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ORIGINAL PAPER

Statistical analyses of inherent variability of soil strength and effects on engineering geology design

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Abstract Clayey soil strata, as all natural deposits, generally show variability in the values of their geotechnical properties. This is due mainly to geological and environmental processes such as deposition and diagenesis, which introduce heterogeneity, anisotropy and variability to soil properties. Other causes of variability, and thus uncertainty, are the representativeness of samples and errors related to testing procedure, measurement and data processing procedures. To improve our knowledge about the inherent variability in the geomechanical properties of clays, this work presents a case study related to the analysis of the strength variability along a log of marine stiff clay deposits, which are apparently quite homogeneous. The analysis was based on pocket penetrometer strength measurement, performed both punctually and across the whole deposit. The adopted testing procedure, which is fast and reliable, provides a really wide dataset of the investigated soil property, with more than 800 data points. These allow for detailed variability analysis, and a reliable estimation of the coefficient of variation as well as research into the best fitting probability density functions, which are key factors for robust design. The presented case study allows discussion of the inherent variability of soil properties, and its influence on the characteristic values of soil strength in geotechnical design.

Keywords Inherent soil variability · Strength · Statistical analysis · Pocket penetrometer · Characteristic value

Introduction

Clayey deposits, like all natural soil deposits, generally show a broad variability in their geotechnical properties and parameters. This is due to geological and environmental processes related to deposition and diagenesis (Kulhawy 1992; Kim et al. 2012), and to shrinking–swelling dynamics (Vogel et al. 2005; Galeandro et al. 2013a), which cause variability of geomechanical properties and uncertainty in their values (Cherubini and Orr 1999; Phoon and Kulhawy 1999). In addition to this inherent soil variability, the variability of soil properties is also associated with other causes of uncertainty, such as the representativeness of samples, measurement errors due to measuring instruments and procedures, procedural-operator variation, random testing effects, and transformation uncertainty due to the adoption of semi-empirical and theoretical models for design parameters (Phoon and Kulhawy 1999; Baecher and Christian 2003; Akbas and Kulhawy 2010; Kim et al. 2012; Di Matteo et al. 2013).

All these uncertainties affect the variability of measured soil strength, and thus the design process (Harr 1987; Cherubini 2000a, b). For this reason, a reliability analysis is needed to choose soil parameters required for the design of geo-works interacting with soils.

Several theoretical frameworks aim to explicitly quantify and process the uncertainty related to geotechnical engineering applications (Harr 1987; Phoon et al. 1995; Cherubini and Orr 1999; Akbas and Kulhawy 2010). These are based on statistical analyses, aimed at reducing the uncertainty, and those errors related to the non-linearity of

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the processes. Therefore, these approaches can accurately estimate geotechnical properties (Que et al. 2008), supporting the evaluation of the reliability of design parameters and of the whole design process in general.

In particular, the values of soil parameters necessary for deterministic geotechnical design should be selected in a reliable way considering a specific safety level. Their evaluation represents a critical step in design processes, especially because of the various sources of uncertainty (Schneider 1997; Cherubini and Orr 1999; Phoon and Kulhawy 1999a; Cortellazzo 2000; Cardoso and Fernandes 2001; Baxter et al. 2008; Bond and Harris 2008; Kim et al. 2012).

A reliable estimation of the inherent variability of soil properties is the key to reliability-based design in geotechnical engineering. For this purpose, the coefficient of variation (COV) and probability distribution function (PDF) are commonly used for quantifying the inherent variability of geotechnical properties (Cortellazzo and Mazzuccato 1996; Phoon and Kulhawy 1999; Cortellazzo 2000; Akbas and Kulhawy 2010). The COV values of a number of soil properties were extensively investigated (Lumb 1974; Lacasse and Nadim 1996; Phoon and Kulhawy 1999; Cherubini and Orr 1999; Baecher and Christian 2003 and others). In particular, starting from a broad literature review, Phoon and Kulhawy (1999) present the COVs of soil properties along a general soil type, and the approximate range of mean values for which the COVs are applicable.

This work focuses on contributing to our knowledge of the local variability of soil strength by examining a case study related to the analysis of variability of the undrained shear strength of a marine stiff clay, measured by a pocket penetrometer. It is noteworthy, that the analysis of local variability of soil strength was pursued here by looking at shear strength data obtained by testing materials from a single borehole. Therefore, we did not face any problem with the spatial variability of soil properties. In fact, the aim was to emphasise the importance of determining the local variability of shear strength data, since a single borehole is usually available for any geotechnical engineering design procedures of small structures.

The use of a pocket penetrometer allows provision of a large dataset of measurements, and the development of detailed statistical analysis of COVs, as well as PDFs that better fit the measured data.

A global statistical analysis was performed, i.e., considering the entire dataset with a measurement step of about 25 cm across the whole investigated clayey deposit (about 25 m), and a local statistical analysis, i.e., analysing the variability of the ten measurements at each investigated depth.

The case study presented shows that the variability of the measured strength values is quite large, both locally and across the whole deposit, even if the tested clay deposit is homogeneous both geologically and visually. In addition, a discussion about the inherent variability of soil properties and its influence on the characteristic values of soil strength for geotechnical engineering design is presented.

Test site

The investigated site is located at the top of a hill close to Grottole (Matera, South Italy, Fig. 1a). The site is characterised by a marine regressive sequence of the lithological terms of Bradanic foredeep domain (Pieri et al. 1996; Tropeano et al. 2002; Galeandro et al. 2013b). The outcropping deposits are coarse-grained terraced deposits (Monte Marano Sands) topping the deposits of the marine sequence of the Bradanic foredeep domain. These are characterised mainly by silty-clayey deposits of clays and consist of stiff and jointed grey-blue marly-silty clays and clayey silts, characterised by the presence of silty and sandy levels up to 10 cm thick, whose frequency and thickness increases upward. These deposits are usually blue-grey silty clays, except for the upward levels; there the deposits assume a yellowish-ocher color, due to weathering processes, which induced drying and caused degradation of the clay bounding. On the one hand this weathering process usually induces a mechanical decay, on the other, diagenesis phenomena could increase soil strength.

Figure 1b shows the schematic stratigraphic log of the investigated site, obtained by continuous rotary sampling of a 30-m-deep borehole. The stratigraphic sequence of silty-clayey deposits, all belonging to the geological formation of the Pleistocenic Sub-Apennine grey-blue clays and clayey silts (Pieri et al. 1996), can be summarised as follows (Fig. 1b):

- Layer A: 6–15 m (yellowish ocher sandy clayey silts);
- Layer B: 15–20 m (grey blue clayey silts);
- Layer C: 20–30 m (grey-blue clays).

Even though the whole deposit seems to be quite homogeneous, it is characterised by inherent variability, as a consequence of the presence of sandy intercalations of variable thickness, sedimentation compression, the weathering process, etc. The variability in strength is also due to small-scale geological variations, i.e., microstructures within geological material (Cafaro and Cherubini 2002). Thus, this geological formation may be an interesting case study for the characterisation of the inherent variability of soil geotechnical properties. In fact, the investigated soil deposit is not affected by geological structure superimposed by tectonics, or by a complex geological history like other tectonically deformed clayey deposits. Soil

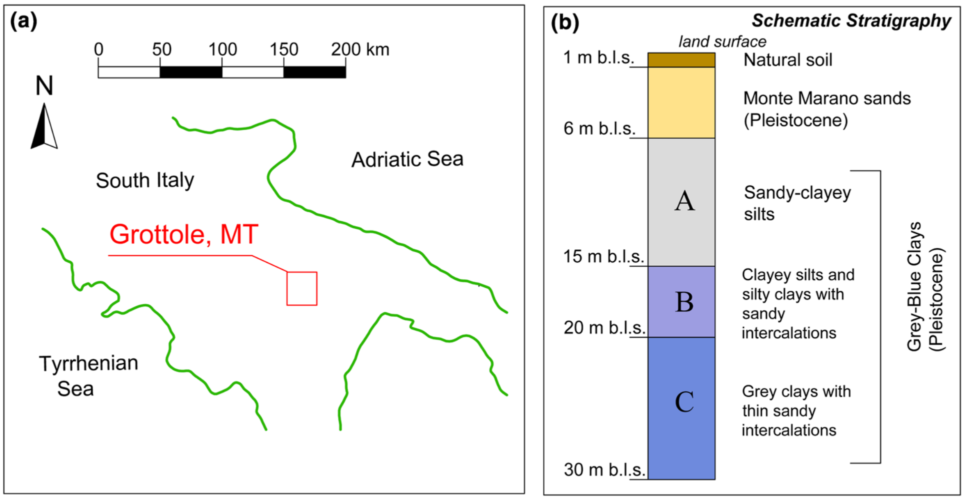


Fig. 1 Location and schematic stratigraphy of the study site

Table 1 Main geotechnical properties of the studied deposit by laboratory test on undisturbed samples

Sample/depth (m)	1/6.10–6.50	2/16.50–16.90
W (%)	18.6	21.9
γ (kN/m^3)	20.1	20.0
G_s	2.70	2.71
S_r (%)	88.1	94.5
Clay (%)	28.8	45.3
Silt (%)	53.2	54.7
Sand	18.0	
w_L	42.4	42.4
w_P	21.5	21.7
IP	20.9	20.7
IC	1.14	0.99
c' (kPa)	10	20
ϕ' (degrees)	29	22
c_u (kPa)	110	155

laboratory tests performed on undisturbed samples (Table 1) showed that the studied deposits are stiff silty clays of medium plasticity, in good agreement with literature values for sub-Apennine clays in the same area (Genevois et al. 1984).

Data collection

To perform a detailed reliable statistical analysis of soil strength variability for engineering purposes, it is necessary to collect a large number of reliable measurements, involving soil strata of relevant thickness, like those commonly investigated for geotechnical design purposes. In order to gather a large dataset of soil strength values along the entire extracted soil column, a hand pocket penetrometer is used, allowing quick determination of approximate values of soil strength. Readings are obtained by pushing the loading piston against the soil sample, and reading the approximate unconfined compressive strength (kg/cm^2) on the permanent scale on the piston barrel. The measurements were performed on the extracted column of sub-Apennine grey-blue clays, after removing the external layer of the samples, likely disturbed by the sampler itself. The extracted soil column was studied by dividing it into vertical segments of 25 cm. Around ten measurements (Fig. 2) were extracted for each increment, obtaining a population of collected data characterised by 870 strength values (kg/cm^2). These were used to perform a local study of the inherent variability of the soil strength, analysing the ten values measured at each measurement step, and to perform a global analysis across the whole investigated strata with steps of 25 cm.

Local analysis of the inherent variability of pocket penetrometer strength

To evaluate the inherent variability in strength of the studied deposit measured by pocket penetrometer, the average value μ (kg/cm^2), the standard deviation σ (kg/cm^2) and the COV of the ten measured compressive strength values (Fig. 3) were estimated for each measurement step.

Locally, COVs range between 0.05 and 0.2, sometimes exceeding this value, peaking at 0.4 or more. These COV values may appear rather high for measurements performed at the same point, if compared with COV literature values (Harr 1987; Cherubini and Orr 1999; Phoon and Kulhawy 1999). However, they refer to a specific point and show how soil strength may be locally affected by a severe variability, related to sedimentation, diagenesis and weathering phenomena.

Fig. 2 Data collection. **a** Removal of the disturbed surficial portion of samples. **b** Execution of measurement on the collected samples. **c**, **d** Some samples subjected to the measurements

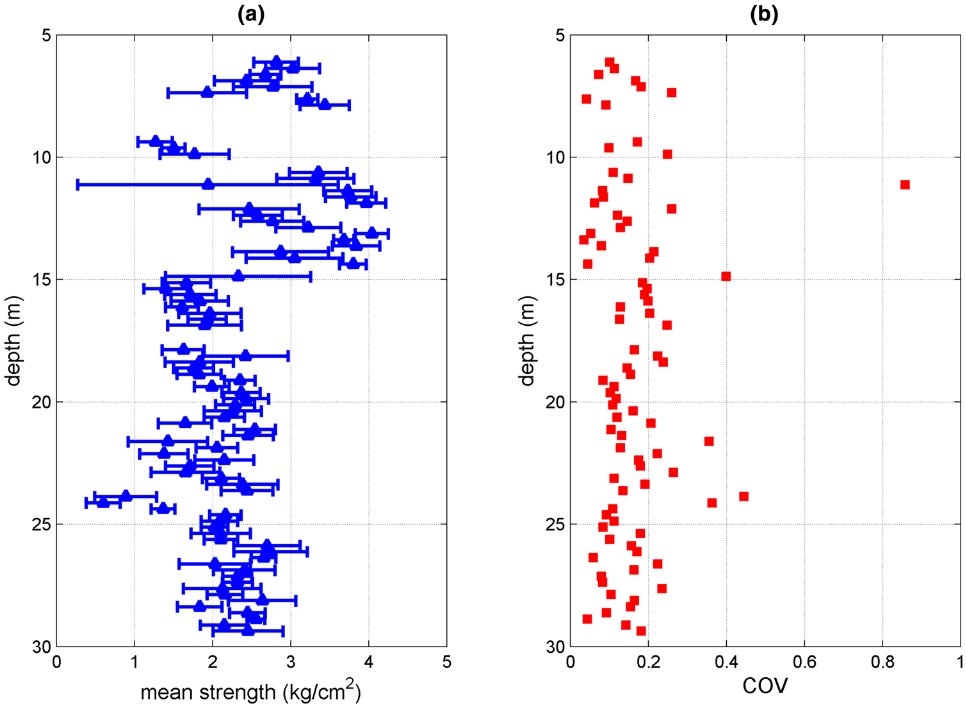


Fig. 3 **a** Average strength and standard deviation vs depth. **b** Coefficient of variation (COV) vs depth

The soil strength profile (Fig. 3a) was very variable along the studied log. The highest values of strength were measured at a depth of 13–14 m, and were due partly to the presence of drier soil. It is interesting to note that average strength values decrease dramatically below a depth of 15 m, at the transition from the yellowish sandy-clayey loams to grey-blue clayey loams (Figs. 1b, 2). This difference is consistent with the different consistence indexes measured on laboratory samples (Table 1). Similar results about the relationship between geotechnical behaviour of weathered and unweathered sub-Apennine clays were reported by Cotecchia (1996) and by the present authors for other sites, where it was reported that weathering may introduce an increase of strength due to drying. Unweathered silty clays below the depth of 15 m, or more, are characterised by average measured values consistent with those depths. The measured local variability is quite high for such apparently homogeneous silty clay deposits.

Analysis of the inherent variability across the whole soil strata

To complete the analysis of the inherent variability of soil strength across all soil strata, an analysis of variability of the entire set of measured strength data was undertaken, based on the assumption that the soil column from 6 m to the base of the borehole at 30 m is a unique soil stratum belonging to the same geological formation.

The outcomes of the analysis are summarised in Tables 2 and 3, where Table 2 summarises the results for all samples, while Table 3 reports the results for each batch of data, representative of single tests. The value of COV is 0.33, which is slightly higher than the local COVs previously evaluated for each set of ten measurements (Fig. 3b), but still considerably lower than COV values from literature (Harr 1987; Phoon and Kulhawy 1999; Cherubini and Orr 1999) for undrained strength: 0.45–0.55. This quite low value can be considered consistent with the presumable homogeneity of the studied geological formation, and with the absence of variability due to testing procedure, measurement and data transformation procedures. In fact, all tests are perfectly homogeneous since they were implemented by the same person and on the same day, without any transformation procedure.

Subsequently, the analysis was detailed as follows: the whole dataset was divided into three layers (A, B and C), corresponding to the three soil levels of the stratigraphic profile. For each layer, the average value, the standard deviation and COV (Table 4) were estimated. We compared the mean local COVs, i.e. the average value of local COVs evaluated for each dataset of ten measurements included in the considered soil column. It was interesting to observe that, in the upper weathered layer A, both average strength values μ (kg/cm^2) and COVs were higher than in the lower unweathered strata (B and C). This is probably due to weathering phenomena affecting the inherent variability of the geological formation, thus, values for unweathered levels were lower than for weathered ones. In addition, it could be due also to a more severe sedimentation variability affecting the upper part of the deposit as consequence of the regressive phases of sedimentation.

It was interesting to observe that COVs evaluated for these three layers were moderately higher than the locally evaluated COV values, considering the ten data points at each depth increment of about 25 cm. It was also interesting to observe that the coefficient of variation evaluated for the layer B (between 15 and 20 m) was slightly smaller than that evaluated for the deeper soil stratum C (between 20 and 30 m). This is reasonable for quite homogeneous strata, showing that the smaller the stratum, the lower the COV value.

A detailed analysis of the inherent variation was performed by dividing each stratigraphic layer into more thin sub-layers (Tables 4, 5, 6). Results are described in the following, grouped by lithotype.

Yellowish-ocher sandy-clayey silt (layer A)

The detailed analysis of the inherent variability across the upper sandy-clayey silts layer A performed by dividing it in thinner levels of 4.5 (6–10.5 and 10.5–15 m) and 3 m (6–9; 9–12; 12–15) (Table 5) shows that COV values are slightly lower for thinner sub-layers. Only levels between 9 and 12 m show high variability, with COV >0.40. This increase of COV between 9 and 12 m is probably due to some disturbances in lithology and consistency index at a depth of about 10–10.5 m, which becomes evident only when considering thin levels. A special zone characterised by significant anomalies in the sedimentation process or in weathering processes may give a relevant variation of measured values and their relevance may be strong, particularly if the sub-layer is thin. This result emphasises the importance of performing a detailed stratigraphic analysis before defining the representativeness of the tested samples and using the results for geotechnical modelling.

Clayey silts and grey-blue clay (layers B and C)

For grey-blue clay (layer B) and clayey silts (layer C), the COVs obtained by dividing the layer into thinner sub-layers are quite low, being almost equal to, or even lower than, the local COV values (Tables 6, 7). This shows the quite good homogeneity of the deposits, and allows the variation across the strata to be compared with the punctual values. Looking at layer C, higher COV values are obtained for levels between 20 m and 22.5 m, and between 22.5 m and 25 m from the ground level, which is also characterised by a low shear strength value, due to the presence of thicker sandy intercalations and a higher water content. COVs for these

Table 2 Mean values, and standard deviations (SD) of the whole dataset

Soil column (m)	Number of data points	μ (kg/cm^2)	σ (kg/cm^2)	COV
6–30	870	2.30	0.77	0.33

Table 3 Mean values, and SD of each batch of measures, grouped according to the average depth of test

Average depth (m)	Number of data	μ (kg/cm^2)	σ (kg/cm^2)	COV
6.12	6	2.82	0.29	0.10
6.37	7	3.03	0.35	0.11
6.62	14	2.68	0.20	0.07
6.87	11	2.43	0.41	0.17
7.12	11	2.77	0.50	0.18
7.37	9	1.93	0.50	0.26
7.62	7	3.21	0.13	0.04
7.87	8	3.44	0.32	0.09
9.37	9	1.27	0.22	0.17
9.62	10	1.50	0.15	0.10
9.87	10	1.77	0.44	0.25
10.62	9	3.36	0.37	0.11
10.87	10	3.32	0.49	0.15
11.12	10	1.94	1.67	0.86
11.37	12	3.73	0.31	0.08
11.62	10	3.77	0.32	0.09
11.87	7	3.97	0.25	0.06
12.12	10	2.47	0.64	0.26
12.37	10	2.58	0.31	0.12
12.62	12	2.77	0.40	0.15
12.87	11	3.23	0.41	0.13
13.12	10	4.04	0.21	0.05
13.37	5	3.68	0.13	0.04
13.62	10	3.84	0.31	0.08
13.87	7	2.87	0.62	0.21
14.12	10	3.05	0.62	0.20
14.37	10	3.80	0.17	0.04
14.87	7	2.33	0.93	0.40
15.12	12	1.67	0.31	0.19
15.37	12	1.40	0.28	0.20
15.62	12	1.72	0.33	0.19
15.87	13	1.84	0.37	0.20
16.12	11	1.61	0.21	0.13
16.37	12	1.97	0.40	0.20
16.62	10	1.93	0.25	0.13
16.87	12	1.90	0.47	0.25
17.87	11	1.63	0.27	0.17
18.12	13	2.42	0.54	0.22
18.37	12	1.83	0.44	0.24
18.62	12	1.76	0.26	0.15
18.87	11	1.83	0.28	0.15
19.12	12	2.35	0.20	0.08
19.37	10	1.99	0.22	0.11
19.62	12	2.37	0.24	0.10
19.87	12	2.43	0.29	0.12
20.12	13	2.29	0.25	0.11
20.37	10	2.26	0.37	0.16
20.62	12	2.15	0.26	0.12
20.87	12	1.65	0.34	0.21

Table 3 continued

Average depth (m)	Number of data	μ (kg/cm^2)	σ (kg/cm^2)	COV
21.12	10	2.54	0.27	0.11
21.37	11	2.45	0.32	0.13
21.62	11	1.43	0.51	0.36
21.87	9	2.06	0.27	0.13
22.12	9	1.38	0.31	0.22
22.37	8	2.15	0.38	0.18
22.62	11	1.71	0.31	0.18
22.87	9	1.66	0.44	0.27
23.12	10	2.11	0.24	0.11
23.37	11	2.38	0.46	0.19
23.62	12	2.44	0.33	0.14
23.87	10	0.89	0.40	0.44
24.12	9	0.60	0.22	0.36
24.37	12	1.37	0.15	0.11
24.62	12	2.17	0.20	0.09
24.87	11	2.09	0.23	0.11
25.12	12	2.03	0.17	0.08
25.37	13	2.11	0.38	0.18
25.62	13	2.11	0.21	0.10
25.87	11	2.70	0.42	0.16
26.12	11	2.75	0.47	0.17
26.37	11	2.65	0.16	0.06
26.62	10	2.03	0.45	0.22
26.87	12	2.41	0.39	0.16
27.12	12	2.32	0.19	0.08
27.37	10	2.32	0.19	0.08
27.62	12	2.13	0.50	0.23
27.87	10	2.16	0.23	0.11
28.12	11	2.64	0.43	0.16
28.37	11	1.84	0.28	0.15
28.62	10	2.45	0.23	0.09
28.87	7	2.56	0.11	0.04
29.12	10	2.15	0.31	0.14
29.37	11	2.45	0.45	0.18

Table 4 Mean values, SD and COV for each stratigraphic level

Layer	Soil column (m)	Number of data points	μ (kg/cm^2)	σ (kg/cm^2)	COV	Mean local COV
A	6–15	262	2.90	0.90	0.31	0.16
B	15–20	199	1.92	0.44	0.23	0.17
C	20–30	409	2.10	0.56	0.26	0.16

Table 5 Mean values, SD and COV for the yellowish ocher sandy clayey loams, layer A

Soil column (m)	Number of data	μ (kg/cm^2)	σ (kg/cm^2)	COV	Mean local COV
6–10.5	102	2.40	0.75	0.31	0.14
10.5–15	160	3.22	0.84	0.26	0.18
6–9	73	2.74	0,55	0.20	0.13
9–12	87	2.73	1.20	0.44	0.21
12–15	102	3.15	0.73	0.23	0.15

Table 6 Mean values, SD and COV for grey blue clayey silts group of data, layer B

Soil column (m)	Number of data	μ (kg/cm^2)	σ (kg/cm^2)	COV	Mean local COV
15–20	199	1.92	0.44	0.23	0.16
15–17.5	94	1.75	0.37	0.21	0.18
17.5–20	105	2.08	0.44	0.21	0.15

Table 7 Mean values, standard deviations and COVs for grey blue clay group of data, layer C

Soil column (m)	Number of data	μ (kg/cm^2)	σ (kg/cm^2)	COV	Mean local COV
20–25	212	1.91	0.60	0.31	0.17
25–30	197	2.31	0.42	0.18	0.16
20–22.5	105	2.04	0.50	0.25	0.17
22.5–25	107	1.78	0.65	0.37	0.20
25–27.5	115	2.33	0.41	0.18	0.13
27.5–30	82	2.28	0.42	0.19	0.14

levels are equal to 0.25 and 0.37, respectively. This result is consistent with the analysis of intraclass correlation coefficient *RI* (Wickremesinghe and Campanella 1991; Phoon et al. 2003) and Bartlett test statistics B_{stat} (Kanji 1993) profiles, generally used for CPT soundings, which allows statistical detection of homogeneous layer boundaries by identifying the peaks of these variables.

The intraclass coefficient index *RI* is generated by moving two contiguous windows containing *m* data points each over a measurement profile and computed as follows:

$$RI = \frac{1}{1 + \frac{1}{\frac{m-1}{m} + \frac{(\mu_1 - \mu_2)^2}{2\left(s_1^2 + s_2^2\right)}}}$$

where μ_{I} and s_{I}^2 represent in the order average value and variance of samples on a window. For the case of variances of two samples, s_1^2 and s_2^2, the Bartlett test statistic can be evaluated as follows (Kanji 1993):

$$B_{stat} = \frac{2.30259(m-1)}{C}\left[2\log s^2 - \left(\log s_1^2 + \log s_2^2\right)\right]$$

where *m* is the number of data points used to evaluate s_1^2 (or s_2^2), the total variance *s* is defined as:

$$s_2 = \frac{s_1^2 + s_2^2}{2}$$

And the constant *C* is given by:

$$C = 1 + \frac{1}{2(m-1)}$$

It is noteworthy that this additional analysis not only identifies statistically homogeneous sections consistent with geological boundaries, but also detects a change of homogeneous section between ca. 20 m and 25 m (Fig. 4). This confirms the importance of a detailed stratigraphic analysis of the layers, in order to gather evidence of variability due to soil characteristics. Then, a more detailed analysis of the soil log shows that the investigated layers are characterised by some anomalies, whereas sandy levels are thicker, as already observed for the previous yellowish-ocher sandy-clayey silt layer.

Statistical distribution of data

For each considered group of data, a statistical analysis was performed, dividing the measured data into 0.5 kg/cm^2 wide classes of strength. Class width was chosen by considering the resolution of the instrument of 0.1 kg/cm^2, in order to obtain classes characterised almost by the same number of values. Data fitting was estimated with some common probability distribution functions used to analyse geotechnical data. In particular, normal distribution, log-normal distribution and Gamma distribution were accounted for (Table 8). In particular, the latter two distributions were considered in order to account for the asymmetrical distribution of samples, with respect to the average value.

Figure 5 shows the fitting of data to the assumed probability distribution functions for each sub-layer (A, B and C) and for the entire soil column. In order to determine the reliable COVs and PDFs for the investigated soil strength, once parameters of distributions are estimated and then fit to the collected data, goodness-of-fit tests, such as the Kolmogorov–Smirnov (K–S) and Pearson (χ^2) tests are performed, assuming a significance level α equal to 0.05 (Benjamin and Cornell 1970; Ang and Tang 1975; Baecher and Christian 2003; Fenton and Griffiths 2008; Kim et al. 2012).

Table 9 shows results of goodness-of-fit tests.

If data from 6 m to 30 m are analysed all together, all PDFs are rejected, according to Pearson test. Dividing the soil strength profile into layers (A, B and C) corresponding

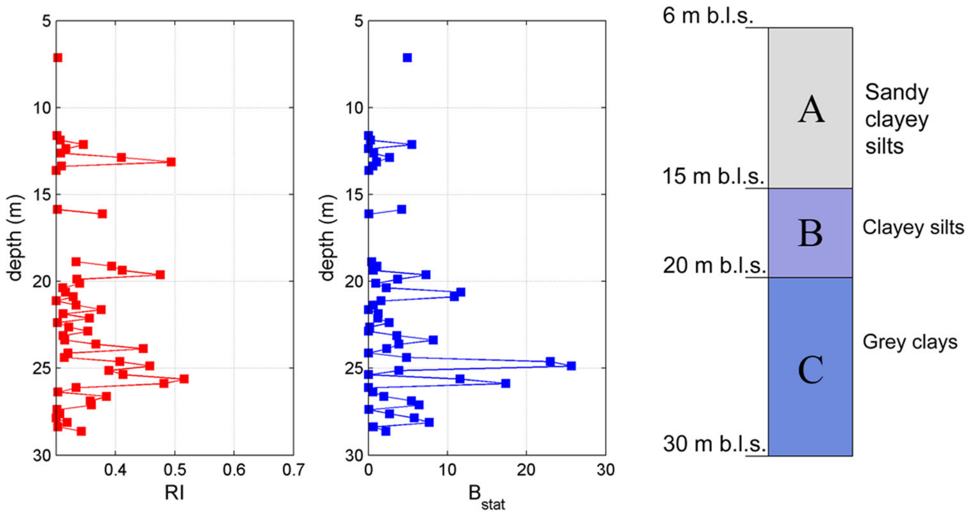


Fig. 4 Identification of soil boundaries using intraclass correlation coefficient index (*RI*) and Bartlett statistic (B_{stat})

Table 8 Probability distribution functions: μ (kg/cm^2) and σ (kg/cm^2) indicate mean value and standard deviation, respectively

Normal distribution (N)	$p(x)=\frac{1}{\sqrt{2\pi}\sigma(x)}e^{\left\{-\frac{1}{2}\left[\frac{x-\mu(x)}{\sigma(x)}\right]^2\right\}}$
Lognormal distribution (L)	$y=\ln(x)$
	$p(x)=\frac{1}{x\sqrt{2\pi}\sigma(y)}e^{\left\{-\frac{1}{2}\left[\frac{y-\mu(y)}{\sigma(y)}\right]^2\right\}}$
	$\mu(y)=\ln\mu(x)-\frac{1}{2}\ln\left[1+\frac{\sigma^2(x)}{\mu^2(x)}\right]$
	$\sigma^2(y)=\ln\left[1+\frac{\sigma^2(x)}{\mu^2(x)}\right]$
Gamma distribution (G)	$p(x)=\frac{1}{b^a\Gamma(a)}x^{a-1}e^{\frac{-x}{b}}$
	$\Gamma(a)=\int\limits_0^{+\infty}e^{-t}t^{a-1}dt$

a and *b* are the shape and scale parameters, respectively, of Gamma distribution to be estimated from samples

to the geological boundaries of the soil profile (Fig. 2), both tests reject all distributions for the layer between 6 m and 15 m, and for the deepest layer, 20–30 m. Assuming thinner layers, it is noteworthy that central layers seem to fit the assumed probability distributions, with no clear prevalence of a distribution over the others. The deepest layers, 25–30 m seems to fit better to Gamma probability distribution, although the Kolmogorov–Smirnov test does not reject the assumption. For the remaining layer, there is a general uncertainty, for which it is not possible to clearly assume a probability distribution among the selected layers; this is likely due to the variability in the measurements.

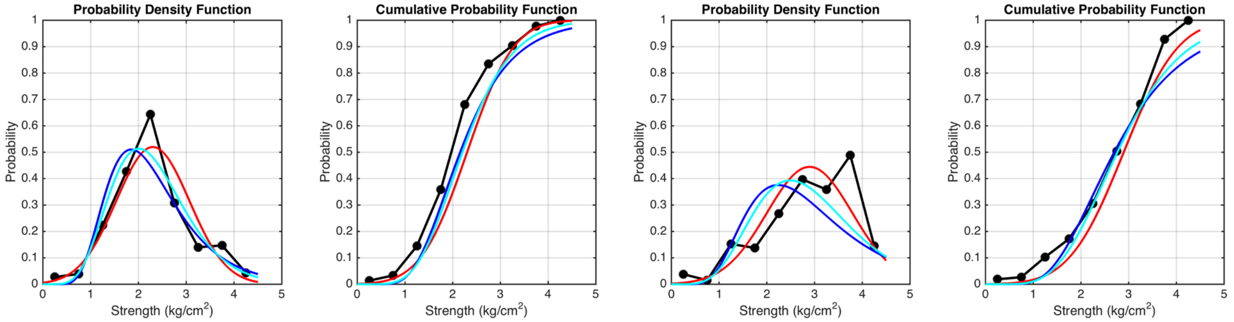
Table 9 summarises the test results and indicates the best distributions, i.e. the distributions verified by one or both of the performed goodness-of-fit tests, considering the best fitting distributions to be those simultaneously verified by both tests. Where no distribution was verified by the tests, the distribution where tests fail less often was chosen. It is clear that none of the considered distributions prevailed in terms of fitness to the measured data. Anyway, a normal distribution, which was also suggested by Eurocode 7, was not rejected at least by one test 8 times out of 17. Gamma distribution was not rejected 7/15 times, being the second best performing distribution. Finally, the lognormal distribution was not rejected at least by one test 5/17 times. It is interesting to note that normal and lognormal distributions constitute the best-fitting distribution for the datasets characterised by relatively low COVs, i.e. lower than 0.25, except the layer at 6–15 m where the COV is equal to 0.31, while the Gamma distribution can be considered the best fitting distribution for datasets with highest COV, i.e. higher than 0.31.

Characteristic values

According to the common practices of geotechnical engineering, soil strength parameters are obtained by applying partial factors to the characteristic values of strength parameters (Potts and Zdravkovic 2012). Characteristic values are defined as a careful estimate of the value affecting the occurrence of the limit state, and are obtained starting from in situ or laboratory observations (Orr 2000; Frank et al. 2004; Baxter et al. 2008). Then, starting from measured data, characteristic values can be evaluated by using an engineering expert approach (Cherubini and Orr 1999). Anyway, a commonly adopted approach is the statistical one, which suggests the following formulation for the characteristic values:

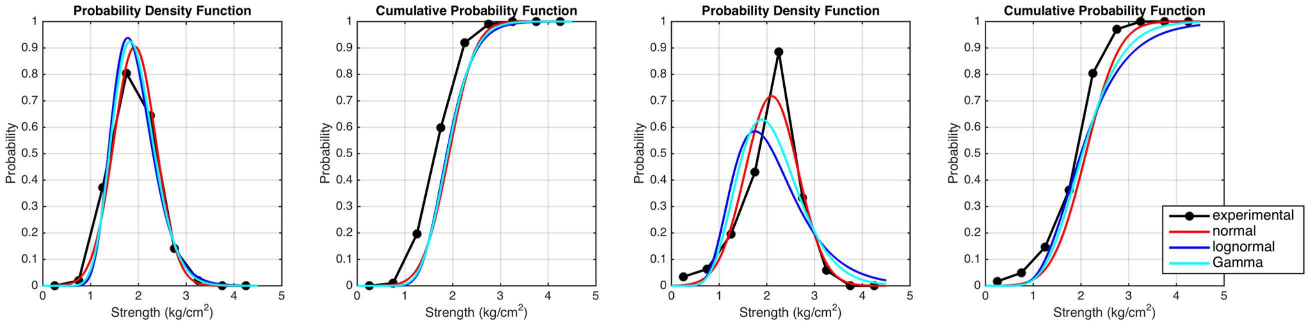
$$x_c=\mu-k_n\sigma=\mu(1-k_nCOV) \quad (1)$$

where k_{n} is a statistical coefficient, depending on the chosen confidence level and on the assumed distribution function. In structural engineering, a confidence level of



(a) 6 - 30 m

(b) Layer A, 6 - 15 m



(c) Layer B: 15 - 20 m

(d) Layer C: 20 - 30 m

Fig. 5 Fitting data to probability distribution functions (PDFs) and cumulative probability functions (CPFs) for soil samples between depths of **a** 6 and 30 m; **b** 6 and 15 m; **c** 15 and 20 m; **d** 20 and 30 m; **e** results of statistical tests

Table 9 Results of statistical tests and determination of reliable probability distribution functions (PDFs)

Layer	Soil column (m)	Number of data	Normal (N) distribution		Lognormal (L) distribution		Gamma (G) distribution		Best distribution	COV
			Test results		Test results		Test results			
			χ^2	K–S	χ^2	K–S	χ^2	K–S		
A + B + C	6–30	870	R[a]	R	R	R	R	R	–	0.33
A	6–15 (Layer A)	262	R	R	R	R	R	R	–	0.31
B	15–20 (Layer B)	199	NR[b]	R	NR	NR	NR	NR	N–L–G	0.23
C	20–30 (Layer C)	409	R	R	R	R	R	R	–	0.26
A	6–10.5	102	R	R	R	R	R	R	–	0.31
	10.5–15	160	R	R	R	R	R	R	–	0.26
A	6–9	73	R	NR	R	R	R	R	N	0.20
	9–12	87	R	R	R	R	R	R	–	0.44
	12–15	102	R	R	R	R	R	R	–	0.23
B	15–17.5	94	NR	NR	NR	NR	NR	NR	N–L–G	0.21
	17.5–20	105	NR	NR	NR	R	NR	NR	N–L–G	0.21
C	20–25	212	R	R	R	R	R	R	–	0.31
	25–30	197	R	NR	R	R	R	R	N	0.18
C	20–22.5	105	R	NR	R	R	R	R	N–G	0.25
	22.5–25	107	R	R	R	R	R	NR	G	0.37
	25–27.5	115	R	NR	R	NR	R	NR	N–L–G	0.18
	27.5–30	82	R	NR	R	NR	R	NR	N–L–G	0.19

Chi square test has been performed considering $\alpha = 0.05$, corresponding to $\chi^2\alpha = 5.991$

[a] The test rejects the distribution

[b] The test does not reject the distribution

Table 10 Scaling coefficient ε_n (from Zupan and Turk 2002)

n	3	4	5	6	7	8	10	100	∞
ε_n	1.128	1.085	1.064	1.051	1.043	1.036	1.028	1.002	1.000

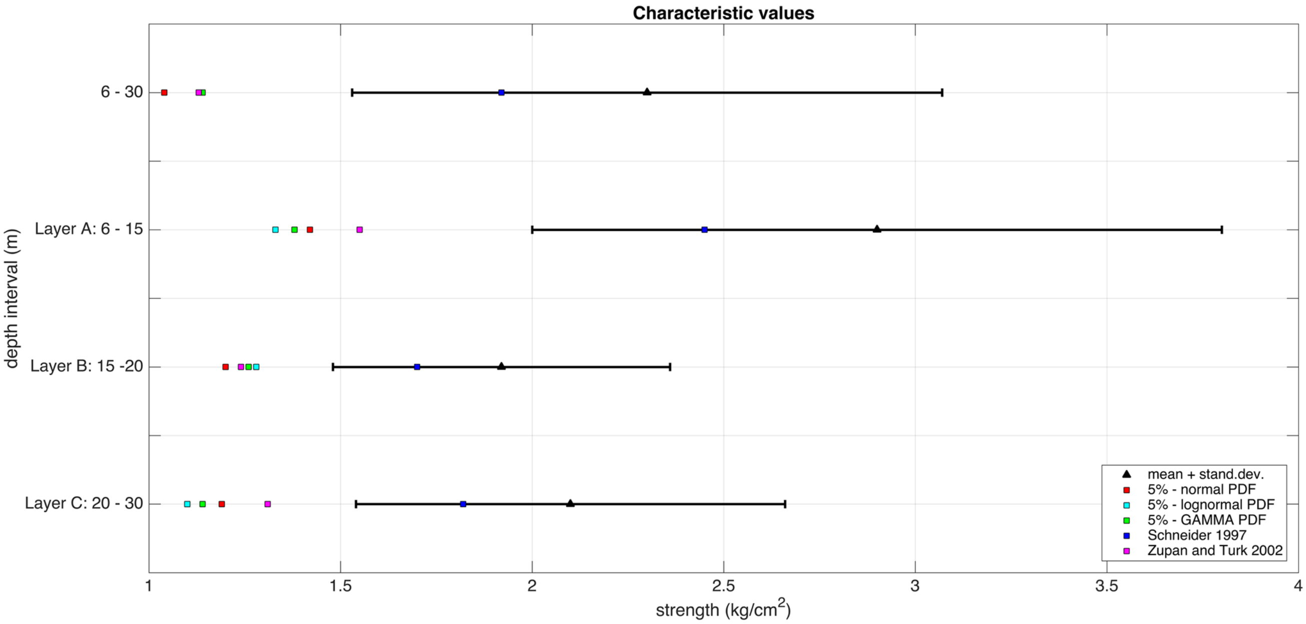


Fig. 6 Characteristic values evaluated according to different statistical distribution and other approaches

Table 11 Characteristic values evaluated according to different statistical distribution and other approaches

Depth interval (m)	μ (kg/cm^2)	σ (kg/cm^2)	COV	Best fitting PDF	Characteristic values				
					Normal	Log-normal	Gamma	Schneider (1997)	Zupan and Turk (2002)
6–30	2.3	0.77	0.33	–	1.04	1.13	1.14	1.92	1.13
6–15 (Layer A)	2.9	0.9	0.31	N	1.42	1.33	1.38	2.45	1.55
15–20 (Layer B)	1.92	0.44	0.23	L	1.20	1.28	1.26	1.70	1.24
20–30 (Layer C)	2.1	0.56	0.26	–	1.19	1.10	1.14	1.82	1.31

95 % is normally assumed, which can be adopted also in geotechnical engineering. However, the use of such a level in engineering geology design may sometimes be too conservative. Statistical methods can be reliably applied if the numerosity of the pool of samples is higher than ten (Schneider 1997); however, in engineering geology practice, such numerosity is unusual. Schneider (1997) proposed an approximation of Eq. (1) assuming a value of $k_n = 0.5$, which is a good approximation for $n > 10$ tests (Schneider 1997). Zupan and Turk (2002) proposed another formulation for the characteristic value at 95 % of confidence level based on the normal distribution, using the following improved unbiased estimate of Eq. (1) evaluating the coefficient k_n as:

$$k_n = 1.645 \cdot \varepsilon_n \tag{2}$$

where ε_n is a sort of scaling coefficient depending on the number of performed tests (Table 10). Figure 6 and Table 11 show the characteristic values for the dataset of the investigated case study, evaluated as the 5 % percentile related to the studied probability distributions, based on Schneiders' (1997) approximation and on the approach of Zupan and Turk (2002), in order to show how different procedures can affect the final result. Results show interesting differences among the estimated characteristic values. The characteristic values evaluated as the 5 % percentile of the three assumed probability distributions were rather similar, despite the depth at which they are estimated. They

Table 12 Characteristic values evaluated as 5 % percentile of normal distribution for some sample size *n* and for some soil layers

	n	3	5	10	20	30	50	80	≥100
6–30	COV	0.35	0.20	0.22	0.30	0.32	0.31	0.33	0.32
	X_c(normal)	0.82	1.68	1.51	1.22	1.12	1.18	1.06	1.09
Layer A 6–15	COV	0.26	0.32	0.27	0.26	0.27	0.29	0.30	0.31
	X_c(normal)	1.30	1.35	1.57	1.82	1.72	1.54	1.48	1.44
Layer B 15–20	COV	0.24	0.30	0.27	0.29	0.24	0.24	0.23	0.24
	X_c(normal)	1.08	0.97	1.04	0.95	1.19	1.16	1.22	1.15
Layer C 20–30	COV	0.11	0.23	0.22	0.19	0.23	0.22	0.26	0.26
	X_c(normal)	1.96	1.30	1.25	1.40	1.29	1.32	1.23	1.18

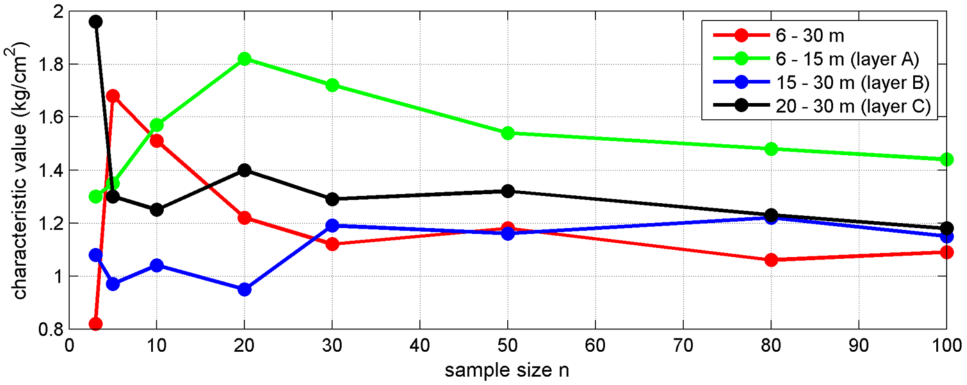


Fig. 7 Characteristic values evaluated as 5 % percentile of normal distribution for some sample size *n* and for some soil layers

are always lower than Schneider values, while remaining comparable with Zumpan and Turk values. This means that values based on probability distributions are less conservative but more economically convenient, since they lead to cheaper solutions in engineering geology design. This is particularly true when looking at the normal distribution, as recommended by Eurocode 7.

In particular, from the engineering geologist and geotechnical designer's point of view, Schneider approximation provides higher characteristic values, and these values are less conservative than those coming from the classical Statistics and Zupan and Turk's (2002) approach. Zupan and Turk (2002) estimated characteristic values, which are really close to the values evaluated as 5 % percentile of normal distribution.

Table 12 and Fig. 7 show characteristic values evaluated as 5 % percentile of normal distribution considering different sample sizes *n*, characterised by *n* random values for some considered layers. The results show that, for sample pool size *n* higher than 20 or 30, it is possible to obtain characteristic values affected by low uncertainty.

Conclusions

The work aimed to make a contribution to knowledge about the inherent variability of soil strength related to geological structures due to sedimentation, diagenesis and weathering phenomena, in order to show the importance of its reliable evaluation for engineering geology design. For this purpose, the paper focussed on characterising and estimating the variability of the undrained strength of a marine stiff clay measured by a pocket penetrometer by means of a statistical analysis, aimed at evaluating the representative coefficients of variation and the PDFs fitting the experimental data.

The soil strength profile showed that the relevant variability of soil strength along the studied log was higher than expected for such homogeneous silty clay deposits. Locally, COVs ranged between 0.05 and 0.2, and rarely exceeded 0.4. For the entire soil column in the same geological formation, the value of the COV was 0.33, higher than the average local value, and lower than literature values of 0.45–0.55. COVs evaluated by dividing the whole log in three layers corresponding to some variation of stratigraphic features of the deposits were slightly higher than the single COVs evaluated locally at each depth increment.

If the soil column is divided in small thickness levels, COVs were lower than the values corresponding to the entire layer and were not particularly high, being equal to ca. 0.20, except for some levels as a consequence of high variability due to weathering phenomena, the presence of thicker sandy intercalations, and higher water content. This result is perfectly consistent with the statistically homogeneous sections detected by evaluating *RI* and B_{stat}

profiles. The analysis shows that an accurate evaluation of geological and grain size differences of soil strata is important for evaluation of the representativeness of samples. A pocket penetrometer is an interesting tool for controlling local variability and the representativeness of samples for laboratory tests.

The fitting analysis of the measured dataset with different statistical distributions showed that none of the considered probability distributions (normal, lognormal and Gamma) may be considered fully reliable, even if the best is the normal, according to Eurocode 7 indications. In any case, normal and lognormal distributions constitute the best fitting distributions for homogeneous datasets with COV <0.25. Gamma distribution gives interesting results for soil layers characterised by different materials and with a COV of the measured dataset larger than 0.31.

Finally, the available dataset was used to check the reliability of different approaches for the evaluation of characteristic values. The characteristic values of soil properties were estimated as 5 % percentile of PDFs and by other approaches, showing that characteristic values evaluated as 5 % percentile of the normal distribution are quite consistent with those evaluated by the approach of Zupan and Turk (2002). The characteristic values evaluated according to the assumed PDFs corresponds to low values, i.e. cheaper solutions, than the Schneider approximation, which provides higher characteristic values and is then less conservative.

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