Review of Economic Design (2024) 28:1–19
https://doi.org/10.1007/s10058-023-00328-8

ORIGINAL PAPER

King Solomon's dilemma: an experiment on implementation in iterative elimination of (obviously) dominated strategies

Makoto Hagiwara[1] · **Fumihiro Yonekura**[2]

Received: 11 January 2022 / Accepted: 13 January 2023 / Published online: 7 February 2023

Abstract

"King Solomon's dilemma" is based on a biblical story that can be considered an allocation problem for an indivisible good among two players. We experimentally compare the performance of the mechanism of Mihara (Jpn Econ Rev 63(3):420–429, 2012) with a modified version of his mechanism that we propose. Mihara's mechanism uses a second-price auction, while we change it to an ascending clock auction. We find that the modified version performs relatively better than Mihara's in terms of the right-player allocations, "resource inefficiency," "wrong-player inefficiency," and the equilibrium strategies of high valuation players. Regarding the first-best allocations and equilibrium strategies of low valuation players, in our experiment, there was a trend for improvement under the modified version relative to Mihara's mechanism.

Keywords King Solomon's dilemma · Mihara's mechanism · Ascending clock auctions · Laboratory experiment

JEL Classification C72 · C91 · D44

We are grateful to two anonymous reviewers, as well as Emiko Fukuda, Ryo Kawasaki, Takehito Masuda, H. Reiju Mihara, Masaki Miyashita, Shigeharu Okajima, Tatsuyoshi Saijo, Akiyoshi Shioura, William Thomson, Hirofumi Yamamura, and Takehiko Yamato for their helpful comments. Hagiwara was partially supported under JSPS KAKENHI Grant Numbers JP17J01520 and JP20K22110. We also thank Wiley Editing Services (http://wileyeditingservices.com) for the English language editing.

✉ Makoto Hagiwara
m.hagiwara@osaka-ue.ac.jp

Fumihiro Yonekura
fumi314@yahoo.ne.jp

1 Faculty of Economics, Osaka University of Economics, 2-2-8, Osumi, Higashiyodogawa-ku, Osaka 533-8533, Japan

2 Department of Industrial Engineering and Economics, School of Engineering, Tokyo Institute of Technology, 2-12-1 Ookayama, Meguro-ku, Tokyo 152-8552, Japan

1 Introduction

"King Solomon's dilemma" is based on a biblical story, where King Solomon assigns a child to one of two women, both of whom claim to be the mother. The King knows that the mother is one of the two women. However, the King does not know the identity of the mother. The woman who is not the mother keeps her true identity a secret, and the King aims to assign the child to the mother without payment. This represents the allocation problem of an indivisible good among two players, and several examples show this story is a realistic description (see Glazer and Ma 1989, footnote 1).[1] A social planner wants to assign the good without payment to the player whose valuation is the highest. We say that such an allocation is the "first-best." Several mechanisms have been proposed to solve this problem.

While Glazer and Ma (1989) and Moore (1992) assume that each player knows the other player's valuation, Perry and Reny (1999), Olszewski (2003), Qin and Yang (2009), and Mihara (2012) do not. We focus on Mihara (2012), who construct a mechanism comprising the following two stages.[2] First, each player chooses whether to participate in a second-price auction. Second, if both players choose to participate in the auction, each player pays a fee and participates in the auction. For details, see Sect. 2. Mihara's mechanism implements the first-best allocation in one round of elimination of weakly dominated strategies, followed by two rounds of iterative elimination of strictly dominated strategies. Mihara's mechanism is simple and has the feature that only the top valuation player chooses to participate in the auction on the equilibrium path, which eliminates the need to hold the auction.

The result of Mihara (2012) relies on a property of a second-price auction, "strategy-proofness." This axiom states that truth-telling is always a weakly dominant strategy for each player under the direct revelation mechanism. Although choosing when to quit in an ascending clock auction is the same as choosing a bid in a second-price auction, ascending clock auctions are easier to understand than second-price auctions. As evidenced by a laboratory experiment, subjects are substantially more likely to play the dominant strategy in an ascending clock auction than in a second-price auction (Kagel et al. 1987).[3,4] Inspired by this observation, Li (2017) formulates what he calls "obvious dominance." That is, for player i, a strategy S_i is obviously dominant in a game if, for any deviating strategy S_i', starting from any earliest information set where S_i and S_i' diverge, the best possible outcome from S_i' is no better than the worst possible outcome from S_i.[5] A mechanism is *obviously strategy-proof* if it has an equilibrium

[1] King Solomon's dilemma is generalized by Qin and Yang (2009) as an allocation problem for k identical and indivisible goods among n players, where $1 \leq k < n$, among at least two players. Mihara (2012) also investigates this generalized problem.

[2] Mihara (2012) briefly compares the mechanisms proposed by Perry and Reny (1999), Olszewski (2003), and Qin and Yang (2009) with Mihara's mechanism. See Section 1 of Mihara (2012).

[3] For second-price auction experiments, see Harstad (2000) and Kagel and Levin (1993). For ascending-clock auction experiments, see McCabe et al. (1990). Note that, relative to Li (2017), the studies cited in this footnote do not directly compare the two formats with the same value distribution and the same subject pool.

[4] For an experiment on multi-unit demand auctions, see Kagel and Levin (2009). They experimentally study Vickrey (1961)'s static auction and the two dynamic auctions of Ausubel (2004).

[5] For the formal definitions of earliest information sets and obviously dominant strategies, see Li (2017).

in obviously dominant strategies. Li (2017) shows that obviously dominant strategies are those considered optimal by a cognitively limited agent who cannot engage in contingent reasoning. This definition allows differentiating between ascending clock auctions and second-price auctions. Ascending clock auctions are *obviously strategy-proof*, while second-price auctions are *strategy-proof*, but not *obviously strategy-proof*. This motivates us to replace a second price auction in Mihara's mechanism with an ascending clock auction.

This study aims to experimentally compare the relative performance of Mihara's mechanism with a modified version of his mechanism that we propose. We modify Mihara's mechanism from the point of view of "iterative elimination of obviously dominated strategies." For details, see Sect. 2.

