ORIGINAL PAPER

In situ determination of hydraulic conductivity in Yangtze Delta deposits using a modified piezocone model

Mingfei Zhang¹ • Liyuan Tong¹ • Yijun Yang² • Hongbo Che¹ • Huangsong Pan¹

Received: 27 May 2016 / Accepted: 26 October 2016 / Published online: 9 November 2016 - Springer-Verlag Berlin Heidelberg 2016

Abstract In order to more precisely determine the in situ hydraulic conductivity of soils as an essential parameter in geotechnical engineering, this article presents a new method based on piezocone tests. In light of results obtained from a series of classical numerical simulations of piezocone dissipation tests and in situ tests, the modified direction and value assumptions of excess pore water pressure distribution are fundamental: (1) the flow surface of pore water is assumed to be cylindrical in shape at larger scales, and (2) the initial state of induced excess pore pressure is assumed to satisfy a negative exponential distribution in dissipating. After detailing the existing approaches, a comparison of data in the Yangtze Delta region between them and the proposed method based on graphical and statistical analysis has been accomplished; the comparison revealed the accuracy and validity of the proposed method, with five indices utilized, including a new relative error index. The reasonable assumptions, logical derivation and mathematical analysis together indicate the academic value and application potential of the proposed method.

Keywords Hydraulic conductivity · CPTU · Modified cylindrical flow model - Negative exponential distribution - Yangtze Delta

 \boxtimes Mingfei Zhang zhangmingfeiyanjiu@163.com

Introduction

Hydraulic conductivity is one of the most important mechanical properties of soil, influencing both long-term consolidation deformation and soil stability (Shen et al. [2003](#page-10-0); Zeng et al. [2011;](#page-11-0) Chai et al. [2011](#page-10-0)). Many challenges in geotechnical engineering are related to the hydraulic conductivity of soil, including the design of foundation pit dewatering (Ma et al. [2014\)](#page-10-0), the estimation of foundation settlement and the analysis of soil consolidation (Shen et al. [2013](#page-10-0); Shen and Xu [2011;](#page-10-0) Xu et al. [2008](#page-10-0), [2012](#page-10-0), [2013](#page-10-0); Horpibulsuk et al. [2011](#page-10-0)). Hitherto, numerous research projects have been dedicated to methods of hydraulic conductivity measurement (Randolph and Wroth [1979](#page-10-0); Clarke et al. [1979](#page-10-0); Baligh and Levadoux [1980](#page-9-0); Leroueil and Jamiolkowski [1991](#page-10-0); Robertson [1990](#page-10-0); Jefferies and Davies [1993;](#page-10-0) Lunne et al. [1997](#page-10-0); Elsworth and Lee [2005](#page-10-0); Elsworth and Lee [2007](#page-10-0); Cai et al. [2007](#page-10-0); Robertson [2009](#page-10-0); Chai et al. [2011;](#page-10-0) Wang et al. [2013;](#page-10-0) Wang and Shen [2013](#page-10-0); Zou et al. [2014\)](#page-11-0). One widely used, economic and efficient method to determine in situ hydraulic conductivity is the piezocone penetration test (CPTU; Campanella and Robertson [1988](#page-10-0); Lunne et al. [1997;](#page-10-0) Mitchell and Brandon [1998](#page-10-0)), providing near-continuous measurements of tip resistance q_t , sleeve friction f_s and pore water pressure u at the shoulder, face or shaft of the cone. The test can provide a quantitative measurement of various soil properties, including soil stratigraphy, soil mechanical properties, soil type and the distribution of soil saturation (Douglas and Olsen [1981;](#page-10-0) Robertson [1990](#page-10-0); Lunne et al. [1997;](#page-10-0) Mitchell and Brandon [1998](#page-10-0); Lu et al. [2004](#page-10-0); Cetin and Ozan [2009](#page-10-0); Cai [2010;](#page-9-0) Shen et al. [2010](#page-10-0), [2015](#page-10-0); Wang et al. [2013;](#page-10-0) Wang and Shen [2013](#page-10-0)).

The utility of equations derived from piezocone soundings and employed to describe the hydraulic

¹ Institute of Geotechnical Engineering, Southeast University, Nanjing 210096, China

² Guangzhou Metro Design & Research Institute Co. Ltd., Guangzhou 510000, China

conductivity of soils can be classified into three types. The first involves introducing a relation for the coefficient of consolidation of soils via the dissipation test, then indirectly deriving a further equation for hydraulic conductivity (Gupta and Davidson [1986;](#page-10-0) Robertson et al. [1992](#page-10-0); Danziger et al. [1997](#page-10-0); Burns and Mayne [1998](#page-9-0); Baligh and Levadoux [1980;](#page-9-0) Leroueil and Jamiolkowski [1991;](#page-10-0) Cai et al. [2007](#page-10-0)), a method that is both time-consuming and labour-intensive. A second possible approach is to apply the soil behaviour index proposed by Robertson [\(1990](#page-10-0), [2009\)](#page-10-0). This index, however, is empirical and may cause large errors in various parameters.

