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ORIGINAL PAPER

# Collusion and turnover in experience goods markets

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**Abstract**

We study an infinite horizon duopoly with identical long-lived firms and a sequence of short-lived consumers. Consumers are willing to pay more for a higher-quality good, but quality is a noisy function of the firm's unobserved effort, and it cannot be observed by consumers prior to purchase. We show that a duopoly can overcome this moral hazard problem, and that it can outperform a monopoly in terms of both efficiency and producer surplus. Specifically, we consider collusive $N$-turnover equilibria, which involve buyers switching firms in perpetuity, i.e., buying from one firm until that firm delivers bad quality $N$ times, and then switching to the other firm. We show (1) that first-best efficiency can be achieved by duopoly in a collusive $N$-turnover equilibrium, even when monopoly cannot avoid deadweight loss, and (2) that monopoly profit in *any* equilibrium is strictly dominated by joint duopoly profit in some collusive $N$-turnover equilibrium.

**Keywords** Experience goods · Duopoly · Tacit collusion · Turnover · Efficiency

**JEL Classification** C7 · D8

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## 1 Introduction

Consumers are sometimes willing to pay a premium for higher quality goods. However, in markets for experience goods, the quality of a good cannot be assessed before purchase. For example, in the market for education, parents are willing to pay high tuition if they expect their children to receive a high-quality education. However, the quality of the education received by their children might be evident only after graduation. Prior to purchase, parents might obtain information about the quality of a school's education from its historical placement records and from the known success of its alumni.

When firms must exert high effort to produce high-quality goods, and this effort is unobservable by consumers, there is a moral hazard problem. In particular, suppose that costly effort by the firm makes better quality more likely, but the resulting quality is non-deterministic. If consumers are certain that the firm will exert the costly effort, then they are willing to pay a higher price. Consumers assume that any history of poor quality was due to chance, and, thus, poor quality by the firm goes unpunished. Then, the firm has no incentive to exert the costly effort in the first place.

We study this moral hazard problem by comparing two market structures, i.e., a monopoly and a duopoly, in markets for experience goods when there is imperfect monitoring. We show that there are cases in which a duopoly can overcome the moral hazard problem and deliver high-quality goods in expectation with first-best efficiency. This can be accomplished, even though a monopolist facing the same circumstance would *never* exert costly effort, and thus would provide only low-quality goods in expectation. Furthermore, we show that a duopoly can generate producer surplus that exceeds the monopoly profit in any equilibrium of the monopoly game.

Specifically, we consider a market with an infinite horizon and two long-lived firms with identical technologies. The quality of a firm's output is a stochastic function of its level of effort, i.e., high quality is more likely when the firm exerts high effort, and high effort incurs a fixed cost. There is a continuum of short-lived homogeneous consumers in every period. Buyers obtain more value from a high-quality good, and thus are willing to pay a higher price when they expect better quality. However, consumers observe neither the effort choice of the firm nor the quality of the good before purchase. The payoff from consuming a high-quality product and the cost of high effort to the firms are such that social surplus is greatest when exactly one firm exerts high effort in each period and sells to the entire market.

In order to establish our results, we first derive an upper bound on monopoly profit, showing that a monopolist can never capture the first-best expected social surplus.[1] Then, we construct duopoly equilibria in which the firms outperform monopoly in terms of efficiency and producer surplus (in any monopoly equilibrium). These constructions involve two key features—price collusion and turnover by consumers. Specifically, we consider $N$-turnover duopoly equilibria in which the firms take turns serving the market as follows. Once it is a firm's turn to sell, the firm keeps its turn

[1] The intuition is as follows. If such an equilibrium existed, then consumers would have to believe that the firm exerts high effort in every period. If the firm captures the entire efficient surplus each period, then it follows that the firm was not punished for realized bad quality. But then there is no incentive for the firm to exert high effort. We will prove this result in Proposition 1, where we calculate the upper bound.

provided it has not delivered low-quality output more than $N$ times during its current turn. When the active firm delivers low-quality output for the $(N+1)$th time during its current turn, consumers switch to the other firm, and this process is repeated indefinitely.[2] The threat of turnover induces high effort by the active firm, and, thus, it supports efficiency. Such turnover is crucial to our result that duopoly can outperform monopoly as it allows consumers to discipline a firm without destroying social surplus or diminishing producer surplus. On the other hand, the threat of a price war supports turnover by dissuading the inactive firm from stealing its competitor's turn. We prove that for discount rates sufficiently close to one, the above strategies constitute an equilibrium when $N \geq 0$ is suitably chosen.

This paper contributes to the literature on the reputation for quality and the literature on tacit collusion. Early examples of these studies are Mailath and Samuelson (2001), Holmstrom (1999) and Hörner (2002), who study the effect of market structure on firms' incentives to maintain a good reputation, and Kranton (2003), who examines the role of tacit collusion in a duopoly. Also related is Bar-Isaac (2005), who shows that competition can have a non-monotonic effect on quality in experience goods markets. In the following discussion, we focus on the works of Rob and Sekiguchi (2006) and Dana and Fong (2011), which are more closely related to our approach. Interested readers are referred to Bar-Isaac and Tadelis (2008) and Mailath and Samuelson (2006) for excellent surveys of the reputation literature, and to Ivaldi et al. (2007) for a review of the literature on tacit collusion.

In Rob and Sekiguchi (2006) a "turnover equilibrium" is one in which consumers punish each instance of bad quality with reduced demand in later periods. The authors examine when turnover in repeated duopoly generates higher welfare than that generated in the static case. In contrast, we focus on comparisons between market structures, and we always consider the dynamic case. Also, we use a more general notion of turnover to establish our results. In particular, our key equilibrium construction in the duopoly game will, in general, involve consumers forgiving bad quality some fixed number of times before punishment by turnover.[3]

Dana and Fong (2011) study the effect of tacit collusion on firms' incentives to produce high-quality goods. An important result in their paper is that the set of discount rates that support an equilibrium with high-effort is largest when there is duopoly (rather than monopoly, and larger oligopolies, including perfect competition in the limit). A key difference between their paper and ours is that quality is not stochastic in their model, i.e., high quality is obtained if, and only if, the firm has exerted costly effort. The high-quality equilibria in Dana and Fong (2011) are supported by a *grim trigger* strategy in which firms threaten to exert low effort and price at marginal cost following any instance of bad quality. In our model, such strategies cannot sustain

[2] The "turnover" nomenclature comes from Rob and Sekiguchi (2006). Their turnover equilibrium corresponds to a 0-turnover equilibrium in the current paper.

[3] Klein and Leffler (1981) provide an early intuition for the role of turnover by consumers. They consider a case in which quality is a deterministic function of firms' effort, and once a firm produces low-quality goods, it is shut out of the market forever. They show that if a firm is sufficiently patient, it will always provide high-quality goods. When quality is stochastic, and when the bad quality occurs with positive probability given any level of effort (as in our paper), their intuition no longer holds, i.e., each firm eventually will produce bad quality and will be driven out of the market.

high effort. In particular, since quality is a noisy function of effort, with probability one, each firm will have instances of low quality. A grim trigger as in Dana and Fong (2011) will then result in price war with probability one along the equilibrium path. In our paper high effort is sustained by turnover rather than the threat of a price war.

## 2 Model

We consider a market with an infinite horizon and two long-lived firms that produce a consumption good. This duopoly case is our benchmark model. We later give results pertaining to a monopoly model that differs from the duopoly model only in that there is one firm rather than two. All the key objects described below apply to the monopoly setup in a straightforward way, and thus we elect to forgo a formal description of the monopoly game.

In each period, $t = 1, 2, \ldots,$ there is a unit mass of short-lived consumers. Each consumer buys at most one unit of the good from one of the two firms. The output of a firm in a given period is of variable quality; it is either of good quality or bad quality. Specifically, the quality of firm $i$'s good in period $t$ is $\omega_{it} \in \{g, b\}$.[4]

The quality of a firm's output is a noisy function of its effort level. In each period, firm $i$ chooses effort level $e_{it} \in \{H, L\}$ , i.e., *High*, or *Low* effort. The firm must pay a fixed cost $c > 0$ to exert high effort. The cost of low effort is normalized to zero, and the marginal cost of producing the good is zero for both firms.[5]

All consumers are identical. Their utility depends on the quality of the good purchased and the price paid for the purchase. If a buyer consumes a good of quality $g$, and pays $p$, then her payoff is $1 - p$. On the other hand, if the buyer pays $p$, but consumes a good of quality $b$, then her payoff is $-p$. Consumers are risk neutral. Thus, if a consumer assigns probability $\gamma$ to the good being of high quality, then her expected payoff from consuming at price $p$ is $\gamma - p$. If she chooses not to consume, her payoff is zero.

The realized quality of the product depends on effort levels, but is i.i.d. across periods, and independent across firms. Each firm's quality is a random variable governed according to the probabilities described in the table below.

| | **g** | **b** |
|---|---|---|
| $e = \mathbf{H}$ | $\alpha$ | $1 - \alpha$ |
| $e = \mathbf{L}$ | $\beta$ | $1 - \beta$ |

[4] That is, in each period the firm's entire production run is either of good quality or bad quality. Our results hold if instead, we model quality as a variable defect rate, where, for example, a high-quality production run implies a lower defect rate than a low-quality run (provided that the realized defect rate of each firm is publicly observable).

[5] One interpretation here is as follows. Before each production run, a firm can invest in a technology that makes higher quality output more likely. The quality enhancing technology has constant returns in quality, however, such that any number of goods can be produced of a given quality at zero marginal cost.

For example, a firm's output quality is $g$ with probability $\alpha \in (0, 1)$ when the firm chooses effort level $e = H$. Good quality output is more likely when the firm chooses high effort, that is, $\alpha > \beta > 0$. We assume, further, that $\alpha - c > \beta$, and thus social surplus is higher when one firm exerts high effort and serves the entire market than when both firms exert low effort. Notice, moreover, that the maximal expected surplus in any given period is $\alpha - c$ and that this expected surplus is realized if, and only if, exactly one firm exerts the costly effort, and sells to the entire market.

The market operates in the following manner. In each period, $t = 1, 2, \ldots,$ consumers arrive on the market. Firm $i$, $i = 1, 2$, chooses effort level $e_{it} \in \{H, L\}$, where chosen effort level is private information. The firms then simultaneously announce prices, $p_{it} \in [0, \infty)$, $i = 1, 2$. Consumers observe the announced prices and a public history of the game, but not the current output quality of the firms. Buyers then decide whether to buy from firm 1 or firm 2, or to buy from neither firm. After purchase, each consumer realizes the quality of her purchased good. Realized quality in period $t$ then becomes public information in all subsequent periods, $\tau > t$. A period $\tau > 1$ public history is then $h_\tau = \{(x_{1t}, x_{2t}, p_{1t}, p_{2t})\}_{t=1}^{\tau-1}$, where $x_{it} \in \{g, b\}$ if firm $i$ sold any goods in period $t$, and $x_{it} = \{\emptyset\}$, otherwise. The set of all period $\tau$ public histories is $\mathcal{H}_\tau$, and $\mathcal{H}$ is the set of all public histories.

The strategy of firm $i$, $i = 1, 2$, is $\sigma_i$. A public strategy of a firm is one that conditions effort and price choices only on the public history, i.e., $\sigma_i : \mathcal{H} \to \{H, L\} \times [0, \infty)$.[6] For simplicity, we assume that all consumers adopt the same strategy. A consumer's public strategy conditions her purchase decision on the public history and on the firms' currently posted prices. Such a consumer strategy consists of a pair of functions $\mu = (s_1, s_2)$. Each $s_i$, $i = 1, 2$, is a function from $\mathcal{H}_t \times [0, \infty)^2$ to $[0, 1]$ such that $s_1(h, p_1, p_2) + s_2(h, p_1, p_2) \leq 1$, where $s_i(h, p_1, p_2)$ is the probability with which the consumer buys the good from firm $i$ when the public history is $h$ and the firms post the prices $p_1$ and $p_2$. We allow $s_1 + s_2 < 1$, which corresponds to cases where consumers reject both offers with positive probability.

Suppose consumers adopt the strategy $\mu = (s_1, s_2)$. The stage game payoff to firm $i$ at history, $h \in \mathcal{H}$, given the price offers $p_1, p_2$, is $s_i(h, p_1, p_2)p_i$, less the cost of effort to the firm. Both firms discount future payoffs according to the discount factor $\delta \in (0, 1)$. The payoff to a firm at a given history is then the average discounted sum of subsequent expected period payoffs. An equilibrium is then:

**Definition 1** *(Perfect Public Equilibrium)* A perfect public equilibrium (PPE) is a public strategy profile $(\sigma_1, \sigma_2, \mu)$ such that, for every public history $h \in \mathcal{H}$ (on and off the equilibrium path), the strategy of firm $i$ at $h$ is a best response to the remaining players' strategies, and furthermore, for each pair of prices, $p_1$ and $p_2$, the consumer strategy $\mu(h, p_1, p_2)$ is a best response for consumers given their beliefs about firms' current expected quality, where these beliefs are consistent with the firms' strategies.

[6] More generally, a firm's strategy can condition, in arbitrary ways, on its own history of effort choices, which is private information. We focus, however, on public equilibria. This makes the subsequent analysis considerably simpler without a significant loss of generality.

## 3 Results

Let $G_D$ denote the duopoly game described in the previous section, and let $G_M$ denote the monopoly game, identical in all respects to $G_D$, except that there is only one firm, rather than two. Our first result establishes an upper bound on the average payoff that can be earned by a monopolist. This upper bound holds in any equilibrium of the monopoly game (not just in a PPE).

**Proposition 1** (Monopoly).

(1) *Suppose* $(\alpha - c) - c\frac{1-\alpha}{\alpha-\beta} < \beta$. *Then, for each discount rate, there is no equilibrium of* $G_M$ *in which the monopoly firm exerts high effort at any history. The average payoff to the monopoly firm is* $\beta$*, the expected value of the good to consumers given low effort by the firm.*
(2) *Suppose* $(\alpha - c) - c\frac{1-\alpha}{\alpha-\beta} \geq \beta$. *Then, the average payoff to the firm at any history, in any equilibrium, is bounded above by* $(\alpha - c) - c\frac{1-\alpha}{\alpha-\beta}$.

The proof is in the "Appendix".

From now on we write $v_M = (\alpha - c) - c\frac{1-\alpha}{\alpha-\beta}$. Recall that $\alpha - c$ is the largest expected surplus that can be generated in any period. Hence, Proposition 1 implies that a monopolist can never capture the first-best expected surplus. The intuition is simple. In order to induce high effort by the monopolist, there must be punishment for delivering bad quality. In the case of monopoly, this punishment must involve diminished producer surplus in the continuation game.

The term $\frac{c}{\alpha-\beta}$, appearing in $v_M$, plays a key role in all of our results. This is the smallest discounted penalty for delivering low quality that induces the firm to exert high effort (this is established formally in the proof of the proposition, and it is also discussed in more detail below). Part (1) follows from Part (2). To see this, recall that consumers are short-lived, and thus myopic. Hence, it is always a best response for consumers to accept any price $p \leq \beta$. If $v_M < \beta$, then the monopolist will never exert high effort since it can guarantee itself a period payoff of $\beta$.

Next, consider our main result on duopoly. (The proof is in the "Appendix").

**Proposition 2** (Duopoly) *Suppose* $v_M > 0$. *Then each of the following are true.*

(1) *If* $\alpha > \frac{c}{\alpha-\beta}$*, then, for each* $\delta$ *sufficiently close to one, the duopoly game has a PPE in which total surplus is* $\alpha - c$ *in each period, and producer surplus dominates monopoly profit in any equilibrium of* $G_M$.
(2) *If* $\alpha \leq \frac{c}{\alpha-\beta}$*, then, for each* $\delta$ *sufficiently close to one, the duopoly game has a PPE that dominates any equilibrium of* $G_M$ *in terms of total surplus and producer surplus.*

We see that the conventional wisdom, that a monopolist can always replicate other market structures, does not hold here. In particular, the result implies that if $v_M > 0$, then a duopoly can outperform monopoly in terms of producer surplus.[7] Part (1) of

[7] Although the case $v_M \leq 0$ is not covered by the claim, in this case monopoly profit is $\beta$ in each period (Proposition 1), for all discount rates. Clearly this can be matched by firms in $G_D$, provided that they are sufficiently patient.

the proposition states that duopoly firms can generate the first-best expected surplus in each period. Part (2) applies to cases in which it might not be possible to induce high effort by the firms in *every* period, and thus to obtain efficiency. Importantly, we find that sometimes duopoly can yield efficiency but a monopoly cannot. A stark contrast between the two market structures is obtained by applying Propositions 1 and 2.

***Example 1*** Suppose that $\alpha = 3/4$, $\beta = 1/4$, and $c \in (1/3, 3/8)$. Then $v_M(1) < \beta$, and thus the monopoly firm never chooses high effort. However, $\alpha > \frac{c}{\alpha-\beta}$, and therefore, in the duopoly game, sufficiently patient firms can generate the efficient expected social surplus, $\alpha - c$, in each period.

In order to prove Part (1) of Proposition 2 we construct an efficient PPE with turnover by consumers. With this in mind, consider the following definition.

**Definition 2** An $N$-turnover equilibrium of $G_D$ is one in which the firms take turns serving the market; when it is a firm's turn to sell, the firm keeps its turn until it delivers bad quality $N + 1$ times during its current turn, whereupon consumers switch to the other firm.

Definition 2 includes the turnover equilibria from Rob and Sekiguchi (2006) as a special case. In particular, Rob and Sekiguchi (2006) focus on equilibria in which the consumers switch firms each time the active firm delivers bad quality. $N$-turnover with $N > 0$ will involve forgiveness of the active firm $N$ times before a switch. In some cases, inducing high effort from duopoly firms will require $N$-turnover with $N > 0$.[8] This is further discussed below.

In the efficient $N$-turnover equilibrium the firm whose turn it is to sell chooses high effort. In order to support turn-taking in the equilibrium, the inactive firm is dissuaded from undercutting its competitor by the threat of a price war. Costly effort by the active firm is then sustained by two distinct mechanisms. The first is the threat of lost market share due to turnover. The second mechanism involves punishment via prices. Specifically, a firm charges the price $p = \alpha$ when it begins its turn to serve the market. During its turn, if the firm delivered low quality in the previous period, then it charges a low price, $p_* < \alpha$. On the other hand, if the firm delivered high quality in the previous period, then it is rewarded with the high price, $p = \alpha$.

In some cases, the duopoly game will have an efficient equilibrium without any price punishment, and, thus, the firms can extract the full efficient surplus in each period. That is, a firm that delivers bad quality is punished only via turnover, and hence the loss to the firm is captured entirely by its competitor. To see when $G_D$ has such an equilibrium consider the following. Suppose consumers turnover after each realization of bad quality, i.e., 0-turnover. Let $V$ denote the expected payoff to the active firm, given that it exerts high effort, and let $W$ denote the expected payoff to

[8] Our $N$-turnover strategies are similar to the review strategies by Radner (1985). Those strategies involve a testing phase as follows. The infinite horizon is divided in infinitely many blocks of $T$ periods each, called review phases. After the $T$ periods have passed, the players will have either passed or failed a test, after which a new phase begins. The continuation strategies depend on whether the agent passed or failed the test. Other papers have subsequently used similar strategies, such as Matsushima (2004), Escobar and Toikka (2013) and Chandrasekher (2015).

the inactive firm. It follows that

$$V = \alpha - c + \delta\alpha V + \delta(1-\alpha)W, \quad \text{and}$$
$$W = \delta\alpha W + \delta(1-\alpha)V.$$

In order for the active firm to exert high effort in the equilibrium we must have

$$V \geq \alpha + \delta\beta V + \delta(1-\beta)W.$$

This holds only if

$$\delta(V-W) \geq \frac{c}{\alpha-\beta}. \tag{1}$$

We have, moreover, that $V - W = \alpha - c + \delta(2\alpha - 1)(V - W)$, and thus

$$V - W = \frac{\alpha - c}{1 - \delta(2\alpha - 1)}.$$

Hence, $V - W$ tends to $\frac{\alpha-c}{2(1-\alpha)}$ as $\delta$ tends to one. It follows that if,

$$\frac{\alpha-c}{2(1-\alpha)} > \frac{c}{\alpha-\beta}, \tag{2}$$

then condition (1) will hold, for $\delta$ sufficiently close to one, and thus the duopoly game will have an efficient 0-turnover equilibrium in which producer surplus is $\alpha - c$.

When Eq. (2) is not satisfied, punishment by turnover is not sufficient to induce high effort by the firms. In these cases, a firm must be punished for bad quality with the low price $p_*$. The condition, $\alpha > \frac{c}{\alpha-\beta}$, from Part (1) of Proposition 2 now plays a key role. To see this, consider an $N$-turnover equilibrium with $N > 0$.

Suppose the active firm has $n > 0$ remaining chances to deliver bad quality before consumers switch to the other firm. Then, the discounted penalty to the firm for delivering bad quality is strictly greater than $\delta(\alpha - p_*)$, i.e., if the firm delivers bad quality, then (1) it must sell at $p_*$ in the subsequent period, and (2) it is also closer to losing its turn. Recall that $\frac{c}{\alpha-\beta}$ is the smallest discounted penalty that induces high effort by the firm. The condition $\alpha > \frac{c}{\alpha-\beta}$ ensures that there is some punishment price, $p_* < \alpha$, that will induce the firm to exert high effort when it has $n > 0$ chances remaining. When the active firm has no remaining chances it will be a best response to choose high effort, provided $N$ is sufficiently large (i.e., if the firm produces low quality output, then it will lose its turn to sell for at least $N$ periods).

To see that producer surplus in the $N$-turnover equilibrium can exceed producer surplus in $G_M$, notice the following. In the monopoly case, in order to induce high effort by the firm, the expected discounted continuation profit of the firm must decrease by $\frac{c}{\alpha-\beta}$ every time the firm delivers bad quality output. This is also true for the duopoly firms. However, for the monopoly this penalty is applied to producer surplus *every* time the firm delivers bad quality. In an $N$-turnover equilibrium, on the other hand, not every penalty decreases producer surplus. Indeed, 1 out of every $N + 1$ penalties are through turnover, and these penalties do not diminish producer surplus.

Next, consider Part (2) of Proposition 2, that is, the case in which $\alpha \leq \frac{c}{\alpha-\beta}$. In this case, the monopoly firm will never exert high effort, and thus producer surplus is $\beta$ in every equilibrium of $G_M$.[9] In the duopoly game, on the other hand, while it may not be possible to induce high effort from the firms in every period, we show that the duopoly can outperform a monopolist in terms of efficiency and producer surplus. In particular, we construct an equilibrium as follows. Only one firm serves the market in each period. Firm 1 is the high-effort firm and firm 2 is the low-effort firm. If firm 1 delivers low-quality output, then consumers switch to firm 2 for $K$ periods. Buyers return to firm 1 after the $K$ period punishment phase. If the punishment phase is sufficiently long, then firm 1 will exert high effort. Since high effort is chosen along the equilibrium path, it follows that total surplus dominates $\beta$, which is the largest achievable surplus in $G_M$, given that $\alpha \leq \frac{c}{\alpha-\beta}$.

We see that the distinctive feature of a duopoly that allows it to outperform a monopoly is that, in duopoly, turnover allows more severe punishment of firms for delivering bad quality. Thus, it is possible to induce high effort from duopoly firms under a broader set of circumstances than is the case when there is a monopoly. To see this, notice that consumers will accept any a price $p \leq \beta$. Hence, a monopolist can always guarantee itself a period payoff of $\beta$. There is no such guarantee of a stage game payoff for a duopoly firm. For example, when all consumers switch firms, the punished firm earns a period payoff of zero until consumers switch again.

Although the producer surplus in a duopoly can dominate $v_M$, the profit of each duopoly firm is bounded above by $v_M$. To see this, recall that in order for a firm to exert high effort it must face an expected discounted loss of at least $\frac{c}{\alpha-\beta}$ for producing low-quality output. Since producer surplus is bounded above by $\alpha - c$, it follows that when a firm chooses high effort, its average expected continuation payoff can never exceed $\alpha - c - (1-\alpha)\frac{c}{\alpha-\beta} = v_M$. However, we have the following result, which is proved in the "Appendix".

**Proposition 3** (1) *Suppose $v_M \geq \frac{\alpha-c}{2}$. Then, for each $\delta$ sufficiently close to one, the duopoly game has a PPE in which each firm earns an average profit of $\frac{\alpha-c}{2}$.*

(2) *Suppose $v_M \in (0, \frac{\alpha-c}{2}]$, and $\alpha > \frac{c}{\alpha-\beta}$. Then, for each $\epsilon > 0$, if the firms are sufficiently patient, the duopoly game has an efficient PPE in which each firm earns an average expected payoff of $v_M - \epsilon$.*

The condition, $v_M > \frac{\alpha-c}{2}$, from Part (1) is equivalent to the condition that ensures there is an efficient 0-turnover equilibrium [see Eq. (2)]. Part (2) is established by constructing an appropriate $N$-turnover equilibrium.

[9] Notice that $\alpha \leq \frac{c}{\alpha-\beta}$ if, and only if, $v_M \leq \beta\frac{c}{\alpha-\beta}$, and recall that $\frac{c}{\alpha-\beta} < 1$, by assumption. Hence, $\alpha \leq \frac{c}{\alpha-\beta}$ implies $v_M < \beta$, and, therefore, Proposition 1 implies the monopoly firm never exerts high effort in an equilibrium.

## 4 Further discussion of the results

### 4.1 Robustness

How robust are our results to perturbations of the model? We have assumed, for example, that both firms can observe the realized quality of the product purchased by consumers. This may not be reasonable in some cases. However, the assumption can be relaxed, provided the firms can correctly anticipate consumer demand.

Recall that an efficient equilibrium involves exactly one firm exerting costly effort in each period, with all consumers purchasing from this high-effort firm. Hence, attaining efficiency in our setting requires that each firm know when it is its turn to serve the market, and thus when to exert high effort. Conversely, the firm that is supposed to be dormant must know that it should choose low effort. However, if each firm could not observe the other's quality and thus could not anticipate when consumers are going to switch demand, our duopoly result will continue to hold under the following reasonable perturbation of the model. In each period the firms post prices, consumers then submit orders to the firms, and firms make effort choices only *after* observing their own orders. Propositions 2 and 3 will hold in this new model. In fact, the results hold even if the firms cannot observe own realized quality, and when consumers obtain only noisy signals of prior realized quality.[10]

### 4.2 Multiplicity and selection of equilibria

As expected, our duopoly game has a large number of equilibria. One empirically plausible equilibrium involves both firms exerting high effort and splitting the market in each period. Other possibilities include segmented equilibria, where one firm specializes in high-quality goods, while the other firm never exerts costly effort. In this section we describe a selection argument based on the robustness of the market structure in an equilibrium.[11]

Consider the following perturbations of the games $G_M$ and $G_D$. Suppose that a firm's cost of effort is private information. Specifically, the cost of effort to the firm is zero with probability $\epsilon$, and $c > 0$ with the probability $1 - \epsilon$. The games $G_M(\epsilon)$ and $G_D(\epsilon)$ are otherwise identical to $G_M$ and $G_D$, respectively. The appropriate solution concept for the perturbed games is Perfect Bayesian Equilibrium (PBE). With this in mind, let $v_M(\epsilon)$ denote the maximum average payoff achievable in a PBE of $G_M(\epsilon)$ by a monopolist with cost of effort $c > 0$.

A straightforward application of the main theorem in Fudenberg and Levine (1992) gives:

**Proposition 4** *For each $\epsilon > 0$, $v_M(\epsilon)$ tends to $\alpha - c$, in the limit, as $\delta$ tends to one.*

[10] A higher $\delta$ will be needed the noisier is the signal obtained by consumers.

[11] Dana and Fong (2011) establish a result on the anti-persistence of monopoly in experience goods markets with perfect monitoring. Specifically, they show that duopoly gives the widest range of discount rates that support high quality in equilibrium. Earlier work by Gilbert and Newbery (1982), Reinganum (1983) and Yi (1995) also discuss the persistence of monopoly, albeit under different conditions.

Now, say that an equilibrium of $G_D(\epsilon)$ is robust to monopoly if payoffs in the equilibrium are such that the "regular" firms would never agree to merge. The idea here is that if we observe a duopoly, that is, if the firms have not elected to merge, then we can reasonably expect their behavior to be consistent with an equilibrium that is robust to monopoly. In view of Proposition 4, we see that, as $\delta$ tends to one, the only robust equilibria of $G_D(\epsilon)$ are those where the $c > 0$ type sellers earn the producer surplus $\alpha - c$ in every period. Notice, however, that the monopolist's payoff in $G_M(\epsilon)$ must be strictly less than $\alpha - c$. On the other hand, sufficiently patient duopoly firms can achieve the producer surplus $\alpha - c$ in $G_D(\epsilon)$. To see this, notice that if the unperturbed game, $G_D$, has an $N$-turnover equilibrium in which the firms extract the efficient surplus in each period, then this must also be true of $G_D(\epsilon)$. In view of Part (1) of Proposition 3, we have:

**Corollary 1** *Suppose $v_M > \frac{\alpha - c}{2}$. Then, if $\delta$ is sufficiently close to one, there is a PBE of $G_D(\epsilon)$ in which the duopoly firms generate the expected surplus $\alpha - c$ in each period and in which the firms extract this entire surplus. Hence, this equilibrium is robust to monopoly.*

## 5 Conclusion

There are many markets, such as health care, fine dining, and education, in which consumers are willing to pay more for high-quality goods, but in which quality cannot be observed before purchase. When producing high-quality goods requires costly effort by firms, there is moral hazard. In this paper, we present a dynamic duopoly model with imperfect public monitoring and argue that price collusion and turnover allow firms in duopoly to overcome this moral hazard and hence to outperform monopoly in terms of efficiency and producer surplus. The argument is simple. Consumers discipline firms by punishing bad quality with low market share (turnover), thus inducing firms to exert costly quality-enhancing effort. Turnover is key as it allows the market to discipline firms without destroying surplus or diminishing producer surplus. Collusion supports a premium for producing high-quality goods, and it also supports turnover by dissuading firms from stealing each other's turns.

## 6 Appendix

In this Appendix, we prove the results stated in Sect. 3.

### 6.1 Proof of Proposition 1

The key step in the proof of Proposition 1 is to establish the following upper bound on the average monopoly payoff, given that the monopolist exerts costly effort.

**Proposition 5** *Suppose the monopolist exerts costly effort in some equilibrium of $G_M$. Then, the average monopoly payoff in any equilibrium of $G_M$ is bounded above by $\alpha - c - (1 - \alpha)\frac{c}{\alpha - \beta}$.*

***Proof*** Suppose there is some equilibrium of $G_M$ along which the monopolist exerts high effort, at some history. Let $\bar{V}$ be the maximum payoff that the monopolist can achieve in an equilibrium of $G_M$. Consider a history $h$, at which this payoff is achieved, and let $\gamma$ denote the consumers' belief that good quality will be realized at $h$. Clearly, $\gamma \leq \alpha$.

If the strategy of the monopolist is such that it chooses high effort at $h$ , then

$$\bar{V} \leq \gamma - c + \delta \left(\alpha V^{+} + (1-\alpha) V^{-}\right),$$

where $V^{+}$ is the monopolist's continuation payoff at $h$ when realized quality is good and $V^{-}$ is its continuation payoff if bad quality is realized instead. If the monopolist's strategy is such that it chooses low effort at $h$ with positive probability, then

$$\bar{V} \leq \gamma + \delta \left(\beta V^{+} + (1-\beta) V^{-}\right).$$

It must be true that the monopolist is weakly better off by choosing high effort at $h$. Suppose not. Then $\bar{V} \leq \beta + \delta(\beta V^{+} + (1-\beta)V^{-})$. Recall that $\bar{V}$ is the highest possible continuation payoff. This implies that

$$\bar{V} \leq \beta + \delta \left(\beta \bar{V} + (1-\beta) \bar{V}\right) = \beta + \delta \bar{V}.$$

Solving for $\bar{V}$ here yields that $\bar{V} \leq \frac{\beta}{1-\delta}$ . Since $\beta$ is the bound on the monopolist's average payoff in $G_M$, this contradicts that the monopolist ever exerts high effort in an equilibrium of $G_M$.

We have established that the monopolist is weakly better off by exerting high effort at $h$. It follows that

$$\gamma - c + \delta(\alpha V^{+} + (1-\alpha)V^{-}) \geq \gamma + \delta \left(\beta(V^{+} + (1-\beta)V^{-}\right),$$

and therefore,

$$V^{+} - V^{-} \geq \frac{c}{\delta (\alpha - \beta)}. \tag{3}$$

Again, write the monopolist's payoff at $h$, and then add and subtract $\delta V^{+}$ to obtain,

$$\begin{aligned} \bar{V} &= \gamma - c + \delta \left(\alpha V^{+} + (1-\alpha) V^{-}\right) + \delta V^{+} - \delta V^{+} \\ &\leq \alpha - c + \delta \left(\alpha V^{+} + (1-\alpha) V^{-}\right) + \delta \bar{V} - \delta V^{+}. \end{aligned}$$

In the last step we used the fact that $V^{+} \leq \bar{V}$ and that $\gamma \leq \alpha$. Rearranging this yields

$$\begin{aligned} &\bar{V} (1-\delta) \leq \alpha - c + \delta \left(\alpha V^{+} - V^{+} + (1-\alpha) V^{-}\right), \text{ and thus} \\ &\bar{V} (1-\delta) \leq \alpha - c - \delta (1-\alpha) \left(V^{+} - V^{-}\right). \end{aligned}$$

Equation (3) then gives:

$$\bar{V}(1-\delta) \leq (\alpha - c) - (1-\alpha) \frac{c}{\alpha - \beta}.$$

Hence, the maximum average payoff achievable by the monopolist in $G_M$, given that it exerts high effort, is $\alpha - c - (1-\alpha)\frac{c}{\alpha-\beta}$. □

This result establishes Part (2) of Proposition 1. To see that it also implies Part (1) of Proposition 1, recall that consumers are myopic and thus accept any price $p \leq \beta$. It follows that if the monopolist exerts low effort in every period, then it will earn $\beta$ in each period. Hence, if $\alpha - c - c\frac{1-\alpha}{\alpha-\beta} < \beta$ the monopolist will never choose high effort in an equilibrium.

### 6.2 Proof of Propositions 2 and 3

Part (1) of Proposition 2 follows from Proposition 3. Hence, we first prove Proposition 3 by establishing Propositions 6 and 7 below. Part (2) of Proposition 2 is proved last (Propsition 8 below).

**Proposition 6** *Suppose $v_M > \frac{\alpha-c}{2}$. Then, for each discount factor sufficiently close to one, the duopoly game has an $N$-turnover equilibrium, with $N=0$, in which the firms generate the maximal expected surplus in each period, and the average joint profit of the firms is $\alpha - c$.*

***Proof*** Consider a 0-turnover equilibrium. Let firm 1 be active in the initial period. An active firm keeps its turn until it delivers bad quality once. Once the active firm delivers bad quality, consumers switch to the other firm. Let there be a price war state. The game enters the price war state if any firm steals the other firm's turn by offering consumers a higher surplus. Once the game enters the price war state it stays there forever.

Consider buyer strategies as follows. Consumers buy from whichever firm provides the highest expected consumer surplus, provided this is positive. Suppose the game is in the price war state. If the firms price to yield the same expected consumer surplus, and this surplus is positive, then buyers mix evenly between the two firms. If the game is not in the price war state and each firm prices to yield the same expected consumer surplus, then consumers buy from the active firm. If both firms price in order to give a negative expected surplus to consumers, then consumers reject both offers.

Let firm strategies be as follows. The active firm chooses high effort and sets the price $p = \alpha$. The inactive firm chooses low effort and sets some arbitrary price $p \geq \beta$. If the game is in the price war state each firm chooses low effort and sets the price $p = 0$.

Obviously, the consumer strategy is a best response to the firm strategies, and the firm strategies are best responses when there is a price war. We thus focus on establishing that the firms are best responding when they are not in a price war. With this in mind, let $V^+$ denote the continuation payoff to the active firm, and let $V^-$ denote the payoff to the inactive firm, given that there is no price war. We have

$$\begin{aligned} V^+ &= \alpha - c + \delta\alpha V^+ + \delta(1-\alpha)V^- \\ V^- &= \delta\alpha V^- + \delta(1-\alpha)V^+. \end{aligned} \tag{4}$$

Subtracting the second line from the first gives

$$V^+ - V^- = \frac{\alpha - c}{1 - \delta(2\alpha - 1)}. \tag{5}$$

Using this in the second line of Eq. (4), and solving for $V^-$, gives

$$V^- = \frac{\delta(1-\alpha)}{1-\delta}\frac{\alpha - c}{1 - \delta(2\alpha - 1)}. \tag{6}$$

It is a best response for the active firm to exert high effort if $V^+ \geq \alpha + \delta(\beta V^+ + (1-\beta)V^-)$. After some algebra we find that this is satisfied when $\delta(V^+ - V^-) \geq \frac{c}{\alpha-\beta}$. The above strategies constitute an equilibrium provided that

$$V^+ - V^- \geq \frac{c}{\delta(\alpha - \beta)}, \text{ and}$$
$$V^- \geq \beta.$$

We assume that in the equilibrium consumers believe a firm has exerted low effort whenever it makes a deviating price offer. Hence, the second condition suffices to ensure that the inactive firm will not undercut its competitor (recall that continuation profit is zero for each firm in the price war state). From Eq. (6) we see that $V^- \geq \beta$ will hold given a sufficiently large $\delta$. Hence, to complete the proof, it suffices to show that for sufficiently large $\delta$ we will have

$$\frac{\alpha - c}{1 - \delta(2\alpha - 1)} \geq \frac{c}{\delta(\alpha - \beta)}.$$

This will be possible provided

$$\frac{\alpha - c}{2} > (1-\alpha)\frac{c}{\alpha - \beta}.$$

By adding $(\alpha - c)/2$ to both sides of this expression, we see that it holds if, and only if, $v_M > (\alpha - c)/2$, which is the hypothesis of the proposition. Recall that the active firm charges the price $p = \alpha$ in each period, and thus the firms extract the surplus $\alpha - c$ in each period. □

**Proposition 7** *Suppose* $v_M \in (0, \frac{\alpha-c}{2})$ *and* $\alpha > \frac{c}{\alpha-\beta}$. *Then, for each* $\epsilon > 0$, *if the firms are sufficiently patient, the duopoly game has an* $N$*-turnover equilibrium in which the firms generate the maximal expected surplus in each period, and the average profit of each firm is at least* $v_M - \epsilon$.

***Proof*** Fix some price $p_* \in [0, \alpha)$, and an $N \in \{0, 1, 2, ..., \}$. We construct an $N$-turnover equilibrium as follows. The firms take turns serving the market. Firm 1 is active first. The active firm always chooses high effort. A firm loses its turn once it has delivered bad quality $N + 1$ times during its turn. While it is active, the firm prices as follows. When the firm begins its turn, it sets the price $p = \alpha$. After this, it posts the price $p = \alpha$ if it delivered good quality in the previous period, and it posts $p_*$ if it delivered bad quality. The inactive firm chooses low effort and posts the price $p' > \beta$. If any firm makes a deviating price offer, then the game enters a price war state and stays there forever. When in the price war state each firm chooses low effort and post the price $p = 0$. Consumers buy from whichever firm prices to yield the highest expected consumer surplus, provided this is positive. In the price war state buyers mix evenly between the two firms when the firms price to yield the same expected consumer surplus, provided this is positive. On the other hand, a consumer rejects both offers if both prices yield a negative expected consumer surplus. When not in the price war state, if each firm prices to yield the same expected consumer surplus, then buyers purchase from the active firm.

Given the above strategy profiles, the firms will generate the efficient expected surplus, $\alpha - c$, in each period. The buyer strategy is a best responses to the firm strategies, and the firms' strategies are best responses in the price war state. In order to verify that we have an equilibrium, it thus suffices to show that the firms' strategies are best responses when they are not in a price war. With this in mind, let $V^+(n)$, $n = 0, 1, 2 \ldots N$, denote the active firm's expected continuation payoff given that it will earn $p = \alpha$ in the current period, and that it has $n+1$ remaining chances to deliver bad quality, before the consumers switch to the other firm. Similarly, let $V^-(n)$ denote the active firm's continuation payoff given that it will earn $p_*$ in the current period, and that it has $n + 1$ remaining chances. Let $W(n)$ denote the inactive firm's expected continuation payoff when the other firm's has $n + 1$ remaining chances to deliver bad quality before losing its turn.

We have

$$\begin{aligned} &V^+(n) = \alpha - c + \delta\alpha V^+(n) + \delta(1-\alpha)V^-(n-1), \quad n = 1, \ldots N, \\ &V^-(n) = p_* - c + \delta\alpha V^+(n) + \delta(1-\alpha)V^-(n-1), \quad n = 1, \ldots N-1, \\ &V^+(0) = \alpha - c + \delta\alpha V^+(0) + \delta(1-\alpha)W(N), \quad \text{and} \\ &V^-(0) = p_* - c + \delta\alpha V^+(0) + \delta(1-\alpha)W(N). \end{aligned} \tag{7}$$

It is a best response for the active firm to choose high effort if

$$\begin{aligned} &V^+(n) \geq \alpha + \delta\beta V^+(n) + \delta(1-\beta)V^-(n-1), \quad n = 1, \ldots N, \\ &V^-(n) \geq p_* + \delta\beta V^+(n) + \delta(1-\beta)V^-(n-1), \quad n = 1, \ldots N-1 \\ &V^+(0) \geq \alpha + \delta\beta V^+(0) + \delta(1-\beta)W(N), \quad \text{and} \\ &V^-(0) \geq p_* + \delta\beta V^+(0) + \delta(1-\beta)W(N). \end{aligned}$$

After some algebra we find that these conditions are

$$\begin{aligned} \delta(V^+(n) - V^-(n-1)) &\geq \frac{c}{\alpha - \beta}, \quad n = 1, \ldots N, \quad \text{and} \\ \delta(V^+(0) - W(N)) &\geq \frac{c}{\alpha - \beta}. \end{aligned} \tag{8}$$

In order for the strategies to constitute an equilibrium we require that the equations in (8) hold, and that $W(n) \geq \beta$, for each $n = 1, \ldots, N$ (otherwise, it is a best response for the inactive firm to undercut its competitor).

Equation (7) implies

$$V^{+}(n) = \frac{\alpha - c}{1 - \alpha\delta} + \frac{\delta(1-\alpha)}{1 - \alpha\delta} V^{-}(n-1), \quad n = 1, \ldots, N.$$

Since $V^{-}(k) = V^{+}(k) - (\alpha - p_{*})$, $k = 1, \ldots N-1$, we have that

$$V^{+}(n) = \frac{\alpha - c - \delta(1-\alpha)(\alpha - p_{*})}{1 - \alpha\delta} + \frac{\delta(1-\alpha)}{1 - \alpha\delta} V^{+}(n-1), \quad n = 1, \ldots, N.$$

Iterating this expression we find that

$$V^{+}(n) = x(\delta)\frac{1 - \xi^{n}}{1 - \xi} + \xi^{n} V^{+}(0), \quad n = 1, \ldots, N,$$

where we have defined $x(\delta) = \frac{\alpha - c - \delta(1-\alpha)(\alpha - p_{*})}{1 - \alpha\delta}$, and $\xi = \frac{\delta(1-\alpha)}{1-\alpha\delta}$. We also have

$$V^{+}(0) = \frac{\alpha - c}{1 - \alpha\delta} + \xi W(N),$$

and thus

$$V^{+}(n) = x(\delta)\frac{1 - \xi^{n}}{1 - \xi} + \xi^{n}\frac{\alpha - c}{1 - \alpha\delta} + \xi^{n+1} W(N), \quad n = 1, \ldots, N. \tag{9}$$

Next, notice that for each $n = 1, \ldots, N$,

$$W(n) = \alpha\delta W(n) + (1-\alpha)\delta W(n-1),$$

and thus $W(n) = \xi W(n-1)$, which yields $W(n) = \xi^{n} W(0), \quad n = 1, \ldots, N$. Notice also that

$$W(0) = \alpha\delta W(0) + (1-\alpha)\delta V^{+}(N),$$

and thus, $W(0) = \xi V^{+}(N)$. Therefore,

$$W(n) = \xi^{n+1} V^{+}(N), \quad n = 1, \ldots, N.$$

Using this in Eq. (9), we obtain

$$V^{+}(n) = x(\delta)\frac{1 - \xi^{n}}{1 - \xi} + \xi^{n}\frac{\alpha - c}{1 - \alpha\delta} + \xi^{2(n+1)} V^{+}(N), \quad n = 1, \ldots, N. \tag{10}$$

Setting $n$ equal to $N$ in the above formula and solving for $V^+(N)$ yields

$$V^+(N) = \frac{x(\delta)}{1-\xi}\left(\frac{1-\xi^N}{1-\xi^{2(N+1)}}\right) + \frac{\alpha - c}{1-\alpha\delta}\left(\frac{\xi^N}{1-\xi^{2(N+1)}}\right). \tag{11}$$

Now, consider the conditions in (8). Equation (7) implies

$$\begin{aligned} V^+(n) - V^+(n-1) &= \delta\alpha(V^+(n) - V^+(n-1)) + \delta(1-\alpha)(V^-(n-1) - V^-(n-2)) \\ &= \delta\alpha(V^+(n) - V^+(n-1)) + \delta(1-\alpha)(V^+(n-1) - V^+(n-2)). \end{aligned}$$

(To obtain the second line, we used the fact that $V^-(k) - V^+(k) = \alpha - p_*, k = 1, \ldots N-1$.) Therefore,

$$\begin{aligned} V^+(n) - V^+(n-1) &= \xi^{n-1}(V^+(1) - V^+(0)), \quad n = 1, \ldots, N. \\ &= \xi^n(V^-(0) - W(N)), \quad n = 1, \ldots, N, \end{aligned}$$

since $V^+(1) - V^+(0) = \xi(V^-(0) - W(N))$. This yields

$$V^+(n) - V^-(n-1) = (\alpha - p_*) + \xi^n(V^+(0) - W(N) - (\alpha - p_*)), \quad n = 1, \ldots N.$$

Since $\xi \le 1$, and $\alpha > p_*$, we obtain

$$V^+(n) - V^-(n-1) \ge \xi^n(V^+(0) - W(N)), \quad n = 1, \ldots, N.$$

Next, $V^+(0) = \frac{\alpha - c}{1-\alpha\delta} + \xi W(N)$, and so we see that

$$\begin{aligned} V^+(n) - V^-(n-1) &\ge \xi^n\left(\frac{\alpha - c}{1-\alpha\delta} - (1-\xi)W(N)\right), \quad n = 1, \ldots, N, \quad \text{and} \\ V^+(0) - W(N) &= \frac{\alpha - c}{1-\alpha\delta} - (1-\xi)W(N). \end{aligned}$$

Now recall that $W(N) = \xi^{N+1}V^+(N)$, and therefore

$$\begin{aligned} V^+(n) - V^-(n-1) &\ge \xi^n\left(\frac{\alpha - c}{1-\alpha\delta} - (1-\xi)\xi^{N+1}V^+(N)\right), \quad n = 1, \ldots, N, \quad \text{and} \\ V^+(0) - W(N) &= \frac{\alpha - c}{1-\alpha\delta} - (1-\xi)\xi^{N+1}V^+(N). \end{aligned} \tag{12}$$

With this in mind, consider the term $(1-\xi)V^+(N)$ in Eq. (12). Equation (11) gives that

$$(1-\xi)V^+(N) = x(\delta)\frac{1-\xi^N}{1-\xi^{2(N+1)}} + \left(\frac{1-\xi}{1-\xi^{2(N+1)}}\right)\frac{\alpha - c}{1-\alpha\delta}\xi^N. \tag{13}$$

Recall that $\xi = \frac{(1-\alpha)\delta}{1-\alpha\delta}$, and thus $\xi$ tends to one as $\delta$ tends to one. Taking the limit in (13), as $\delta$ tends to one, we find (by applying L'Hospital's rule):

$$\lim_{\delta \longrightarrow 1} \left\{(1-\xi)V^{+}(N)\right\} = x(1)\frac{N}{2(N+1)} + \frac{1}{2(N+1)}\frac{\alpha - c}{1-\alpha}.$$

Recall that $x(1) = \frac{\alpha - c - (1-\alpha)(\alpha - p_*)}{1-\alpha}$, and thus

$$\lim_{\delta \longrightarrow 1} \left\{(1-\xi)V^{+}(N)\right\} = \frac{\alpha - c}{2(1-\alpha)} - (\alpha - p_*)\frac{N}{2(N+1)}. \tag{14}$$

Using this in Eq. (12) we see that

$$\begin{aligned} \lim_{\delta \longrightarrow 1} \left\{V^{+}(n) - V^{-}(n-1)\right\} &\geq \frac{\alpha - c}{2(1-\alpha)} + (\alpha - p_*)\frac{N}{2(N+1)}, \quad \text{and} \\ \lim_{\delta \longrightarrow 1} \left\{V^{+}(0) - W(N)\right\} &= \frac{\alpha - c}{2(1-\alpha)} + (\alpha - p_*)\frac{N}{2(N+1)}. \end{aligned} \tag{15}$$

Now recall the hypothesis, $v_M < \frac{\alpha - c}{2}$. Since $v_M = \alpha - c - (1-\alpha)\frac{c}{\alpha - \beta}$, it follows that $v_M < \frac{\alpha - c}{2}$ if, and only if, $\frac{\alpha - c}{2(1-\alpha)} < \frac{c}{\alpha - \beta}$. On the other hand, the assumption that $\alpha > \frac{c}{\alpha - \beta}$ implies there is some $p_* \in (0, \alpha]$, and $N$ such that, for a sufficiently small $\epsilon > 0$,

$$\frac{\alpha - c}{2(1-\alpha)} + (\alpha - p_*)\frac{N}{2(N+1)} = \frac{c}{\alpha - \beta} + \epsilon. \tag{16}$$

To see this, notice that if $\alpha > \frac{c}{\alpha - \beta}$, and $v_M = (\alpha - c) - (1-\alpha)\frac{c}{\alpha - \beta} > 0$, then

$$\frac{\alpha - c}{2(1-\alpha)} + \frac{\alpha}{2} > \frac{\alpha - c}{2(1-\alpha)} + \frac{c}{2(\alpha - \beta)} > \frac{c}{\alpha - \beta}.$$

For each $N$ and $p_*$, such that Eq. (16) holds, we have that it is a best response for the active firm to exert high effort, provided the firms are sufficiently patient [recall the conditions in (8)]. Since $1 - \xi = \frac{1-\delta}{1-\alpha\delta}$, Eq. (14) implies that $(1-\delta)V^{+}(N)$ tends to

$$v^* = \frac{\alpha - c}{2} - (1-\alpha)(\alpha - p_*)\frac{N}{2(N+1)}.$$

The $(1-\delta)W(n)$ terms tends to this limit as well, for each $n = 0, 1 \ldots, N$ (thus $W(n) > \beta$, for all $n = 0, 1, \ldots,$ as is required so that the inactive firm will not undercut its competitor). It follows that the average discounted profit of each firm is $v^*$. Notice, however, that by choice of $p_*$ and $N$,

$$\begin{aligned} v^* &= \frac{\alpha - c}{2} - (1-\alpha)\left(\frac{c}{\alpha - \beta} + \epsilon - \frac{1}{2}\frac{\alpha - c}{1-\alpha}\right) \\ &= \alpha - c - (1-\alpha)\frac{c}{\alpha - \beta} - \epsilon = v_M - \epsilon. \end{aligned}$$

This completes the proof of the proposition. □

In order to complete the proof of Proposition 2 we now establish Part (2) of the proposition.

**Proposition 8** *Suppose $v_M > 0$ and $\alpha \leq \frac{c}{\alpha-\beta}$. Then, for each sufficiently large $\delta$, there is a PPE of $G_D$ that dominates every equilibrium of $G_M$ with respect to producer surplus and total surplus.*

***Proof*** If $\alpha \leq \frac{c}{\alpha-\beta}$, then the monopolist never exerts high effort in an equilibrium of $G_M$. To see this, recall that $v_M = (\alpha - c) - (1-\alpha)\frac{c}{\alpha-\beta}$. Hence,

$$\begin{aligned} v_M &= \alpha - \frac{c}{\alpha-\beta} + \alpha\frac{c}{\alpha-\beta} - c \\ &\leq \alpha\frac{c}{\alpha-\beta} - c \\ &= \beta\frac{c}{\alpha-\beta} < \beta, \end{aligned}$$

since $\frac{c}{\alpha-\beta} < 1$, by assumption. Hence, Proposition 1 implies that the monopoly firm never exerts high effort in an equilibrium. In order to prove the result it then suffices to show that $G_D$ has a PPE in which there is high effort by at least one firm and, moreover, the firms extract the entire surplus in each period. Consider strategies as follows.

The firms take turns serving the market. When firm 1 is active it exerts high effort and charges the price $p = \alpha$. When firm 2 is active it exerts low effort and charges the price $\beta$. The inactive firm always chooses low effort and sets some price $p' > \beta$. If firm 1 delivers bad quality, then buyers switch to firm 2 for $K$ periods. Buyers return to firm 1 after each such $K$ period punishment phase. If an inactive firm undercuts the active one, then the game enters a price war state. In the price war state each firm chooses low effort and sets the price $p = 0$. Consumers buy from whichever firm provides the highest expected consumer surplus, provided this is positive. In the price war state if each firm prices to yield the same expected consumer surplus, and this surplus is positive, then consumers mix evenly between the firms. When the firms are not in a price war, and each firm prices to yield the same expected consumer surplus, consumers buy from the active firm. If each firm prices to yield negative expected consumer surplus then consumers reject both offers.

Fix $K$. Let $V(K)$ denote the continuation payoff to firm 1 when it is active. We have

$$V(K) = \alpha - c + \delta\alpha V(K) + \delta(1-\alpha)\delta^K V(K).$$

Hence,

$$V(K) = \frac{\alpha - c}{1 - \delta[\alpha + (1-\alpha)\delta^K]}. \tag{17}$$

It is a best response for firm 1 to choose high effort if $V(K) - \delta^K V(K) \geq \frac{c}{\delta(\alpha-\beta)}$. