\rm sl}}{125.2}\right)^3 + \left(\frac{V_{\rm sl}}{158.5}\right)^4 - 4.8\right] \tag{4}
$$

Figure 2 shows the liquefaction triggering correlation given in Eq. (4) along with 618 V_s -based case histories from 49 earthquakes, in which the r_d and M_w values from the correlations of Idriss [\(1999\)](#page-11-0) and Idriss and Boulanger [\(2006,](#page-11-0) [2010\)](#page-11-0) are adopted. As seen in Fig. 2, the red triangle plots by Saygili [\(2005\)](#page-12-0) seem to dominate the location of the boundary curve. These case histories were from the town of Yuanlin during the 1999 Chi-Chi, Taiwan $M_{\rm w}$ 7.6 earthquake. The town of Yuanlin is located approximately 15 km from the Chelungpu fault rupture. The geometric mean peak ground acceleration (PGA) is 0.18 g. The town is situated on a thick alluvial deposit of the Tsosui River at the foothill of Baguashan. The ground water table (GWT) is quite shallow, ranging from about 0.5 to 4.0 m below the ground surface. There exist layers of silt fine sand to silty medium fine and very loose sands that are very susceptible to liquefaction (MAA [2000](#page-12-0); Juang et al. [2002b](#page-11-0)). Of the 23 available case histories developed by Green et al. [\(2011\)](#page-11-0) from the 2010–2011 Christchurch, New Zealand earthquakes, 18 cases indicated surface manifestation of liquefaction, three cases indicated no evidence of

Fig. 2 V_s -based 618 case histories of the liquefaction performance from 49 earthquakes and the verification of the 23 available cases developed by Green et al. [\(2011](#page-11-0)) from 2010 to 2011 Christchurch earthquakes

surface manifestation, and three case were in the margin between liquefaction and non-liquefaction. These cases are not included in the calibration database, and thus provide a more objective way to validate the developed model. These cases are also plotted in Fig. 2, which shows that the predictions by Eq. (4) match well with the observations.

Also shown in this figure are the existing V_s -based liquefaction triggering correlations developed by Andrus and Stokoe ([2000\)](#page-11-0), Zhou and Chen [\(2007\)](#page-12-0), and Kayen et al. [\(2013\)](#page-12-0). As shown in Fig. 2, in the region of $V_{s1} = 95$ to 240 m/s, the liquefaction triggering correlation given in Eq. (4) envelops the liquefaction triggering correlations by Andrus and Stokoe ([2000](#page-11-0)) and Kayen et al. [\(2013\)](#page-12-0), and appears to be reasonably conservative. On the other hand, unlike the correlation by Zhou and Chen ([2007](#page-12-0)), Eq. (4) is not overly conservative in the high V_{s1} region (V_{s1} > 225 m/s). Therefore, Eq. (4) is deemed suitable for use in civil engineering projects with high levels of risk potential.

Sensitivity of the proposed liquefaction correlation to various components of the simplified procedure

All the stress-based simplified approaches for evaluating the liquefaction potential of soil, initiated by Seed and Idriss [\(1971](#page-12-0)), are based on an assessment of the r_d (shear stress reduction factor) of the CSR. As a result, all existing correlations suffer from moderately biased estimates of in situ CSRs, especially at shallow depths. Fortunately, the case histories at shallow depth compose a large portion of the database. To this end, three different r_d models were used to study the effect of different r_d models on the interpretation of the liquefaction case history database. Figure [3](#page-6-0) shows the distribution of the

Normalized shear - wave velocity, $V_{\text{sl}}(m/s)$

 CSR and V_{s1} of the case history data, in which the CSR values are computed using three different r_d models presented by Cetin et al. ([2004\)](#page-11-0), Kishida et al. ([2009\)](#page-12-0), and Youd et al. [\(2001\)](#page-12-0), respectively. Also plotted in Fig. 3 is the proposed liquefaction triggering correlation given in Eq. [\(4](#page-5-0)). The results shown in Fig. 3 indicate that the position of the proposed liquefaction triggering correlation (Eq. [4](#page-5-0)), relative to the expanded database, is not sensitive to the formulation of r_d .

