

SPH-based numerical simulation of catastrophic debris flows after the 2008 Wenchuan earthquake

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Received: 30 March 2014 / Accepted: 5 November 2014 / Published online: 25 November 2014
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Abstract Post-earthquake debris flows that have occurred in Sichuan Province in southwestern China following the Wenchuan earthquake on May 12, 2008, have caused significant damage and casualties. Previous earthquake-induced landslides produced large amounts of loose material that remained on the steep slopes and in the gullies. As a consequence of heavy rainstorms during the rainy seasons, the existing loose material was transformed into numerous debris flows. Research has shown that the debris flows in the Wenchuan earthquake disaster areas have been characterized by their large scale, high speed, long run-out, and destructive impact. In order to identify the areas potentially at risk and to predict the flow severity, an accurate numerical method is needed to simulate these debris flows. In this paper, we have proposed a smoothed particle

hydrodynamics (SPH) modeling technique—a meshfree particle method—to simulate the post-earthquake debris flows in the Wenchuan earthquake disaster areas. The SPH modeling technique introduces a Bingham model to analyze the relationship between material stress rates and particle motion velocity. Compared to traditional numerical methods, the SPH modeling technique is a true meshfree method of a pure Lagrangian nature. It can instantaneously track the motion of each particle, accurately predict the velocity, and naturally handle problems with extremely large deformations. In addition, the SPH method is based on continuum mechanics, and is therefore an efficient method to simulate large-scale debris flows. In this work, first, a viscoplastic fluid was simulated and verified with experimental results in order to evaluate the accuracy of the SPH model. Then propagation analysis of two typical post-earthquake debris flows in earthquake-hit areas was carried out, applying the SPH model. The simulation results showed good agreement with the limited field observation data. Our proposed SPH numerical modeling is able to capture the fundamental dynamic behavior of post-earthquake debris flows and can partially explain these complex phenomena. These simulation results can provide a preliminary scientific basis for hazard assessment and site selection for reconstruction in earthquake-prone areas.

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Keywords Wenchuan earthquake · Post-earthquake
debris flow · Smoothed particle hydrodynamics ·
Long run-out · Hazard assessment

Introduction

On May 12, 2008, a destructive Ms 8.0 earthquake occurred in Wenchuan in Sichuan Province of China. Because of the

strength of the earthquake, the long duration of shaking, and the fragile mountain landscape, over 46,000 landslides were triggered in steep mountainous terrain (Huang and Li 2009a; Wu et al. 2008; Xu et al. 2009; Yin et al. 2011). Furthermore, the massive co-seismic landslide deposition in gully drainage areas provided numerous loose solid materials, which were easily eroded and developed into debris flows (Shieh et al. 2009). Significant debris flows have been triggered by intense rainfall events during three rainy seasons since the Wenchuan earthquake, resulting in more than 2,000 deaths/missing persons, and destroying numerous houses, bridges, and traffic facilities in the earthquake-affected areas (Tang et al. 2009, 2012; Xu 2010; Xu et al. 2012). Previous studies have concluded that debris flows in earthquake-hit areas will continue for 5–20 years post-earthquake, and in particular during the first 5–10 years (Cui et al. 2011; Tang et al. 2009). For example, the 1999 Chi-Chi earthquake triggered numerous co-seismic landslides, which consequently provided a great deal of loose sediment, promoting heavy debris flows for the next 10 years (Chen and Hawkins 2009). Therefore, effective methods for performing analysis of debris flow run-out and related intensities (e.g., impact force and velocity) are needed in order to understand the dynamic behavior of these phenomena.

Over recent decades, the dynamic behavior of debris flows has been a major research focus. Empirical and analytical methods based on field investigations and theoretical derivation have been proposed for estimating the dynamic characteristics (e.g., impact force, velocity, and discharge) of debris flows (Hungri 1995; Rickenmann 1999). In addition, model tests have also been adopted to analyze the mechanical parameters of the flows (Mazzanti and De Blasio 2011; Xie et al. 2014). With respect to the accuracy and efficiency of these methods, numerical simulations have the significant advantage of performing run-out analysis for the debris flow over irregular terrains. Moreover, numerical methods can calculate the related intensities of debris flows, which can be coupled with vulnerability functions for quantitative risk assessment.

Numerical simulations based on conventional Eulerian coordinates have been widely used in the dynamic analysis of debris flow. O'Brien et al. (1993) and Jakob et al. (2013) applied a finite difference method (FDM), the FLO-2D model, to numerical simulations of debris flows. Medina et al. (2008) developed a 2D finite volume method (FVM), FLATModel, to calculate the dynamic characteristics of the flows. In addition, Chen and Lee (2000) proposed a 3D dynamic model with a Lagrangian frame and a finite element method (FEM) to simulate the sliding process of debris flows. Notably, all of these mesh-based methods have promoted the dynamic analysis of debris flows. However, as debris flows are usually characterized by a long run-out, mesh-based numerical methods will have several limitations. For example, mesh

generation takes a large amount of time. In addition, with the development of large deformations, winding, twisting, and distortion of mesh will occur, and consequently, remeshing is necessary during the calculation. This is often tedious and time-consuming, and may introduce additional inaccuracy into the solution, thus highlighting new demands for numerical simulation of debris flow.

Smoothed particle hydrodynamics (SPH) is a meshfree particle method based on the Lagrangian description, which was originally applied to solve astrophysical problems in 3D open space (Lucy 1977). Compared with traditional numerical methods, the proposed SPH has the following advantages in simulating the post-earthquake debris flows: (a) a purely Lagrangian description; particles in the SPH method carry field variables, such as mass, density, and stress tensor, and move with a material velocity, which can instantaneously track the motion of each particle and accurately predict the velocity and long run-out distance of the debris flows; (b) a mesh-free method; the problem domain in the SPH method is represented by a set of arbitrarily distributed particles, which can thus avoid mesh distortions and can perform large deformations (Liu and Liu 2010); (c) continuum mechanics; the SPH method is based on continuum mechanics, so it is an efficient approach for simulating the entire process of large-scale irregular deformations in soil material (Bui et al. 2008). Over the past few years, the SPH method has been widely applied in the field of fluid dynamics (Monaghan 1994; Shao and Lo 2003). Recently, Laigle et al. (2007), Minatti and Pasculli (2011), and Pasculli et al. (2013) have applied the SPH method to simulate a mudflow model that promoted the dynamic analysis of the debris flows. However, to the best of our knowledge, research on post-earthquake debris flows using SPH has not been published.

In this paper, we first analyze the characteristics of the catastrophic debris flows after the Ms 8.0 Wenchuan earthquake. We then present the basic features of the SPH modeling technique. Here, we choose to apply the Bingham constitutive model in the proposed SPH numerical method to simulate the rheological behavior of the debris flow. To verify the reliability of the proposed SPH model, experimental results obtained from Laigle et al. (2007) are compared with numerical results of the SPH approach implemented in our code. Finally, we apply the SPH model to simulate two post-earthquake debris flows. The results of the run-out and the pattern of deposition show good agreement with the limited field observation data.