The final method involves theoretical analysis based on a combination of dislocation analysis, Darcy's law and cavity expansion theory (Elsworth and Lee [2005](#page-10-0); Elsworth and Lee [2007;](#page-10-0) Chai et al. [2011;](#page-10-0) Wang et al. [2013;](#page-10-0) Wang and Shen [2013;](#page-10-0) Zou et al. [2014\)](#page-11-0). Elsworth and Lee ([2005,](#page-10-0) 2007) first proposed a semi-rigorous method and an explicit equation. After that, Chai et al. [\(2011](#page-10-0)) presented a modified method based on a half-spherical flow assumption. Yet, numerical simulations of piezocone dissipation tests (Yi et al. [2012a,](#page-10-0) [b](#page-11-0); Mahmoodzadeh et al. [2014](#page-10-0), [2015;](#page-10-0) Ceccato and Simonini [2016](#page-10-0)) indicate that the distribution of excess pore pressures is more suitable for cylindrical flow in the horizontal direction, while the negative exponent distribution of initial excess pore water pressure satisfy the test results closely (Baligh and Levadoux [1980;](#page-9-0) Roy et al. [1981;](#page-10-0) Zhu and Tang [1986](#page-11-0); Tang et al. [2002](#page-10-0); Zhu et al. [2005;](#page-11-0) Ma et al. [2007](#page-10-0)).

The objective of this paper is to propose an approach in order to estimate the hydraulic conductivity. First, existing approaches are briefly reviewed and discussed. A new method is thereby presented in detail and compared with existing approaches using piezocone data from Yangtze Delta deposits through a graphical and statistical methodology.

Modification methods for predicting hydraulic conductivity

Brief review of Elsworth's method

In order to evaluate the hydraulic conductivity directly from piezocone tests, Elsworth and Lee [\(2005](#page-10-0)) presented a method (hereafter referred to as Elsworth's method) based on a dislocation model (Elsworth [1991\)](#page-10-0), as shown in Fig. 1a, where a is the radius of the cone, i_a is the hydraulic gradient at radius $r = a$, k is hydraulic conductivity, U is the rate of cone penetration, u_a is the absolute pore water pressure measured by the piezocone and u_s is the initial static pore water pressure. The following assumptions are substantially adopted: (1) during piezocone penetration, 'dynamic steady' spherical

Fig. 1 Basic concept of a Elsworth's method (recreated from Chai et al. [2011](#page-10-0)) and b Chai's method (modified from Chai et al. [2011\)](#page-10-0)

flow of pore water will form around the tip of the cone; (2) excess pore water pressure in the soil around the cone has a power function distribution for radial distance; and (3) the diameter of the spherical cavity is assumed to be the same as the diameter of the cone, while the rate of spherical flow of pore water through the periphery of the cavity is assumed to be equal to the rate of volume penetration $\triangle V$ of the cone. An explicit equation was derived to calculate the hydraulic conductivity, using Darcy's law and assuming that zero excess pore water pressure exists at an infinite distance from the cone.

In an infinite porous medium, on the condition that excess pore water pressure is zero for radial distance $r \to \infty$, the distribution of pore water pressure u can be expressed as:

$$
u - u_s = (u_a - u_s)a/r.
$$
\n⁽¹⁾

Then, the hydraulic gradient at radius $r = a$ may be deduced by way of

$$
i_a = \frac{1}{\gamma_w} \frac{du}{dr}|_{r=a} = \frac{u_a - u_s}{a\gamma_w} = B_q Q_t \frac{\sigma'_{v0}}{a\gamma_w},\tag{2}
$$

where γ_w is the unit weight of water, and B_q and Q_t are the dimensionless pore water pressure ratio and dimensionless tip resistance, respectively, defined as (Wroth [1984\)](#page-10-0)

$$
B_{\rm q} = (u_2 - u_0)/(q_{\rm t} - \sigma_{\rm v0}), \tag{3}
$$

$$
Q_{t} = (q_{t} - \sigma_{v0})/(\sigma'_{v0}),
$$
\n(4)

In which q_t is the static point resistance, u_0 is hydrostatic pressure, u_2 is pore water pressure on the cone shoulder, σ_{v0} represents the total overburden stress (Teh and Houlsby [1991;](#page-10-0) Lu et al. [2004\)](#page-10-0) and σ'_{v0} is the initial vertical effective stress.

Spherical radial flow around the cone per unit time can be obtained using:

$$
q = 4\pi a^2 i_a k. \tag{5}
$$

Cone penetration amount per unit time is given by

$$
\Delta \dot{V} = \pi a^2 U \tag{6}
$$

Substituting Eq. ([2\)](#page-1-0) into Eq. (5), and assuming $\Delta \dot{V} = q$, one can obtain the equation:

$$
4k B_{\mathbf{q}} Q_{\mathbf{t}} \frac{\sigma_{\mathbf{v}0}'}{a\gamma_{\mathbf{w}}} = U. \tag{7}
$$

A dimensionless hydraulic conductivity coefficient is defined herein as:

$$
K_{\rm D} = 1/B_{\rm q} Q_{\rm t}.\tag{8}
$$

Combining the above equations, the in situ value of k is expressed by:

$$
K_{\rm D} = \frac{4k\sigma'_{\rm v0}}{a\gamma_{\rm w}U} \text{ or } k = \frac{\gamma_{\rm w}K_{\rm D}U}{4\sigma'_{\rm v0}}.\tag{9}
$$

Subsequently, the line $K_{\text{D}} = 1/B_{\text{q}}Q_{\text{t}}$ does not provide the best fit to the measured data. As a consequence, Els-worth and Lee [\(2007](#page-10-0)) modified the $K_{\rm D} - B_{\rm q} Q_{\rm t}$ relation:

$$
K_{\rm D} = \frac{\alpha}{\left(B_{\rm q} Q_{\rm t}\right)^{\beta}},\tag{10}
$$

Where α and β are constants. Elsworth and Lee ([2007\)](#page-10-0) suggested suitable values for α and β of 0.62 and 1.6, respectively.

