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Assessment of basic friction angles of various rock types from Turkey under dry, wet and submerged conditions and some considerations on tilt testing

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Abstract The shear strength of discontinuities has a prime importance in the assessment of structurally controlled stability problems. Natural discontinuity surfaces are usually not smooth and show irregularities, which have an important effect on their shear strength. The peak shear strength of natural, unfilled, rough discontinuities is estimated by a non-linear failure criterion and one of the main inputs of this criterion is the basic friction angle, which is determined via a tilt test or direct shear test. The limited number of studies on tilt testing and the need to develop a standard and/or suggested method for estimating a reliable basic friction angle value from tilt tests have given rise to some recent research efforts on different aspects of this test. In this study, an attempt was made to contribute to previous works on the tilt test with the aid of an experimental program on a wide range of rock types. The study mainly focused on investigating the dependence of the basic friction angle on different testing conditions (dry, wet and submerged), the suitable number of tests (repetitions of sliding on the same surface) which does not cause a wear effect, and a more detailed assessment on the influence of water head on the basic friction angle determined under submerged conditions. Additional considerations in the study include a general evaluation of the influence of the testing surface's mineralogical composition on test results. For this purpose, 22 different rock types were collected from different parts of Turkey and their mineralogical

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compositions were determined via a total of 415 tilt tests under three different conditions. The results obtained from the experimental studies are presented and compared with the results of previous studies and some recommendations are made.

Keywords Basic friction angle · Tilt testing · Tilting table - Mineralogy - Wear effect

Introduction

Rock masses are composed of intact rock pieces and discontinuities such as bedding planes, joints faults, shear zones, etc. Since the stresses are low at shallow depths, failure of intact rock material does not commonly occur and the behaviour of the rock mass is mainly controlled by the movement along discontinuities. Therefore, the shear strength of the discontinuities, which separate the rock blocks, has a prime importance in the assessment of structurally controlled stability problems. For planar discontinuity surfaces, shear strength can be estimated using the linear Mohr–Coulomb criterion. However, natural discontinuity surfaces are usually not smooth and show irregularities, which are called asperities and have an important effect on their shear strength. Ripley and Lee [\(1961](#page-16-0)), Patton [\(1966](#page-16-0)) and Goldstein et al. ([1966\)](#page-16-0) were the earlier pioneers regarding the shear strength of discontinuities with asperities. The simplest peak-shear strength model for such rock discontinuities is perhaps Patton's model (Patton [1966](#page-16-0)). Patton ([1966\)](#page-16-0) and Goldstein et al. [\(1966](#page-16-0)) presented the original idea that the basic friction angles of discontinuities and their relations were basically the same. Their attempts to explain and predict the shear strength of non-planar rock discontinuities is based on

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Eq. 1, which was first recommended by Newland and Allely ([1957\)](#page-16-0) for the observed behaviour of granular material such as sand. Patton ([1966\)](#page-16-0) mentioned that the shear strength of the sawtooth-shaped discontinuities can be explained by Eq. 1 for a low range of normal stresses where the asperity degradation during shearing is negligible (Fig. 1).

$$
\tau = \sigma_n \, \tan(\phi_b + i) \tag{1}
$$

where ϕ_b is the basic friction angle of the discontinuity surface and i is the angle of the sawtooth face. Equation 1 corresponds to initial frictional part of the graph in Fig. 1 together with a rigid body motion of the upper block sliding over the asperities. It means that this equation is valid at low normal stresses where shear displacement is due to sliding along the inclined surfaces and the asperities are not broken. With increasing normal stress and displacement, the asperities will be sheared off and the friction angle will progressively decrease to a minimum value of the basic friction angle, as shown in the second broken part of the failure envelope in Fig. 1.

By considering that Eq. 1 does not reflect the reality that changes in shear strength (τ) with increasing normal stress (σ_n) are gradual and the failure envelope of such surfaces is non-linear, Barton and his co-workers (Barton [1973,](#page-16-0) [1976](#page-16-0); Barton and Choubey [1977](#page-16-0); Barton and Bandis [1980,](#page-16-0) [1990\)](#page-16-0) studied the shear behaviour of rough natural discontinuities. For the first time, Barton [\(1973](#page-16-0)) proposed the following empirical criterion for the shear strength of rough and unfilled rock discontinuities:

Fig. 1 Effect of surface roughness and normal stress on friction angle of a discontinuity surface and strength envelope for sawtooth asperity of inclination, i [redrawn from Patton ([1966\)](#page-16-0) and Wyllie [\(1992](#page-16-0))]

$$
\tau = \sigma_n \cdot \tan \left[\phi_b + \text{JRC} \cdot \log_{10} \left(\frac{\text{JCS}}{\sigma_n} \right) \right] \tag{2}
$$

where τ is the peak shear strength of the unfilled discontinuity (rock-to-rock contact across the plane), σ_n is the effective normal stress acting on the discontinuity, JRC is the joint roughness coefficient, JCS is the joint wall compressive strength of the rock immediately surrounding the discontinuity and $\phi_{\rm b}$ is the basic friction angle which is determined on dry and wet sawn surfaces for dry and saturated discontinuities, respectively. A few years later, Barton and Choubey ([1977\)](#page-16-0) modified Eq. 2 for the general case of weathered and unweathered discontinuities and proposed the equation given below.

$$
\tau = \sigma_n \cdot \tan\left[\phi_r + \text{JRC} \cdot \log_{10}\left(\frac{\text{JCS}}{\sigma_n}\right)\right]
$$
 (3)

where ϕ_r is the residual friction angle of the discontinuity, that is, the one controlling the strength of a planar joint $JRC = 0$ and the other parameters are the same parameters given in Eq. 2. The residual friction angle, ϕ_r , is estimated according to Barton and Choubey ([1977\)](#page-16-0) as follows:

$$
\phi_r = (\phi_b - 20^{\circ}) + 20 \cdot \binom{r}{R} \tag{4}
$$

where r is the Schmidt hammer rebound number recorded for weathered and wet discontinuity surfaces and R is the Schmidt hammer rebound number recorded for unweathered sawn surfaces of the same rock. In Eq. 4, the basic friction angle, ϕ_b , represents fresh surfaces that are neither weathered nor wet. This equation also means that in the case of an unweathered rock, ϕ_r is equal to ϕ_b and r is equal to R.

