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ORIGINAL PAPER

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Cohesion of unsaturated residual soils as a function of volumetric water content

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Abstract Soil cohesion changes with the moisture state of a soil. This paper presents an empirical equation to predict the cohesive component in the shear strength of unsaturated residual soils as an exponential function of volumetric water content. The formulation originated from a multiple linear-regression analysis for data sets obtained from shear tests using undisturbed soils with varying moisture contents. The empirical equations can realistically predict the reduction in soil cohesion due to wetting ($R^2 = 0.88$, 0.93). The methodology described in this paper provides a convenient alternative to the quantitative estimation of unsaturated shear strength, especially in an engineering practice such as a slope stability analysis as no matrix suction data are required.

Keywords Unsaturated soil · Shear strength · Cohesion · Volumetric water content

Résumé La cohésion des sols change avec leur teneur en eau. L'article présente une relation empirique permettant d'exprimer la valeur de cohésion de sols résiduels non saturés comme une fonction exponentielle de la teneur en eau volumétrique. La formulation résulte d'une analyse par régression linéaire de séries de données provenant d'essais de cisaillement sur des sols non remaniés présentant diverses teneurs en eau. Les équations obtenues sont capables de représenter correctement la diminution de la cohésion résultant de l'humidification d'un sol. La méthodologie présentée dans l'article permet d'obtenir une estimation de la résistance au cisaillement en conditions non saturées, en particulier pour des applications relatives à des analyses de stabilité des pentes, sans faire appel à des données concernant l'état de succion du sol.

Mots clés Sol non saturé · Résistance au cisaillement · Cohésion · Teneur en eau volumétrique

Introduction

The cohesive strength of an unsaturated soil plays an important role in the stability of both natural and artificial soil slopes. Reduction of the soil cohesion due to wetting can cause shear deformation of the slopes at a previously unsaturated shallow layer (Krahn et al. 1989; Rahardjo et al. 1995; Rao 1996; Kim et al. 2004). For this reason, determination of the critical condition for sliding requires a slope stability analysis including an effect for the loss of cohesion.

For prediction of the shear strength of an unsaturated soil, two approaches had been proposed by Bishop (1959—effective stress approach) and Fredlund et al. (1978—independent stress variables approach). As a consequence, optimization of the parameters and/or modifications to the formulae have become important issues. Many researchers have demonstrated both theoretical and empirical formulations to estimate unsaturated shear strength, e.g. the verification of the non-linear change in cohesion of an unsaturated soil (Escario and Sáez 1986; Gan et al. 1988); an empirical formulation based on Bishop's concept (Öberg and Sällfors 1997; Khalili and Khabbaz 1998); an analytical model based on a soil–water retention curve (Fredlund et al. 1995; Vanapalli et al. 1996); the prediction of soil cohesion using hyperbolic equation (Miao et al. 2002; Lee et al. 2003).

All of these established models or equations have employed a function of matrix suction (soil pore-air pressure minus pore-water pressure) as a parameter of soil moisture condition. As a consequence, a prediction of unsaturated shear strength inevitably requires a value for the matrix suction of the soil. However, in engineering practice, generally no reliable values for matrix suction are available.

In this paper, an empirical equation is suggested for the prediction of unsaturated shear strength as a function of volumetric water content. Measurement of volumetric water content generally involves a much easier procedure than that for matrix suction and allows the estimation of unsaturated shear strength without any advanced soil testing such as a suction-controlled triaxial tests.

The equation was obtained from a multiple linear-regression analysis of the results of shear box tests using undisturbed residual soils with varying moisture contents. Indeed the equation lacks a theoretical validity from a physical standpoint, but has practical advantages especially for geotechnical engineering purposes. The equation makes it possible to analyse the stability of slopes even if no matrix suction data are available.

Soils and experimental procedure

Soils for experiments

Two undisturbed residual soils obtained from natural hillslopes in Mt Kanozan, Boso Peninsula, central Japan, were examined. These soils originated from weakly consolidated mid-Pleistocene sandstone and siltstone referred to as the Ichijuku Formation and Awakura Formation, respectively. The hilly terrain from which they were taken includes several slopes with scars from the landslides which occurred following torrential rainfall on 1 August 1989 (Matsushi and Matsukura 2004).