Furthermore, both the magnitude scaling factor (MSF) and the shear stress reduction factor r_d are dependent on the moment magnitude M_w . However, the M_w values of the same earthquake events could be assessed somewhat differently in different databases; for example, see Table [2](#page-7-0) for the $M_{\rm w}$ values of nine earthquake events listed in Idriss and Boulanger [\(2010](#page-11-0)) versus those reported in Kayen et al. [\(2013\)](#page-12-0). The variations in the earthquake magnitude could affect the *MSF*, the r_d , and therefore the *CSR*. However, such effect on the position of the proposed liquefaction triggering correlation (Eq. [4\)](#page-5-0), relative to the expanded database, appears to be quite modest, as reflected in Fig. [4](#page-7-0), in which the M_w values from Kayen et al. [\(2013\)](#page-12-0) and those from Idriss and Boulanger ([2010](#page-11-0)) are adopted.

Similarly, the effect of different formulations of MSF is examined. Figure [5](#page-8-0) shows the proposed liquefaction trigger-ing correlation (Eq. [4\)](#page-5-0) along with the data of CSR and V_{s1} of the expanded database, in which MSF values are computed based on the formulations in Seed and Idriss [\(1982\)](#page-12-0), Andrus and Stokoe [\(2000](#page-11-0)), and Kayen et al. ([2013\)](#page-12-0), respectively. Again, the results show that the position of the proposed liquefaction triggering correlation (Eq. [4\)](#page-5-0), relative to the expanded database, is not sensitive to different MSF formulations.

Finally, the effect of fines content (FC) on the proposed liquefaction correlation, relative to the expanded database. Figure [6](#page-8-0) shows the proposed liquefaction triggering correla-tion (Eq. [4](#page-5-0)) along with the data of CSR and V_{s1} for three FC scenarios, $FC \le 5\%$, $5\% < FC \le 35\%$, and $FC > 35\%$. As can be seen in Fig. [6,](#page-8-0) the position of the proposed liquefaction correlation (Eq. [4\)](#page-5-0) is quite satisfactory in delineating the liquefaction cases from non-liquefaction cases in all three FC scenarios. Considering the uncertainty (variation) in the measured V_s values in the database, which likely out-weights the effect of FC, it appears adequate to adopt a single liquefaction triggering curve for all SC scenarios. The liquefaction triggering correlation (Eq. [4\)](#page-5-0) is deemed reasonably and conservatively positioned, relative to the expanded database.

Mapping function: from nominal safety factor to liquefaction probability

In a deterministic approach, the soil liquefaction is predicted to occur if the factor of safety ($F_s = CRR/CSR$) against the initiation of liquefaction is less than or equal to 1.

Because of the uncertainties existing in the adopted model and the input data, the computed F_s is a random variable. Juang et al. [\(2013\)](#page-11-0) used the term "nominal" factor of safety \tilde{F}_s , a fixed value, which is computed using nominal values of the input parameters. The term "nominal"

Normalized shear-wave velocity V_{s1} (m/s)

Fig. 3 The distribution of the liquefaction case history data processed with different r_d relationships compared to the liquefaction triggering of Eq. [\(4](#page-5-0))

Table 2 Difference of M_W values from the databases of Idriss and Boulanger [\(2010\)](#page-11-0) and Kayen et al. ([2013](#page-12-0))

value is used herein as a fixed value estimate of a random variable.

Furthermore, Juang et al. ([2002a](#page-11-0)) suggested a simplified probabilistic model expressed in the form of a mapping function that maps the nominal safety factor (F_S) to the probability of soil liquefaction (P_L) .

$$
P_{\rm L} = \frac{1}{1 + \left(\tilde{F}_{\rm s}/a\right)^b} \tag{5}
$$

where *a* and *b* are the model parameters.