In addition, Aydan et al. ([1994\)](#page-16-0) also proposed a yield function for rock discontinuities and it was extended to evaluate the anisotropic shear strength of rock discontinuities by Aydan et al. [\(1996](#page-16-0)) as given below.

$$
\tau = A_1 \left(1 - e^{-B_1 \sigma_n} \right) + \sigma_n \left[\tan \phi_i + A_2 e^{-B_2 \sigma_n} \right] \tag{5}
$$

where τ is shear stress, σ_n is normal stress, A_1 , A_2 , B_1 and B_2 are coefficients, ϕ_i is the friction angle of a wall rock, ϕ_{α} is the friction angle of an asperity wall. Provided that $\phi_{\alpha} \ge \phi_i$, coefficients A_1 , A_2 , B_1 and B_2 are given as $A_1 = c$, $A_2 = \tan \phi_\alpha - \tan \phi_i$, $B_1 = (\tan \alpha + \phi_\alpha) - \tan \alpha$ $(\phi_i)/c$, $B_2 = (\tan (\phi_\alpha) - \tan (\phi_i))/\sigma_t$.

As seen from the above given equations, the basic friction angle has an important input in the estimation of the shear strength of rough discontinuity surfaces. The basic friction angle is a measurement of the shear strength along an artificially planar saw-cut surface. This property, which represents the strength of planar, unfilled, nonweathered, non-polished rock discontinuities is determined by two methods in the laboratory, by the tilt test and from

the direct shear test. Some researchers (Hencher and Richards [1989](#page-16-0); Hencher [1995](#page-16-0), [2012\)](#page-16-0) mention that the strength for saw-cut surfaces or other artificially smoothed surfaces determined from a tilt test is usually lower than the ϕ _b measured from direct shear tests on natural joints where corrections are made for sample-specific roughness causing dilation. However, the tilt test is a very practical, cheap and commonly used method for the estimation of ϕ_b as an input of the Barton's shear strength criterion unless saw-cut rock surfaces with any kind of polishing are not used (Alejano et al. [2012a](#page-15-0), [b\)](#page-15-0).

The early attempts to estimate the basic friction angles of joints from the tilt test were made in the 1970s by some researchers such as Coulson ([1971\)](#page-16-0) and Barton and his coworkers (Barton [1973](#page-16-0), [1976;](#page-16-0) Barton and Choubey [1977](#page-16-0)). At the end of the 1980s, Cruden and co-workers made some attempts to estimate $\phi_{\rm b}$ using a portable tilting table they developed (Cruden and Hu [1988](#page-16-0); Bruce et al. [1989;](#page-16-0) Hu and Cruden [1992\)](#page-16-0), but their studies do not provide full indications for normalizing tilt testing. There is not a standard method recommended by the international standard institutions [such as the American Society for Testing and Materials (ASTM), British Standards Institution (BSI), etc.) or a suggested method recommended by the International Society for Rock Mechanics (ISRM) for tilt testing. Only a few methods proposed for the tilt test (Stimpson [1981](#page-16-0); USBR [2009\)](#page-16-0) are available in literature. Alejano et al. [\(2012a](#page-15-0)), González et al. [\(2014](#page-16-0)) and Ruiz and Li ([2014\)](#page-16-0) reported that there exists an erratum in the equation of the basic friction angle given in Stimpson's method, which uses rock cores to perform the tilt test with a core cylinder shaped specimen placed over other two core cylinder-shaped specimens (Fig. 2a). Based on their experimental studies, Alejano et al. [\(2012a\)](#page-15-0) also showed that the Stimpson's method (sliding along generatrixes) overestimate the values of ϕ _b and recommended that tilt tests should not be performed on generatrixes for obtaining

Fig. 2 Different set ups for the tilt test: a Stimpson-type test, b test with disks, c test performed on a cylindrical sample longitudinally cut and d test on square-based slabs (rearranged from Alejano et al. [2012a\)](#page-15-0)

In the tilt tests, some supplementary repetitions (tests) are performed on the same saw-cut surface and the mean value from the successive tests is used as an estimate of $\phi_{\rm b}$. In the literature, different numbers of repetitions applied in the tilt tests have been reported (i.e. Cruden and Hu [1988](#page-16-0); Bruce et al. [1989;](#page-16-0) Hu and Cruden [1992](#page-16-0); USBR [2009](#page-16-0); Alejano et al. [2012a;](#page-15-0) González et al. [2014](#page-16-0); Ruiz and Li [2014](#page-16-0); Kveldsvik et al. [2008\)](#page-16-0) or no information on this number has been given (i.e. Barton and Choubey [1977](#page-16-0); Aydan et al. [1995](#page-16-0); Aydan [1998;](#page-16-0) Wines and Lilly [2003](#page-16-0)). However, the number of tests mostly ranges between three and five. Recently, based on a huge number of tilt tests performed on slab samples, Alejano et al. ([2012a](#page-15-0)) recommended that three repetitions (tests) are sufficient, but a fourth supplementary repetition should be performed when the maximum deviation between one of the results and the median is larger than 3°. Most recently, González et al. [\(2014](#page-16-0)) suggested that the average value of the first successive five tilt tests performed on recently saw-cut specimens of appropriate dimensions can be used as the basic friction angle in the Barton's criterion. However, first Hencher $(1976, 2012)$ $(1976, 2012)$ $(1976, 2012)$ and then González et al. (2014) (2014) and Pérez-Rey et al. [\(2015](#page-16-0)) reported that repeated tilt testing of

saw-cut and rock sliders affects the basic friction angle (underestimation), and, therefore, the effect of wear on sliding angle (basic friction angle) in tilt tests should also be taken into account. Pérez-Rey et al. (2015) (2015) , who considered the wear effect and conducted a series of tilt tests up to a distance of a few meters (i.e. 5 m) on some pairs of saw-cut samples, proposed a tentative approach to estimate the basic friction angle. But in this approach, a huge number of tests on the same surface are necessary.