Soil samples for the shear tests were taken from the scars of landslides typical in this area. Hereinafter, the soil samples originating from the sandstone and the siltstone are referred to as 'sand-soil' and 'silt-soil', respectively. For sampling, iron rings with a 60 mm inside diameter and 20 mm high were horizontally inserted into the excavated soil layer to extract the undisturbed soil cores. Twenty eight samples of sand-soil and 24 of silt-soil were obtained from depths of between 0.7 and 0.85 m and 0.35 and 0.4 m, respectively, roughly corresponding to the depths of the slip surfaces.

Table 1 shows the physical properties of the soils. It can be seen that they vary in their grain size distribution; the sand-soil having 83.6% sand and 16.4% silt and clay, whereas the silt-soil has 34.6% sand and 65.4% silt and clay. Reflecting the grain size, the sand-soil has a greater dry unit weight (12.8 kN/m^3) and hence smaller porosity (51.6%) than the silt-soils (11.0 kN/m^3 and 58.3%, respectively).

Adjustment of moisture content and shear test

The moisture contents of the samples were adjusted to six conditions by drying in a low-temperature oven and by wetting with a given amount of water. After this preparation, the samples were left sealed in plastic bags for at least 1 week to allow moisture equilibration.

The prepared soil specimens were tested in an isometric circular shear-box which horizontally deforms the specimen at the height of 10 mm (i.e. the specimen is sheared in half). Normal stress was applied vertically to the specimen by hanging a weight on a loading ram.

Single stage shear box tests (consolidated-drained shearing) with a strain control method (1 mm/min strain rate) were conducted in four different conditions, i.e.

Table 1 Physical properties of the soils

	Sampling depth (cm)	Grain-size distribution (%)			Specific gravity (–)	Dry unit weight (kN/m^3)	Porosity (%)
		Clay	Silt	Sand		Av. ± SD[a]	Av. ± SD[a]
Sand-soil	70–85	8.4	7.9	83.6	2.69	12.8 ± 0.4	51.6 ± 1.4
Silt-soil	35–40	19.4	46.0	34.6	2.68	11.0 ± 0.6	58.3 ± 2.3

[a] $\pm 1\sigma$ ($n = 28$ for the sand-soil, $n = 24$ for the silt-soil)

normal stresses of 10, 20, 30 and 40 kPa. The volumetric water content and degree of saturation of the specimens were calculated from the difference in weight between a specimen just after the shear test and in a completely dried condition.

Results

The shear strength of the soils clearly decreased with an increase in moisture content. Table 2 summarizes the test results for the specimens with varied moisture