Overview of the catastrophic debris flows after the 2008 Wenchuan earthquake

The May 12, 2008 Wenchuan earthquake endangered surrounding regions, and the most severely hit areas were

Beichuan, Wenchuan, Mianzhu, Dujiangyan, Pengzhou, and Shifang (Liu and Sun 2009). Not only did the devastating earthquake destroy buildings, roads, and bridges, but it also triggered numerous secondary geological hazards (e.g., rock falls, landslides, debris flows, and barrier lakes), causing large numbers of casualties and aggravating the social and economic impacts of the earthquake (Huang and Li 2009b; Huang et al. 2012a). Furthermore, the earthquake and the secondary geological hazards severely damaged the mountain terrain. It is thought that co-seismic landslides greatly increase sediment supply on slopes and in channels (Tang et al. 2009). Unfortunately, the earthquake-affected areas are situated in a subtropical humid monsoon climate zone, where the high-intensity rainfall events are concentrated from July to September. With the effects of the heavy rainfall, numerous debris flows have been triggered in the gullies of earthquake-affected areas since the Wenchuan earthquake event. During the five years following the earthquake, the two most powerful large-scale clusters of debris flows occurred on September 24, 2008, and on August 13, 2010. A heavy rainstorm in Beichuan County on September 24, 2008, triggered 72 debris flows in gullies, which buried most of the old Beichuan County (Tang et al. 2009). This cluster of flows is known as the '924' debris flow. More recently, from August 12 to 14, 2010, heavy rainstorms occurred in parts of areas affected by the 2008 Wenchuan earthquake, triggering three regional catastrophic debris flows near the towns of Qingping, Yingxiu, and Longchi (Xu et al. 2012). This disaster has become known as the '813' debris flow in Sichuan Province. In this study, along with several post-earthquake debris flows, four typical large-scale debris flow gullies in the earthquake-hit areas were selected in order to analyze the characteristics of the catastrophic debris flows after the 2008 Wenchuan event.

The Wenchuan earthquake transformed the Niumian Gully into an area prone to debris flows. In total, seven large-scale debris flows have been triggered since the earthquake (Yang et al. 2011). The Niumian Gully is located at the meizoseismal area of the Wenchuan earthquake, about 2 km from Yinxu. The gully mouth lies at an altitude of 880 m and the highest peak at 2,282 m. The catchment area is approximately 11.1 km². Simultaneously with the Wenchuan earthquake, a large-scale landslide was induced in the upstream of the Niumian Gully. Approximately 750 × 10⁴ m³ of solid material flowed down and transformed into debris flows, burying the entire 3.9 km length of the Niumian Gully. The solid material was approximately 30 m thick, which provided a great deal of loose material for debris flows (Yin 2009). From July to September 2008, a number of debris flows were triggered that flowed out of the gully and buried the G213 highway. For example, affected by the continuous rainfall, a large

slide of loose solid material from previous landslides was initiated in the Niumian Gully during the '924' debris flows, seriously affecting post-earthquake rebuilding (Xie et al. 2008). In addition, triggered by the rainstorm, approximately 55 × 10⁴ m³ of solid material was transformed into another debris flow in the Niumian Gully on August 13, 2010 (Yang et al. 2011). The debris blocked the Min Jiang River, directly threatening the newly built town of Yingxiu in the upstream. Figure 1 shows the large accumulations of loose solid material remaining in the Niumian Gully.

The Hongchun Gully, an ancient debris-flow gully located opposite Yingxiu, was near the epicenter of the Wenchuan earthquake. The length of the steep main gully is approximately 3.6 km, with an average gully gradient of 35.8 %. The altitude of the gully ranges from 880 to 2,168 m, and the catchment area is approximate 5.35 km². Before the Wenchuan earthquake, around 200 × 10⁴ m³ of loose deposits were distributed along the sides of the gully. After the earthquake, the environment and stability of the entire gully was severely damaged, inducing numerous rock falls and landslides. In total, approximately 150 × 10⁴ m³ of new loose deposits were formed along both sides of the gully (Xu et al. 2012), providing a large amount of material for debris flows. Furthermore, the deposits blocked the drainage channel, increasing the risk of additional debris flows. The heavy rainstorm on August 13, 2010, over Yingxiu almost instantaneously triggered numerous debris flows in surrounding gullies. The loose



Fig. 1 Niumian Gully



Fig. 2 Hongchun Gully

materials in the main gully and three upstream branches (Ganxipu, Dashuigou, and Xindianzi Gullies) were mobilized simultaneously. As a result, this devastating debris flow of approximately $70 \times 10^4 \text{ m}^3$ blocked the gully channel (Fig. 2). Approximately 60 % of the debris blocked the Min River and formed a debris dam, which changed the course of the river and caused flooding in Yingxiu (Xu 2010).

Wenjia Gully is located north of Qingping, 3.6 km from the Yinxiu–Beichuan Fault. The gully mouth of the 5.2-km main channel lies at an altitude of 883 m and the highest peak at an altitude of 2,402 m, with an average longitudinal channel gradient of 46.74 %. The catchment area is approximate 7.81 km^2 . Prior to the 2008 Wenchuan earthquake, no debris flows had been observed or recorded in the Wenjia Gully, which was covered with vegetation. During the earthquake, a limestone rock mass with a volume of approximately $5,000 \times 10^4 \text{ m}^3$ slid from an elevation of between 1,780 and 2,340 m (Xu et al. 2012). The massive sliding material moved at a high speed along the downstream gully and transformed into a rock avalanche. After a series of collisions and scraping events, approximately $2,260 \times 10^4 \text{ m}^3$ of coarse and loose solid material accumulated on the Hanjia Plain, as well as along the downstream gully and the 1,300 m platform. The combination of abundant source material and the steep average longitudinal gradient render the Wenjia Gully prone to debris flows. Five such events have occurred in the gully since the Wenchuan earthquake. The first debris flow occurred on September 24, 2008. Approximately $90 \times 10^4 \text{ m}^3$ of debris flowed down and buried the road. The most serious event, however, which was affected by a rainstorm, occurred on August 13, 2010. A mass of solid material composed of limestone rock fragments, clay, and silty clay, approximately $450 \times 10^4 \text{ m}^3$ in volume, was set in motion from the 1,300 m platform (Xu et al. 2012). As density is normally used to classify debris flows, if the density of a debris flow is greater than 1.8 g/cm^3 , it is defined as a viscous debris flow (Yu 2008). Field studies



(a) Overview of Wenjia gully after the ‘813’ debris flow



(b) Large deposits of debris remain over the Wenjia gully

Fig. 3 Wenjia Gully

and calculations show that the density of the Wenjia Gully debris flow was $1.97\text{--}2.19 \text{ g/cm}^3$, characterized by viscous debris flow (Sichuan Province Geological Engineering Complex 2010). The debris finally flowed into the Mianyuan River, arriving 1,500 m downstream and burying Qingping. This was also the most serious event of the 11 debris flows that occurred in Qingping on August 13, 2010. Figure 3a shows an aerial view of Wenjia Gully after the ‘813’ debris flow, and Fig. 3b depicts the large accumulations of loose solid material remaining throughout the gully.

Because of the torrential rainfall on August 13, 2010, approximately 50 debris flows occurred along the two sides of the Longxi River in Longchi, causing devastating losses. It was estimated that more than $1,000 \times 10^4 \text{ m}^3$ of solid material flowed into the Longxi River, burying most of the surrounding houses (Xu 2010). The most serious event occurred in Bayi Gully, which is a debris flow gully located on the right bank of the Longxi River. The gully mouth of the 4.5-km main channel lies at an altitude of 850 m and the highest peak at 2,456 m. The average longitudinal gradient of the channel is 37.67 %, and the catchment area

is approximate 8.63 km^2 (Ma et al. 2011). Prior to the Wenchuan earthquake, no debris flows had occurred in this area for 50 years, and the earthquake induced rock falls and landslides on both sides of the gully, creating large masses of loose solid materials. More than $100 \times 10^4 \text{ m}^3$ of solid materials were continuously mobilized to debris flows from May to July 2008 (Zhang et al. 2010). In the most serious event of the ‘813’ debris flows in Longchi, approximately $200 \times 10^4 \text{ m}^3$ of loose solid materials composed of quarternary deposits of marlstone, sandstone, and mudstone were simultaneously triggered from the main gully and three upstream branches (Xiaowan, Xiaogan, and Dagan Gullies). Ma et al. (2011) and Zhou et al. (2011) identified the Bayi Gully debris flow as a viscous debris flow, with a density of $1.82\text{--}1.88 \text{ g/cm}^3$. The destructive debris flowed through the long main gully, destroying all of the block engineering structures and burying the surrounding road and houses.

Here, we focus primarily on the evolution of debris flow gullies affected by the 2008 Wenchuan earthquake. For more details regarding the main features of debris flows in the Wenchuan earthquake-hit areas, readers are referred to the exhaustive reviews published by Tang et al. (2009, 2012) and Xu et al. (2012). As described above, the debris flow became the major secondary geological disaster in the earthquake-affected areas. Compared with other debris flows, the post-earthquake debris flows have specific characteristics, which are their large scale, long distance, high frequency, rapid movement, and destructive impact. Therefore, a high-performance computing method is required to perform simulations of such debris flows.

With its distinctive features of pure meshfree Lagrangian and continuum mechanics, the SPH method can accurately predict the long run-out of the debris flows at each instant, and therefore can provide preliminary estimates of the destructive impacts of such high-frequency debris flows in earthquake-affected areas.

The SPH method

Basic equations for SPH

Smoothed particle hydrodynamics is a pure meshfree Lagrangian numerical technique originally applied to the continuum scale. The state of the meshfree systems are represented by a set of arbitrarily distributed particles. In addition, as a pure Lagrangian method, the moving particles possess material properties (e.g., mass, density, and stress tensor) (Lucy 1977). Based on the material properties carried by the moving particles, the field variables (e.g., density, velocity, and energy) can be obtained in SPH form through two kinds of approximation: kernel and particle.

By introducing a smoothing kernel function W , the kernel approximation is used to approximate the function of field variables through an integral representation. In this way, the field variables can be presented in a continuous form. The kernel approximation is expressed as follows:

$$\langle f(x) \rangle = \int_{\Omega} f(x') \cdot W(x - x', h) \cdot dx' \tag{1}$$

where $f(x)$ is a function of the particle position vector x , the angle brackets $\langle \rangle$ denote a kernel approximation, Ω is the support area of a particle with position vector x , x' is a neighboring particle position vector in Ω , and W is the smoothing kernel function. In this work, the cubic B-spline function (Monaghan and Lattanzio 1985) was selected as the smoothing kernel function:

$$W(R, h) = \alpha_d \times \begin{cases} \frac{2}{3} - R^2 + \frac{1}{2}R^3 & 0 \leq R < 1 \\ \frac{1}{6}(2 - R)^3 & 1 \leq R < 2 \\ 0 & R \geq 2 \end{cases} \tag{2}$$

where α_d is the normalization factor; in two-dimensional space, $\alpha_d = 15/7\pi h^2$. R is the normalized distance between particles x and x' , $R = |x - x'|/h$, and h is the smoothing length defining the size of the influence area of W , which has significant influence on the numerical accuracy and efficiency. Based on a parametric study in this work, a smoothing length of $h = 2 \cdot \Delta x$ was found to provide satisfactory results, and is selected throughout this paper, where Δx is the initial particle spacing.

As mentioned previously, the problem domain in SPH is represented by a set of particles associated with material properties. By estimating field variables on these arbitrarily distributed particles within the influence domain, the particle approximation discretizes the continuous form of the SPH kernel approximation into a summation of the neighboring particles as follows:

$$\langle f(x) \rangle = \sum_{j=1}^N m_j \frac{f(x_j)}{\rho_j} W(x - x_j, h) \tag{3}$$

where N is the total number of particles within the influence domain of the particle at point x , m_j is the mass of neighboring particles, and ρ_j is the density of neighboring particles. Particle approximation indicates that the value of field variables at a particle can be estimated by the average value of all of the particles within the influence domain.

Governing equations for SPH

Based on the Lagrangian description, the governing equations (i.e., conservation equations of mass and momentum)

for dynamic fluid flows can be written as a set of partial differential equations. With the kernel and particle approximation techniques mentioned above, the partial differential equations for dynamic fluid flows can be rewritten in many forms of the SPH formula.

Generally, there are two approaches to evolve density in the SPH method: summation density and continuity density (Liu and Liu 2010). The continuity density, which was widely applied in the previous work, has high computing efficiency and is suitable for the algorithms of parallel processors. However, it is affected by high-frequency density oscillations (Pasculli et al. 2013). In this article, directly derived from Eq. (3), the density at particle i was obtained using the summation density approach. This approach conserves mass exactly, in which the density of the central particle is calculated by summing over the contributions of the neighboring particles as follows:

$$\rho_i = \sum_{j=1}^N m_j W_{ij}. \quad (4)$$

However, the summation density approach will suffer from particle deficiency at the edge of the fluid domain, and will eliminate the density of the concerned particles and thus affect the accuracy of the results. To remedy this problem, a normalization formulation for the density approximation was applied to improve the accuracy of this approach (Randles and Libersky 1996):

$$\rho_i = \frac{\sum_{j=1}^N m_j W_{ij}}{\sum_{j=1}^N \frac{m_j}{\rho_j} W_{ij}}. \quad (5)$$

The motion equation in SPH form applied here to estimate the acceleration is expressed as follows:

$$\frac{du_i^\alpha}{dt} = \sum_{j=1}^N m_j \left[\frac{\sigma_i^{\alpha\beta}}{(\rho_i)^2} + \frac{\sigma_j^{\alpha\beta}}{(\rho_j)^2} \right] \frac{\partial W_{ij}}{\partial x_j^\beta} + F_i \quad (6)$$

where u^α is the velocity component, $\sigma^{\alpha\beta}$ is the stress tensor, and F is the external force vector (i.e., gravity force). The superscript values α and β denote the different coordinate directions.

The total stress tensor in Eq. (6) is made up of two parts, defined as:

$$\sigma^{\alpha\beta} = -p\delta^{\alpha\beta} + \tau^{\alpha\beta} \quad (7)$$

where p represents the isotropic pressure, and $\tau^{\alpha\beta}$ represents the viscous stress tensor.

Here, we employed a strict incompressibility condition for fluids, and a pressure Poisson equation obtained by Shao and Lo (2003) was adopted to obtain the pressure p :

$$\nabla \left(\frac{1}{\rho_*} \nabla P_{t+1} \right) = \frac{\rho_0 - \rho_*}{\rho_0 \Delta t^2} \quad (8)$$

where ρ_0 and ρ_* are the initial and temporal fluid density of each particle, and P_{t+1} is the particle pressure of time $t+1$.