Another issue associated with the basic friction angle is the effect (capillarity and lubrication) of water (wet and submerged conditions) on the tilt test results. Barton and Choubey ([1977\)](#page-16-0) recommended that the basic friction angle, $\phi_{\rm b}$, in Eq. [4](#page-1-0) represents fresh surfaces that are neither weathered nor wet. But ϕ_b should also be considered for submerged and wet surfaces. Some test results, obtained in the 1960s and 1970s and compiled by Barton ([1973](#page-16-0) and [1976\)](#page-16-0), indicated that ϕ_b values determined from wet surfaces of different rock types are smaller than those determined from dry surfaces. However, in this compilation given in the literature, it is not clear that the water condition reported for each rock type represents a wet or submerged surface. Aydan et al. ([1995\)](#page-16-0) compared the basic friction angles determined under dry and wet conditions and reported that wet ϕ_b values are generally smaller than dry ϕ_b values probably due to the uncertainty of the effective normal stress over the shearing area. Aydan et al. [\(1995](#page-16-0)) and Aydan [\(1998](#page-16-0)) also performed a series of tilt tests on natural discontinuities, drill cores, saw-cut surfaces and polished surfaces in order to check which effective stress law (Biot or Terzaghi) holds for discontinuities. The effective normal stress of the Biot type is written as:

$$
\sigma'_n = \sigma_n - \alpha p \tag{6}
$$

where p is the liquid pressure acting on the discontinuity, and α is a non-dimensional parameter. If α is equal to one, the above effective stress law coincides with that of Terzaghi. Most of the experimental results obtained by the above-mentioned researchers lie around the line corresponding to $\alpha = 1$, which implies that the effective stress law of Terzaghi holds for rock discontinuities. However, this conclusion is only valid provided that rock constituents adjacent to the planar surface are not prone to water absorption, which cause softening and lubrication of the planar surface. Aydan ([1998\)](#page-16-0), who also performed tilt tests on four rock types under submerged conditions, observed that the basic friction angles determined under submerged conditions seem not to be influenced by the variation of water head at all and the maximum variation is only restricted to 2–3 %. Zandarin [\(2010](#page-16-0)) investigated the suction effect for the artificially prepared joints of claystone which have different roughness profiles. The researcher used a wetting cycle on the samples by using a vapour equilibrium technique and performed direct shear tests with suction control. Zandarin ([2010\)](#page-16-0) observed that the shear strength of joints generally increased with an increase in the suction.

Although there are some studies (e.g. Horn and Deere [1962](#page-16-0); Morrow et al. [2000](#page-16-0); Moore and Lockner [2004;](#page-16-0) Tembe et al. [2010\)](#page-16-0) on the variation of the coefficient of friction of minerals under dry and saturated conditions, the effect of rock mineralogy on the basic friction angle determined from the tilt test was investigated by Cruden and Hu ([1988\)](#page-16-0) on only dolomitic carbonate rocks selected from Kananaskis Country, Canada. These researchers reported that the basic friction angle of the investigated carbonate rocks are controlled by mineralogy and grain sizes. For pure carbonate rocks, they observed that increasing dolomite content decreases the basic friction angles while big grain sizes increase the basic friction angles.

The above-given brief background indicates that due to the presence of limited studies on tilt testing and a need to develop a standard and/or a suggested method to estimate a reliable value for the basic friction angle from the tilt test, there are some recent efforts on different aspects of this test. In this study, an attempt was made to contribute to previous works on the tilt test with the aid of an experimental program on a wide range of rock types. The study mainly focused on investigating the dependence of the basic friction angle on testing conditions (dry, wet and submerged), a suitable number of tests (repetitions of sliding on the same surface) which will not cause a wear effect, and a more detailed assessment on the influence of water head on the basic friction angle determined under submerged conditions. Additional considerations include a general evaluation of the influence of a testing surface's mineralogical composition on test results. For these purposes, 22 different rock types were collected from different parts of Turkey and their mineralogical compositions were determined based on thin section studies and X-ray diffraction (XRD) analyses. Then, they were cut into mainly square and partly prismatic-based slabs and subjected to tilt tests under dry, wet and submerged conditions by considering the comments and recommendations mentioned, particularly in the most recent studies regarding tilt tests.

Rock types tested and their mineralogical properties

In this study, a total of 22 rock types of sedimentary, metamorphic and magmatic origin were collected from some outcrops and stone processing plants in different parts of Turkey (Fig. [3\)](#page-4-0). Prime attention was paid in selecting the rock blocks and slabs to be free from macroscopic defects. Each rock type used in this study was investigated mineralogically and petrographically based on thin section

studies and XRD analyses, which were carried out at the laboratories of the Geological Engineering Department at Hacettepe University. For this purpose, thin sections prepared from the collected rock types were investigated under a polarizing microscope. The XRD analyses were performed on each rock type to assess the relative quantity of clay minerals and whole sample mineralogy. The XRD

study

diffractograms were obtained using a Philips PW-1140 model diffractometer with a goniometer speed of $2^{\circ}/$ min. Semi-quantitative estimates of the minerals were determined from the powder diffractograms following an external standard method developed by Gundogdu [\(1982](#page-16-0)).

A summary of the semi-quantitative mineralogies of the rock samples based on XRD and the petrographical names of the rock types are given in Tables 1 and [2,](#page-5-0) respectively. According to these results, the rocks tested can be categorized into three groups such as sedimentary and metamorphic carbonate rocks (different limestones and travertines, and marble), volcanic and volcano-sedimentary rocks (different andesites, basalts, ignimbrites), and magmatic rocks (granite and carbonated serpentinite). Whole sample mineralogy (Table 1) demonstrated that, except for carbonate rocks consisting of calcite and dolomite, all rocks mainly included feldspar, quartz and clay minerals in different amounts.

Experimental studies

Sample preparation and tilting table

Since the core-based methods for tilt testing are not Fig. 3 General view of the 22 different rock types employed in the strongly recommended by the authors of most recent

Table 1 Summary of semi-quantitative whole rock mineralogy of 22 rock types based on XRD analyses											
Rock type no.	Quartz	Feldspar	Clay	Mica	Dolomite	Calcite	Heu.	Amp.	Pyr.	Liz.	TMag
MET ₂	24	57	19								
MET3	16	44	34	6							
FC1	14	47	38			1					
FC4	22	23	48	5		\overline{c}					
1		$\overline{}$				100					
\overline{c}						100					
3					41	14				45	
4						100					
5						100					
6						100					
8						100					
9					52	48					
10	61	21	15	3	-						
11	-	67			-				27		6
12	15	76		3				6			
13	43	37		20							
14	8	11	13				68				
15		-			67	32					
16						100					
17	48	3	44		$\mathbf{1}$	4					
19						100					
20	62	21	17								
Heu heulandite (Zeolite group), Amp amphibole, Pyr pyroxene, Liz lizardite, TMag titanomagnetite											

Table 2 Petrographic descriptions and micrographs of the samples used in the study

studies, in this study, the use of square-based or prismatic slabs was preferred. In order to use a large tilt surface and to assure that contact stresses are compressive when sliding occurs, the areas of all slabs were at least 50 cm^2 and their length-to-height ratios were greater than two as recommended by Alejano et al. [\(2012a\)](#page-15-0). For this purpose, squarebased slabs with dimensions between $100 \times 100 \times 20$ mm and $75 \times 75 \times 20$ mm were cut by a diamond-tipped and water-lubricated saw in the laboratory. However, a few samples were prepared as rectangular slabs with dimensions ranging between $100 \times 70 \times 20$ mm and $95 \times 65 \times 20$ mm (Fig. [3](#page-4-0)). When the basic friction angles obtained from saw-cut surfaces are compared with those of polished surfaces, there is always a large difference between them; those of saw-cut surfaces are greater (i.e. Hencher [1977](#page-16-0); Ohta and Aydan [2010](#page-16-0)). In other words, it depends on the condition of the testing surface, and the use of polished surfaces in tilt tests should be avoided. Therefore, in this study, the surfaces of the square and rectangular slabs were not polished and saw-cut surfaces were used in the tilt tests. However, the frictional resistance of planar rock surfaces changes with textural roughness (Coulson [1971](#page-16-0)) and the saw-cut surfaces may involve some microscopic roughness. The microscopic surfaces cannot be determined using a pin-type profilometer and more sensitive profilometers are needed (Shimizu et al. [1996\)](#page-16-0). USBR [\(2009](#page-16-0)), suggesting that the surfaces of the contact planes of the test specimen may be polished with either No. 80- or 200-grit plates, or they may retain their natural roughness; however, the sliding surfaces should not contain abrupt irregularities. Except for this general suggestion, there are no methods/ approaches or comments related to the cutting process and the estimation of microscopic roughness of the test surfaces. Due to these reasons, in this study, after cutting each slab, a straight edge was placed diagonally across the surface from corner to corner and irregularities were found visually and the sample surfaces was lapped using a lapping table, if necessary. Finally, the length, width and height of each slab were measured along two directions perpendicular to each other and the average of each measurement was calculated.

In this study, a simple self-fabricated metal tilting table, shown in Fig. 4a, was used. It is a hand-operated tilt table and weighs about 26 kg. The device consists of a rigid frame supporting a hinged table and a manually rotated arm with a screw feed that rotates the table from 0° to about 85 $^{\circ}$. A metal specimen holder is mounted in front of the table to prevent the movement of the lower specimen during tilting and the height of the holder can be arranged by pushing it up or down depending on the height of the lower specimen. The horizontality of the table is measured using a bubble level and the tilting angle is measured by an inclinometer with an

Fig. 4 The tilt test apparatus a before and b during the test and c water tank, which is mounted on the tilt test apparatus for submerged tests $(A$ upper sample, B lower sample, C holder for the lower specimen, D side holders, E tilting table, F bubble level, G inclinometer with an accuracy of $\pm 0.1^{\circ}$, H inclinometer with an accuracy of $\pm 0.5^{\circ}$, *I* vertical upright metal rods to establish the horizontality of the tilt test, J handle)

accuracy of $\pm 0.1^\circ$ (Fig. 4a, b). A hard, plastic water tank can be mounted on the tilting table to estimate the basic friction angle of saw-cut surfaces under submerged conditions as seen in Fig. 4c. A fibre specimen holder is also mounted at the bottom of the water tank to hold the lower specimen during submerged tilt tests.

The methodology employed and test conditions

Since no standard or a suggested method for the tilt test is available, in this study, some recommendations based on the most recently performed tilt tests (i.e. Alejano et al. [2012a](#page-15-0); González et al. [2014;](#page-16-0) Ruiz and Li 2014; Pérez-Rey et al. [2015\)](#page-16-0) were taken into consideration and the procedure given below was followed in the experimental studies.

In literature, different tilting rates ranging between 2.5° / min (USBR [2009](#page-16-0)) and 24°/min (Alejano et al. [2012a](#page-15-0); Gon-zález et al. [2014](#page-16-0); Pérez-Rey et al. [2015](#page-16-0)) were recommended and/or selected. 8°/min (Cruden and Hu [1988](#page-16-0); Bruce et al. [1989;](#page-16-0) Hu and Cruden [1992\)](#page-16-0) and $15^{\circ}/$ min (Ruiz and Li 2014) are the other tilting rates applied between the upper and lower bounds mentioned above. The effect of tilting rate on the tilt test results was investigated only by Bruce et al. [\(1989](#page-16-0)). They compared the results of the tilt tests conducted under two different tilting rates controlled by a motor at 2.5° / min and $8^{\circ}/$ min, and the experimental results indicated that there was only a little difference, such as 0.3° , between the results obtained from the tests at these two different tilting rates. These researchers also reported that the samples tested at $8^{\circ}/$ min by hand and by motor displayed an average increase of 0.9° from motor to hand tilts and vibrations from the motor depress the sliding angle by about 1° . By considering this ignorable amount of difference between the basic friction angles determined under different tilting rates reported by Bruce et al. [\(1989](#page-16-0)) and the recommendation on the tilting rate by Alejano et al. $(2012a)$ $(2012a)$ and González et al. [\(2014](#page-16-0)), who conducted more than 20,000 tilt tests, in this study, a tilting rate of 24°/min was applied.

After the horizontality of the tilting table was checked using both a bubble level and inclinometer mounted on the table before each test, the lower specimen was placed just behind the specimen holder on the tilting table and the upper specimen was put on top of it (Fig. [4a](#page-6-0)). Then the table was progressively tilted at the rate of $24^{\circ}/\text{min}$ (0.4 \textdegree /s) until the upper specimen began to slide and the tilt angle was recorded (Fig. [4b](#page-6-0)). As recommended by Alejano et al. [\(2012a\)](#page-15-0), in this study, if displacement of the upper specimen was at least about 10 % of the sample length, the test was accepted as valid. Each test was repeated three times on the same surface, the specimen surface tested was wiped with a dry cloth after each test to avoid powder and both lower and upper specimens were placed in the initial same positions. In the calculation of tilt angles along a saw-cut surface, the following approaches (steps) were followed for each test alternatively:

(a) If the deviation between three repetitions (tests) is less than or equal to $\pm 3^{\circ}$, the mean of the three tests is calculated and taken as the basic friction angle of the surface.

- (b) If the deviation between one of the first three tests is greater than $\pm 3^{\circ}$, a fourth test (repetition) is performed and if the deviation of the fourth test is less than $\pm 3^{\circ}$, the mean of the four tests is calculated. Then one of the four tests, which has the largest deviation from the calculated mean, is omitted, and the mean of the rest of them is calculated and taken as the basic friction angle.
- (c) If the deviation between the fourth test and one of the three previous tests is still greater than $\pm 3^{\circ}$, a fifth test is performed and the two of the five tests which have the largest deviations from their mean are omitted and the mean of the three is taken as the basic friction angle.

In addition, in order to assess the variability of the results obtained from different saw-cut surfaces of the same rock, the tests were conducted on the opposite faces of pairs of some selected specimens along the same direction, as illustrated in Fig. [5](#page-8-0). The mean of the measurements obtained from both original and opposite surfaces are taken as the basic friction angle.

In order to assess the effect of wetness or humidity on the basic friction angle, the tilt tests were conducted on all rock types under three different conditions, namely dry, wet and submerged. In the tilt tests conducted on wet surfaces, water was introduced on the surfaces of the upper and lower samples using a squeeze bottle filled with water. A series of submerged tilt tests on samples by mounting a water tank on the tilting table were also carried out to determine the basic friction angle under submerged conditions and to assess the influence of the variation of water head on the basic friction angle using different rock types and to compare the results with those obtained by Aydan [\(1998](#page-16-0)) who tested four rock types under submerged conditions. For this purpose, three different water heads (75, 100 and 150 mm) were marked on the water tank (Fig. [6\)](#page-8-0) and the tank was filled with water up to these levels and submerged tests were separately conducted under each water head.

In all tests, the above-mentioned procedures were followed. A total of 415 tilt tests (115, 103 and 197 tests under dry, wet and submerged conditions, respectively) were conducted and their results are discussed and compared in the following section.

Assessments of the test results and comparisons

The results of the tilt tests conducted on 22 rock types under three different conditions are presented in Table [3,](#page-9-0) and the results obtained from the submerged tests based on different water heads are given separately in Table [4.](#page-10-0)

Table 3 Tilt test results obtained under three different conditions

* Tests performed on opposite surfaces of the sample

Tables 3 and [4](#page-10-0) indicate that the standard deviations calculated for three different test conditions are small and range between 0.12° and 1.58° , 0.36° and 1.97° , and 0.40° and 2.23° for dry, wet and submerged conditions, respectively. The basic friction angles determined after at least three tests for each sample generally show a normal distribution. The histogram showing the values of basic friction angle obtained from rock type number 12 under a submerged condition is given in Fig. [7](#page-11-0)a as an example. In addition, the standard deviations obtained from dry, wet and submerged tilt tests on different rock types are shown in the form of histograms according to one and two standard deviations in Fig. [7b](#page-11-0), c. The probability of observing a variable which shows a normal distribution within the range of one standard deviation from the mean is 68.27 %. Since the values of the standard deviation calculated for all test conditions are smaller than one standard deviation of 3° shown by the dashed line in Fig. [7b](#page-11-0), 68.27 % of the basic friction angles determined by the tilt tests will deviate from the mean less than 3° . On the other hand, the probability of observing a basic friction angle in the range of two standard deviations is 95.45 % and Fig. [7c](#page-11-0) suggests that 57 of the 66 total data sets show a good agreement with this probability. In other words, 95.45 % of the basic friction angles have a standard deviation less than 3° from the mean. These results show an agreement with those obtained by Alejano et al. [\(2012a\)](#page-15-0) who considered the median instead of the mean.

Some tilt tests were also conducted under dry and wet conditions on the opposite faces of pairs of some selected samples along the same direction (see Fig. [5\)](#page-8-0) and the differences between the basic friction angles estimated from the opposite surfaces of the slabs were compared. The comparisons given in Table [5](#page-12-0) indicate that, except for two samples tested under dry conditions (FC1 and FC4), the differences between the basic friction angles obtained from the opposite surfaces of the samples are smaller than 3° . Therefore, it can be concluded that the variability or deviation in the basic friction angle determined from the two opposite surfaces of the same sample are considerably similar. Therefore, in order to increase the number of tests, tilt tests can be carried out on the opposite surfaces of a

Rock type no.	Water head (7.5 cm)			Water head (10 cm)			Water head (15 cm)		
	Number of tests	Mean	Standard deviation	Number of tests	Mean	Standard deviation	Number of tests	Mean	Standard deviation
MET ₂	\ast	\ast	\ast	3	28.20	1.47	$6+$	26.37	0.72
MET3	\ast	\ast	\ast	3	25.37	1.10	3	25.17	0.67
$FC-1$	3	26.63	1.00	3	25.63	0.42	3	27.87	0.85
$FC-4$	3	29.20	1.08	3	28.93	1.53	3	27.93	1.36
1	\ast	\ast	\ast	3	35.67	0.42	3	36.43	1.31
$\mathfrak{2}$	3	28.07	2.41	3	28.23	1.30	3	27.73	0.21
3	3	32.17	1.10	3	30.77	0.83	3	30.97	1.53
4	3	29.70	0.95	3	30.97	1.79	3	28.33	1.30
5	3	33.27	1.48	3	35.10	0.26	3	34.67	1.47
6	3	33.63	0.65	3	34.23	1.10	3	32.63	1.33
8	3	24.73	1.52	3	28.30	2.07	3	28.33	1.74
9	3	25.63	0.45	3	25.73	0.65	3	25.43	0.61
10	3	25.57	0.40	3	27.13	0.65	3	25.83	0.59
11	3	28.23	0.55	3	28.50	0.62	3	25.73	0.23
12	3	18.33	1.27	3	19.13	1.12	3	18.93	0.93
13	3	28.00	1.66	3	27.93	1.68	3	29.77	2.08
14	3	27.60	1.06	3	27.70	0.44	3	28.97	1.27
15	3	22.03	1.59	3	22.87	1.48	3	22.37	1.81
16	3	32.67	1.16	3	31.37	1.06	3	33.20	1.25
17	3	27.13	1.22	3	25.80	0.98	3	27.43	0.40
19	3	25.60	0.36	3	25.63	0.70	3	25.57	0.49
20	3	26.60	0.26	3	26.23	0.75	3	26.33	1.47

Table 4 Submerged tilt test results under different water heads

* No tests were conducted due to limited number of samples

? Tests were conducted on the opposite surfaces of the sample

sample and the mean of the values obtained from both original and opposite surfaces can be taken as the basic friction angle of the sample.

The effect of wear on the sliding angle (basic friction angle), which is mentioned in the literature, was also assessed. For this purpose, variation in the basic friction angle of each specimen with the number of tests (repetitions) was compared. As seen from Tables [3](#page-9-0) and 4, the number of tests on saw-cut surfaces ranges between 3 and 5 (6, 7 and 8 in these tables represent the total numbers of repetitions/tests on the two opposite surfaces). When the sliding angles from these repetitions for each sample are compared, it is understood that there is an ignorable amount of difference between the sliding angles determined in each repetition. Figures [7](#page-11-0) and [8](#page-12-0) showing the basic friction angle varying with the number of tests (repetitions) for all samples in dry, and some samples selected from the wet and submerged tests, respectively, suggests that if a maximum of five tests are conducted and the rock flour on the surface is removed after each test, no or an ignorable wear effect occurs on the test surfaces. This observation supports the number of repetitions (four or five tests) rec-ommended by Alejano et al. ([2012a](#page-15-0)) and González et al. [\(2014](#page-16-0)). Most recently, Pérez-Rey et al. ([2015\)](#page-16-0) recommended that their approach, which is based on a number of tests to estimate the basic friction angle as the logarithmic fit associated to the first test, can be more representative of natural joints than averages of a few tests. When the approach of Pérez-Rey et al. (2015) (2015) and the observation of the authors on the wear effect (Figs. 8 and 9) are compared, application of the maximum four or five repetitions in the tilt tests seems more practical.

The effect of water head on the basic friction angle under submerged conditions was assessed only by Aydan [\(1998](#page-16-0)). In his experiments, Aydan [\(1998](#page-16-0)) used only four rock types (sandstone, tuff, granite and andesite) and reported that the basic friction angle obtained from the tilt tests under submerged conditions seems to not be influenced by the variation of water head. In this most recent study, 22 rock types were tested in a tank under submerged conditions and the results obtained from the tilt tests conducted under three different water heads are shown in

Fig. 7 a Histogram of the basic friction angle values obtained from rock type no. 12 under submerged conditions, **b** 1 standard deviation and c 2 standard deviations for the tests conducted under dry, wet and submerged conditions

Fig. [10](#page-13-0) in the form of water head versus basic friction angle of all samples tested. It is clear from the plots in Fig. [10](#page-13-0), for all samples, the basic friction angles are not affected by water head in the tilt test and the results of this study confirm a previous study.

Figure [11](#page-14-0) shows plots of dry versus submerged and wet basic friction angles. Figure [11](#page-14-0)a indicates that the basic friction angles of all samples determined under submerged conditions are smaller than those determined under dry conditions, and the drop in the submerged basic friction angle ranges between 0.19° and 8.64° , which is due to the lubrication effect developed on minerals in different degrees. Although the wet basic friction angle is smaller than the dry basic friction angle for more than half of the rock types tested in this study, nine rock types have wet basic friction angles greater than their dry basic friction

Rock type no.	Dry		Wet			
	Difference in basic friction angle $(°)$	Difference in basic friction angle $(\%)$	Difference in basic friction angle $(°)$	Difference in basic friction angle $(\%)$		
$MET-2$	1.87	6.47	0.70	2.36		
$MET-3$	0.83	2.98	0.20	0.66		
17	0.10	0.29	1.20	3.79		
$FC-1$	3.97	11.67	2.00	6.83		
$FC-4$	3.10	10.58	1.37	4.90		
20	0.10	0.33	1.47	5.47		
13	0.03	0.11	0.03	0.11		
11	1.40	4.43				

Table 5 The differences in the basic friction angle estimated from the tests conducted on the opposite surfaces of the samples under dry and wet conditions

Fig. 8 Variation of the basic friction angles of different rock types with the number of tests under dry conditions

angles (Fig. [11b](#page-14-0)). When compared to dry basic friction angles, the increase in the wet basic friction angle is due to the capillary effect (suction). This effect was also investigated by Zandarin [\(2010\)](#page-16-0) and he observed that shear strength of artificially prepared joints of claystone increased with an increase in suction. It is also noted from Fig. [11](#page-14-0)a that, in general terms, when the basic friction angle in wet conditions is larger than in dry conditions, the difference between both tend to be smaller when the opposite occurs.

The differences between the dry basic friction angles and wet and submerged angles shown in Fig. [12](#page-14-0) are due to the differences between the suction effect on testing surfaces and the degree of lubrication of each mineral

involved as per the rock type. It can be concluded that if the magnitude of increase in the basic friction angle due to a suction effect is greater than the magnitude of the decrease in the basic friction angle associated with a lubrication effect, basic friction angles of the samples determined under wet conditions become greater than those determined under dry conditions. In the study by Horn and Deere [\(1962](#page-16-0)), it was observed that the coefficients of friction of the minerals quartz, feldspar and calcite generally increase under wet conditions, while they decrease for mica minerals (muscovite, biotite) and talc under the same conditions. Morrow et al. ([2000\)](#page-16-0) reported decreases in different degrees (between 5 and 70 %) in the coefficients of friction of some minerals,

 (a) Specimen no. 35 \bullet 3 .±..........._±.............+ 32.5 \mathbf{D} 10 <u>..............................</u> $\phi_{\rm b}$ ($^{\circ}$) ugu. 30 $\bullet 15$ $+17$ 27.5 \mathbf{a} \mathbf{Q}_{20} 25 $\overline{2}$ $\overline{3}$ $\overline{4}$ $\sqrt{5}$ Ω $\overline{1}$ Number of tilt test Specimen no. (**b**) 40 \bullet FC-1 35 \blacksquare 1 ._≱...............∡. 30 \bullet 3 \mathcal{C} $\phi_{\rm b}$ ($\triangle 6$ 25 $+12$ 20t.................t... Q 14 15 $\overline{0}$ $\overline{1}$ $\overline{2}$ $\overline{3}$ $\overline{4}$ Number of tilt test

such as calcite, clay minerals, muscovite, talc, chlorite and lizardite, depending on degree of saturation. In this study, the feldspar content of the samples with wet basic

Fig. 9 Variation of the basic friction angle values of some selected specimens with the number of tests under a wet and b submerged conditions

friction angles greater than dry basic friction angles (Table [3;](#page-9-0) Fig. [12](#page-14-0), samples MET 2, MET 3, 4, 8, 9, 10, 11, 12) are generally higher than those in other samples (see Table [1\)](#page-4-0). In other words, in rock types with a high feldspar content, the suction effect may be pronounced. Meanwhile, the presence of a high amount of clay minerals generally results in a decrease in the basic friction angle depending on wetness (Moore and Lockner [2004\)](#page-16-0) and there is little or no difference between the dry and wet or saturated coefficient of friction of quartz (Moore and Lockner [2004](#page-16-0)). Morrow et al. [\(2000](#page-16-0)) reported that the coefficient of friction of calcite, which is exceptionally strong when dry (0.85), drops when it becomes wet and/or saturated. Therefore, the majority of the carbonate rocks, used in this study (Tables [1](#page-4-0) and [3](#page-9-0); Fig. 12), have wet basic friction angles smaller than their dry ones. However, these are preliminary assessments on the effect of mineralogical composition on the basic friction angle and should not be generalized and further studies are needed.

In order to more sensitively assess the effect of mineralogy on the basic friction angle, the effect of all minerals involved as per the tested rock types were considered and the variation in the basic friction angle depending on saturation was assessed using multivariate regression analysis. In the regression analyses, except for minor minerals only observed in a few rock types (such as heulandites, amphibole, pyroxene, lizardite and titanomagnetite), all main mineral types given in Table [1](#page-4-0) were included as independent variables. The following regression equation with a very high coefficient of correlation $(r = 0.954)$

Fig. 10 Variation of the basic friction angle values with water head under submerged conditions

helps the estimation of the drop ($\Delta\phi_b$) in the basic friction angle under submerged conditions using the mineral content of rocks.

Fig. 11 Comparison of the dry basic friction angles with wet a and b submerged basic friction angles

The significance level of regression coefficients of Eq. 7 were evaluated by F and t tests (Tables 6, [7\)](#page-15-0). The F value of Eq. 7 was calculated as 26.427 and the significance level of this value is 0.000. Therefore, it can be concluded that there is a significant relationship between dependent and one or more independent variables in Eq. 7. On the other hand, all t values found for the multivariate equation have a significance level less than 0.05, meaning that the multivariate relationship given in Eq. 7 is significant. The drops in the basic friction angle calculated from experimental studies and those estimated from Eq. 7 for all rock types used in this study are compared in Fig. [13.](#page-15-0) It is clear from this figure that, except for a few, the observed versus predicted data plots fall between the 2:1 and 1:2 lines and generally close to the 1:1 diagonal line.

Conclusions

In this study, an attempt was made to contribute to previous works on tilt testing with the aid of an experimental program on a wide range of rock types under different conditions. The main conclusions drawn from the study are given below.

The test results indicate that the standard deviations calculated for three different test conditions (dry, wet and submerged) are small and range between 0.12° and 1.58° , 0.36° and 1.97° , and 0.40° and 2.23° for dry, wet and submerged conditions, respectively. Statistical assessments on the test data also indicated that, except for a few rock

Table 6 F test results **determined from Eq. 7** Model Sum of squares df Mean square F Significance Regression 1743.804 5 348.761 26.427 0.000 Residual 171.561 13 13.197 Total 1915.365 18

Table 7 t Test results for regression coefficients determined by Eq. [7](#page-14-0)

Model		Unstandardized coefficients	Standardized coefficients	t	Significance	
	B	Standard error	Beta			
(Constant)	-57.498	9.420		-6.104	0.000	
Clay and mica	0.657	0.108	1.146	6.067	0.000	
Calcite	0.631	0.095	2.983	6.650	0.000	
Dolomite	1.269	0.131	2.160	9.685	0.000	
Ouartz	0.696	0.104	1.466	6.666	0.000	
Feldspar	0.619	0.106	1.432	5.833	0.000	

Fig. 13 Comparison of the basic friction angles predicted from Eq. [7](#page-14-0) and experimentally determined

types, 95.45 % of the basic friction angles have a deviation less than 3° from the mean. These results suggest that a fourth or fifth supplementary repetition (test) should be performed when the maximum deviation between the test results is greater than 3° ; these results also show an agreement with those obtained by Alejano et al. (2012a) who considered the median instead of the mean.

Based on the test results, it can be concluded that the variability or deviation in the basic friction angle determined from the two opposite surfaces of the same sample are considerably similar. Therefore, in order to increase the number of tests, tilt tests can be carried out on the original and opposite surfaces of a sample and the mean of the values obtained from both surfaces can be taken as the basic friction angle of the sample.

The variation of the basic friction angle with the number of tests (repetitions) for the samples tested under dry, wet and submerged conditions suggests that if a maximum of five tests are conducted and the rock flour on the surface is removed after each test, no or ignorable wear effect occurs on the test surfaces. This observation supports the number of repetitions (four or five tests) recommended by Alejano et al. (2012a) and González et al. ([2014](#page-16-0)). When the approach of Pérez-Rey et al. [\(2015\)](#page-16-0), which is based on a number of tests to estimate the basic friction angle as the logarithmic fit associated to the first test, and the observation of the authors on the wear effect are compared, application of a maximum of four or five repetitions in the tilt tests seems more practical.

Submerged tilt tests conducted on 22 different rock types showed that the basic friction angles are not affected by water head in the tilt test and confirm the results of a previous study based on a limited number of rock types. The differences between the dry and wet basic friction angles are due to the differences between the suction effect on testing surfaces and degree of lubrication of each mineral involved as per the rock type.

In this study, the feldspar content of the samples with wet basic friction angles greater than dry basic friction angles are generally higher than those in other samples, indicating that the suction effect may be more pronounced in rock types with high content of feldspar minerals. Minerals other than feldspar do not have any explicit effect (increasing or decreasing) on the variation of basic friction angle due to wetting. A regression equation with a very high coefficient of correlation $(r = 0.954)$ was developed for the estimation of the drop in the basic friction angle $(\Delta \phi_h)$ under submerged conditions using the mineral content of rocks. As can be seen from Eq. [7,](#page-14-0) all the minerals (quartz, feldspar, clay and mica, dolomite and calcite) used in the regression analyses have a decreasing effect on the basic friction angle depending on transition from the dry test condition to the submerged condition.

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References

- Alejano LR, González J, Muralha J (2012a) Comparison of different techniques of tilt testing and basic friction angle variability assessment. Rock Mech Rock Eng 45:1023–1035
- Alejano LR, Veiga M, Gómez-Márquez I, Taboada J (2012b) Stability of granite drystone masonry retaining walls (II):

relevant parameters and analytical and numerical studies of real walls. Géotechnique 62(11):1027-1040

- Aydan \ddot{O} (1998) Simple tests on the validation of the effective stress law for rock discontinuities. In: Thimus JF, Abousleiman Y, Cheng AHD, Coussy O, Detournay E (eds) Proceedings of poromechanics, Belgium, pp 539–544
- Aydan Ö, Akagi T, Okuda H, Kawamoto T (1994) The cyclic shear behaviour of interfaces in rock anchor systems and their effect on the long-term performance of rock anchors. Int. Symp. on New Developments of Rock Mechanics and Rock Engineering, Shenyang, pp 15–22
- Aydan \ddot{O} , Shimizu Y, Kawamoto T (1995) A portable system for in situ characterization of surface morphology and frictional properties of rock discontinuities. In: Proceedings of 4th international symposium field measurements in geomechanics, pp 463–470
- Aydan \ddot{O} , Shimizu Y, Kawamoto T (1996) The anisotropy of surface morphology characteristics of rock discontinuities and its evaluation. In: Auberttn A, Hassani F, Mitri H (eds), Proceedings of NARMS'96, Balkema, Rotterdam, 1391–1398
- Barton N (1973) Review of a new shear-strength criterion for rock joints. Eng Geol 7:287–332
- Barton N (1976) The shear strength of rock and rock joints. Int J of Rock Mech MinSci Geomech Abstr 13:255–279
- Barton N, Bandis S (1980) Some effects of scale on the shear strength of joints. Int J Rock Mech Min Sci Geomech Abstr 17:69–73
- Barton N, Bandis S (1990) Review of predictive capabilities of JRC– JCS model in engineering practice. In: Proc. Int. Conf. on Rock Joints, pp 603–610
- Barton N, Choubey V (1977) The shear strength of rock joints in theory and practice. Rock Mech 10:1–54
- Bruce IG, Cruden DM, Eaton TM (1989) Use of a tilting table to determine the basic friction angle of hard rock samples. Canadian Geotech. J. 26:474–479
- Coulson JH (1971) Shear strength of flat surfaces in rock stability of rock slopes. In: Cording EJ (ed.) Proc. of the 13th Symp. on Rock Medch., Am. Soc. Civ. Eng., New York, pp 77–105
- Cruden DM, Hu XQ (1988) Basic friction angles of carbonate rocks from Kananaskis country, Canada. Bull of the Int Association of Eng Geol 38:55–59
- Goldstein M, Goosev B, Pyrogovsky N, Tulinov N, Turovskaya A (1966) Investigation of mechanical properties of cracked rock.In: Proc. of the 1st Int. ISRM Congress, Lisbon vol. 1: I–18
- González J, González-Pastoriza N, Castro U, Alejano LR, Muralha J (2014) Considerations on the laboratory estimate of the basic friction angle of rock joints. In: Alejano LR, Perucho A, Olalla C, Jimenez R (eds), Rock Mechanics and Rock Engineering: Structures on and in Rock Masses, EUROCK 2014, Vigo, Rotterdam, Balkema, pp 98–99 (on CD)
- Gundogdu MN (1982) Geological, mineralogical and geochemical investigation of the Neogene aged Bigadic sedimentary basin. PhD Thesis, Geological Engineering Department, Hacettepe University, Ankara (in Turkish)
- Hencher SR (1976) A simple sliding apparatus for the measurement of rock friction. Discussion. Géotechnique 26(4):641-644
- Hencher SR (1977) The effect of vibration on the friction between planar rock surface. Unpublished PhD Thesis, Imperial College of Science and Technology, London University
- Hencher SR (1995) Interpretation of direct shear tests on rock joints. In: Proceedings of the 35th US symposium on rock mechanics, Lake Tahoe, pp 99–106
- Hencher SR (2012) Discussion of the Paper ''Comparison of Different Techniques of Tilt Testing and Basic Friction Angle Variability Assessment''. Rock Mech Rock Eng 45:1141–1143
- Hencher SR, Richards LR (1989) Laboratory direct shear testing of rock discontinuities. Ground Eng 22(2):24–31
- Horn HM, Deere DU (1962) Frictional characteristics of minerals. Geotechnique 12:319–335
- Hu XQ, Cruden DM (1992) A portable tilting table for on-site tests of the friction angles of discontinuities in rock masses. Bull Int Ass Eng Geol 46:59–62
- Kveldsvik V, Nilsen B, Einstein HH, Nadim F (2008) Alternative approaches for analyses of a $100,000 \text{ m}^3$ rock slide based on Barton-Bandis shear strength criterion. Landslides 5:161–176
- Moore DE, Lockner DA (2004) Crystallographic controls on the frictional behaviour off dry and water-saturated sheet structure minerals. J Geophys Res 109:1–16
- Morrow CA, Moore DE, Lockner DA (2000) The effect of mineral bond strength and adsorbed water on fault gouge frictional strength. Geophys Res Lett 27:815–818
- Newland PL, Allely BH (1957) Volume changes in drained triaxial tests on granular materials. Geotechnique 7:17–34
- Ohta Y, Aydan $\ddot{\mathrm{O}}$ (2010) The dynamic response of geo-materials during fracturing and slippage. Rock Mech Rock Eng 43:727–740
- Patton FD (1966) Multiple modes of shear failure in rock and related materials. PhD Thesis, University of Illinois, Urbana
- Pérez-Rey I, Alejano LR, González-Pastoriza N, González J, Arzúa J (2015) Effect of time and wear on the basic friction angle of rock discontinuities. In: Schubert W (ed) EUROCK 2015 & 64th Geomechanics Colloquium, Salzburg, Austria, ÖGG
- Ripley CF, Lee KL (1961) Sliding friction tests on sedimentary rock samples. Trans. 7th Int. Congress on Large Dams, Rome, Italy, vol. 4: 657–671
- Ruiz J, Li C (2014) Measurement of the basic friction angle of rock by three different tilt test methods. In: Alejano L, Perucho A, Olalla C, Jimenez R (eds) EUROCK2014: Rock Mechanics and Rock Engineering: Structures on and in Rock Masses, Vigo, Spain, Rotterdam: Balkema (on CD)
- Shimizu Y, Aydan Ö, Kawamoto T (1996) The effect of measuring methods on surface morphology characteristics. Korea-Japan Joint Symposium on Rock Engineering, 231–236
- Stimpson B (1981) A suggested technique for determining the basic friction angle of rock surfaces using core. Int J Rock Mech MinSci Geomech Abstr 18:63–65
- Tembe S, Lockner DA, Wong TF (2010) Effect of clay content and mineralogy on frictional sliding behavior of simulated gouges:binary and ternary mixtures of quartz, illite and montmorillonite. J Geophys Res, Solid Erath, 115 (B3) B03416, 1–22. doi:[10.](http://dx.doi.org/10.1029/2009JB006383) [1029/2009JB006383](http://dx.doi.org/10.1029/2009JB006383)
- USBR (2009) Procedure for determining the angle of basic friction (static) using a tilting table test (Designation USBR 6258-09). <http://www.usbr.gov>
- Wines DP, Lilly PA (2003) Estimates of rock joint shear strength in the Fimiston open pit operation in Western Australia. Int J Rock Mech Min Sci 40:929–937
- Wyllie DC (1992) Foundations on Rock. Chapman & Hall, London
- Zandarin MT (2010) Thermo-hydro-mechanical analysis of joints a theoretical and experimental study. PhD Thesis, Department of Geotechnical Engineering and Geosciences, Universitat Politècnica de Catalunya, Barcelona