Table 2 Moisture contents and shear test results of each specimen group

Group	Moisture condition	Volumetric water content (m^3/m^3)		Degree of saturation (%)		Normal stress (kPa)	Shear strength (kPa)
Sand-soil							
A	Oven-dried (40°C)	0.03	Av. (0.04)	6.7	Av. (7.6)	10	33.2
		0.03		4.8		20	40.5
		0.03		6.4		30	53.8
		0.07		12.4		40	65.6
B	Air-dried (25°C)	0.11	(0.11)	19.6	(20.7)	10	32.5
		0.13		24.3		20	36.8
		0.10		20.0		30	40.6
		0.09		18.8		40	40.1
C	Natural water content	0.18	(0.16)	35.7	(30.3)	10	20.4
		0.16		31.6		20	26.2
		0.15		28.8		30	29.9
		0.14		25.2		40	40.1
D	Add 6 ml water	0.24	(0.24)	47.9	(47.2)	10	13.3
		0.25		49.4		20	20.0
		0.24		46.2		30	25.9
		0.24		45.4		40	32.3
E	Add 13 ml water	0.37	(0.36)	73.3	(70.2)	10	12.6
		0.36		69.5		20	15.1
		0.36		67.2		30	22.8
		0.36		70.9		40	25.5
F	Capillary saturation	0.40	(0.43)	79.2	(84.4)	10	11.3
		0.42		85.1		20	16.6
		0.43		85.0		30	21.2
		0.40		77.6		40	26.8
		0.44		83.6		10	8.9
		0.46		89.4		20	15.9
		0.41		80.6		30	20.2
		0.46		94.9		40	27.6
Silt-soil							
G	Oven-dried (40°C)	0.15	(0.14)	25.0	(22.0)	10	45.1
		0.06		9.9		20	75.0
		0.15		24.9		30	77.0
		0.18		28.3		40	88.3
H	Air-dried (25°C)	0.28	(0.28)	45.8	(47.0)	10	32.1
		0.29		46.6		20	35.2
		0.29		48.8		30	41.2
		0.28		46.8		40	66.0
I	Natural water content	0.40	(0.38)	68.8	(66.6)	10	15.5
		0.37		66.3		20	22.6
		0.36		64.8		30	29.9
		0.37		66.3		40	32.5
J	Add 5 ml water	0.44	(0.45)	75.1	(75.9)	10	13.7
		0.45		77.4		20	19.3
		0.45		77.6		30	24.6
		0.43		73.6		40	29.5
K	Add 10 ml water	0.47	(0.48)	81.4	(83.3)	10	13.4
		0.48		84.0		20	17.8
		0.49		83.4		30	24.2
		0.47		84.4		40	28.3
L	Capillary saturation	0.53	(0.52)	94.9	(93.1)	10	10.9
		0.52		95.8		20	15.9
		0.53		92.7		30	20.2
		0.49		89.2		40	26.8

conditions (the six moisture conditions referred to as A–F in the sand-soil samples and G–L in the silt-soils). Average volumetric water contents ranged from 0.04 to 0.43 for the sand-soil (7.6–84.4% saturation) and from 0.14 to 0.52 for the silt-soil (22.0–93.1% saturation). Figure 1 shows the results with a simple linear-regression for each of the groups. Table 3 lists the values of *y*-intercept and the inclination of the regression lines (i.e. cohesive strength and angle of shearing resistance in terms of simple linear-regression).

The inclinations of the regression lines are largest in the driest conditions and drastically decrease for the wetter samples, converging at 25–33° in the sand-soil and 27–31° in the silt-soil (Fig. 1; Table 3). In other words, the angle of shearing resistance of the moist soils seems to be constant, independent of volumetric water content, except in the driest condition.

The *y*-intercepts of the regression lines decreased with increasing moisture content and approached a minimum value at the saturated condition (from 25.4 to 4.4 kPa in the sand-soil, 37.8 to 5.2 kPa in the silt-soil, Table 3). Figure 2 shows the relationships between the average volumetric water content in each group and the *y*-intercept of the regression lines. The values of the *y*-intercept tend to decrease linearly on the semi-log scale (i.e. the cohesive strength of the soils exponentially decreases with an increase in volumetric water content).

Table 3 Shear strength parameters obtained by a simple linear-regression for the data set of each specimen group

Sand-soil

Group	A	B	C	D	E	F
y-intercept (kPa)	20.1	25.4	13.2	6.9	7.2	4.4
Inclination (°)	48.4	30.5	32.6	32.7	25.3	29.6

Silt-soil

Group	G	H	I	J	K	L
y-intercept (kPa)	37.8	16.2	10.3	8.4	7.9	5.2
Inclination (°)	53.3	47.7	30.7	28.2	27.5	27.9

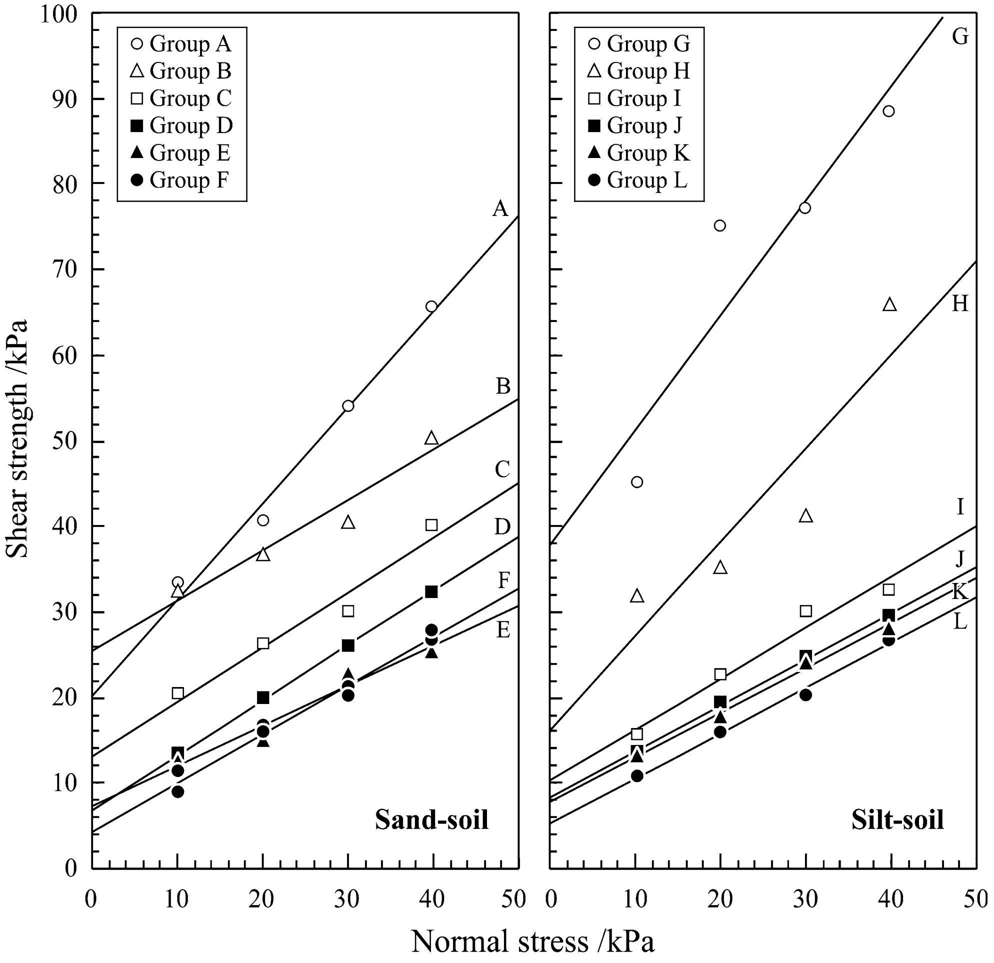

Fig. 1 Results of the shear tests. The *solid lines* indicate the simple linear-regression lines for each specimen group

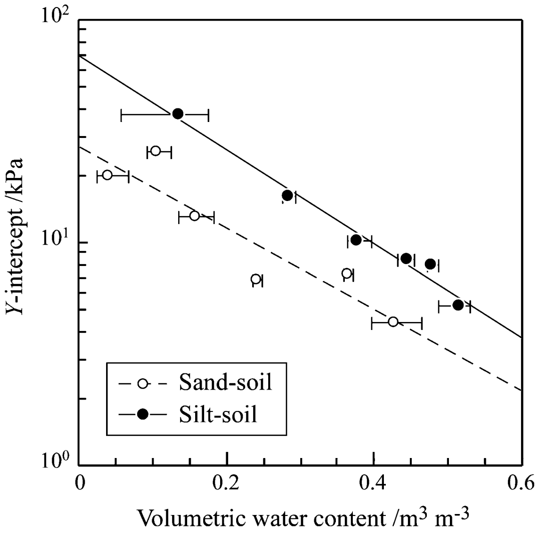

Fig. 2 Relationships between the average volumetric water content and y-intercept of the regression lines shown in Fig. 1

Discussion

Formulation of shear strength as a function of volumetric water content

On the basis of the shear strength reduction characteristics (Figs. 1 and 2), it was assumed that: (1) the angle of shearing resistance takes a constant value; (2) an exponential function is valid to express the relationship between apparent cohesion and volumetric water content.