Chai's method

On the basis of Elsworth's method, Chai et al. ([2011\)](#page-10-0) presented a half-spherical flow approach (hereafter referred to as Chai's method), as shown in Fig. [1](#page-1-0)b. In this approach, the following hypotheses are assumed: (1) a half-spherical flow of pore water covers the tip of the cone because pore

water cannot flow into the cone; and (2) the rate of halfspherical flow of pore water through the periphery of the cavity is linearly proportional to the rate of volume penetration of the cone (Shen et al. [2015\)](#page-10-0). Chai et al. ([2011\)](#page-10-0) modified Elsworth's method in terms of a bi-linear relation defined by

$$
K_{\rm D}^{'} = \begin{cases} 1/B_{\rm q}Q_{\rm t}, & B_{\rm q}Q_{\rm t} < 0.45\\ 0.044/(B_{\rm q}Q_{\rm t})^{4.91}, & B_{\rm q}Q_{\rm t} > 0.45 \end{cases}
$$
(11)

Compared to Elsworth's method, it was deduced:

$$
K_{\rm D}' = \frac{2k\sigma_{\rm v0}'}{a\gamma_{\rm w}U} = \frac{K_{\rm D}}{2}.
$$
\n(12)

Moreover, Chai's approach is applicable to both normally or lightly over-consolidated clayey deposits as well as loose sandy deposits.

Zou's method

Owing to the stratification commonly observed in natural soil deposits, the hydraulic conductivity in the horizontal direction, k_h , is often larger than that in the vertical direction, k_v (Leroueil et al. 1990). In fact, whereas most laboratory consolidation tests are conducted with the samples cut in the vertical direction with respect to the in situ condition, pore pressure dissipation occurs mainly in the horizontal direction. As a result, Zou et al. ([2014\)](#page-11-0) proposed an explicit equation (see Fig. [2a](#page-3-0)), assuming radial flow normal to an improved cylindrical surface, which is given by

$$
K_{\rm D}^{''} = \begin{cases} 1/B_{\rm q}Q_{\rm t}, & B_{q}Q_{\rm t} < 0.35\\ 0.017/(B_{\rm q}Q_{\rm t})^{4.64}, & B_{\rm q}Q_{\rm t} > 0.35 \end{cases}
$$
(13)

Assuming that the rate of 'dynamic steady' flow through the periphery of the cavity with a radius a of a cylinder is linearly proportional to the rate of volume penetration of the cone, one can obtain:

$$
2\pi a \cdot h \cdot k \cdot i_a = \pi a^2 U = \Delta \dot{V}.
$$
 (14)

The proposed method

The direction and value assumptions of excess pore water pressure distribution are fundamental and substantial for a reliable method, yet, the previous methods emphasized pore water pressure distribution assumptions in a finite local area, hence, the improved assumptions are as follows (shown in Fig. [2](#page-3-0)b):

• A dynamic steady cylindrical flow of pore water will form around the tip of the cone.

Fig. 2 Basic concept behind: a Zou's method (Zou et al. [2014\)](#page-11-0); b the method proposed in the present paper

- The diameter of the cylindrical cavity is assumed to be the same as the diameter of the cone.
- Excess pore water pressure in the soil around the cone has a negative exponential function distribution for radial distance, and there is no excess pore water pressure at an infinite distance.

Based on the assumption $q = \Delta V$ ($=\pi a^2 U$), the mathematical function adopted in this case is:

$$
2\pi a \cdot \eta h \cdot k_h \cdot i_a = \pi a^2 U = \Delta \dot{V}.
$$
 (15)

As shown in Fig. 3, based on the numerical simulation of piezocone dissipation tests (Ceccato and Simonini [2016\)](#page-10-0), the scope of excess pore water pressure dissipation is not confined to the filter thickness h , but instead extends to a larger range ηh (where η is a parameter calibrated with the experimental and simulation values). Yet, it could not be considered in previous methods.

It is essential to determine the distribution of initial excess pore water pressure during penetration, hence, a number of laboratory and field tests (Fig. 4) are carried out

Fig. 3 Excess pore pressure distribution around the cone (Ceccato and Simonini [2016](#page-10-0))

Fig. 4 Fitting curve between initial pore water pressures with negative exponent distribution (modified from Wang and Shen [2013](#page-10-0))

which revealed that the negative exponent distribution of initial excess pore water pressure near the tip fit the test results closely (Baligh and Levadoux [1980;](#page-9-0) Roy et al. [1981](#page-10-0); Zhu and Tang [1986;](#page-11-0) Gupta and Davidson [1986](#page-10-0); Tang et al. [2002;](#page-10-0) Zhu et al. [2005](#page-11-0); Ma et al. [2007](#page-10-0)). Therefore, under the condition that excess pore water pressure is zero for radial distance $r \to \infty$, the distribution of pore water pressure u can be expressed as:

$$
u - u_{s} = (u_{2} - u_{s})e^{-\theta(r/a - 1)},
$$
\n(16)