With the pressure term in hand, the stress tensor can be obtained through Eq. (7). The acceleration of the particles can then be calculated using motion equation, and the particle velocity and position can be updated using the leapfrog (LF) iterative scheme.

Constitutive model for debris flow

The distinctive feature of debris flows that makes them different from other natural unconfined flows is that they are a mixture of water, clay, and granular materials (Laigle and Coussot 1997). Atkin and Craine (1976) and Iverson (1997) proposed a mixture theory that used different rheological models for the liquid and solid phases. Although the mixture theory can accurately reflect the characteristics of debris flow, such highly theoretical models are too complex to be applicable in numerical solutions. In order to better apply the rheological models for simulating debris flow, an equivalent single-phase homogeneous rheological model was proposed. Chen (1988) simulated debris flows with a generalized viscoplastic fluid (GVF) model, which considered the problem as a single-phase material. Laigle and Coussot (1997) found that a debris flow mixture with high clay fraction behaved as viscoplastic fluids, and that a single-phase model was a good approximation to predict this behavior. It has been shown that non-Newtonian rheological models can replicate the behavior of debris flows (Rickenmann et al. 2006; Shieh et al. 1996). The most commonly used are the Bingham and Herschel–Bulkley models. Laigle et al. (2007), Minatti and Pasculli (2011), and Pasculli et al. (2013) successfully applied the Herschel–Bulkley for mudflow simulation. Liu and Mei (1989) used the Bingham fluid theory in their research on fluid mud with a high concentration of cohesive clay particles. Laboratory studies by Kaitna et al. (2007) found that the Bingham rheological model was an appropriate tool for describing the flow behavior of coarse-grained debris flows. Marr et al. (2002) and Beguería et al. (2009) applied the Bingham fluid theory in numerical simulations of the dynamic behavior of debris flows. In this work, we chose the Bingham model as the constitutive law, into which the Mohr–Coulomb yield criterion can easily be introduced.

For Bingham fluid, the relationship between the shear strain rate and shear stress is given by:

$$\tau = \eta \dot{\gamma} + \tau_y. \quad (9)$$

To simplify the calculation, the concept of equivalent viscosity (Uzuoka et al. 1998) was introduced:

$$\eta' = \eta + \tau_y / \dot{\gamma} \tag{10}$$

$$\dot{\gamma} = \sqrt{\frac{\dot{\gamma}^{\alpha\beta} \dot{\gamma}^{\alpha\beta}}{2}} \tag{11}$$

where τ is the shear stress, τ_y and η are the Bingham yield stress and viscosity, η' is the equivalent viscosity, $\dot{\gamma}$ is the shear strain rate, and $\dot{\gamma}^{\alpha\beta}$ is the shear strain rate tensor.

Combined with the Mohr–Coulomb yield criterion, Eq. (9) can be rewritten as:

$$\tau = \eta \dot{\gamma} + \sigma \tan \varphi + c \tag{12}$$

where σ is the pressure, φ is the frictional angle, and c is the cohesion.

From Eq. (10), it can be deduced that an infinite equivalent viscosity coefficient arises when the shear strain rate tends to zero, and therefore, a maximum for equivalent viscosity is required to avoid such an infinite value. In this work, the maximum equivalent viscosity is set to be a large value ($\eta_{\max} = 1.0 \times 10^8$ Pa s). The equivalent viscosity can then be expressed as:

$$\begin{aligned} \eta' &= \eta + \tau_y / \dot{\gamma} & \text{if } \eta' > \eta_{\max} \\ \eta' &= \eta_{\max} & \text{if } \eta' \leq \eta_{\max} \end{aligned} \tag{13}$$

where η_{\max} is the maximum of the equivalent viscosity.

Treatment of boundary conditions and free surfaces

Boundary treatment is still challenging in the context of the SPH method and requires special care. In this paper, a general method following Randles and Libersky (1996) was adopted to treat the 2D geometric boundary, which used ghost particles as a mirror image of the interior particles with respect to the boundary. The ghost particles on the boundary line carry the same field variable as the interior SPH particles, and the values are then calculated by smooth interpolation. This boundary treatment method was applied by our research team in previous work (Huang et al. 2012b, 2013). Another common method, the no-slip boundary condition (Morris et al. 1997), has also been applied by Pasculli et al. (2013) and our research team (Dai et al. 2014).

Due to the lack of particles in the outer region of the free surface, the density of particles on the free surface decreased abruptly. The free surface in the SPH method was thus identified by the following criterion, and the zero-pressure boundary condition was then applied:

$$(\rho_*)_i < 0.99 \rho_0. \tag{14}$$

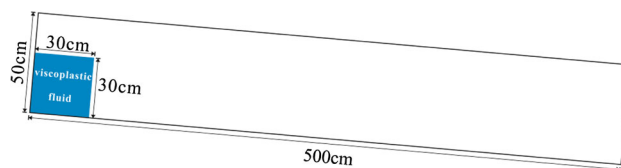


Fig. 4 Model of viscoplastic fluid

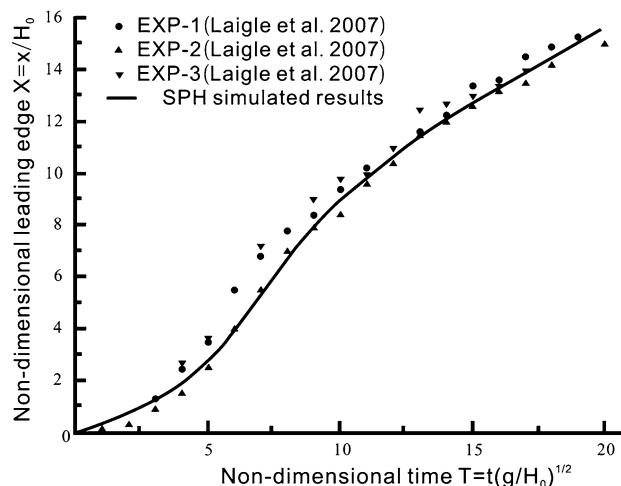


Fig. 5 Comparison of SPH simulation and test in viscoplastic fluid

Table 1 Parameters used in the SPH simulation of the experimental viscoplastic fluid flow

Density ρ (kg/m ³)	1,410
Cohesion c (kPa)	0.2
Angle of internal friction φ (°)	18
Viscosity coefficient η (Pa s)	21.0
Acceleration of gravity g (m/s ²)	9.8
Unit time step Δt (s)	0.0025
Time step (n)	1,500

Program validation

We have previously addressed the classic free surface problem in the fluid dynamics of a dam break (Huang et al. 2012b). The results of the SPH simulation showed excellent agreement with the dam break test results, reflecting the accuracy of the proposed SPH modeling technique. In addition, the SPH modeling technique proposed here was also previously applied by the authors (Huang et al. 2011, 2012b, 2013) for flow analysis of liquefied soils, run-out analysis of flow-like landslides, and flow slide prediction for landfills. The simulation results all demonstrate good agreement with the data obtained from laboratory tests and field observations. Prior to the application of the SPH model for the simulation of debris flows as described in this

paper, a SPH simulation of viscoplastic fluid was also validated against experimental results by Laigle et al. (2007). The model fluid of this experiment was a mixture of kaolin clay and water, which was used to measure the propagation of the viscoplastic fluid flow. Figure 4 shows the experimental modeling of the viscoplastic fluid, which was also used in the SPH simulation. Table 1 shows the parameters used in the SPH simulation of the viscoplastic fluid. The numerical model includes 7,540 particles in total, with 3,364 particles for the moving debris flow and 4,176 particles for the fixed boundary. The space between

two particles is 0.005 m. As shown in Fig. 5, there is good agreement between the SPH simulation and the viscoplastic fluid flow experiment results. The notations represented in this paper are normalized time $T = t(g/H_0)^{1/2}$ and leading edge $X = x/H_0$, where H_0 is the initial height of the viscoplastic fluid, and x and t are the distance of the surge front from the axis and the flow time, respectively. The benchmark studies that were conducted not only demonstrate the accuracy of the SPH program, but also indicate that the proposed SPH model is capable of simulating debris flows with satisfactory accuracy.