From this, the regression function can be postulated as follows:

$$\tau = \sigma' \tan \phi' + C\,\mathrm{e}^{-\mu\theta} \tag{1}$$

where τ is shear strength, σ' is net normal stress, θ is volumetric water content, ϕ' is effective angle of shearing resistance (the most suitable constant for each soil), C is a hypothetical maximum value of cohesion (when $\theta = 0$) and μ is a coefficient related to susceptibility of strength reduction ($\mu > 0$).

To determine the values of unknowns (i.e. ϕ', C and μ), Eq. 1 should be linearized as follows:

$$\ln[\tau - \sigma' \tan \phi'] = -\mu\theta + \ln C. \tag{2}$$

The left side of the equation represents a logarithm of the cohesive component of the shear strength. This equation allows a multiple linear-regression analysis, as follows:

1. Assign an arbitrary value to ϕ'.
2. Compute the term $\ln[\tau - \sigma' \tan \phi']$ for every data set of τ and σ'.
3. Conduct a simple linear-regression for the data sets of $\ln[\tau - \sigma' \tan \phi']$ and θ.
4. Repeat 1–3 to seek the most suitable value of ϕ'.
5. Determine the value of ϕ' which gives the highest correlation coefficient.
6. Compute the value of C and μ from the y-intercept ($\ln\ C$) and inclination ($-\mu$) of the regression line.

Regression analyses

In the analyses reported here, the test results of the driest condition were excluded (groups A and G, Table 2) as the angle of shearing resistance in these conditions was significantly larger than in wetter conditions (Fig. 1; Table 3). The reason for this phenomenon is not clear, but in an excessively dry condition, cementation of fines and/or shrinkage of the soil mass may contribute to a greater frictional resistance of the soil aggregates.

Figure 3 illustrates the results of the regression analyses. Figure 3a shows the change in the correlation coefficient of Eq. 2 with respect to the varying values of ϕ'. Substituting the most suitable values of ϕ' (peak values in Fig. 3a: 28.3° for the sand-soil and 27.7° for the silt-soil), the relationships between $\ln[\tau - \sigma' \tan \phi']$

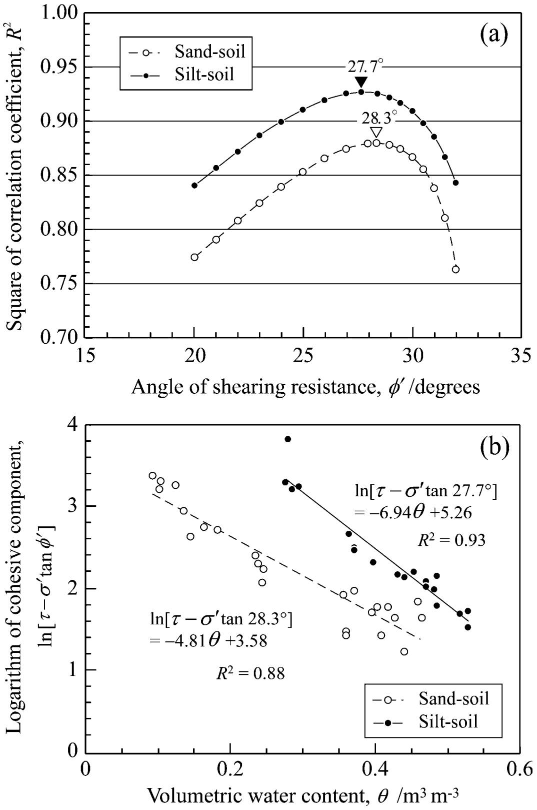

Fig. 3 Change in square of correlation coefficient of Eq. 2 with respect to the varied value of ϕ' (**a**), and the relationships between logarithm of cohesive shear strength and volumetric water content substituting the most suitable values of ϕ' (**b**)

Table 4 Optimum parameter values for Eq. 1

	ϕ' (°)	C (kPa)	μ (–)	R^2
Sand-soil	28.3	35.8	4.81	0.88
Silt-soil	27.7	192.9	6.94	0.93

and θ can be obtained as shown in Fig. 3b. The values of the parameters obtained by the regression are listed in Table 4. The shear strength of both the sand-soil and the silt-soil was well represented by the regression functions: $\tau = \sigma' \tan 28.3° + 35.8\ e^{-4.81\theta}$ for the sand-soil ($R^2 = 0.88$), and$\tau = \sigma' \tan 27.7° + 192.9\ e^{-6.94\theta}$ for the silt-soil ($R^2 = 0.93$).