Although Li (2017) observed that subjects frequently selected obviously dominant strategies,[6] the notion of iterative elimination of obviously dominated strategies has not been necessarily supported by other experimental studies. For example, Beard and Beil (1994) observed the failure of a two-step iterative elimination of obviously dominated strategies using a simple game.[7] Players 1 and 2 move in sequence, each chooses an action from their only two strategies, and a two-step iterative elimination of obviously dominated strategies is needed to achieve the unique equilibrium outcome. In five of seven games, less than half of the players who moved first chose the equilibrium strategy. These results were replicated by Goeree and Holt (2001).[8] However, Masuda et al. (2014) report a positive result regarding iterative elimination of obviously dominated strategies.[9] For public good provision, they construct a mechanism to achieve a symmetric Pareto-efficient outcome in iterative elimination of obviously dominated strategies, and report that their mechanism works well in their experiment, as cooperation is observed frequently. [10] Based on these previous studies, it is not easy to predict whether our mechanism will perform better than Mihara's mechanism.

We experimentally compare the relative performance of Mihara's mechanism with ours, and observed the following. Our mechanism allocates the object to the right player more frequently than does Mihara's; the number of inefficient payments under our mechanism is lower than under Mihara's; and the equilibrium strategies of high val-

[6] For further experimental evidence on obviously dominant strategies, see Breitmoser and Schweighofer-Kodritsch (2021), who replicate Li's experimental study and add three intermediate auction formats.

[7] Note that Beard and Beil (1994) do not state the notion of iterative elimination of obviously dominated strategies.

[8] For a review of several studies on the failure of the finite-step iterative elimination of dominated strategies, see Katok et al. (2002); Nagel (1995); Schotter et al. (1994), and Sefton and Yavas (1996). For a survey regarding the failure of iterative elimination of dominated strategies, see Chapter 12 of Dhami (2016). Further, for experimental studies on the assumption of common knowledge of rationality, see Costa-Gomes et al. (2001) and Kneeland (2015).

[9] For other positive experimental results on iterative elimination of obviously dominated strategies, see Saijo and Shen (2018) and Saijo et al. (2018). Note that Masuda et al. (2014), Saijo and Shen (2018), and Saijo et al. (2018) do not state the feature of the implementation of iterative elimination of obviously dominated strategies in their mechanisms.

[10] Regarding the experiment of Masuda et al. (2014), in Stage 1, players simultaneously and independently choose their contributions to the public good as integers from 0 to 24. In Stage 2, players simultaneously and independently decide on whether to approve the other player's choice. Although our mechanism includes more steps to eliminate obviously dominated strategies than Masuda et al. (2014), we frequently observed the equilibrium outcome in our mechanism. For details on our results, see Sect. 4.

uation players are more frequently selected under our mechanism than under Mihara's. In our experiment, there were not significant differences between Mihara's mechanism and ours regarding the first-best allocations and the equilibrium strategies of low valuation players. However, there were trends for improvement on these under our mechanism relative to Mihara's.

Ascending clock auctions themselves perform well in several experiments, such as that of Li (2017), since subjects play such a simple game as they sequentially choose to quit or stay in an auction.[11] The experimental results on ascending clock auctions suggest that subjects find the obviously dominant strategy while making a decision in the auction. However, our experimental results suggest that subjects may find the best strategy for themselves before participating in the auction in our mechanism since most pairs of subjects do not proceed to the auction stage and select the first-best allocation immediately (see Sect. 4.3.1).

The rest of this paper proceeds as follows. Section 2 presents the model and two mechanisms. Sect. 3 describes our experimental design. Section 4 reports the results. Section 5 discusses the results and three open questions.

2 The model and two mechanisms

There are only two players, player 1 and player 2. A single indivisible good is to be allocated to the player with the highest valuation. It is common knowledge that, for each pair of players' valuations, there is a player whose valuation exceeds the other's. Each player knows her own valuation and whose valuation is the highest. A social planner knows neither the players' valuations nor whose valuation is highest. Each player's payoff for obtaining the object with the payment $p \in \mathbb{R}$ and the initial capital balance $w \in \mathbb{R}$ is $v + w - p$, where v is the player's valuation of the object. Each player's payoff for obtaining no object and payment p is $w - p$. We assume that the two players and the planner know that a gap exists between the two valuations, which is greater than a positive real number $M > 0$. The planner wants to allocate the object without payment to the player with the highest valuation. Hereafter, we say that such an allocation is the **first-best**.

In the allocation problem, the following mechanism has been designed by Mihara (2012).

A Second-Price Auction with Participation Stage (hereafter, SPAPS)
Participation stage: The two players simultaneously choose either "auction" or "no auction."

If only one player chooses "auction," then the object is assigned to this player without payment, while the other player gets no object and pays nothing, and the game ends. If no player chooses "auction," then no player gets the object and pays, and the game ends. If both players choose "auction," then each player pays a participation fee equal to M and we move to a second-price auction.

[11] This feature of ascending clock auctions is generalized by Pycia and Troyan (2019), who show that *obvious strategy-proofness* is characterized by clinch-or-pass games, which they call "millipede games."

Auction stage: The two players simultaneously and independently bid for the object. The object is assigned to the player with the highest bid, at a price equal to the other player's bid. The other player gets the object and pays nothing, and the game ends.

Regarding SPAPS, Mihara (2012) presents the following result:

Proposition 1 *SPAPS implements the first-best allocation in one round of elimination of weakly dominated strategies, followed by two rounds of iterative elimination of strictly dominated strategies.*

Strategy-proofness requires that in the direct revelation mechanism, truth-telling is a weakly dominant strategy for each agent.[12] While the second-price auction is *strategy-proof*, it is not *obviously strategy-proof*. Hence, SPAPS does not implement the first-best allocation in iterative elimination of obviously dominated strategies. Then, we modify SPAPS as follows:

An Ascending Clock Auction with Participation Stage (hereafter, ACAPS)
The participation stage: This stage is the same as SPAPS.
The auction stage: The price of the object begins at 0 and increases the price of the object. Each participant will be regarded as an active bidder for the object until the player ceases to bid on the object. Each player can drop out of the auction. The exit from the auction is not reversible, that is, no player can re-enter once she is out. The only player who is still an active bidder (i.e., did not drop out) is eligible to acquire the object at the price at which the other bidder exited.

We will now explain how ACAPS implements the first-best allocation. In the first round, we focus on the auction stage. Assume that, in the participation stage, each player says "auction." In this case, for each player, quitting when the price is her own valuation is the unique obviously dominant strategy, as shown by Li (2017). In the second round, we focus on the player whose valuation is highest in the participation stage. If each player quits when the price is her own valuation in the auction stage, choosing "auction" is the unique obviously dominant strategy for the player whose valuation is highest. In the third round, we focus on the player whose valuation is lowest in the participation stage. If each player quits when the price is her own valuation at the auction stage and the player whose valuation is highest selects "auction" in the participation stage, choosing "no auction" is the unique obviously dominant strategy for the player whose valuation is lowest. In this sense, we say that the remaining strategy profile is an **equilibrium in the iterative elimination of obviously dominated strategies**. By the above three steps, we have the following statement of Proposition 2.

Proposition 2 *ACAPS implements the first-best allocation in iterative elimination of obviously dominated strategies.*

3 Experimental design

We experimentally compare the performance of ACAPS relative to SPAPS. Our experimental design is essentially the same as that of Elbittar and Di Giannatale (2017).

[12] For the definition of *strategy-proofness*, see, for example, Barberà (2011).

Note that Elbittar and Di Giannatale (2017) experimentally compare the relative performance of two mechanisms proposed by Moore (1992) and Perry and Reny (1999) which are different mechanisms from ours. The differences between Elbittar and Di Giannatale (2017) and our study are mentioned when needed from the next paragraph.

The details of the experimental design are as follows:

3.1 Subjects

For each session, the subjects were undergraduate and graduate students at Tokyo Institute of Technology. The subjects were informed of an opportunity to earn money. None of them had prior experience with second-price-auction experiments, ascending-clock-auction experiments, or King-Solomon-dilemma experiments.

Two sessions were conducted under each of SPAPS and ACAPS, and 20 subjects participated in each session (80 subjects in total). For convenience, we consider Sessions 1 and 2 (resp. Sessions 3 and 4) the ones regarding SPAPS (resp. ACAPS). No subject attended more than one session. Our experiment was conducted at Tokyo Institute of Technology in December 2019. The study employed computers with the experimental software z-Tree (Fischbacher 2007). In each session, 20 subjects were seated at computer stations, separated with partitions in the Experimental Economics Laboratory.

3.2 Flow of the experiment

Upon arrival, the subjects were randomly assigned to a computer terminal. Each subject received an instruction, a record sheet, and a post-experimental questionnaire in Japanese.[13] After the subjects confirmed having received all experimental materials, the experimenter read the instructions aloud to ensure that all subjects understood them. Subjects were allowed to ask questions. Then, a computer initiated the experiment. At the end of the experiment, each participant answered a questionnaire and was paid in cash.

3.3 Practice and true periods

To familiarize the subjects with the procedures, there were three practice periods, and then 20 true periods.[14]

[13] For the English translations of the documents, see the supplementary material.

[14] In contrast to our study, Elbittar and Di Giannatale (2017) used two practice periods and 10 true periods. We changed the number of practice periods for subjects to understand all cases of SPAPS or ACAPS and we changed the number of true periods to ascertain whether the equilibrium outcome is more frequently achieved as a period progresses.

3.4 Matching procedure

After the three practice periods, each subject was designated either as a high valuation player (HVP) or as a low valuation player (LVP). These types remained fixed for the entire session for each type of player, so that they would identify themselves with a type. In each period, an HVP was randomly paired with an LVP. However, they were never paired with the same subject more than twice, nor were they ever paired with the same subject in two consecutive periods. Furthermore, they did not know with whom they were paired with in any given period.

3.5 Valuations

Let θ_H be the valuation of an HVP and θ_L the valuation of an LVP. Players' valuations for each period were integers drawn randomly from the interval $[0, 200]$, with the following restriction: $\theta_H - \theta_L > M \equiv 50$.

3.6 Information setting

Both players were informed whether they had the higher or lower valuation and of the restriction that $\theta_H - \theta_L > 50$. However, they were not told the exact amount of the opponent's valuation.

3.7 Initial capital

All players were endowed with an initial capital balance per their type. The initial capital balances were 30 for the HVPs and 70 for the LVPs, as in the experimental design of Elbittar and Di Giannatale (2017).[15] The difference between the initial capital balance of the LVPs and that of the HVPs compensates for the asymmetry in their valuations. Each subject knew her own initial capital balance but not the opponent's balance.

3.8 Information feedback

During the decision process, some of the possible payoff calculations were privately assigned by the computer to each subject at different decision nodes. If players chose to participate in an auction, after the bids were submitted for a second-price auction or the first drop-out occurred for an ascending clock auction, the payoffs of the players were calculated. Finally, at the end of each period, each subject received her payoff.

[15] Section 5 discusses the initial capital balances.

3.9 Payoffs and final payments

The final payoff was determined by selecting one period randomly out of the 20 true periods played for the cash reward. Subjects were also informed that any winnings would be added to their initial capital balances, and any losses would be subtracted from it. The initial capital balances were considered players' possible payoffs for each period. Moreover, upon entry in the experiment, each subject was paid JPY 1,013 per hour, with a payment independent of performance.[16]

Under SPAPS, the final payment was, on average, JPY 3,054; for HVPs, it was JPY 3,279; for LVPs, it was JPY 2,828; and the experimental time was approximately 2.5 hours. Under ACAPS, the final payment, on average, was JPY 3,702; for HVPs, it was JPY 4,002; for LVPs, it was JPY 3,403; and the experimental time was approximately 3 hours.[17] The experimental time was different between SPAPS and ACAPS because of the waiting time in an ascending clock auction.

3.10 Bidding restrictions

For the ascending clock auction, subjects were informed that the price had reached a maximum level of 270 without a subject dropping out, the sale price for the object would be 270, and the object would be sold to one of the claimants (chosen randomly by the computer) at this price. Accordingly, the other bidder would pay nothing. This setting is the same as that of Elbittar and Di Giannatale (2017). To compare the ascending clock auction with the second-price auction, subjects were not allowed to bid above 270 in the second-price auction.

4 Results

4.1 Efficiency

We investigate whether the first-best and right-player allocations were more frequently achieved under ACAPS than under SPAPS.[18] Table 1 provides the proportions of the first-best and right-player allocations in each session for periods 1–20, periods 1–10, and periods 11–20.

We measure how players' valuations and the mechanisms affect the first-best and right-player allocations. In the following three-level logit model using clustering at the

16 Elbittar and Di Giannatale (2017) paid MXN 50 to each subject as entry payment. In December 2019, when our sessions were run, the exchange rate was approximately JPY 5.665 per MXN 1 (and JPY 108.873 per USD 1). In our study, the JPY 1,013 hourly payment was based on the minimum wage in Tokyo, Japan in 2019. For details, see the website of the Japanese of the Ministry of Health, Labour and Welfare: https://jsite.mhlw.go.jp/tokyo-roudoukyoku/news_topics/houdou/20190830chinginka.html.

17 Although the experimental time is different between the SPAPS and ACAPS sessions, the difference regarding the average final payment between SPAPS and ACAPS is a result of both the experimental time and players' behavior in the game. See the equation regarding how to calculate the final payment in the supplementary materials.

18 The right-player allocation means that the good is allocated to the player whose valuation is the highest.

Table 1 Proportions of the first-best and right-player allocations in each session

		SPAPS		ACAPS	
		Session 1	Session 2	Session 3	Session 4
First-best	Periods 1–20	72.0%	76.0%	78.0%	95.0%
	Periods 1–10	57.0%	73.0%	71.0%	91.0%
	Periods 11–20	87.0%	79.0%	85.0%	99.0%
Right-player	Periods 1–20	87.0%	89.5%	96.0%	98.0%
	Periods 1–10	81.0%	86.0%	94.0%	97.0%
	Periods 11–20	93.0%	93.0%	98.0%	99.0%

session level and the subject level, SPAPS is the baseline model,[19] which is compared to ACAPS. The specifications can be formulated, in general, as follows:

$$y_{ijt} = 1\{intercept + \beta_1 \theta_{H,ijt} + \beta_2 \theta_{L,ijt} + \beta_3 period + \gamma d_{ACAPS} + u_i + v_j + \epsilon_{ijt} \geq 0\},$$

where

- y_{ijt} represents either the first-best or the right-player allocation for player i in period t of session j;
- $y_{ijt} = 1$ (resp. $y_{ijt} = 0$) means that either the first-best or the right-player allocation is achieved (resp. not achieved);
- $1\{.\}$ is an indicator function that takes the value of one if the left-hand side of the inequality inside the parentheses is greater than or equal to zero, and zero otherwise;
- θ_H is the valuation of the HVP;
- θ_L is the valuation of the LVP;
- $period$ represents the period;
- d_{ACAPS} is a dummy variable that takes the value of one for ACAPS;
- u_i is the subject-specific random effect;
- v_j is the session-specific random effect; and
- ϵ_{ijt} is the usual error term.

Table 2 provides the regression results for the first-best and right-player allocations.

From Table 2, we derive the following results for each of the first-best and right-player allocations.

Result 1. (First-best allocations) *When HVPs' valuations increase or LVPs' valuations decrease, the first-best allocation is achieved more frequently.*

Result 2. (Right-player allocations)

1. *Right-player allocation under ACAPS is achieved more frequently than under SPAPS.*

[19] As in Sect. 3, we divided the 80 subjects into four sessions, each with 20 periods. As such, a three-level model is appropriate. For the three-level model by clustering at the session and subject levels, see for example Chapter 4 of Moffatt (2015).

Table 2 Regression results for the first-best and right-player allocations

	First-best	Right-player
θ_H	0.007**	0.025***
	(0.003)	(0.004)
θ_L	– 0.025***	– 0.028***
	(0.003)	(0.004)
$Period$	0.099***	– 0.086***
	(0.013)	(0.019)
d_{ACAPS}	1.233	1.654***
	(0.684)	(0.321)
$Intercept$	0.464	0.714
	(0.6)	(0.53)
Observations	1600	1600

The numbers between parentheses below each coefficient represent that coefficient's standard error.
$^{*}p < 0.05$; $^{**}p < 0.01$; $^{***}p < 0.001$

2. *When HVPs' valuations increase or LVPs' valuations decrease, the right-player allocation is frequently achieved.*

In our experiment, there was not significant difference regarding the first-best allocations between SPAPS and ACAPS. However, there was a trend for improvement under ACAPS relative to SPAPS ($p = 0.072 < 0.1$).

In contrast to our theoretical predictions, the subjects' valuations affect the achievement of the first-best and right-player allocation, as in Results 1 and 2(2). For HVPs, if the valuation of an HVP increases, the player may have a stronger incentive to claim the object and participate in the auction. For LVPs, if the valuation of an LVP for the object is higher than the participation fee, the subjects may imagine that she will obtain the object with payment. The subjects then participated in the auction. An additional experiment is necessary to gauge the prediction accuracy for LVPs, in which the participation fee changes from 50 to a higher or lower fee.

4.2 Sources of inefficiency

We consider the two possible sources of inefficiency introduced by Elbittar and Di Giannatale (2017): **resource inefficiency** (RI) and **wrong-player inefficiency** (WPI). The former is obtained by dividing the sum of the two players' payments by the sum of the high valuation and the two players' initial capitals. The latter is obtained by dividing the high valuation minus the winner's valuation by the sum of the high valuation and the initial capitals of the HVP and LVP. In other words, RI indicates the inefficiency in monetary terms from paying the participation fee and the winning bid. Additionally, WPI indicates the inefficiency from allocating the object to an LVP. Note that, initial capital is included in the definitions of RI and WPI, and this amount affects the measures.[20]

[20] In Sect. 5, we discuss that one may change the amount of initial capital. It is thus better to exclude initial capital from the definitions of RI and WPI to measure the sources of inefficiency when the amount of initial capital changes.

Table 3 Net mean efficiency, resource inefficiency, and wrong-player inefficiency

		NME[a]	RI[b]	WPI[c]
SPAPS	Periods 1–20	73.8%	17.9%	8.3%
	Periods 1–10	64.8%	23.0%	12.1%
	Periods 11–20	82.7%	12.7%	4.5%
ACAPS	Periods 1–20	93.9%	5.1%	1.0%
	Periods 1–10	91.2%	7.2%	1.6%
	Periods 11–20	96.6%	3.0%	0.4%

[a] NME=(Winner's valuation − Players' payments + Players' initial capital)/ (Highest valuation + Players' initial capital)
[b] RI = Players' payments/(Highest valuation + Players' initial capital)
[c] WPI=(Highest valuation−Winner's valuation)/(Highest valuation+ Players' initial capital)

Before analyzing these two sources of inefficiency, let us introduce the notion of **net mean efficiency** (NME), defined as the total of the players' net gains as a proportion of the total surplus. At equilibrium, NME should be 100% of the total surplus, otherwise there would be two types of inefficiency: RI and WPI.

Table 3 reports the NME, RI, and WPI under SPAPS and ACAPS for the 20 periods, periods 1–10, and periods 11–20.

We measure the effect of players' valuations and the mechanisms of the sources of inefficiency. In the following three-level linear model using clustering at the session level and the subject level, SPAPS is the baseline model, and we compare it to ACAPS. We can formulate the specifications, in general, as follows:

$$y'_{ijt} = intercept + \beta_1\theta_{H,ijt} + \beta_2\theta_{L,ijt} + \beta_3 period + \gamma d_{ACAPS} + u'_i + v'_j + \epsilon'_{ijt},$$

where y'_{ijt} is either RI or WPI for player i in period t of session j. Table 4 provides the regression results for the sources of inefficiency.

from Table 4, we derive the following results for RI and WPI.

Result 3. (Resource inefficiency)

1. *Resource inefficiency under ACAPS is lower than under SPAPS.*
2. *When LVPs' valuations increase, resource inefficiency increases.*

Result 4. (Wrong-player inefficiency)

1. *Wrong-player inefficiency under ACAPS is lower than under SPAPS.*
2. *When HVPs' valuations decrease or LVPs' valuations increase, wrong-player inefficiency increases.*

From Results 3(1) and 4(1), ACAPS performs relatively better than SPAPS in both RI and WPI.

In contrast to our theoretical predictions, the subjects' valuations affect the sources of inefficiency, as in Results 3(2) and 4(2). We have the same conjecture as in the final paragraph of Sect. 4.1 regarding the reason why LVPs and HVPs are affected by their valuations. To understand accurately the reason for the observations in Results 3(2) and 4(2), additional experiments are necessary.

Table 4 Regression results for the sources of inefficiency

	RI	WPI
θ_H	−0.0001	−0.0004***
	(0.0002)	(0.00008)
θ_L	0.003***	0.0002**
	(0.0002)	(0.00008)
$Period$	− 0.007***	− 0.002***
	(0.001)	(0.0004)
d_{ACAPS}	− 0.082*	− 0.033***
	(0.041)	(0.007)
$Intercept$	0.137**	0.111***
	(0.043)	(0.012)
Observations	1600	1600

The numbers between parentheses below each coefficient represent that coefficient's standard error

$^{*}p < 0.05$; $^{**}p < 0.01$; $^{***}p < 0.001$

Table 5 Players' behaviors in the participation stage

	SPAPS	ACAPS
(1) HVPs' decision		
(1a) "auction"	96.5% = 386/400	99.3% = 397/400
(1b) "no auction"	3.5% = 14/400	0.7% = 3/400
(2) LVPs' decision		
(2a) "auction"	24.3% = 97/400	12.8% = 51/400
(2b) "no auction"	75.8% = 303/400	87.3% = 349/400

4.3 Players' behaviors

4.3.1 The participation stage

Table 5 describes the proportions of the players who choose "auction" or "no auction" at the participation stage for the 20 periods under SPAPS and ACAPS. Under SPAPS, 96.5% of HVPs select "auction" and 75.8% of LVPs select "no auction." Under ACAPS, 99.3% of HVPs select "auction" and 87.3% of LVPs select "no auction." Therefore, ACAPS performs relatively better than SPAPS in the participation stage. Figure 1 (resp.Fig. 2) describes the transition regarding the proportion of the players who say "auction" in the participation stage for each period under SPAPS (resp. ACAPS). Additionally, 51 pairs proceeded to the auction stage under ACAPS, compared to the 90 pairs did under SPAPS. Under ACAPS, since most pairs of players do not proceed to the auction stage and select the first-best allocation immediately, players may find the best strategy for themselves before participating in the auction.

We statistically measure the effects of players' valuations and the mechanisms on players' behaviors regarding whether they participate in the auction. In the following

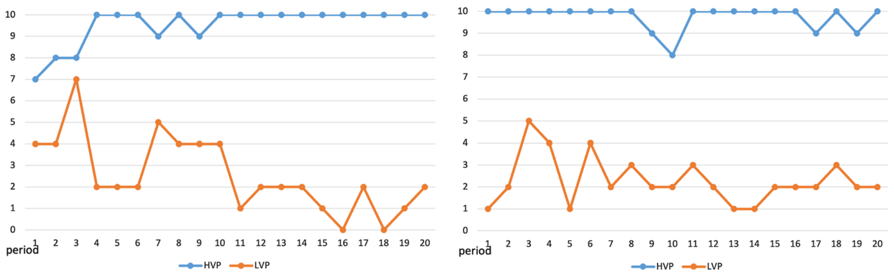

Fig. 1 Participation rate for each period in the participation stage under SPAPS (Left: Session 1; Right: Session 2)

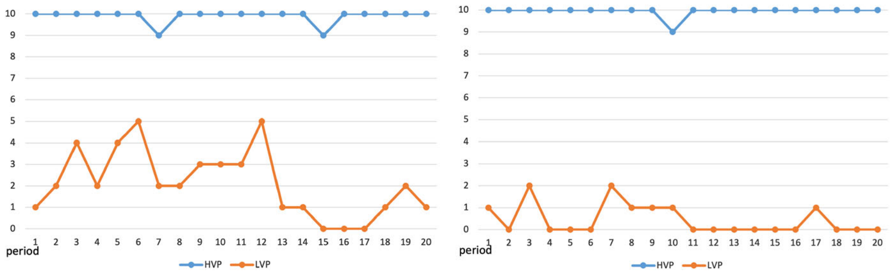

Fig. 2 Participation rate for each period in the participation stage under ACAPS (Left: Session 3; Right: Session 4)

three-level logit model using clustering at the session and the subject levels, SPAPS is the baseline model to which ACAPS is compared. We can formulate the specifications, in general, as follows:

$$y''_{ijt} = 1\{intercept + \beta''_1\theta_{ijt} + \beta''_2 period + \gamma'' d_{ACAPS} + u''_i + v''_j + \epsilon''_{ijt} \geq 0\},$$

where y''_{ijt} represents either “auction” or “no auction”, as chosen by player i in period t of session j and $y''_{ijt} = 1$ (resp. $y''_{ijt} = 0$) means saying “auction” (resp. “no auction”) and θ_i is the valuation of player i. Table 6 provides the regression results for HVPs’ and LVPs’ behaviors in the participation stage.

From Table 6, we have the following results for players’ behavior at the participation stage.