Fig. 6 Simulated propagation process of the Bayi Gully debris flow

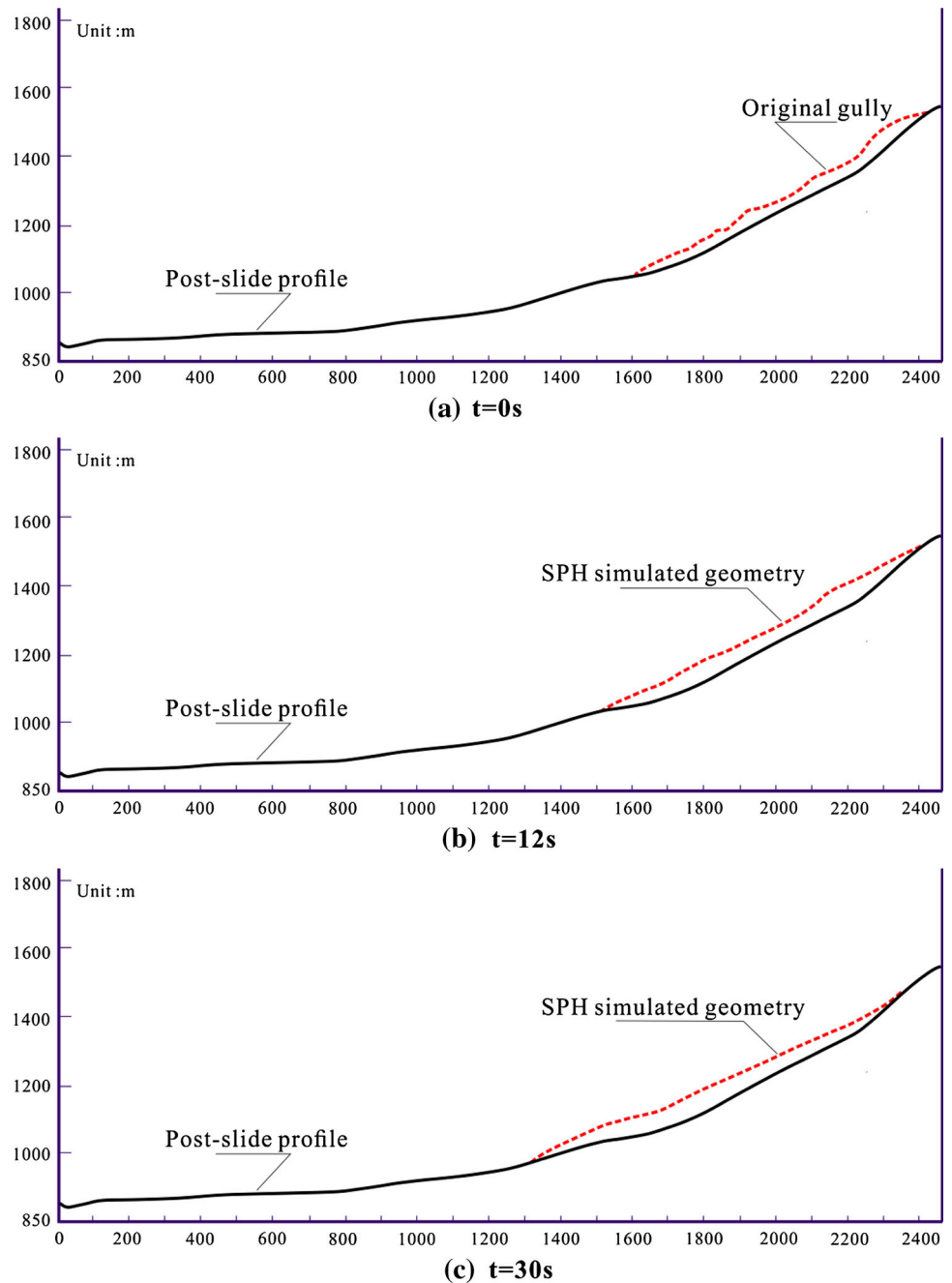
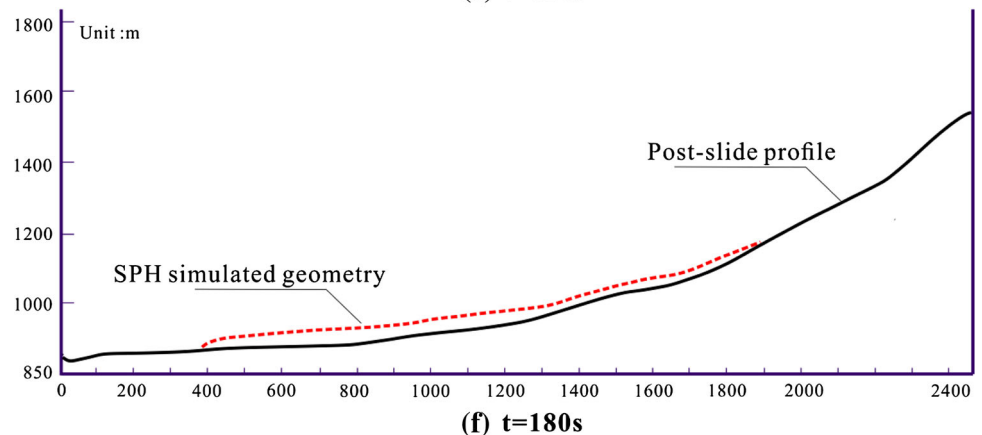
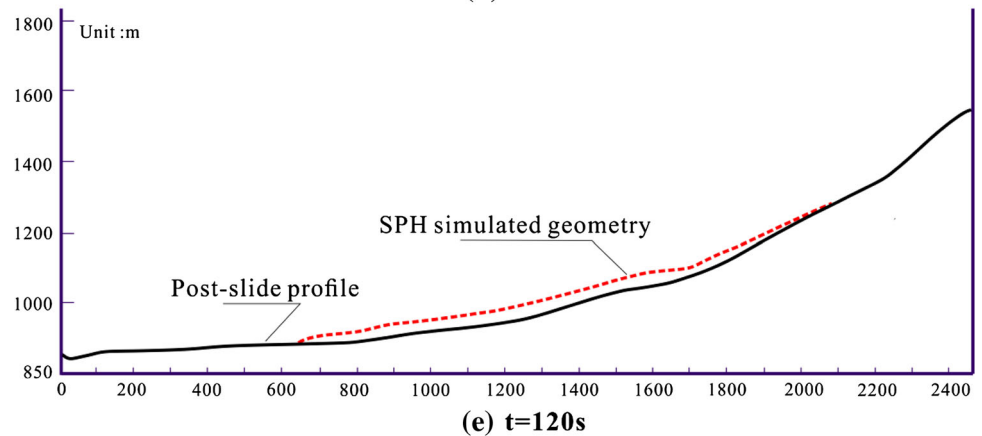
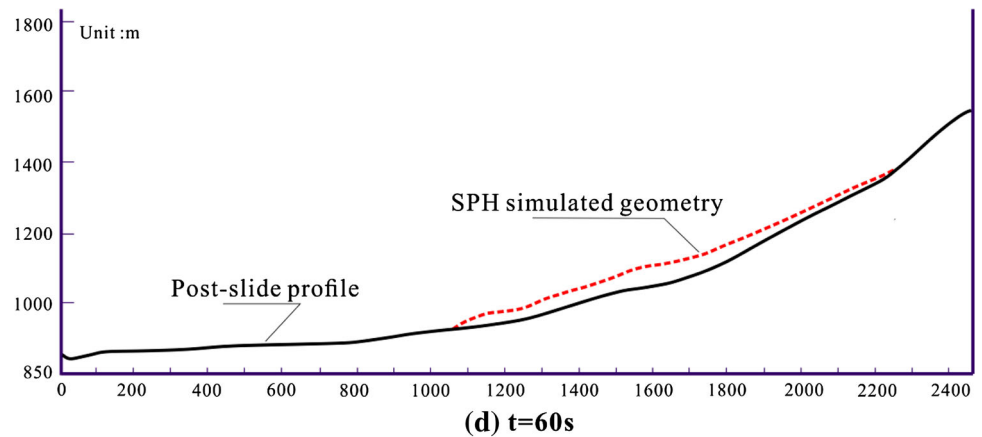