The regression variables in the equations include the liquefaction characteristics of the soils, effects of lubrication and susceptibility to changes in pore-water pressure during the shear deformation. For this reason, the values should be considered purely as empirical parameters for the soils, each of which has inherent geotechnical behaviours.

Accuracy of the prediction

Figure 4 demonstrates the comparisons of the measured and calculated shear strength. In the diagram, values of SE (standard error of estimate) are defined as a standard deviation of the predictive residuals. The smaller value of SE indicates the more accurate prediction.

As shown in Fig. 4, the values of SE were 2.67 kPa for the sand-soil and 4.25 kPa for the silt-soil. Note that the SE in the silt-soil was significantly affected by an anomalous shear strength value (an arrowed plot in Fig. 4; normal stress of 40 kPa in the group H, Table 2). Ignoring this, the value of SE in the silt-soil reduces to 1.29 kPa.

The magnitude of these errors indicates that the method can provide a 'rule of thumb' estimation of shear strength in unsaturated residual soils. The methodology described in the paper may be an efficient means of assessing the continuous variation of slope stability, using field measurements of subsurface volumetric water content (e.g. using dielectric soil-moisture sensors) or with mass-balance based seepage analysis.

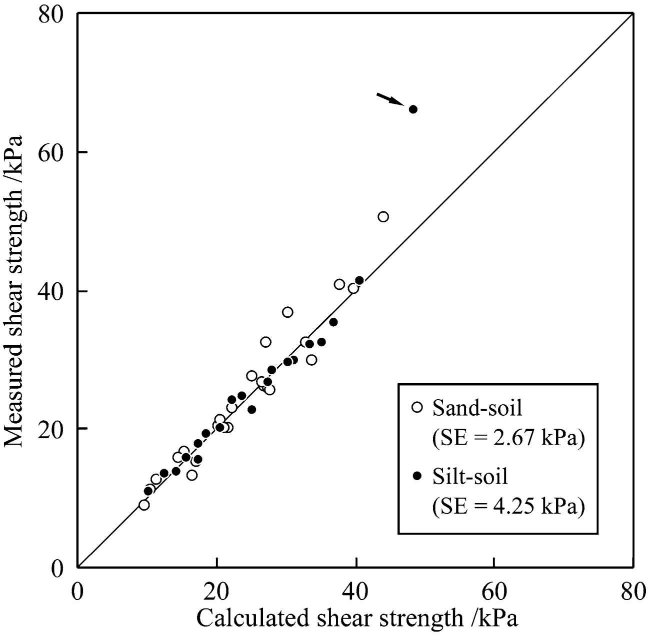

Fig. 4 Comparison of measured and calculated shear strength. SE = standard error of estimate: a standard deviation of the predictive residuals. An *arrowed outlier* is a datum with normal stress of 40 kPa within the group H in Table 2

Conclusions

The cohesive strength of an unsaturated soil was formulated as an exponential function of volumetric water content. In the formulation, shear strength τ was expressed as $\tau = \sigma' \tan \phi' + C\ e^{-\mu\theta}$. Where σ' is net normal stress, ϕ' is effective angle of shearing resistance, C is maximum cohesion, μ is a susceptibility coefficient and θ is volumetric water content of soil. An advantage of this formulation is that all the parameters required are available without any elaborate soil testing. The variables can be obtained by a basic shear test and a subsequent regression analysis. In the case of the two undisturbed residual soils reported here, the predictive errors of the equation are less than a few kilopascal. It is considered that this empirical method provides a convenient alternative for engineering practice.

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