The model parameters a and b may be estimated based on a liquefaction case history database D with the principle of the maximum likelihood. The weighted likelihood function $L(a, b | D)$ to account for the data imbalance (i.e., uneven number of liquefaction cases and non-liquefaction cases in a database) using weighting factors w_L and w_{NL} may be written as follows (Cetin et al. [2004](#page-11-0); Moss et al. [2006;](#page-12-0) Juang et al. [2012,](#page-11-0) [2013](#page-11-0); Kayen et al. [2013;](#page-12-0) Chen et al. [2015](#page-11-0)):

$$
L(a,b|D) = \left\{ \prod_{i=1}^{380} \frac{1}{1 + (\tilde{F}_{S}/a)^{b}} \right\}^{w_{L}}
$$

$$
\times \left\{ \prod_{j=1}^{234} \left(1 - \frac{1}{1 + (\tilde{F}_{S}/a)^{b}} \right) \right\}^{w_{NL}}
$$
 (6)

Here, $D = \left\{ \left(L^{(i)}, \tilde{F}_{\rm S}^{(i)} \right); i = 1, ..., 614 \right\}$, where $L^{(i)}$ indicates the *i*th case, and $L^{(i)} = 1$ for the liquefaction case and is 0 otherwise. The values of w_L and w_{NL} are the weights assigned to the liquefaction and non-liquefaction cases, respectively.

The adopted global database is dominated by positive liquefaction observations (380 liquefaction cases), with a minority of test sites with no surface manifestation (234 non-liquefaction cases), as depicted in Fig. [2](#page-5-0). This unavoidable sampling disparity will result in a bias that may impact the statistical analysis and can produce a skewed

Fig. 4 Distribution of the liquefaction case history data processed using different moment magnitude $M_{\rm w}$ relationships compared to the liquefaction triggering correlation of Eq. [\(4](#page-5-0))

Fig. 5 Distribution of the liquefaction case history data processed using different MSF relationships compared to the liquefaction triggering correlation of Eq. ([4\)](#page-5-0)

Normalized shear-wave velocity V_{s1} (m/s)

Fig. 6 Distribution of case history data with different fine contents (FC) compared with the liquefaction triggering correlation of Eq. ([4\)](#page-5-0)

prediction, if unchecked (Kayen et al. [2013\)](#page-12-0). In their investigation of this type of bias, Cetin et al. ([2004](#page-11-0)) recommended a methodology to account for this choice-based sampling, and developed a weighting factor (w; $w_{\text{NL}}/w_{\text{L}}$) for application to the likelihood function of Eq. ([6\)](#page-7-0). Based

on the work of Cetin et al. (2004) , all "liquefied" data are weighted by a factor of $w_L = 0.8$, and all "non-liquefied" data are weighted by a factor of $w_{NL} = 1.2$, resulting in a ratio of $w_{\text{NL}}/w_{\text{L}} = 1.5$.

Intuitively, the weighting factors w_L and w_{NL} may be determined based on the number of liquefaction cases and the number of non-liquefaction cases in the database. The number of liquefaction cases is approximately equal to 1.62 times the number of non-liquefaction cases in the adopted database. Based on this intuition, Juang et al. [\(2013\)](#page-11-0) developed a procedure for estimating the weighting factors, and with the adopted database, the weighting factors are quite consistent with those obtained by Cetin et al. ([2004\)](#page-11-0) and Kayen et al. ([2013](#page-12-0)). Therefore, considering the consistency with these previous studies, the weighting factor is adopted as: $w_{\text{NL}}/w_{\text{L}} = 1.5$ ($w_{\text{L}} = 0.8$ and $w_{\text{NL}} = 1.2$) for the adopted global database of case histories. Although the true value of $w_{\text{NL}}/w_{\text{L}}$ is not entirely known, these weighting factors appear to be reasonable. Furthermore, the work of Boulanger and Idriss ([2012\)](#page-11-0) suggested that the effect of sampling bias perceived in the database is quite modest.