Fig. 6 continued



Applications of SPH modeling to the post-earthquake debris flows in Wenchuan earthquake-affected areas

The Bayi and Wenjia Gully debris flows are two typical large-scale '813' debris flows in the Wenchuan earthquake-affected area. Propagation analysis of these debris flows was conducted to apply the SPH method to the complex large-scale flows. This work may provide a preliminary estimate of the destructive impact of such large-scale debris flows in the areas affected by the earthquake.

Bayi Gully debris flow

Based on the proposed modeling technique, an SPH simulation of the propagation of the Bayi Gully debris flow was conducted to analyze the dynamic characteristics of such post-earthquake debris flows. The pre- and post-earthquake topography is shown in Fig. 7 (Ma et al. 2011). The numerical model included 7,784 particles in total, comprising 5,177 particles for the moving debris flow and 2,607 for the fixed boundary. The space between two particles is 3 m. Just as in the real situation, the debris flow

Fig. 6 continued

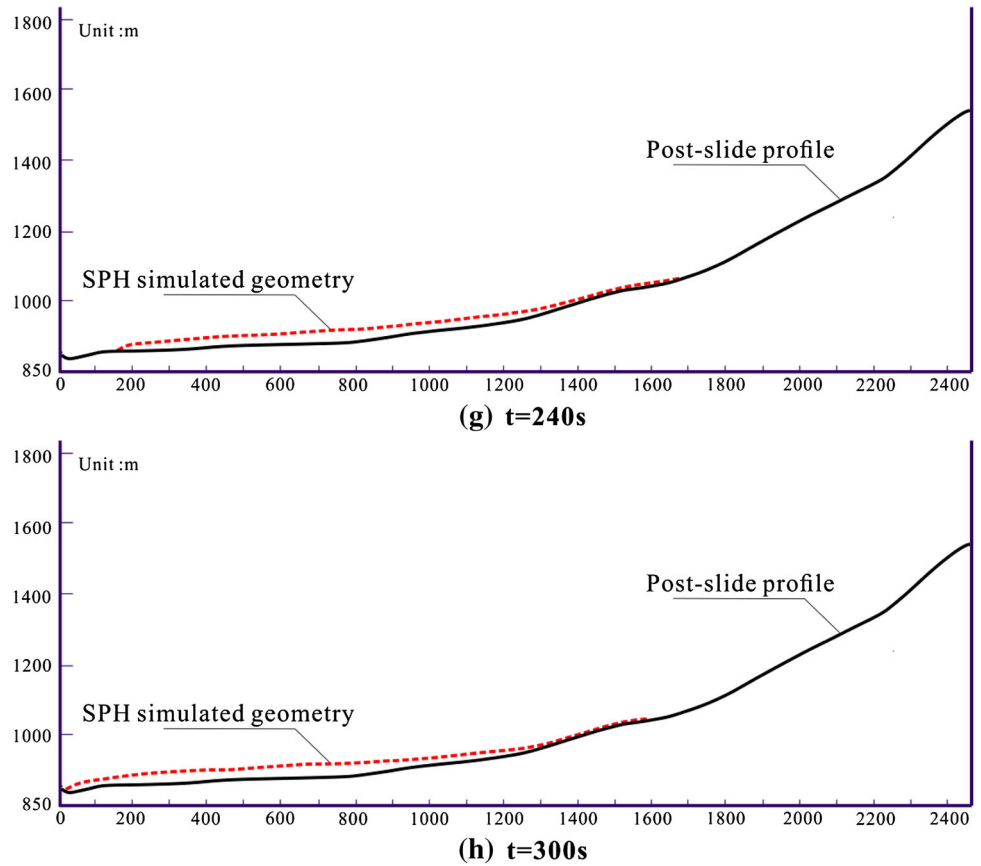


Fig. 7 Pre- and post-failure profile and comparison of SPH simulation and surveyed data for the Bayi Gully debris flow

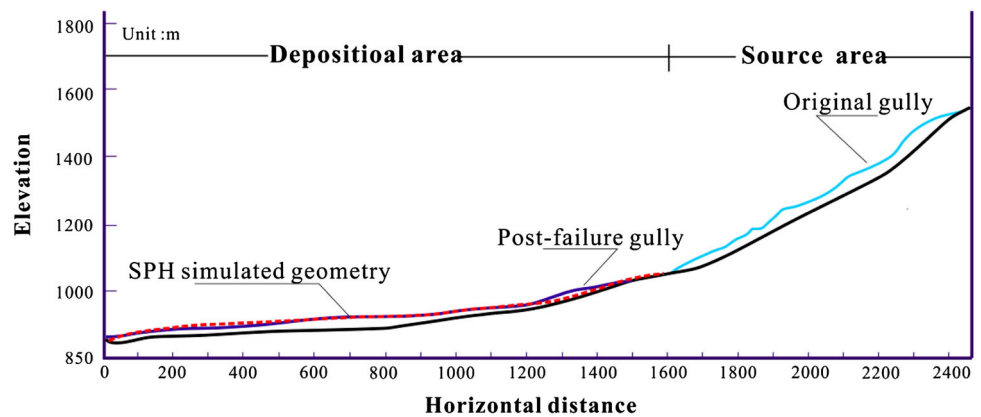


Table 2 Parameters used in the SPH simulation of the Bayi Gully debris flow

Density ρ (kg/m ³)	1,850
Cohesion c (kPa)	0.8
Angle of internal friction φ (°)	34
Viscosity coefficient η (Pa s)	112.0
Acceleration of gravity g (m/s ²)	9.8
Unit time step Δt (s)	0.03
Time step (n)	10,000