Where θ is a soil parameter: $0.35 < \theta \le 1.5$ for clay, $0.3 < \theta \le 0.35$ for silt and $0.1 < \theta \le 0.3$ for sand (Zhu and Tang [1986](#page-11-0); Ma et al. [2007;](#page-10-0) Shen et al. [2015](#page-10-0)). According to Darcy's law, the hydraulic gradient on the surface of the cylinder i_a can be expressed by,

$$
i_a = \theta \frac{u_2 - u_s}{a\gamma_w} e^{-\theta(r/a - 1)}|_{r=a} = \theta B_q Q_t \frac{\sigma'_{v0}}{a\gamma_w}.
$$
 (17)

Chai et al. (2011) (2011) considered that K_D values (and, thus, the value of k) deduced from CPTU tests mainly represent the hydraulic conductivity of a natural deposit in the horizontal direction. The results obtained from a series of classical numerical simulations of piezocone dissipation tests and in situ tests (see Figs. [3,](#page-3-0) 5a, b for conventional CPTU), indicated the surface area for water flow could be cylindrical in shape around the cone at larger scales, even though this area seems spherical in shape only in a finite local area, whereas the half-spherical surface area assuredly is more suitable for spudcans (Fig. 5c) and piezoballs (Fig. 5d). More conformity of the surface area and the distribution of initial excess pore water pressure with actual conditions increase the possibility of accuracy. Hence, the cylindrical surface is more suitable for conventional CPTU primarily.

Substituting Eq. (17) (17) into Eq. (15) (15) , one can obtain:

Fig. 5 Distribution of excess pore pressures: a Yi et al. [\(2012a\)](#page-10-0); b Mahmoodzadeh et al. [\(2015](#page-10-0)); c Yi et al. [\(2012b](#page-11-0)); d Mahmoodzadeh et al. ([2014\)](#page-10-0)

$$
k_h = \frac{Ua}{2\eta h} \cdot \frac{1}{i_a} = \frac{a}{2\eta h} \cdot \frac{1}{B_q Q_t \theta} \cdot \frac{Ua\gamma_w}{\sigma'_{v0}}.
$$
 (18)

Defining $K_D^{\prime\prime\prime} = 1/B_q Q_t$, k_h is expressed as:

$$
k_{\rm h} = \frac{a}{2\eta h} \cdot \frac{K_{\rm D}''}{\theta} \cdot \frac{U a \gamma_{\rm w}}{\sigma_{\rm v0}'}.
$$
\n(19)

Comparing with Elsworth's method, the relationship is given by

$$
K_{\rm D}^{'''} = \theta \eta h K_{\rm D} / 2a. \tag{20}
$$

Based on previous methods (Elsworth and lee [2005](#page-10-0); Chai et al. [2011;](#page-10-0) Wang et al. [2013;](#page-10-0) Wang and Shen [2013](#page-10-0); Zou et al. [2014](#page-11-0)), it follows that:

$$
K_{\rm D}^{"'} = \begin{cases} 1/B_{\rm q}Q_{\rm t}, & B_{\rm q}Q_{\rm t} < \varepsilon \\ \alpha/(B_{\rm q}Q_{\rm t})^{\beta}, & B_{\rm q}Q_{\rm t} > \varepsilon \end{cases}
$$
 (21)

where ε is a constant parameter. According to international standards for CPTU cones, their height should be equal to 5 mm and their radius to 17.85 mm. Considering Ma et al. [\(2007](#page-10-0)) found the value of θ equal to 0.3, the data provided by Elsworth and Lee [\(2005](#page-10-0); see Fig. 6) can be employed to obtain values of $\eta = 8$ and $\varepsilon = 0.98$. Equation (21) can then be expressed as follows:

$$
K_{\rm D}^{"} = \begin{cases} 1/B_{\rm q}Q_{\rm t}, & B_{\rm q}Q_{\rm t} < 0.98\\ 0.87/(B_{\rm q}Q_{\rm t})^{7.81}, & B_{\rm q}Q_{\rm t} > 0.98 \end{cases}.
$$
 (22)

Data

The area of the Yangtze River Delta (including Shanghai, Suzhou, Wuxi, Changzhou and other cities) is located in $t = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}$ $t = \begin{bmatrix} 2 & 3 & 4 & 5 & 6 \end{bmatrix}$

Normalised radial distance r/R

Radial distance (m) and the eastern part of China, with seven sites across the region

Fig. 6 Relationship between the proposed bi-linear $K''_{\text{D}} - B_q Q_t$ (data from Elsworth and Lee [2007](#page-10-0))

(four in Suzhou: Yushan Station, Xinghui Road Station, Hongzhuang Station and Zhuhui Road Station; two in Nanjing: Jiangbei work well and the fourth Yangtze River Bridge; and the Yangtze Bridge in Taizhou) selected for the present study, as shown in Fig. 7. A summary description of these sites is also provided in Table 1. Typical profiles of CPTU measurements, including cone tip resistance (q_t) , side friction resistance (f_s) , and pore water pressure (u_2) versus depth recorded at Hongzhuang Station in Suzhou are presented in Fig. [8](#page-6-0). At each of the investigated sites, high-quality piston samples were taken at different depths that corresponded to the depths of piezocone dissipation tests undertaken in comprehensive laboratory testing. Soil samples were collected by means of a stationary piston sampler, 76 mm in diameter, at 1.0-m intervals below ground level. Once the stationary piston sampler was withdrawn from the borehole, the soil at both ends of the tube was excavated for wax sealing. Horizontal permeability tests were carried out in the laboratory on undisturbed samples of cohesive soils obtained from high-

Fig. 7 Map of the Yangtze River Delta showing the seven CPTU site locations

Table 1 Soil properties and description of sites

quality thin-wall samplers, with field pumping tests also performed in boreholes located on cohesionless soils. Groundwater tables at the sampling sites are located at 0–5 m and with depths ranging from 12 to 40 m.