Using the maximum likelihood procedure described above, the likelihood function based on all 618 cases in the database is maximized to yield the best estimates $({\tilde{a}}, {\tilde{b}})$ of the parameters a and b in Eq. ([5\)](#page-7-0). The results of the maximum likelihood analysis using different ratios of w_{NL} to w_L are listed in Table 3. The models are shown in Fig. 7 in the form of $P_L = f(\tilde{F}_S)$. With the adopted $w_{NL}/w_L = 1.5$, $a = 0.509$ and $b = 2.511$, and the relationship between the liquefaction probability P_L and the nominal safety factor \tilde{F}_s is expressed as:

$$
P_{\rm L} = \frac{1}{1 + \left(\tilde{F}_{\rm S} \middle/ 0.509\right)^{2.511}}\tag{7}
$$

Alternatively, the P_L - \tilde{F}_s relationship may be obtained using an empirical approach described in Juang et al. [\(2013\)](#page-11-0). For each of the 618 cases in the expanded database adopted in this study, the nominal safety factor \tilde{F}_s is computed using Eqs. ([2\)](#page-4-0) and ([4\)](#page-5-0). These cases are then placed in different bins based on their \tilde{F}_s values. Taking a bin width of 0.1 as an example, the first bin covered all cases with \tilde{F}_s in the range of 0–0.1; the second bin covered all cases with \ddot{F}_s in the range of 0.1–0.2, and so on. Thus, a set of (P_L , \ddot{F}_s)

) data points can be obtained using the weighted empirical probability approach described by Juang et al. ([2013\)](#page-11-0), and the empirical relationship of P_L versus F_s can be established, as shown in Fig. 7. As shown in Fig. 7, the results of this empirical approach are quite consistent with those obtained with the weighted maximum likelihood technique.

Probabilistic version of the new V_s -based liquefaction triggering correlation

Combining Eqs. [\(4](#page-5-0)) and (7) allows the probabilistic version of the calibrated empirical liquefaction triggering correlation of Eq. [\(4](#page-5-0)) for the expanded database to be expressed as:

$$
CRR = \tilde{F}_{S}(P_{L}) \cdot \exp\left[\frac{V_{s1}}{86.4} + \left(\frac{V_{s1}}{134.0}\right)^{2} - \left(\frac{V_{s1}}{125.2}\right)^{3} + \left(\frac{V_{s1}}{158.5}\right)^{4} - 4.8\right]
$$
\n(8)

where $\tilde{F}_s(P_L)$ denotes that \tilde{F}_s is a function of P_L and its value can be calculated by inverting Eq. (7) for a given P_L .

According to Eq. (7) and Eq. (8), the deterministic liquefaction triggering correlation of Eq. [\(4\)](#page-5-0) corresponds to a liquefaction probability of approximately 15%. The liquefaction triggering curves for liquefaction probabilities of 9%, 15%, 25%, 45%, 65%, and 95% based on the expanded database are graphed in Fig. [8.](#page-10-0) For comparison, the deterministic liquefaction triggering curve recommended by Kayen et al. [\(2013\)](#page-12-0) is also given in Fig. [8](#page-10-0). According to the results shown in Fig. [8](#page-10-0), the concept of a limiting upper bound of normalized velocity V_{s1} (previously set at 215 m/s by Andrus and Stokoe [2000\)](#page-11-0) appears to be inconsistent with the expanded database.

Fig. 7 Empirical relationship between P_L and \tilde{F}_s derived for different values of $w_{\text{NI}}/w_{\text{L}}$ using the weighted maximum likelihood technique and the weighted empirical approach

Fig. 8 Liquefaction triggering correlations for liquefaction probabilities of 9%, 15%, 25%, 45%, 65%, and 95%, the deterministic liquefaction triggering curve by Kayen et al. ([2013](#page-12-0)) compared to the expanded case history database and the verification of the 23 available cases developed by Green et al. ([2011\)](#page-11-0) from 2010 to 2011 Christchurch earthquakes

Normalized shear-wave velocity, V_{s1} (m/s)