Result 5. (HVPs’ behaviors in the participation stage)

1. *Saying “auction” at the participation stage for HVPs under ACAPS is more frequent than under SPAPS.*
2. *When HVPs’ valuations increase, choosing “auction” in the participation stage is more frequent.*

Result 6. (LVPs’ behaviors in the participation stage) *When LVPs’ valuations decrease, choosing “no auction” in the participation stage is more frequent.*

Table 6 Regression results for HVPs' and LVPs' behaviors in the participation stage

	HVPs	LVPs
θ_H	0.033***	–
	(0.005)	
θ_L	–	0.024***
		(0.002)
$Period$	0.095**	− 0.098***
	(0.034)	(0.014)
d_{ACAPS}	1.724***	− 1.222
	(0.464)	(0.7)
$Intercept$	−1.897**	− 1.66**
	(0.689)	(0.516)
Observations	800	800

The numbers between parentheses below each coefficient represent that coefficient's standard error.
$^{*}p < 0.05$ $^{**}p < 0.01$; $^{***}p < 0.001$

In our experiment, there was not significant difference regarding the equilibrium strategies of LVPs between SPAPS and ACAPS. However, there was an improvement trend under ACAPS relative to SPAPS ($p = 0.081 < 0.1$).

In contrast to our theoretical predictions, the subjects' valuations affect the equilibrium strategies as in Results 5(2) and 6. We have the same conjecture as in the final paragraph of Sect. 4.1regarding the reason why LVPs and HVPs are affected by their valuations. To understand accurately the reason for the observations in Results 5(2) and 6, we need to conduct additional experiments.

4.3.2 The auction stage

Table 7 provides the distributions of players' bids under SPAPS and ACAPS. To analyze whether HVPs and LVPs bid rationally, we divide the regions of the second-highest bid under SPAPS (or the first drop-out price under ACAPS), b_{SHB}, into three cases:

Case 1. b_{SHB} is smaller than the loser's valuation minus 2;

Case 2. b_{SHB} is in the interval between the loser's valuation plus 2 and minus 2; and

Case 3. b_{SHB} is larger than the loser's valuation plus 2. Note that we aim to analyze whether a loser's bid is their own valuation (or quitting when the price is their own valuation) of the equilibrium price within 2 as in Elbittar and Di Giannatale (2017) and Li (2017).

First, we observe the behavior of LVPs. Since we observe only the second-highest bid under SPAPS (or the first drop-out price under ACAPS), we focus on the cases where HVPs win the auction to determine the bidding behavior of LVPs. Of all HVPs, 63.3% win the auction under SPAPS and 82.4% win it under ACAPS. The proportion of LVPs in Case 2 under ACAPS (45.2%) is greater than that under SPAPS (19.3%).

Table 7 Players' behavior in the auction stage

	SPAPS	ACAPS
(1) HVPs win the auction	$63.3\% = 57/90$	$82.4\% = 42/51$
$(1a)$ and $b_{SHB} \in [0, \theta_L - 2)$[a]	$28.1\% = 16/57$	$21.4\% = 9/42$
$(1b)$ and $b_{SHB} \in [\theta_L - 2, \theta_L + 2]$	$19.3\% = 11/57$	$45.2\% = 19/42$
$(1c)$ and $b_{SHB} \in (\theta_L + 2, 270]$	$52.6\% = 30/57$	$33.3\% = 14/42$
(2) LVPs win the auction	$36.7\% = 33/90$	$17.6\% = 9/51$
$(2a)$ and $b_{SHB} \in [0, \theta_H - 2)$	$69.7\% = 23/33$	$88.9\% = 8/9$
$(2b)$ and $b_{SHB} \in [\theta_H - 2, \theta_H + 2]$	$18.2\% = 6/33$	$0.0\% = 0/9$
$(2c)$ and $b_{SHB} \in (\theta_H + 2, 270]$	$12.1\% = 4/33$	$11.1\% = 1/9$

[a] b_{SHB}: the second highest bid (or the first drop-out price)

Therefore, LVPs chose the equilibrium strategy more frequently in the auction stage under ACAPS than SPAPS, which is the same as the results regarding second-price-auction and ascending-clock-auction experiments (e.g., Li 2017).

Second, we observe the behavior of HVPs. We focus on the cases where LVPs win the auction to observe the bidding behavior of HVPs. Of all LVPs, 36.7% win the auction under SPAPS and 17.6% win it under ACAPS. Note that most LVPs do not select "auction" in the participation stage under both mechanisms, as in Sect. 4.1. In contrast to the results for LVPs, the proportions of HVPs in Case 1 under SPAPS (69.7%) and ACAPS (88.9%) are relatively large. This result is different from that of the experiments comparing the relative performance between second-price and ascending clock auctions (e.g., Li 2017), which may be explained by the difference that two types of high values and low values are assigned and each player knows their own and the opponent's type.

Figure 3 (resp. Fig. 4) presents the scatterplot between HVPs' and LVPs' valuations and their bids under SPAPS (resp. the first drop-out prices under ACAPS). Under SPAPS, the correlation coefficient between HVPs' valuations and their bids is 0.376, and that between LVPs' valuations and their bids is 0.328. By contrast, when we exclude two extreme points, the correlation coefficient between HVPs' valuations and their bids under ACAPS is 0.582 and that between LVPs' valuations and their bids under ACAPS is 0.622.[21] Therefore, for each type of HVP and LVP, the positive correlation under ACAPS is stronger than that under SPAPS.

5 Conclusions

We first modified the mechanism SPAPS designed by Mihara (2012) on King Solomon's dilemma. SPAPS is constructed based on a second-price auction. We then changed it into an ascending clock auction and we call the modified mechanism ACAPS. We experimentally compared the performance of ACAPS with that of

[21] If we include the two extreme points, the correlation coefficient between HVPs' valuations and their bids under ACAPS is 0.344, and that between LVPs' valuations and their bids under ACAPS is 0.467.