Table 3 Sensitivity analysis of the parameters used in the Bayi Gully

Case	Viscosity coefficient η (Pa s)	Shear strength		Run-out L (m)
		c (kPa)	φ (°)	
1	112	0.8	34	1,596
2	22.4	0.8	34	1,653
	560	0.8	34	1,515
3	112	0.4	17	2,254
	112	1.6	68	1,074

Table 4 Parameters used in the SPH simulation of the Wenjia Gully debris flow

Density ρ (kg/m ³)	2,100
Cohesion c (kPa)	0.8
Angle of internal friction φ (°)	34
Viscosity coefficient η (Pa s)	136.0
Acceleration of gravity g (m/s ²)	9.8
Unit time step Δt (s)	0.03
Time step (n)	20,000

particles can be deformed both horizontally and vertically with gravitational force applied in a vertical direction only. For the simulation of the observed debris flow events, it was not possible to measure the rheological properties of the flow directly. As a result, correct application the Bingham flow model and its parameters became a major challenge. The shear strength parameters and the frictional angle c and cohesion φ for debris flows have been studied in previous technical literature (Shieh et al. 1996; Van Asch et al. 2004; Chen and Ou 2006; Hsiao et al. 2011). The shear strength parameters c and φ adopted in the SPH simulations are based on the average values of experimental results from the above literature and are shown in Table 2. Marr et al. (2002) used the Bingham model to simulate debris flows considering a range of dynamic viscosities from 30 to 300 Pa s. In similar simulations by Beguería et al. (2009), the results showed that viscosity coefficients (η) of 525 and 67 Pa s were the best parameters for the debris flow event in the Wartschenbach torrent on August 16, 1997, and in the Faucon torrent on August 5, 2003. Based on previous research, $\eta = 112$ Pa s was taken as the most appropriate parameter for the debris flow event in Bayi Gully. As shown in Fig. 6, the SPH simulation reproduced the entire propagation process of the debris flow in the Bayi Gully. Figure 7 shows a comparison of the SPH-simulated geometry and gully configuration after the debris flow investigated by Ma et al. (2011). It is evident that the simulated run-out, coverage, and thickness of the sediment have a high degree of similarity with those surveyed on-site.

In order to identify the parameter with the greatest affect on the run-out, sensitivity analyses of the parameters were conducted with a different cohesion, angle of internal friction, and viscosity coefficient. Table 3 shows the three cases with different sets of parameters. Parameters in case 1 were those adopted in this manuscript, and these parameters were then set as the benchmark. The other two cases were considered for comparison. c and φ in case 2 were the same as those for the benchmark case 1. Only the viscosity coefficient η was varied by increasing and decreasing it fivefold. In the Bingham flow constitutive

model, c and φ contribute to the shear strength of the material. Therefore, for case 3, both c and φ were increased and decreased twofold, whereas η was the same as in the benchmark. In case 2, the run-out was reduced slightly, by 5.1 %, when η was increased fivefold. The run-out increased by 3.6 % as η was reduced fivefold. However, when c and φ were doubled in case 3, the run-out dropped significantly, by 26.7 %, while when c and φ were reduced by half in case 3, the run-out increased dramatically, by 41.2 %. It can therefore be concluded that the shear strength parameters in the Bingham flow have the greatest effect on the simulated run-out. However, the viscosity coefficient is sensitive to the simulated run-out, which is consistent with the conclusions of previous research by the authors (Huang et al. 2013).

Wenjia Gully debris flow

As performed previously for the Bayi Gully, a numerical simulation of the whole propagation process of the Wenjia Gully was conducted using the SPH method. Based on a site investigation of the Wenjia Gully carried out by the authors, the pre-earthquake topography combined with the post-earthquake topography is shown in Fig. 9. The numerical model includes 7,762 particles, comprising 4,126 particles for the moving debris flow and 3,636 for the fixed boundary. As in the previous case, a calibration procedure was followed to simultaneously determine the best set of parameters, and $\eta = 136$ Pa s was found to be the best parameter for the debris flow event in the Wenjia Gully. The parameters used in the SPH simulation of Wenjia Gully debris flow are listed in Table 4, and the simulation results of the propagation of the debris flows are depicted in Fig. 8. Figure 9 shows the comparison of the post-failure profile calculated from the SPH simulation and the measured configuration. This was used to evaluate the accuracy of SPH simulation for the Wenjia Gully debris flow, which clearly showed a high degree of similarity with the run-out, the coverage, and thickness of the sediment surveyed on-site.

Conclusions

The 2008 Ms 8.0 Wenchuan Earthquake triggered a large number of co-seismic landslide depositions in the gullies of the mountain areas, endangering the surrounding regions. Induced by heavy rainfall, a large amount of loose solid material was mobilized into numerous debris flows during the rainy seasons, resulting in serious economic losses and many casualties. As a result, debris flows are considered the most significant secondary geological disaster in the earthquake-affected areas, characterized by their large

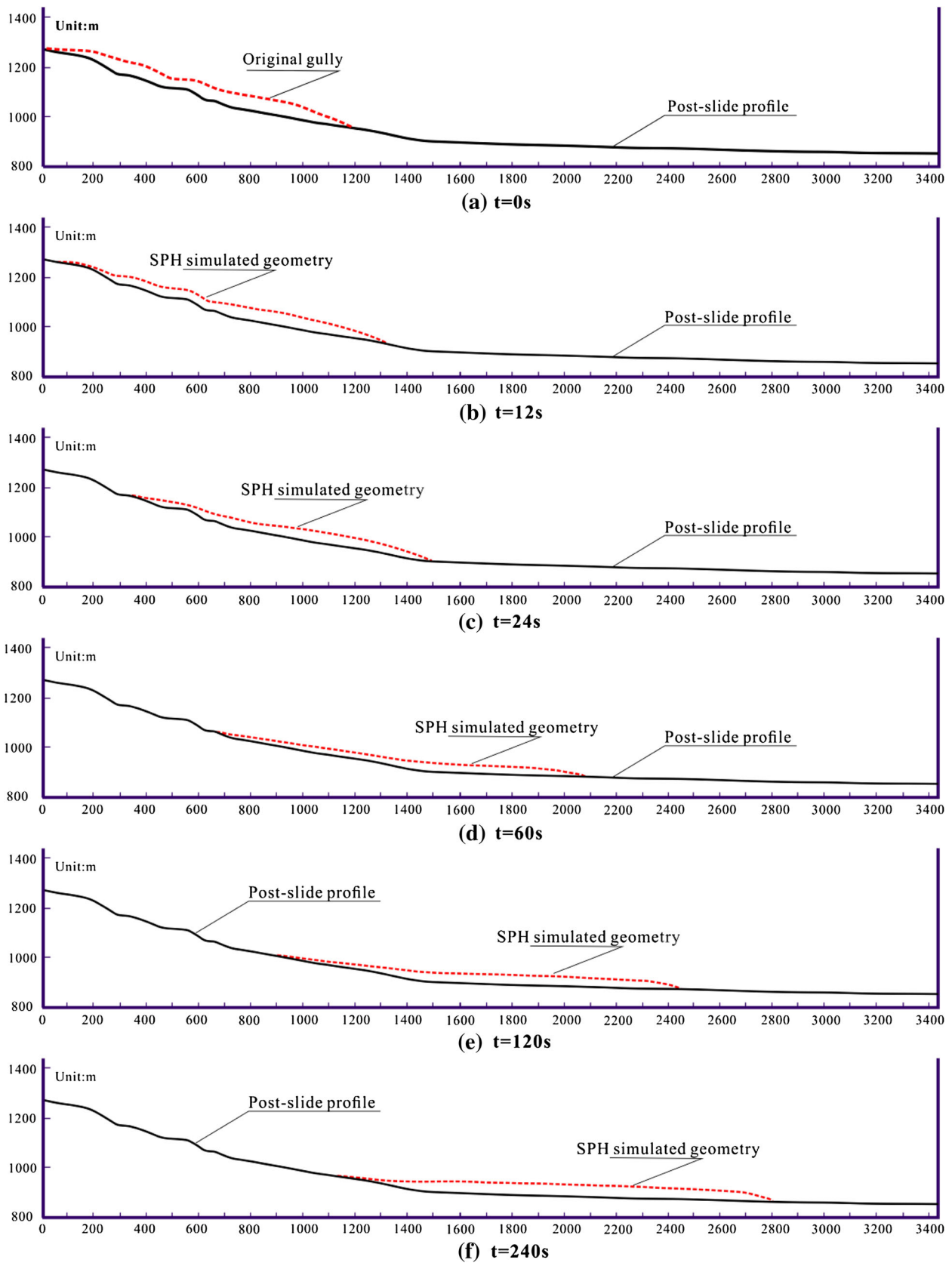


Fig. 8 Simulated propagation process of the Wenjia Gully debris flow

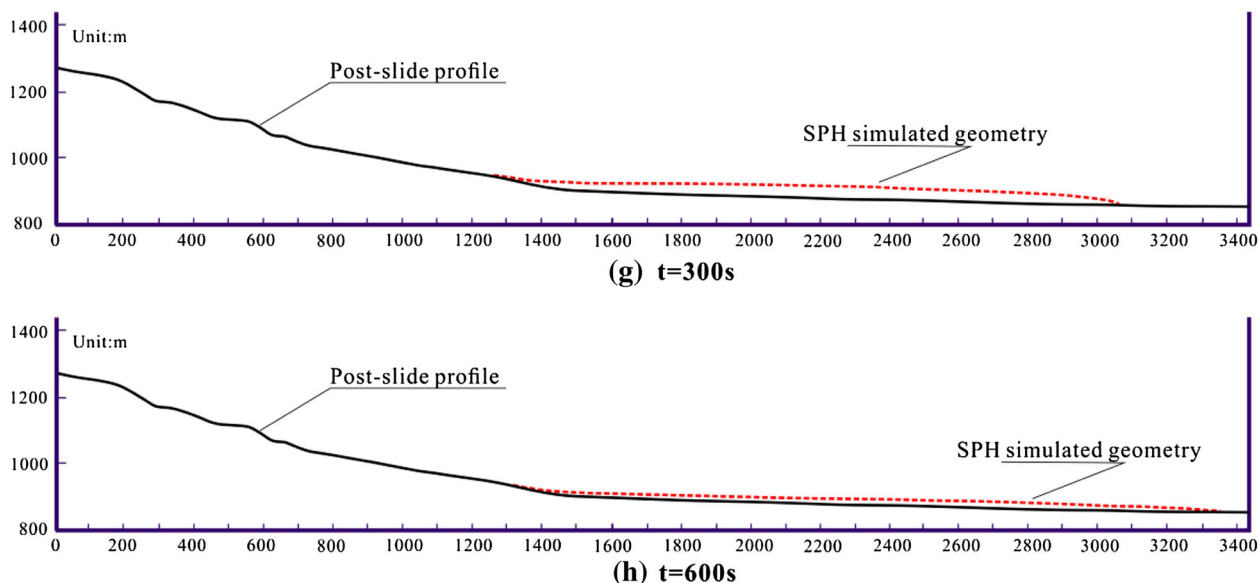


Fig. 8 continued

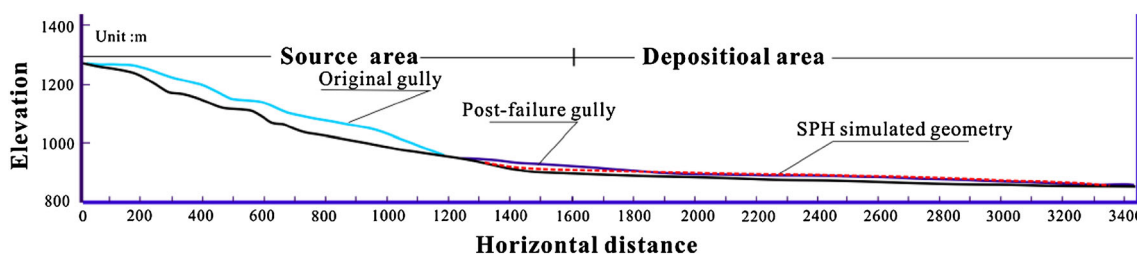


Fig. 9 Pre- and post-failure profile and comparison of SPH simulation and surveyed data for the Wenjia Gully debris flow

scale, long run-out, high frequency, rapid movement, and destructive impact. Therefore, the dynamic analysis of such post-earthquake debris flows is important for hazard assessment and post-disaster reconstruction in earthquake-affected areas.

In this paper, we proposed a high-performance SPH modeling technique to simulate the propagation process of post-earthquake debris flows. The primary advantage of SPH is that it is a pure meshfree method that can bypass the need for a numerical mesh, and thereby avoid severe mesh distortion caused by large deformations. Furthermore, as this is a true Lagrangian method, the moving particles possess material properties (e.g., mass, density, and stress tensor), and can track the motion characteristics of post-earthquake debris flows. The governing equations, including conservation equations of mass and momentum, were framed in SPH formulations with kernel and particle approximation. In addition, the Bingham rheological model was applied to describe the relationship between material stress rates and particle motion velocity. Together, these

features contribute to the potential of the SPH method for post-earthquake debris flow research.

Benchmark analysis was conducted to validate the accuracy of the proposed SPH model. The SPH simulation results of a viscoplastic fluid flow were compared with experimental results from previous literature. The good agreement of the SPH simulation results and experimental results of viscoplastic fluid flow suggests that the proposed SPH method can be used for the simulation of debris flows. Following the benchmark studies, the SPH modeling technique was also used to analyze the propagation process of two typical post-earthquake debris flows in the Bayi and Wenjia Gullies. For the two flows, the simulation results had a high degree of similarity with those of surveys conducted on-site, which indicates that the proposed SPH method is capable of simulating such post-earthquake debris flows. Furthermore, these simulation results also provide preliminary scientific information for hazard assessment and post-disaster reconstruction in the earthquake-affected areas.

Acknowledgments This work was supported by the National Natural Science Foundation of China and the Japan Society for the Promotion of Science under the China-Japan Scientific Cooperation Program (Grant No. 41211140042), the National Basic Research Program of China (973 Program, Grant No. 2012CB719803), the National Science Fund for Distinguished Young Scholars of China (Grant No. 41225011), and the Chang Jiang Scholars Program of China.

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