The deterministic boundary curve of Kayen et al. ([2013](#page-12-0)) is characterized with probabilities ranging from 15% to 45% for V_{s1} above 115 m/s. Moreover, the intersection of the deterministic boundary curve of Kayen et al. ([2013\)](#page-12-0) with the 25% liquefaction probability contour in this study is located approximately at $V_{s1} = 218$ m/s. The main difference between the two liquefaction triggering curves appears to lie within the region of V_{s1} below 150 m/s for CSR below 0.09 and of V_{s1} approximately 230 m/s above where there are little or no liquefaction case histories. The deterministic boundary curve of Kayen et al. [\(2013\)](#page-12-0) was also characterized with liquefaction probabilities ranging from 15% to 25% for V_{s1} above 218 m/s. The newly established deterministic boundary curve (Eq. [4\)](#page-5-0) appears to be slightly more conservative than those recommended in the past studies. However, whether a model is conservative should be viewed in the probability sense. For example, the available cases from the 2010–2011 Christchurch earthquakes (Green et al. [2011\)](#page-11-0) are plotted in Fig. 8 along with the various probability contours. As seen in Fig. 8, some liquefaction cases are located within the liquefaction probabilities ranging from 45% to 65%, one liquefaction case lies on the 15% liquefaction probability contour, and the two margin cases are slightly below the 9% liquefaction probability contour. These results indicate that the newly established deterministic boundary curve and its probability version are quite reasonable.

The zone below the liquefaction triggering correlation for liquefaction probabilities of approximately 9% indicates a non-liquefaction zone for clean sands, the zone above the liquefaction triggering correlation for liquefaction probabilities of 95% indicates a liquefaction zone for clean sands, and the middle gray zone enclosed by the liquefaction triggering correlations for liquefaction probabilities of 9% and 95% is defined as a transition zone for clean sands. The liquefaction

triggering correlations for liquefaction probabilities of 9%, 15%, and 25%, respectively, represent the "very low", "low", and "moderate" probability events of mistaking a liquefiable soil for a non-liquefiable soil.

Conclusions

In this paper, through the use of an expanded database of the Vs-based liquefaction case histories of Andrus et al. [\(1999\)](#page-11-0), Chu et al. [\(2004\)](#page-11-0), Saygili ([2005](#page-12-0)), and Kayen et al. ([2013](#page-12-0)), a new liquefaction triggering correlation equation and its simplified probabilistic version are developed and presented. The following conclusions are drawn:

- (1) The five parameters of the new deterministic liquefaction triggering correlation in Eq. [\(4](#page-5-0)) were determined by trial and error using the following two principles: (1) the liquefaction triggering correlation for $V_{s1} \leq 200$ m/s is generally controlled by the lower bound of the liquefaction case history data and (2) the liquefaction triggering correlation for $V_{s1} > 200$ m/s is generally controlled by the upper bound of the non-liquefaction case history data. Based on calibration with the expanded database, this deterministic liquefaction triggering correlation (Eq. [4](#page-5-0)) corresponds to a probability of liquefaction of approximately 15%, which is consistent with those recommended by Cetin et al. ([2004](#page-11-0)), Boulanger and Idriss [\(2012\)](#page-11-0), and Kayen et al. [\(2013](#page-12-0)) for liquefaction triggering correlation.
- (2) The new calibrated liquefaction triggering correlation can envelop the lower bound of the liquefaction triggering correlations published by Andrus and Stokoe [\(2000\)](#page-11-0), Youd et al. ([2001](#page-12-0)), and Kayen et al. [\(2013\)](#page-12-0). The new
- (3) An empirical equation for determining the probability of liquefaction is expressed in the form of a mapping function that maps the nominal factor of safety to the probability of liquefaction. The mapping function, in the form of Eq. [\(7](#page-9-0)), obtained by the empirical approach is consistent with that obtained using the weighted maximum likelihood technique.
- (4) A probabilistic version of the new calibrated liquefaction triggering correlation is presented. Based on the probability contour curves, in relation to the expanded database, three zones of liquefaction potential, namely the non-liquefaction, transition, and liquefaction zones, are delineated with liquefaction probabilities of ≤9%, 9%– 95%, and ≥95%, respectively. The deterministic liquefaction triggering correlation of Eq. ([4](#page-5-0)), which is generally biased toward the conservative side, corresponds to a liquefaction probability of approximately 15%.

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