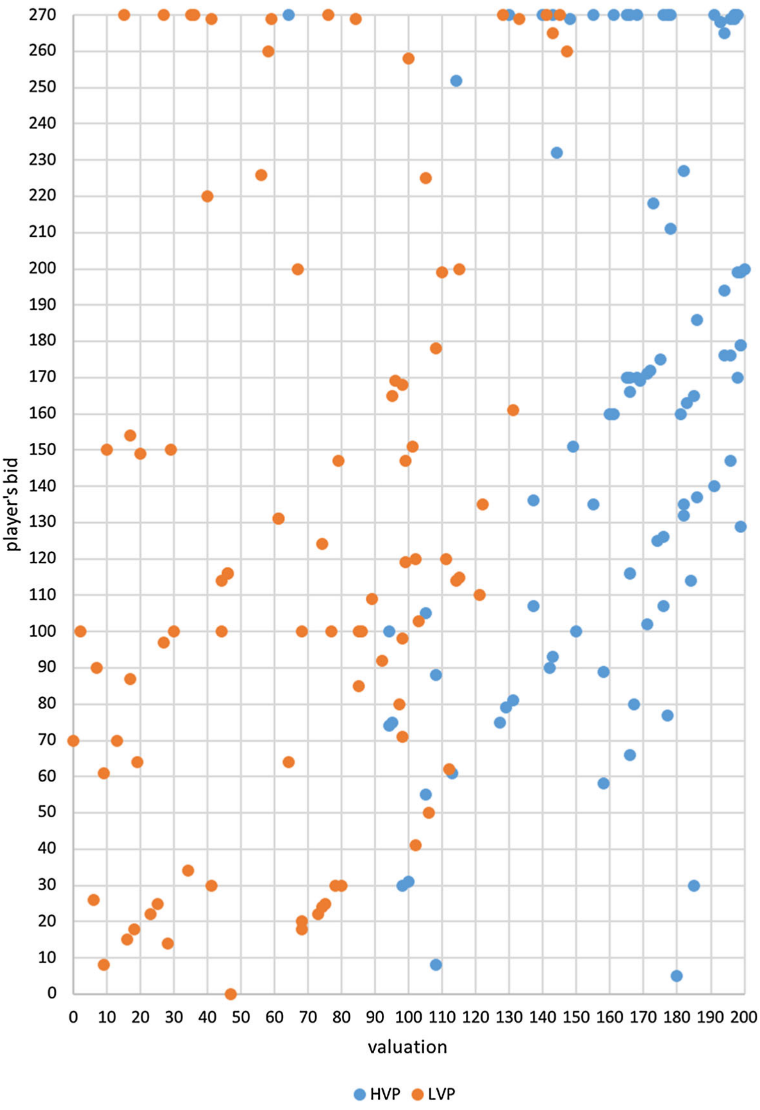

Fig. 3 The scatterplot between HVPs' or LVPs' valuations and their bids under SPAPS

SPAPS. We found that ACAPS performed relatively better than SPAPS in terms of the right-player allocations, "resource inefficiency," "wrong-player inefficiency," and the equilibrium strategies of high valuation players. Regarding the first-best allocations and equilibrium strategies of low valuation players, in our experiment, there was a trend for improvement under ACAPS relative to SPAPS.

Although, in our experiment, the performance of ACAPS was relatively better than that of SPAPS, there remain open questions, such as participation fee, number of players and objects, and types of players.

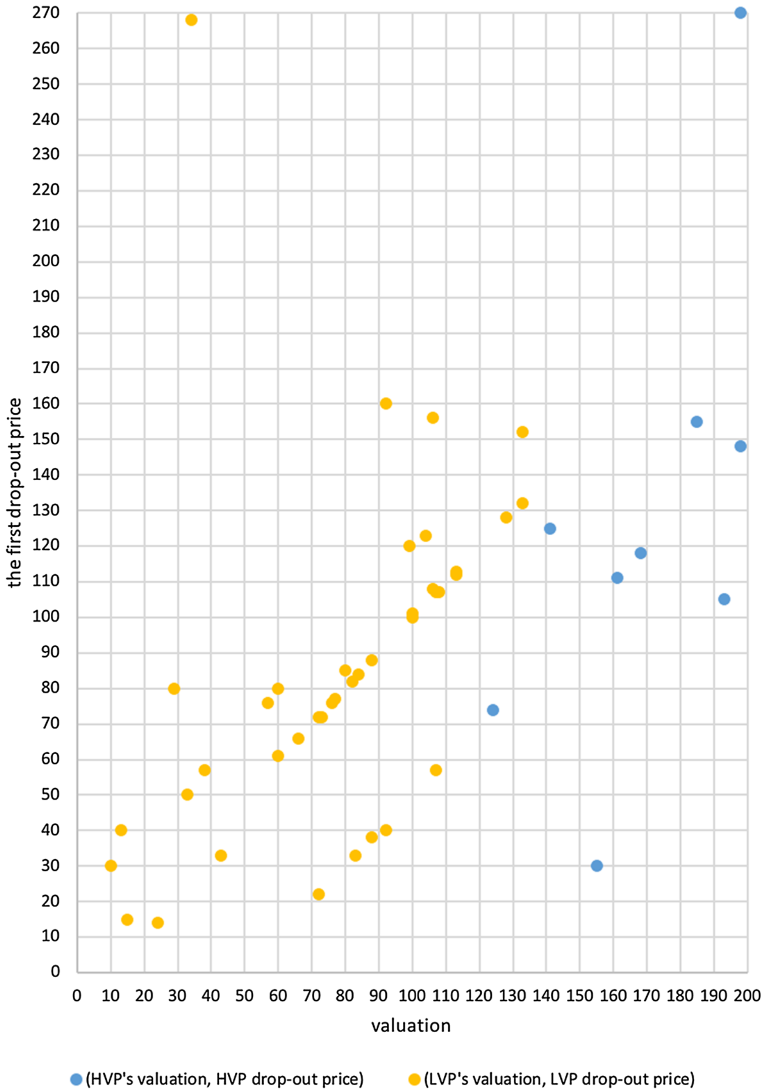

Fig. 4 The scatterplot between HVPs' or LVPs' valuations and their bids under ACAPS

First, in our experiment, when LVPs' valuations increase, resource inefficiency and wrong-player inefficiency increase, as in Sect. 4.2. As discussed in Sect. 4.1, if the valuation of a low valuation player for the object is higher than the participation fee, the player may imagine that she obtains the object with payment and thus participate in the auction. Therefore, increasing the participation fee may give a disincentive to low valuation players to participate in the auction. Using a similar reasoning, future studies can investigate whether conversely decreasing the participation fee may increase the rate of low valuation players selecting "auction" in the participation stage.

Second, our experiment has only two players and one object. If the numbers of players and objects increase, it may be harder for players to find equilibrium strategies. Even in this case, we expect ACAPS to perform relatively better than SPAPS. As discussed in Sect. 1, in a laboratory experiment, subjects are substantially more likely to play the dominant strategy in an ascending clock auction than in a second-price auction, and this observation is discussed from the viewpoint of obvious dominance. If players can more easily find the equilibrium strategy in the auction stage under ACAPS than under SPAPS, they may also find the equilibrium strategy in the participation stage more easily, even if the numbers of players and objects increase.

