Bull Eng Geol Environ (2017) 76:1075–1084
DOI 10.1007/s10064-016-0933-z

ORIGINAL PAPER

A macro-structural constitutive model for partially saturated expansive soils

Wugang Li[1] · **Qing Yang**[1]

Received: 2 February 2016 / Accepted: 6 September 2016 / Published online: 7 December 2016

Abstract The mechanical behaviour of unsaturated expansive soils is an important factor in the design of barriers for nuclear waste disposal. Double structural constitutive models for expansive soil have been developed based on the distinction between micro-structure and macro-structure of the expansive soil. Nevertheless, the micro-parameters and the coupling function of these models are hard to determine. To eliminate the limitations of these models, the concept of a macro-structural neutral loading line was introduced by adopting a simple assumption concerning the plastic swelling potential of unsaturated expansive soil. A macro-structural volume change equation of unsaturated expansive soil was derived using a macro-structural neutral loading line. The model was based on an existing model for non-expansive unsaturated soils. Only one new macro-parameter was introduced into the constitutive model compared to the original model. Finally, a comparison between predicted results and experimental data indicates that the proposed model has a remarkable ability to reproduce the mechanical and hydraulic responses of unsaturated expansive soils.

Keywords Unsaturated expansive soils · Constitutive model · Suction · Swelling behaviour · Microstructure · Macrostructure

✉ Qing Yang
qyang@dlut.edu.cn

[1] Institution of Geotechnical Engineering, Dalian University of Technology, Dalian 116024, China

Introduction

The mechanical behaviour of expansive soils is a key factor in the design of barriers for nuclear waste disposal. Alonso et al. (1990) presented the Barcelona basic model (BBM), which describes the stress–strain behaviour of unsaturated soils of low activity. Since then, lots of constitutive models have been developed (Cui and Delage 1996; Wheeler and Sivakumar 1995; Chiu and Ng 2003; Georgiadis et al. 2005; Thu et al. 2007). These models are able to reproduce the basic mechanical behaviour of unsaturated soils, such as increasing shear strength with suction, and increasing irreversible collapse strains along wetting paths under low confining stress state. However, these models usually fall within the same framework as that of Alonso et al. (1990), and have limitations in predicting the large swelling strains exhibited by expansive soils. Due to their mineralogical composition, unsaturated expansive soils usually exhibit more complicated mechanical behaviour compared to unsaturated soils of low activity. Therefore, building a constitutive model for unsaturated expansive soils that will reproduce their mechanical behaviour is a daunting task. Starting with the double-structural framework (G-A model) proposed by Gens and Alonso (1992), the Barcelona expansive model (BExM) (Alonso et al. 1999) was proposed. This latter model distinguishes micro-structural from macro-structural deformation. The micro-structural level controls the physicochemical phenomena occurring at active clay minerals. The macro-structural deformation is responsible for major structural rearrangements of the soil skeleton. Coupling these two distinct structural deformations produces the final deformations of expansive soils. However, the coupling function of the BExM is difficult to determine. Besides, the micro-structure parameters are difficult to quantify through conventional soil mechanics experiments. A model for unsaturated expansive soils would be more convenient to use if the model parameters could be determined through conventional soil mechanics experiments. Therefore, this paper puts forward a macroscopic constitutive model for partially saturated expansive soils based on the experimental results of expansive soils in the literature.

Macro-structural neutral loading line

Gens and Alonso (1992) proposed the concept of a neutral loading line of unsaturated expansive soils to describe the micro-structure properties where physicochemical and other phenomena occurring at particle level take place. According to the double-structure framework proposed by Gens and Alonso (1992), the micro-structural volumetric elastic strain will be zero if a stress path coincides with the neutral loading line. When a stress path crosses the neutral loading line via suction reduction, micro-structural swelling will take place. The increase in the macro-structural strain is found by multiplying the coupling function of BExM by the increase in the micro-structural strain. However, the coupling function of BExM is difficult to determine and usually depends on the empirical formula. Besides that, it is not easy to determine the micro-structural parameters used to calculate the micro-structural volumetric elastic strain. Therefore, it is necessary to solve the problem in other ways in order to make the constitutive model of unsaturated expansive soils more convenient to use.

When an unsaturated expansive specimen is dried, the micro-structural level remains saturated, as proposed by Gens and Alonso (1992). Elegant tests performed by Graham et al. (1992) support this assumption. The effective stress of the micro-structural level increases as the suction increases. According to the effective stress principle, the soil particles will join together and form aggregates. It is clear that the aggregates will be compressed, which will result in larger pore spaces and an increase in effective stress. The final result is that the volume of the unsaturated expansive specimen decreases along the drying path. On the contary, if an unsaturated expansive specimen is wetted, the effective stress decreases with the decrease in suction. When the effective stress is smaller than the repulsive force produced by the positive charge on the surface of the soil particles, swelling will take place in the micro-structure of the expansive soil according to the effective stress principle. The micro-structural swelling in expansive soil affects the structural arrangement of the macro-structure, thus increasing the irreversible void ratio of the expansive soil. The final result is that the unsaturated expansive soil experiences large volumetric swelling. Based on the analysis above, it can be seen that the change in micro-structure induces a rearrangement of macro-structure, and that this rearrangement in macro-structure reflects the change in micro-structure. So it is possible to develop a constitutive model for expansive soils from a macro-structural level.