The CPTU device used throughout the study was produced by Vertek-Hogentogler and Co., USA, and comprised a versatile piezocone system equipped with advanced digital cone penetrometers fabricated with a 60° tapered, 10 -cm² tip area cone, which provided measurements of q_t , f_s and u_2 with a 5-mm-thick porous filter located just behind the cone tip. The rate of penetration for all tests was 20 mm/s, enabling one set of readings to be obtained for every 50-mm penetration. Shear wave velocities were measured at intervals of 1.0 m, corresponding to successive rod additions during advancement of the penetrometer.

Analysis and discussion

Qualitative analysis

Hydraulic conductivity values obtained using the aforementioned methods were subsequently compared with the laboratory and field pumping results (see Fig. [9](#page-6-0) through Fig. [16](#page-7-0)). The fact that more than 90% of data points are scattered above the bi-linear line in Figs. [9](#page-6-0) and [11](#page-6-0) and below the perfect line displayed in Figs. [10](#page-6-0) and [12](#page-6-0) indicates that both Elsworth's method and Chai's method substantially underestimate the hydraulic conductivity of saturated soils. In contrast, more than half of the data points obtained using Zou's method lie above the bi-linear in Fig. [13](#page-6-0) and below the perfect line shown in Fig. [14](#page-7-0), which implies that the predicted accuracy of this particular

Fig. 8 Typical CPTU soundings recorded at Hongzhuang Station, Suzhou

Fig. 9 Bi-linear $K_D - B_q Q_t$ relationship for Elsworth's method

Fig. 10 Measured versus predicted k_h values for Elsworth's method

Fig. 11 Bi-linear $K'_{\text{D}} - B_q Q_t$ relationship for Chai's method

Fig. 12 Measured versus predicted k_h values for Chai's method

Fig. 13 Bi-linear $K''_{\text{D}} - B_q Q_t$ relationship for Zou's method

Fig. 14 Measured versus predicted k_h values for Zou's method

Fig. 15 Bi-linear $K''_{\text{D}} - B_q Q_t$ relationship for the modified method

method is higher. However, the results shown in Figs. 15 and 16 reveal that the proposed modified method is most applicable to Yangtze Delta soils due to the proximity of the line and data points. It is obvious that the values of k_h for soils ranging from partially drained silt through to undrained clay may be continuously and reasonably evaluated from piezocone sounding records using the modified method. As can be seen from Fig. 16, the data points are evenly distributed around the perfect line $y = x$, which implies that the proposed method produces the best agreement.

Quantitative analysis

The reliability of the adopted correlations is assessed on the basis of four existing and one new criteria: root mean square error (RMSE; Grima and Babuška [1999\)](#page-10-0), the first (mean) and second moment (standard deviation) statistics of the ratio of the estimated to test-determined shear wave

Fig. 16 Measured versus predicted k_h values for the modified method

velocity $(K; Briaud and Tucker 1988)$ $(K; Briaud and Tucker 1988)$, ranking index $(RI;$ Briaud and Tucker [1988](#page-9-0)), ranking distance (RD; Cherubini and Orr [2000\)](#page-10-0) and a new relative error index (RE).

RMSE is the square root of the average of the squared difference between true values and the corresponding observed values. Errors in RMSE are squared before they are averaged; consequently, a relatively high weight is given to large errors. This means that the RMSE is most useful when large errors are particularly undesirable, with the lower the RMSE value, the better the model performance. RMSE is determined via the following equation:

RMSE =
$$
\sqrt{\frac{1}{n} \sum_{i=1}^{n} (h_1 - h_c)^2}
$$
, (23)

where *n* is the number of data points, h_c is the hydraulic conductivity calculated from empirical equations and h_1 is the hydraulic conductivity measured directly from tests.

The first (mean μ) and second moment (SD standard deviation) statistics of the ratio of estimated to measured hydraulic conductivity is denoted by K and determined via the following equation:

$$
K = h_{\rm c}/h_{\rm l} \tag{24}
$$

whereas the accuracy of a method refers in this case to its ability to predict the measured hydraulic conductivity and is represented by the mean of K (Briaud and Tucker [1988](#page-9-0); Cherubini and Orr [2000;](#page-10-0) Giasi et al. [2003](#page-10-0)), method precision refers to the scatter around the mean and is quantified by the standard deviation of K (Briaud and Tucker [1988](#page-9-0); Cherubini and Orr [2000](#page-10-0); Giasi et al. [2003](#page-10-0); Onyejekwe et al. 2015). Theoretically, K ranges from 0 to infinity with an optimum value of one; this results in the nonsymmetric distribution of K around the mean and, thus, also an unequal weight of underprediction and overprediction (Briaud and Tucker [1988\)](#page-9-0).

The ranking index (RI) is also one of the two methods proposed by Briaud and Tucker [\(1988](#page-9-0)) with which to alleviate the problem of nonsymmetrical distribution of K data. Hence, RI values can be used to express an overall judgment regarding the quality of a correlation whilst simultaneously accounting for the mean value and SD of all K data. The ranking index is obtained via the following equation (Briaud and Tucker [1988](#page-9-0)):

$$
RI = \left| \mu_{\ln(K)} \right| + \sigma_{\ln(K)},\tag{25}
$$

where μ and σ represent the mean and SD of the series of analysed data, respectively. RI has been used by several investigators (e.g. Briaud and Tucker [1988;](#page-9-0) Cherubini and Orr [2000;](#page-10-0) Giasi et al. [2003\)](#page-10-0) to evaluate the performance of empirical equations.

RD, first proposed by Cherubini and Orr [\(2000](#page-10-0)), is another method enabling users to make an overall judgment as to the quality of a calculation method, again taking into consideration the mean value and standard deviation of all K data. RD represents, on a plot with mean (μ) values on the x axis and SD (σ) on the y axis, the distance of the point representing a computation using a particular correlation from the point representing the optimum condition ($\mu = 1$) and $\sigma = 0$). RD is determined as follows (Cherubini and Orr [2000](#page-10-0)):

$$
RD = \sqrt{(1 - \mu_K)^2 + \sigma_K^2}.
$$
 (26)

RD and RI provide different evaluations of the suitability of a given correlation equation to fit a measured value (Cherubini and Orr [2000\)](#page-10-0). For correlation equations where the precision (as indicated by the SD), mean value and accuracy are similar, RD gives a better result than RI, while for those that are either very accurate or very precise, RI provides the best result. RD gives equal weight to accuracy and precision, and has been used by several investigators (e.g. Cherubini and Orr [2000](#page-10-0); Giasi et al. [2003\)](#page-10-0) to evaluate the performance of empirical equations.

Relative error (RE), a new statistical criterion proposed in the present paper, is the ratio of the absolute difference between the measured value and the estimate of the measured hydraulic conductivity; the lower the RE value, the better the model performance. RE is mainly used to assess the pros and cons of different methods and is expressed as follows:

$$
RE = |h_1 - h_c|/h_1 = K - 1.
$$
\n(27)

Results and discussion

In the following section, both the observed and calculated data are presented in logarithmic form due to the fact that the obtained values of hydraulic conductivity varied by up to six orders of magnitude; with the more logarithmic the result, the lower the original hydraulic conductivity (that is always less than 1). A summary of the RMSE, K, RE, RI and RD values obtained for the Jiangsu clays is presented in Table 2. RMSE, RI and RD data are also illustrated in Fig. 17 in log form, while those for RE are shown graphically in Fig. [18](#page-9-0).

In terms of RMSE, the best performing correlation was provided by the proposed method $(RMSE = 0.940)$. However, regarding general overestimation $(K > 1$ or $RE > 0$, all methods produced more than 50% of K values greater than 1, indicating that all underestimated the hydraulic conductivity of the studied saturated soils. In terms of accuracy (based on the closeness of the mean of K to 1 or RE to 0), the proposed method again produced the best performance, with a K mean of 1.003. Similarly,

Fig. 17 Results for RMSE, RI and RD

Fig. 18 Results for RE

regarding RI values, as well as the σ of K and RE, the proposed method provided the most accurate evaluation $(RI = 0.070)$. However, both RMSE and RI have certain shortcomings, with the former highlighting only substantial errors (i.e. low accuracy) and the latter considering both accuracy and precision but not assigning an equal weight to them. In contrast, RD, which gives equal weight to accuracy and precision, is a better parameter with which to compare the suitability of different correlation equations (Onyejekwe et al. [2015\)](#page-10-0). In terms of RD, the best method for the analysed Jiangsu soft clays was the proposed method ($RD = 0.068$). Considering a common allowable limit of relative error (ALE) of 5%, the percentage relative error less than ALE (PRELA) is shown graphically in Fig. 18 for each method, with the higher the PRELA value, the better the correlation performance. Again the proposed method achieved the best performance (PRELA $= 78\%$), followed by Zou's method (64%). In summary, the most efficient of the four studied methods is that proposed in the present paper.

Conclusions

Despite several recent improvements, evaluation using conventional CPT-based methods continues to underestimate the in situ hydraulic conductivity of soil. In order to obtain more accurate values of this parameter, the present paper has outlined a new method based on the analysis and comparison of existing methods. A comparison of the results obtained by the proposed method and existing approaches using piezocone, field pumping and laboratory data for Quaternary deposits in the Yangtze Delta region was conducted. The most important findings of our study can be summarized as follows:

- 1. Existing methods mainly assume spherical or halfspherical radial flow around the cone or/and a power functional distribution. Yet, the surface area for water flow seems to be cylindrical rather than spherical in shape around the cone, with the exception of regions very close to the cone, while the surface area seems to be a half ellipsoid or spherical shape for spudcans and piezoballs according to the numerical simulation of piezocone dissipation tests. A number of laboratory and field tests were also carried out demonstrating that the negative exponent distribution of initial excess pore water pressure near the tip fits these results closely. These new hypotheses yielded a new method that builds on existing approaches.
- 2. Obtained using a graphical method for the analysis of field data, the results of measured versus predicted k_h values for the four analysed methods indicate that the proposed method can evaluate the hydraulic conductivity of soil (based on the CPTU test) more accurately.
- 3. In terms of the general overestimation $(K > 1$ or RE > 0) of k_h , the methods of Elsworth and Chai fundamentally underestimate the hydraulic conductivity of soils. In terms of RI and RD, Zou's method $(RI = 0.101$ and $RD = 0.078$ and the newly proposed method $(RI = 0.070$ and $RD = 0.068$) provide more accurate evaluations regarding the allowable limits of relative error (ALE) (5%); however, the proposed method achieved a percentage RE less than ALE (PRELA) value 14% greater than that of Zou (at 78 and 64%, respectively). Generally speaking, the most efficient method is the proposed one in the present paper. This method has been selected thanks to its larger accuracy and truthfulness compared with other possible approaches.

Acknowledgements Much of the research work described herein was funded by the National Natural Science Foundation of China (NSFC; grant no. 4157020433) and Project of the National Twelve-Five Year Research Program of China (grant no. 2012BAJ01B02). These financial supports are gratefully acknowledged.

References

- Baligh MM, Levadoux JN (1980) Pore pressure dissipation after cone penetration. Massachusetts institute of technology, department of civil engineering, constructed facilities division, Massachusetts
- Briaud J, Tucker LM (1988) Measured and predicted axial response of 98 piles. J Geotech Eng 114(9):984–1001
- Burns SE, Mayne PW (1998) Monotonic and dilatory pore pressure decay during piezocone tests in clay. Can Geotech J 35(6):1063–1073
- Cai GJ (2010) Study on theory and engineering application of digital multifunctional piezocone penetration test (CPTU). Ph.D. dissertation, Institute of Geotechnical Engineering, Southeast University
- Cai GJ, Liu SY, Tong LY et al (2007) Study on consolidation and permeability properties of Lianyungang marine clay based on piezocone penetration test. Chin J Rock Mech Eng 26(4):846–857 (in Chinese)
- Campanella RG, Robertson PK (1988) Current status of the piezocone test. In: Proceedings of the first international symposium on penetration testing, ISOPT, Balkema, Rotterdam, The Netherlands, pp 93–116
- Ceccato F, Simonini P (2016) Numerical study of partially drained penetration and pore pressure dissipation in piezocone test. Acta Geotechnica 1–15
- Cetin KQ, Ozan C (2009) CPT-based probabilistic soil characterization and classification. J Geotech Geoenviron Eng 135(1):84–107
- Chai JC, Agung PMA, Hino T et al (2011) Estimating hydraulic conductivity from piezocone soundings. Géotechnique 61(8):699–708
- Cherubini C, Orr TLL (2000) A rational procedure for comparing measured and calculated values in geotechnics. Coast Geotech Eng Pract Yokohama 1:261–265
- Clarke BJ, Cater JP, Wroth CP (1979) In situ determination of the consolidation characteristics of saturated clays. Proceedings of 7th European Conference on Soil Mechanics and Foundation Engineering. British Geotechnical Society, London, pp 207–227
- Croney D, Coleman JD (1961) Pore pressure and suction in soil. Butterworth, London
- Danziger FAB, Almeida MSS, Sills GC (1997) The significance of the strain path analysis in the interpretation of piezocone dissipation data. Géotechnique 47(5):901-914
- Douglas BJ, Olsen RS (1981) Soil classification using electric cone penetrometer, cone penetration testing and experience. ASCE, St. Louis, pp 209–227
- Elsworth D (1991) Dislocation analysis of penetration in saturated porous media. J Eng Mech 117(2):391–408
- Elsworth D, Lee DS (2005) Permeability determination from on-thefly piezocone sounding. J Geotech Geoenviron Eng 131(5):643–653
- Elsworth D, Lee DS (2007) Limits in determining permeability from on-the-fly uCPT sounding. Géotechnique 57(8):679-685
- Giasi CI, Cherubini C, Paccapelo F (2003) Evaluation of compression index of remolded clays by means of Atterberg limits. Bull Eng Geol Environ 62(4):333–340
- Grima MA, Babuška R (1999) Fuzzy model for the prediction of unconfined compressive strength of rock samples. Int J Rock Mech Min 36(3):339–349
- Gupta R, Davidson J (1986) Piezoprobe determination coefficient of consolidation. Soils Found 26(3):12–22
- Horpibulsuk S, Yangsukaseam N, Chinkulkijniwat A, Du YJ (2011) Compressibility and permeability of Bangkok clay compared with kaolinite and bentonite. Appl Clay Sci 52(1):150–159
- Jefferies MG, Davies MP (1993) Use of CPTU to estimate equivalent SPT N60. Geotech Test J 16(4):458–468
- Koizumi Y, Ito K (1967) Field tests with regard to pile driving and bearing capacity of piled foundations. Soils Found 7(3):30–53
- Leroueil S, Jamiolkowski M. (1991) Exploration of soft soil and determination of design parameters. In: Proceedings Geo-Coast'91, Orlando, Vol. 2, pp 969–998
- Lu Q, Randolph MF, Hu Y, Bugarski CA (2004) Numerical study of cone penetration in clay. Géotechnique 54(4):257–267
- Lunne T, Robertson PK, Powell JJM (1997) Cone penetration testing in geotechnical practice. Chapman and Hall, London
- Ma SZ, Tang YC, Meng GT et al (2007) Piezocone penetration test mechanism, methods and its engineering application. The China University of Geosciences Press, Wuhan
- Ma L, Xu YS, Shen SL, Sun WJ (2014) Evaluation of the hydraulic conductivity of aquifers with piles. Hydrogeol J 22(2):371–382
- Mahmoodzadeh H, Randolph MF, Wang D (2014) Numerical simulation of piezocone dissipation test in clays. Géotechnique 64(8):657–666
- Mahmoodzadeh H, Wang D, Randolph MF (2015) Interpretation of piezoball dissipation testing in clay. Géotechnique 65(10):831–842
- Mitchell JK, Brandon TL (1998) Analysis and use of CPT in earthquake and environmental engineering. Geotechnical site characterization, vol 1. Balkema, Rotterdam, pp 69–96
- Onyejekwe S, Kang X, Ge L (2015) Assessment of empirical equations for the compression index of fine-grained soils in Missouri. Bull Eng Geol Environ 74(3):705–716
- Randolph MF, Wroth CP (1979) An analytical solution for the consolidation around a driven pile. Int J Numer Anal Met 3(3):217–229
- Robertson PK (1990) Soil classification using the cone penetration test. Can Geotech J 27(1):151–158
- Robertson PK (2009) Estimating in situ soil permeability from CPT and CPTU. Can Geotech J 46(1):442–447
- Robertson PK, Sully JP, Woeller DJ et al (1992) Estimating coefficient of consolidation from piezocone test. Can Geotech J 29(4):539–550
- Roy M, Blanchet R, Tavenas F et al (1981) Behaviour of a sensitive clay during pile driving. Can Geotech J 18(1):67–85
- Shen SL, Xu YS (2011) Numerical evaluation of land subsidence induced by groundwater pumping in Shanghai. Can Geotech J 48(9):1378–1392
- Shen SL, Han J, Huang XC, Du SJ (2003) Laboratory studies on property changes in surrounding clays due to installation of deep mixing columns. Mar Georesour Geotech 21(1):15–35
- Shen SL, Ma L, Xu YS et al (2013) Interpretation of increased deformation rate in aquifer IV due to groundwater pumping in Shanghai. Can Geotech J 50(11):1129–1142
- Shen SL, Wang J P, Ma L (2010) Identification of soil stratigraphy of soft deposit in Shanghai from CPTU test. In Soil Behavior and Geo-Micromechanics, Proceedings of Sessions of Geoshanghai. ASCE, pp. 384–391
- Shen SL, Wang JP, Wu HN et al (2015) Evaluation of hydraulic conductivity for both marine and deltaic deposits based on piezocone testing. Ocean Eng 110(2015):174–182
- Tang SD, He LS, Fu Z (2002) Excess pore water pressure caused by an installing pile in soft foundation. Rock Soil Mech 23(6):725–732
- The CI, Houlsby GT (1991) An analytical study of the cone penetration test in clay. Geotechnique 41(1):17–34
- Wang JP, Shen SL (2013) Determination of permeability coefficient of soil based on CPTU. Rock Soil Mech 11:3335–3339 (in Chinses)
- Wang JP, Xu YS, Ma L, Shen SL (2013) An approach to evaluate hydraulic conductivity of soil based on CPTU test. Mar Georesour Geotech 31(3):242–253
- Wroth CP (1984) Interpretation of in situ soil tests. Géotechnique 34(4):449–489
- Xu YS, Shen SL, Cai ZY, Zhou GY (2008) The state of land subsidence and prediction activities due to groundwater withdrawal in China. Nat Hazards 45(1):123–135
- Xu YS, Ma L, Shen SL, Sun WJ (2012) Evaluation of land subsidence by considering underground structures that penetrate the aquifers of Shanghai, China. Hydrogeol J 20(8):1623–1634
- Xu YS, Shen SL, Du YJ, Chai JC, Horpibulsuk S (2013) Modelling the cutoff behavior of underground structure in multi-aquiferaquitard groundwater system. Nat Hazards 66(2):731–748
- Yi JT, Goh SH, Lee FH, Randolph MF (2012a) A numerical study of cone penetration in fine-grained soils allowing for consolidation effects. Géotechnique 62(8):707-719
- Yi JT, Lee FH, Goh SH, Zhang XY, Wu JF (2012b) Eulerian finite element analysis of excess pore pressure generated by spudcan installation into soft clay. Comput Geotech 42(2012):157–170
- Zeng LL, Hong ZS, Cai YQ, Han J (2011) Change of hydraulic conductivity during compression of undisturbed and remolded clays. Appl Clay Sci 51(1):86–93
- Zhu XL, Tang SD (1986) Theoretical analysis of the coefficient of consolidation in soft clay estimated by pore water pressure-cone of cone penetration test. Geotech Invest Surv 6:8–12
- Zhu XR, He YH, Xu CF, Wang ZL (2005) Excess pore water pressure caused by single pile driving in saturated soft soil. Chin J Rock Mech Eng 24(2):725–732 (in Chinese)
- Zou HF, Cai GJ, Liu SY et al (2014) Evaluation of coefficient of permeability of saturated soils based on CPTU dislocation theory. Chin J Geotech Eng 36(3):519–528 (in Chinese)