Third, in our experiment, the two types of players were fixed to follow the setting of Elbittar and Di Giannatale (2017). Under this setting, a few subjects with low valuations (two out of the 40 subjects with low valuations under each SPAPS and ACAPS) participated in the auctions to spite their counterparts with high valuations.[22] This may be because subjects' types were fixed, so that subjects with low valuations envied their counterparts.[23] In Ponti et al. (2003) that experimentally investigate two mechanisms proposed by Glazer and Ma (1989) and Ponti (2000) for King Solomon's dilemma, the two types of players were not fixed. In this setting, regarding SPAPS and ACAPS, each subject may change his/her strategy because they can be either a high valuation player or a low valuation player.

Supplementary Information The online version contains supplementary material available at https://doi.org/10.1007/s10058-023-00328-8.

References

Ausubel LM (2004) An efficient ascending-bid auction for multiple objects. Am Econ Rev 94(5):1452–1475
Barberà S (2011) Strategyproof social choice. In: Handbook of social choice and welfare, vol 2. Elsevier, Amsterdam, Netherlands, pp 731–831
Beard TR, Beil RO (1994) Do people rely on the self-interested maximization of others? An experimental test. Manag Sci 40(2):252–262
Breitmoser Y, Schweighofer-Kodritsch S (2021) Obviousness around the Clock. Exp Econ 25:483–513
Costa-Gomes M, Crawford VP, Broseta B (2001) Cognition and behavior in normal-form games: An experimental study. Econometrica 69(5):1193–1235
Dhami S (2016) The foundations of behavioral economic analysis. Oxford University Press, Oxford
Elbittar A, Di Giannatale S (2017) "Neither I nor You Shall Have Him": An experimental study of the King Solomon's Dilemma. J Behav Exp Econ 70:55–69
Fischbacher U (2007) z-Tree: Zurich toolbox for ready-made economic experiments. Exp Econ 10(2):171–178
Glazer J, Ma C-TA (1989) Efficient allocation of a "Prize"-King Solomon's Dilemma. Games Econom Behav 1(3):222–233

[22] This observation stems from the post-experimental questionnaires. While the subjects who spited their counterparts understood that selecting "no auction" is the best response for themselves, they participated in the auctions, as their counterparts had to pay money, including the participation fee.

[23] The subjects who spited their counterparts might not have considered their own payoffs much. As in our instructions, if subjects received negative points, the minimum hourly payment of JPY 1,013 was guaranteed. However, if they did not participate in the auction, they could obtain a larger amount of money. Therefore, our setting on calculating the final payment from subjects' payoffs gave the subjects incentives to obtain much larger payoffs. In short, while they had the chance to obtain a larger amount of money, they might have decided to spite their counterparts, as the loss of money via this behavior is not large.

Goeree JK, Holt CA (2001) Ten little treasures of game theory and ten intuitive contradictions. Am Econ Rev 91(5):1402–1422
Harstad RM (2000) Dominant strategy adoption and Bidders' experience with pricing rules. Exp Econ 3(3):261–280
Kagel JH, Harstad RM, Levin D (1987) Information impact and allocation rules in auctions with affiliated private values: a laboratory study. Econometrica 55(6):1275–1304
Kagel JH, Levin D (1993) Independent private value auctions: bidder behavior in first-, second- and third-price auctions with varying numbers of bidders. Economic Journal 103(419):868–879
Kagel JH, Levin D (2009) Implementing efficient multi-object auction institutions: An experimental study of the performance of boundedly rational agents. Games Econom Behav 66(1):221–237
Katok E, Sefton M, Yavas A (2002) Implementation by iterative dominance and backward induction: an experimental comparison. J Econ Theory 104(1):89–103
Kneeland T (2015) Identifying higher-order rationality. Econometrica 83(5):2065–2079
Li S (2017) Obviously strategy-proof mechanisms. Am Econ Rev 107(11):3257–3287
Masuda T, Okano Y, Saijo T (2014) The minimum approval mechanism implements the efficient public good allocation theoretically and experimentally. Games Econom Behav 83:73–85
McCabe KA, Rassenti SJ, Smith VL (1990) Auction institutional design: theory and behavior of simultaneous multiple-unit generalizations of the Dutch and English auctions. Am Econ Rev 80(5):1276–1283
Mihara HR (2012) The second-price auction solves King Solomon's dilemma. Jpn Econ Rev 63(3):420–429
Moffatt PG (2015) Experimetrics: econometrics for experimental economics. Macmillan International Higher Education
Moore J (1992) Implementation, contracts, and renegotiation in environments with complete information. In: Laffont J-J (ed) Advances in economic theory: sixth world congress, vol 1. Cambridge University Press, Cambridge, United Kingdom, pp 182–281
Nagel R (1995) Unraveling in guessing games: an experimental study. Am Econ Rev 85(5):1313–1326
Olszewski W (2003) A simple and general solution to King Solomon's problem. Games Econ Behav 42(2):315–318
Perry M, Reny PJ (1999) A general solution to king Solomon's dilemma. Games Econ Behav 26(2):279–285
Ponti G (2000) Splitting the baby in two: solving Solomon's dilemma with boundedly rational agents. J Evol Econ 10(4):449–455
Ponti G, Gantner A, López-Pintado D, Montgomery R (2003) Solomon's dilemma: an experimental study on dynamic implementation. Rev Econ Design 8(2):217–239
Pycia M, Troyan P (2019) A theory of simplicity in games and mechanism design, CEPR Discussion Paper No. DP14043
Qin C-Z, Yang C-L (2009) Make a guess: a robust mechanism for King Solomon's dilemma. Econ Theor 39(2):259–268
Saijo T, Masuda T, Yamakawa T (2018) Approval mechanism to solve Prisoner's dilemma: comparison with Varian's compensation mechanism. Soc Choice Welfare 51(1):65–77
Saijo T, Shen J (2018) Mate choice mechanism for solving a quasi-dilemma. J Behav Exp Econ 72:1–8
Schotter A, Weigelt KW, Wilson C (1994) A laboratory investigation of multiperson rationality and presentation effects. Games Econom Behav 6(3):445
Sefton M, Yavas A (1996) Abreu–Matsushima mechanisms: experimental evidence. Games Econom Behav 16(2):280–302
Vickrey W (1961) Counterspeculation, auctions, and competitive sealed tenders. J Finance 16(1):8–37