The swelling mechanical behaviour of unsaturated expansive soil is associated with the swelling pressure of the expansive soil. Gouy-Chapman double-layer theory (Gouy 1910, 1917; Chapman 1913) is used widely to compute the swelling pressure of unsaturated expansive soils. But the double-layer theory is poorly developed in that predictions cannot be exact because the parallel clay-particle arrangement is too simple to represent the actual soil structures (Yong et al. 1984). And other factors, such as the effect of ion size, anion adsorption, and the existence of an attractive force, that are also not considered in the theory (Gens and Alonso 1992). Typical swelling pressure paths for unsaturated expansive clay with constant volume are shown in Fig. 1 (Blight 1965). It can be observed that the shape of two stress paths are similar to each other. In addition, the angle between stress paths and the horizontal axis is about 45° in the low suction state. If the stress path for an unsaturated expansive soil specimen coincides with a swelling-pressure stress path, the volume change of the soil sample will be zero during the test. It can be assumed that, for unsaturated expansive soils, stress paths exist along which no macro-structural plastic swelling deformations takes place if a stress unloading takes place. These stress paths are identified as macro-structural neutral loading lines. The distinction between the macro-structural neutral line and the neutral loading line proposed by Gens and Alonso (1992) is that the macro-structural neutral line is a macroscopic swelling potential surface and the neutral loading line responsible for micro-structure of expansive soils. The relative position between the macro-structural neutral line (MNL) and neutral loading line (NL) in the stress space is illustrated in Fig. 2. The plastic volumetric strain is zero if point A travels to point B along the macro-structural neutral line as shown in Fig. 2.

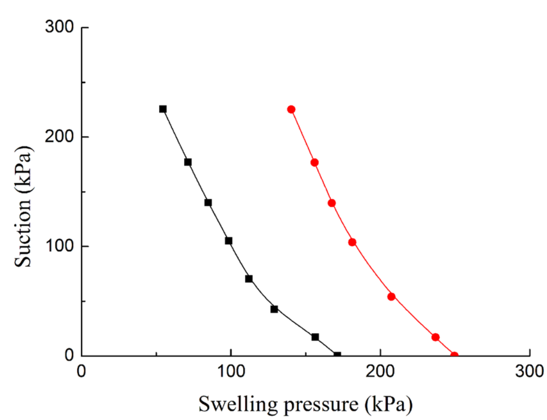

Fig. 1 Typical swelling-pressure stress paths for unsaturated expansive clay, extracted from Blight (1965)

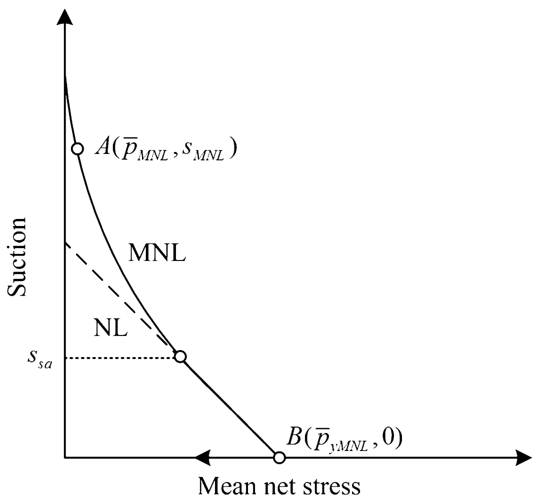

Fig. 2 Macro-structural neutral loading line in the space of mean net stress and suction

During a swelling-pressure test, external stress was increased to prevent any volume change when the sample was wetted, as shown in Fig. 1. The increment of elastic volume change induced by a suction reduction, and the decrement of elastic volume change induced by increment of external stress counterbalanced each other. Hence, the elastic volume change along the swelling-pressure stress path should be small. The value of plastic strain along the swelling-pressure stress path was small as well because the volume was kept constant during the test. Therefore, the swelling-pressure stress-paths were similar to the macro-structural neutral loading lines because great plastic strain would be induced if it were quite different between swelling-pressure stress-paths and macro-structural neutral loading lines. If the elastic volume change along the swelling-pressure stress path is so small that elastic volume change is negligible, the plastic volumetric strain would be zero because the volume was kept constant. In this situation, the swelling-pressure stress paths could be referred as macro-structural neutral loading paths.

Constitutive equations

Changes in volume behaviour is one of the most significant properties of soils. It is a common view that plastic volumetric strain can be adopted as the hardening parameters in elastoplastic constitutive model for saturated soils and unsaturated soils. For saturated soils, the increment of volume change is usually expressed as follows:

$$dv = -\lambda \frac{dp'}{p'} \tag{1}$$

where p' is the mean effective stress, λ is the slope of the normal consolidation line for saturated soils, and v is the specific volume. To describe the large swelling volume change experienced by unsaturated expansive soils, Eq. (1) can be extended to the unsaturated state in the following ways: (1) the net stress and suction approach, (2) the generalized effective stress approach (Sheng 2011), and (3) the SFG approach (Sheng et al. 2008). The advantage of the net stress and suction approach is that the compressibility due to changes in net stress and suction can be treated separately. However, the net stress and suction approach suffers from limitations. A significant limitation of the net stress and suction approach is that the volume change equation becomes undefined at the transition suction between saturated and unsaturated state. And the volume change caused by suction changes is independent of the stress state, which is not compatible with the experimental observation (Delage and Graham 1995). Besides, the expansion induced by suction reduction will be smaller if a larger net stress is applied for unsaturated expansive soils. So the net stress and suction approach is not appropriate to model the mechanical behaviour of unsaturated expansive soils. The limitation of the effective stress approach is that it is difficult to address differences in compressibility due to net stress and suction changes. Sheng et al. (2008) proposed a third way to model the volume change for unsaturated soils under isotropic stress states. The SFG approach represents a middle ground between approach 1 and approach 2 and is expressed as follows:

$$d\varepsilon_v = \frac{dv}{v} = \lambda_{vp} \frac{d\bar{p}}{\bar{p}+s} + \lambda_{vs} \frac{ds}{\bar{p}+s} \tag{2}$$

where $\bar{p} = p - u_a$ is mean net stress, p is total mean stress, u_a is the pore air stress, s is the suction, and λ_{vp} is the slope of the normal consolidation line for saturated soils. λ_{vs} is a function of suction. It is similar to approach 1 in that the SFG approach is defined in terms of net stress and suction and separates the compressibilities due to suction and stress changes. Besides, similar to approach 2, it combines the mean net stress and suction in the denominator. Equation (2) recovers Eq. (1) at transition suction between saturated and unsaturated states. The SFG approach was employed here to develop a constitutive model for unsaturated expansive soils. Sheng et al. (2008) adopted a simple but not unique approximation for λ_{vs} which takes the form

$$\lambda_{vs} = \begin{cases} \lambda_{vp} & s < s_{sa} \\ \lambda_{vp} \dfrac{s_{sa}}{s} & s \geq s_{sa} \end{cases} \tag{3}$$

where s_{sa} is the saturation suction, which is the transition suction between saturated and unsaturated states and is the same as the air-entry value when the soil is dried from full saturation, but is usually smaller than the air-entry value when the soil is wetted from an unsaturated state. However, Eq. (2) is not appropriate to calculate the volume change when unsaturated expansive soils experience large swelling strains. According to the SFG approach, it is assumed that the incremental form takes the following expression when unsaturated expansive soils experience large swelling strains:

$$\mathrm{d}\varepsilon_v = \lambda'_{\mathrm{vp}} \frac{\mathrm{d}\bar{p}}{\bar{p}+f(s)} + \lambda'_{vs} \frac{\mathrm{d}s}{\bar{p}+f(s)} \tag{4}$$

where $f(s)$ is the suction function, and λ'_{vp} is the swelling index for unsaturated expansive soils. Similar to λ_{vs}, λ'_{vs} takes the following expression:

$$\lambda'_{\mathrm{vs}} = \begin{cases} \lambda'_{\mathrm{vp}} & s < s_{\mathrm{sa}} \\ \lambda'_{\mathrm{vp}} \dfrac{s_{\mathrm{sa}}}{s} & s \geq s_{\mathrm{sa}} \end{cases} \tag{5}$$

Although the swelling index λ'_{vs} takes a simple expression, it satisfies the experimental results. The function $f(s)$ needs to be determined first in order to obtain the incremental form for unsaturated expansive soils experiencing large swelling strains.

A macro-structural neutral loading line is illustrated in Fig. 2 in the space of mean net stress and suction. If the stress paths of unsaturated expansive soils coincide with the macro-structural loading lines, no macro-structural plastic strain will take place. Although the macro-structural plastic strain is zero, the elastic strain develops along the macro-structural neutral loading lines. Therefore the elastic volume change has to be considered. The expression for the increment of elastic volumetric swelling strain caused by stress and suction changes can be expressed as follows:

$$\mathrm{d}\varepsilon_v^e = \kappa_{\mathrm{vp}} \frac{\mathrm{d}\bar{p}}{\bar{p}+f(s)} + \kappa_{\mathrm{vs}} \frac{\mathrm{d}s}{\bar{p}+f(s)} \tag{6}$$

where κ_{vp} is the swelling index, and is assumed to be independent of suction. Similar to λ_{vs}, κ_{vs} takes the following expression:

$$\kappa_{\mathrm{vs}} = \begin{cases} \kappa_{\mathrm{vp}} \; s < s_{\mathrm{sa}} \\ \kappa_{\mathrm{vp}} \dfrac{s_{\mathrm{sa}}}{s} \; s \geq s_{\mathrm{sa}} \end{cases} \tag{7}$$

It is now possible to derive the increment of plastic swelling strain along the macro-structural loading line with Eqs. (4) and (7). The increment of plastic swelling strain can be written as

$$\mathrm{d}\varepsilon_v^p = (\lambda'_{\mathrm{vp}} - \kappa_{\mathrm{vp}}) \frac{\mathrm{d}\bar{p}}{\bar{p}+f(s)} + (\lambda'_{\mathrm{vs}} - \kappa_{\mathrm{vs}}) \frac{\mathrm{d}s}{\bar{p}+f(s)} \tag{8}$$

Since no macro-structural plastic strain takes place along the macro-structural neutral loading lines, the value of Eq. (8) should be zero, giving rise to the following equation:

$$\frac{\mathrm{d}\bar{p}}{\mathrm{d}s} = -\frac{\lambda'_{\mathrm{vs}} - \kappa_{\mathrm{vs}}}{\lambda'_{\mathrm{vp}} - \kappa_{\mathrm{vp}}} \tag{9}$$

Equation (9) is the trajectory of macro-structural neutral loading line in the space of mean net stress and suction. Stress points A and B lie on the macro-structural neutral loading line as shown in Fig. 2. In order to get the function of the macro-structural neutral loading, Eq. (9) can be integrated along the macro-structural neutral loading line from point A to point B, leading to

$$\bar{p}_{\mathrm{MNL}} = \begin{cases} \bar{p}_{y\mathrm{MNL}} - s_{\mathrm{MNL}} & s < s_{\mathrm{sa}} \\ \bar{p}_{y\mathrm{MNL}} - s_{\mathrm{sa}} - s_{\mathrm{sa}} \ln \dfrac{s_{\mathrm{MNL}}}{s_{\mathrm{sa}}} & s \geq s_{\mathrm{sa}} \end{cases} \tag{10}$$

where $\bar{p}_{y\mathrm{MNL}}$ is the intersection of a macro-structural neutral loading line and the mean net stress axis. Stress state $(\bar{p}_{\mathrm{MNL}}, s_{\mathrm{MNL}})$ is any point on the macro-structural neutral loading line. Equation (10) is the expression for macro-structural neutral loading line in the space of mean net stress and suction. Taking a specimen as an example, and the initial stress state of the specimen at point A as shown in Fig. 2, plastic swelling takes place via a suction reduction. As a result, the new macro-structural neutral loading line moves left along the mean net stress axis as illustrated in Fig. 2, i.e. the arrow on the mean net stress axis points to the left. The new intersection of the macro-structural neutral loading line and the mean net stress axis will be located on the left side of the point B. Therefore the total increment of expansion strain can be written as

$$\mathrm{d}\varepsilon_v = \lambda'_{\mathrm{vp}} \frac{\mathrm{d}\bar{p}_{y\mathrm{MNL}}}{\bar{p}_{y\mathrm{MNL}}} \tag{11}$$

Equation (12) is derived by substituting Eq. (10) into Eq. (11) and is derived as follows:

$$\mathrm{d}\varepsilon_v = \begin{cases} \lambda'_{\mathrm{vp}} \dfrac{\mathrm{d}\bar{p}_{\mathrm{MNL}}}{\bar{p}_{\mathrm{MNL}} + s_{\mathrm{MNL}}} + \lambda'_{\mathrm{vp}} \dfrac{\mathrm{d}s_{\mathrm{MNL}}}{\bar{p}_{\mathrm{MNL}} + s_{\mathrm{MNL}}} & s_{\mathrm{MNL}} < s_{\mathrm{sa}} \\ \lambda'_{\mathrm{vp}} \dfrac{\mathrm{d}\bar{p}_{\mathrm{MNL}}}{\bar{p}_{\mathrm{MNL}} + s_{\mathrm{sa}} + s_{\mathrm{sa}} \ln \frac{s_{\mathrm{MNL}}}{s_{\mathrm{sa}}}} + \lambda'_{\mathrm{vp}} \dfrac{s_{\mathrm{sa}}}{s_{\mathrm{MNL}}} \dfrac{\mathrm{d}s_{\mathrm{MNL}}}{\bar{p}_{\mathrm{MNL}} + s_{\mathrm{sa}} + s_{\mathrm{sa}} \ln \frac{s_{\mathrm{MNL}}}{s_{\mathrm{sa}}}} & s_{\mathrm{MNL}} \geq s_{\mathrm{sa}} \end{cases} \tag{12}$$

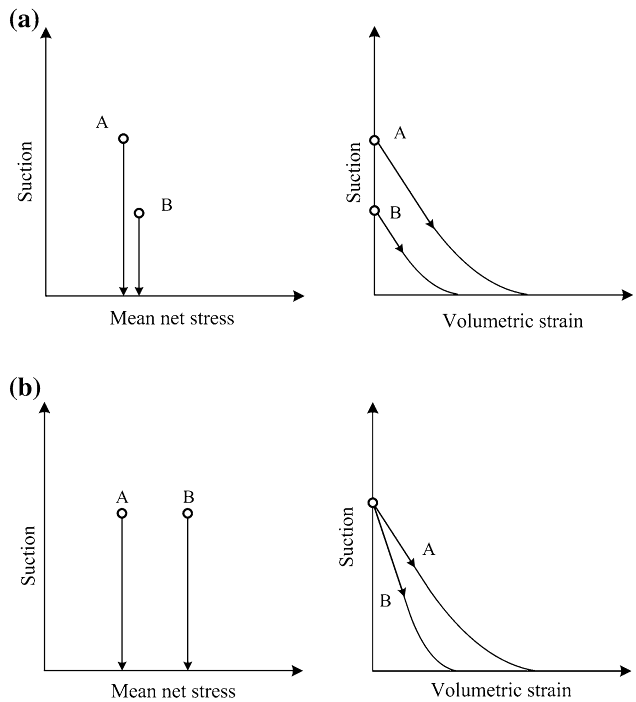

Fig. 3 Behaviour of expansive soils. **a** Dependence of the swelling strain on suction variation. **b** Dependence of the swelling strain on applied mean net stress variation

It is now possible to derive the expression for function $f(s)$ with Eqs. (4), (5), (11) and (12). Function of $f(s)$ takes the following form:

$$f(s) = \begin{cases} s_{\mathrm{MNL}} & s_{\mathrm{MNL}} < s_{\mathrm{sa}} \\ s_{\mathrm{sa}} + s_{\mathrm{sa}} \ln \dfrac{s_{\mathrm{MNL}}}{s_{\mathrm{sa}}} & s_{\mathrm{MNL}} \geq s_{\mathrm{sa}} \end{cases} \tag{13}$$

When unsaturated expansive soils experience large swelling, the expression for increment of swelling strain is derived from Eqs. (4) and (13). For unsaturated expansive soils, there is agreement on the swelling tendency of unsaturated expansive soils under different stress states as shown in Fig. 3. The suction of point A is higher than that of point B, and the mean net stress of the two points are equal with each other as shown in Fig. 3a. The swelling strains of point A induced by suction reduction are larger than that of point B. We can judge the swelling tendency easily by using Eq. (4). If the mean net stress of point B is larger than that of point A, and the suction of the two points are equal to each other as shown in Fig. 3b, the swelling strains of point A induced by suction reduction are larger than that of point B. Likewise, we can judge the swelling tendency easily by using Eq. (4). If the G-A model is adopted here to judge the swelling tendency, we first need to determine the ratio of the current applied mean net stress over the apparent pre-consolidation stress. Therefore, this model is more convenient than the G-A model in practical computation.

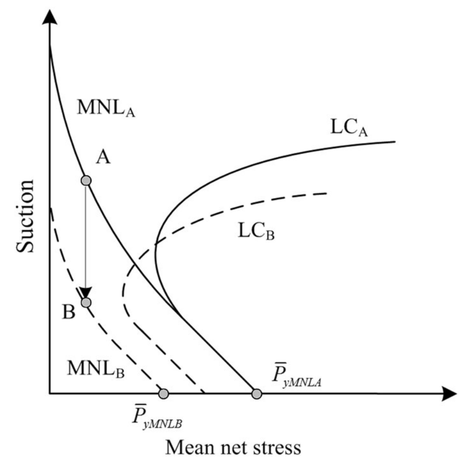

Fig. 4 Coupling between the macro-structural neutral loading line and the loading collapse yield curve

For unsaturated expansive soils, micro-structural swelling takes place if suction decreases with a constant value of the applied mean net stress. In this case, micro-structural swelling will affect the soil skeleton. Since this model does not distinguish between the micro-structural strain and macro-structural strain, the macro-structural neutral loading line plays a role analogous to that of the swelling potential surface. Taking a specimen as an example, and if the initial stress state of the specimen is at point A as shown in Fig. 4, macro-structural swelling strain will take place via a suction reduction from point A to point B. The soil skeleton of the specimen is rearranged due to the plastic swelling strain. The macro-structural swelling strain increases the void ratio of the specimen. The loading collapse yield surface then moves to the left (dotted line in Fig. 4) in response to the new soil skeleton arrangement.

Water retention behaviour of unsaturated expansive soils

The relationship between swelling strain and stress state variables has been established as discussed above. For unsaturated expansive soils, reproducing the relationship between the degree of saturation and stress variables is an important aspect of the model. Sun et al. (2007) and Masin (2010) concluded that the degree of saturation depends on suction, void ratio and stress paths. For unsaturated soils, the change in the void ratio usually follows a change in the stress state. Therefore, an increase in the degree of saturation can be caused by a decrease of suction and an increase in void ratio. Hence, the increment of degree of saturation can be written as Sheng and Zhou (2011)

$$\mathrm{d}S_r = \frac{\partial S_r}{\partial s}\mathrm{d}s + \frac{\partial S_r}{\partial e}\mathrm{d}e \tag{14}$$

Extensive research focussing on soil water characteristic curves has been performed, and great success has been achieved (Mualem 1976; Fredlund and Xing 1994; Van Genuchten 1980). As a first approximation, the relationship between degree of saturation and logarithmic suction under constant value of volume is assumed to be a piecewise linear. Therefore, the first term on the right of the equal sign in Eq. (14) takes the following expression:

$$\frac{\partial S_r}{\partial s}\mathrm{d}s = -\alpha\frac{\mathrm{d}s}{s} \tag{15}$$

where

$$\alpha = \begin{cases} \lambda_s & \text{main drying curve or main wetting cure} \\ \kappa_s & \text{scanning curve} \end{cases} \tag{16}$$

λ_s and κ_s are the slope of the main drying/wetting curve and the scanning curve, respectively. The scanning curve is a series of parallel lines that represent recoverable changes in degree of saturation between the main drying and main wetting curves. Sun et al. (2007) found a linear relationship between the variation in degree of saturation and void ratio under constant suction. Hence, the second term to the right of the equal sign in Eq. (14) can be written as

$$\frac{\partial S_r}{\partial e}\mathrm{d}e = -\lambda_e \mathrm{d}e \tag{17}$$

where the λ_e is the slope of the $S_r - e$ curve under a constant suction larger than the air-entry value.

Comparison of model predictions with experimental data

In this section, a series of experimental results from the literature are used to verify the performance of the proposed constitutive model for unsaturated expansive soils. In the proposed model, only one new parameter, λ'_{vp}, is added to the mechanical part of the proposed model compared to the original SFG model. The numerical examples in this section deal with swelling-pressure stress path behaviour, the volumetric behaviour and the suction-saturation behaviour of the proposed model. Shear strength behaviour, which can be considered by incorporating existing models such as Modified Cam-Clay Model (Roscoe and Burland 1968) into the proposed model, are beyond the scope of this article.

Swelling pressure of expansive clay

Kassiff and Shalom (1971) conducted a series of experiments to investigate the swelling pressure behaviour of unsaturated expansive soils. The clay used in their experiment was a typical Israel expansive clay. The liquid limit of the clay ranges from 78% to 85%, and the plastic limit varies from 58% to 63%, and the specific gravity is 2.75. The volume of the specimens was kept constant during their tests. An external pressure was applied to the specimen in order to keep the volume of the specimen constant due to suction reduction. The experimental results are reproduced in Fig. 5. If the stress paths of tests coincide with the macro-structural neutral loading line, it is obvious that the plastic volume change during the tests will be zero as discussed above. The elastic volume change along the stress paths is quite small because the increment of elastic volume change induced by a suction reduction, and the decrease in the elastic volume change induced by the external pressure increasing are counterbalanced with each other. As an approximation, the elastic volume change was very small during the tests and could be ignored. Therefore, the macro-structural neutral loading line can be verified by the experimental results as shown in Fig. 5. Only one parameter, i.e. the saturation suction, needs to be determined in order to obtain the numerical results of the macro-structural neutral loading lines. However, the saturation suction is not provided in the Kassiff and Shalom's (1971) paper, so we first need to determine the saturation suction. We can then substitute one set of experiment results into Eq. (10) to back calculate the saturation suction. By substituting the experiment results of the specimen with an initial water content of 27% into Eq. (10), the saturation suction of the clay is computed and is about 50 kPa. The parameters used are given in Table 1. Predictions regarding the swelling pressure stress paths are illustrated in Fig. 5. The angles between stress paths and horizontal axis are approximately 45° at low suction state. The angles become larger when the suction increases. The predicted results resemble the experimental data well as shown in Fig. 5. It

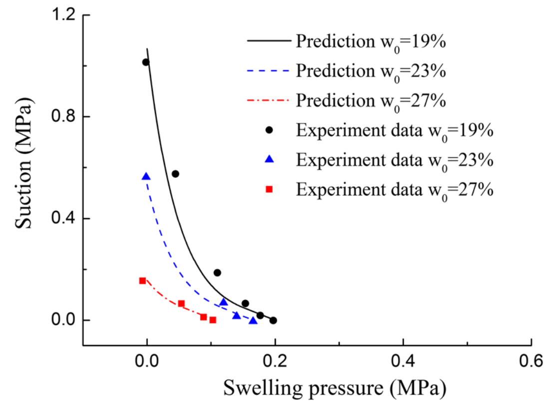

Fig. 5 Predicted and experimental results on swelling pressure stress paths for Israel expansive clay, extracted from the work by Kassiff and Shalom (1971)

Table 1 Material parameters of Israel expansive clays

Initial water content (%)	Saturation suction (kPa)	$\bar{p}_{yMNL}$ (kPa)
19	50	102
23	50	163
27	50	200

can be seen that the proposed model is capable of reproducing the swelling-pressure stress paths.

Swelling behaviour of expansive soils

Lloret et al. (2003) studied the mechanical behaviour of compacted bentonite. The testing program was designed to apply a very large range of suction. Suction up to 550 MPa could be reached by applying a controlled atmosphere where the relative humidity was fixed by a solution of sulphuric acid or salts. Therefore, the tests provide the opportunity to examine the swelling behaviour of bentonite over a wide range of suction. The bentonite used in the test is from the Cortijo de Archidona deposit in Almetia, south-eastern Spain. The montmorillonite content of the bentonite is higher than 90%. The liquid limit of the bentonite is102 ± 4% and the plastic limit is 53 ± 3%. The saturation suction is about 8 MPa according to the soil water characteristic curve presented by Lloret et al. (2003). Based on the test data in Lloret et al. (2003), the material parameters of the bentonite are given in Table 2. The initial vertical stress and suction of the two specimens are 5.1 and 460 MPa, 0.1 and 520 MPa, respectively. The experimental data of the volume change via a suction reduction and the predicted results are shown in Fig. 6. It can be seen that the macro-structural swelling rate is slow at high suction state according to the experiment results. The rate of increase of swelling strains with suction became larger when lower values of suction were reached. The difference between the two sets of experimental data is that the slope of suction-void ratio curve of the S1 specimen is more gentle than that of the S5 specimen due to the higher vertical stress applied to the S1 specimen during the tests. The comparison between the predicted results and the experimental data indicates that the proposed model is capable of reproducing the swelling behaviour of bentonite.

Table 2 Material parameters of bentonite

Property	Value
Liquid limit, LL (%)	102 ± 4
Plastic limit, PL (%)	53 ± 3
s_{sa} (MPa)	8
λ_{vp}	0.08
λ'_{vp}	0.05
κ_{vp}	0.005

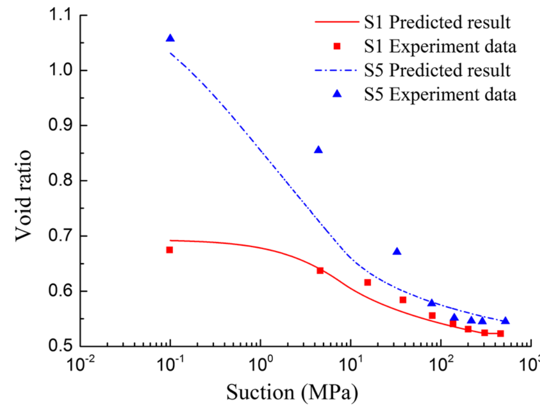

Fig. 6 Computed and experimental variation of void ratio via the suction reduction for test S1 and S5, extracted from the work by Lloret et al. (2003)

Table 3 Material parameters of sand-bentonite mixture

Property	Value
λ_e	0.5
λ_s	0.15
κ_s	0.03
s_{sa} (MPa)	1.1
λ_{vp}	0.14
λ'_{vp}	0.03
κ_{vp}	0.02

Sun et al. (2009) conducted a series of laboratory tests on a heavily compacted sand-bentonite mixture using a suction-controllable oedometer. Bentonite and sand were mixed with 3:7 by weight. The suction during the test varied from 0 kPa to 2500 kPa. The value of the suction was measured by the filter paper method when the value of the applied suction was larger than 1300 kPa. The initial suctions of four specimens were 2500 kPa. Then the four specimens were wetted to respective targeted suctions 300 kPa, 600 kPa, 1200 kPa, and 1500 kPa under 10 kPa vertical net stress. Based on the experimental results of the sand-bentonite mixture, the parameters used for prediction were determined, and are given in Table 3. The test results of the void ratio and the degree of saturation before and after the wetting progress are shown in Fig. 7, as well as the predictions. It can be seen that the macro-structural swelling takes place during the wetting progress, and the rate of increase of swelling strains with suction became larger when lower values of suction were reached. The results predicted by our proposed model are in good agreement with those obtained in the experiment. It is obvious that the proposed model shows a good ability to predict the stress–strain relationship, as well as the hydraulic behaviour of unsaturated expansive soils.

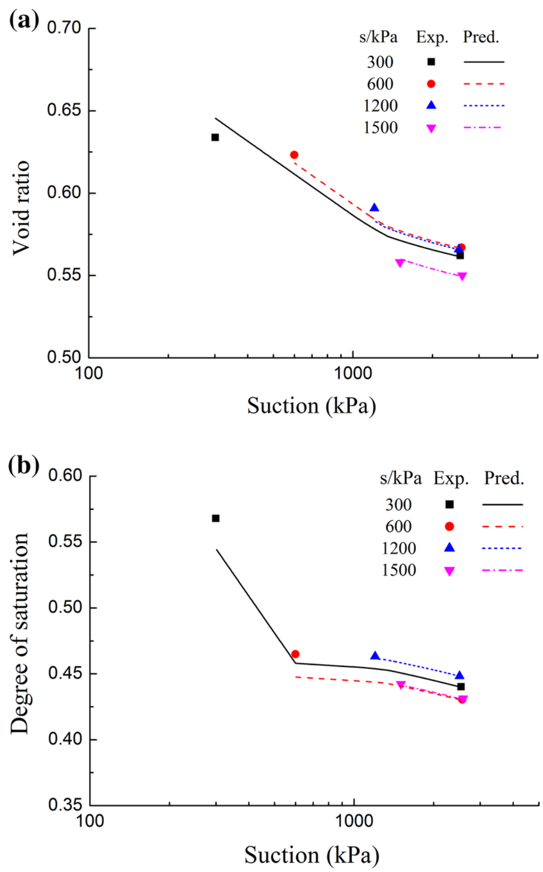

Fig. 7 Comparison between predictions and experimental results. **a** Relationships between suction and void ratio and **b** relationships between suction and degrees of saturation, extracted from the work by Sun et al. (2009)

Swelling and isotropic compression of expansive soils

Zhan (2003) conducted a series of laboratory tests on compacted expansive specimens to investigate the mechanical behaviour of expansive soils. The clay contained 16% illite, 21% montmorillonite, and 4% kaolinite. The clay used in the experiment belongs to medium expansive soil. The liquid limit and plasticity index were 50.5 and 31, respectively. The specimens were statically compacted, and had an initial water content of about 18.5%. The initial suctions of the four specimens were measured at about 540 kPa. During the tests, the four specimens were first wetted to the target suctions about: 25 kPa, 50 kPa, 100 kPa and 200 kPa, respectively, under an isotropic constant net stress of 20 kPa. After the wetting tests, four specimens were isotropic compressed to a mean net stress of 200 kPa. Based on the experimental data in Zhan (2003), the parameters used in the model for predicting the mechanical behaviour, and the water retention curve of the unsaturated expansive soils, were determined and are given in Table 4. Figure 8 shows the experimental results and the predictions of the changes in the void ratio and degree of saturation during the wetting progress. From the results as shown in Fig. 8, it can be seen that swelling takes place, and the swelling rate was slow at a high suction state. The predicted results agree well with the experimental data. Figure 9 shows the experimental results and the predictions of the changes in void ratio and degree of saturation during the compression progress after wetting. The comparison between the test data and the predicted results indicates that the proposed model possesses a remarkable ability to reproduce the mechanical and hydraulic behaviour of unsaturated expansive soils.

Table 4 Material parameters of expansive soils

Property	Value
λ_e	0.83
λ_s	0.1
κ_s	0.03
s_{sa} (kPa)	25
λ_{vp}	0.05
λ'_{vp}	0.035
κ_{vp}	0.01

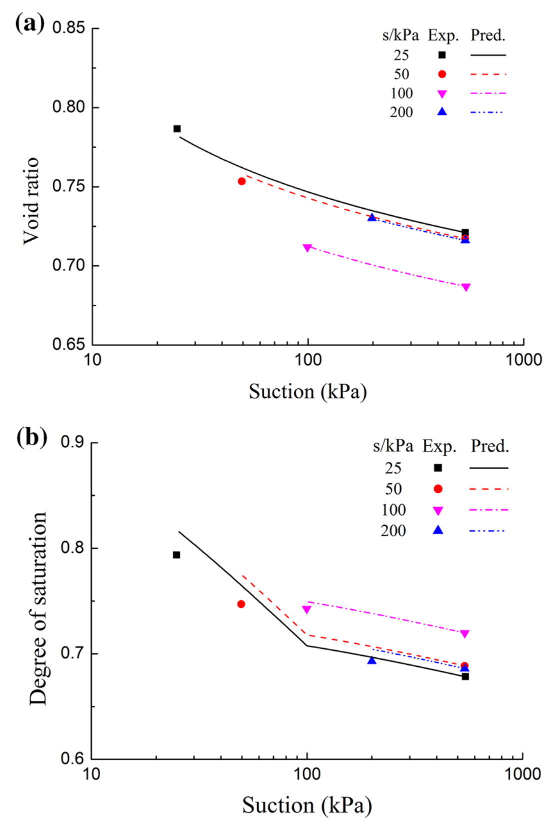

Fig. 8 Comparison between experimental results and predictions. **a** Relationships between suction and void ratio and **b** relationships between suction and degrees of saturation, extracted from the work by Zhan (2003)

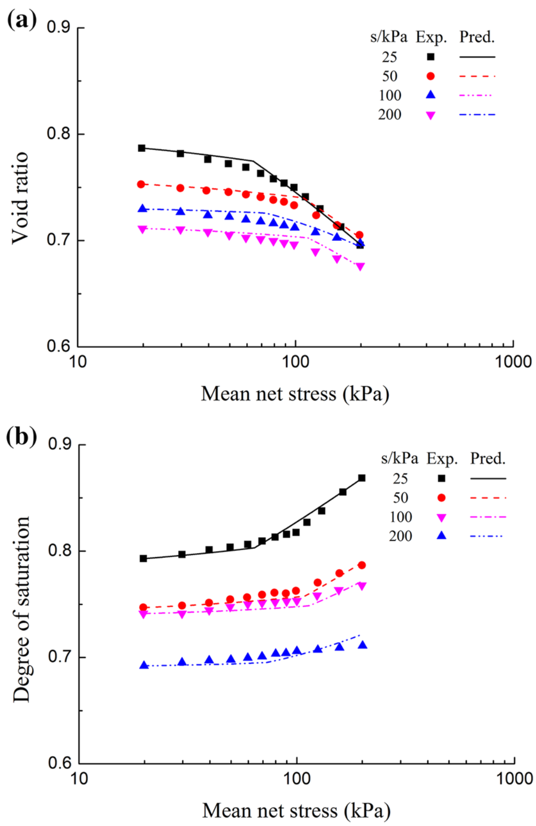

Fig. 9 Comparison between experimental results and predictions. **a** Relationships between mean net stress and void ratio and **b** relationships between mean net stress and degrees of saturation, extracted from the work by Zhan (2003)

Conclusions

This paper presents a macro-structural constitutive model for unsaturated expansive soils using independent stress state variables. The concept of a macro-structural neutral loading line was introduced based on experimental data. This concept eliminates the limitations of the double structural constitutive model for unsaturated expansive soils. The macro-structural neutral loading line plays a role analogous to that of the swelling potential surface. The strain–stress relationship is derived by incorporating the macro-structural neutral loading line into the SFG model for non-expansive soils. The advantage of the proposed model is that no distinction needs to be made between micro-structural and macro-structural deformations. And the mechanical part of the proposed model introduces only one new parameter compared to the original model for non-expansive soils. Hence, it is convenient to employ the proposed model to predict the mechanical response of unsaturated expansive soils. The water retention behaviour of unsaturated expansive soils in the proposed model takes the influence of void ratio into consideration. Comparisons between predictions and experimental results suggest that the relationship between void ratio and stress is well captured by the model. This research has shown that the proposed model possesses a great ability to reproduce the mechanical and hydraulic response of unsaturated expansive soils.

Acknowledgements The authors appreciate the financial support of the National Science Foundation of China, No. 41572252.

References

Alonso EE, Gens A, Josa A (1990) A constitutive model for partially saturated soils. Géotechnique 40(3):405–430

Alonso E, Vaunat J, Gens A (1999) Modelling the mechanical behaviour of expansive clays. Eng Geol 54(1):173–183

Blight G (1965) The time-rate of heave of structures on expansive clay. In: Aithison GD (ed), Moisture equilibria and moisture changes in soils beneath covered areas. Butterworths, Syndey, pp 78–88

Chapman DL (1913) LI. A contribution to the theory of electrocapillarity. Phil Mag 25(148):475–481

Chiu CF, Ng CWW (2003) A state dependent elasto-plastic model for saturated and unsaturated soils. Géotechnique 53(9):809–929

Cui Y, Delage P (1996) Yielding and plastic behaviour of an unsaturated compacted silt. Géotechnique 46(2):291–311

Delage P, Graham J (1995) Understanding the behaviour of unsaturated soils requires reliable conceptual models: state of the art report. In: Alonso EE, Delage P (eds) Proceedings of the first international conference on unsaturated soils. UNSAT '95, Paris, France, pp 1223–1256

Fredlund DG, Xing A (1994) Equations for the soil–water characteristic curve. Can Geotech J 31(4):521–532

Gens A, Alonso E (1992) A framework for the behaviour of unsaturated expansive clays. Can Geotech J 29(6):1013–1032

Georgiadis K, Potts DM, Zdravkovic L (2005) Three dimensional constitutive model for partially and fully saturated soils. Int J Geomech 5(3):244–255

Gouy G (1910) Sur la constitution de la charge electrique a la surface d'un electrolyte. Physique 9(1):457–468

Gouy G (1917) Sur la fonction électrocapillaire. Annales de Physique 7(9):129–184

Graham J, Oswell JM, Gray MN (1992) The effective stress concept in saturated sand-clay buffer. Can Geotech J 29:1033–1043

Kassiff G, Shalom AB (1971) Experimental relationship between swell pressure and suction. Géotechnique 21(3):245–255

Lloret A, Villar MV, Sanchez M, Gens A, Pintado X, Alonso E (2003) Mechanical behaviour of heavily compacted bentonite under high suction changes. Géotechnique 53(1):27–40

Masin D (2010) Predicting the dependency of a degree of saturation on void ratio and suction using effective stress principle for unsaturated soils. Int J Numer Anal Methods Geomech 34:73–90

Mualem Y (1976) A new model for predicting the hydraulic conductivity of unsaturated porous media. Water Resour Res 12(3):513–522

Roscoe KH, Burland J (1968) On the generalized stress-strain behaviour of wet clay. In: Heyman J, Leckie FA (eds) Engineering plasticity. Cambridge University Press, Cambridge, pp 535–609

Sheng D (2011) Review of fundamental principles in modelling unsaturated soil behaviour. Comput Geotech 38(6):757–776

Sheng D, Zhou AN (2011) Coupling hydraulic with mechanical models for unsaturated soils. Can Geotech J 48(5):826–840

Sheng D, Fredlund DG, Gens A (2008) A new modelling approach for unsaturated soils using independent stress variables. Can Geotech J 45(4):511–534

Sun DA, Sheng D, Cui H, Sloan S (2007) A density-dependent elastoplastic hydro-mechanical model for unsaturated compacted soils. Int J Numer Anal Methods Geomech 31(11):1257–1279

Sun DA, Sun WJ, Yan W (2009) Hydraulic and mechanical behaviour of sand-bentonite mixture. Proceedings of the international symposium on unsaturated soil mechanics and deep geological nuclear waste disposal, Shanghai, 90–97

Thu TM, Rahardjo H, Leong EH (2007) Elastoplastic model for unsaturated soil with incorporation of the soil–water characteristic curve. Can Geotech J 44(1):67–77

Van Genuchten MT (1980) A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. Soil Sci Soc Am J 44(5):892–898

Wheeler SJ, Sivakumar V (1995) An elasto-plastic critical state framework for unsaturated soil. Géotechnique 45(1):35–53

Yong R, Sadana M, Gohl W (1984) A particle interaction energy model for assessment of swelling of an expansive soil. Proceedings of the 5th international conference on expansive soils. Adelaide, Australia, pp 4–12

Zhan L (2003) Field and laboratory study of an unsaturated expansive soil associated with rain-induced slope instability. PhD thesis. The Hong Kong University of Science and Technology, Hong Kong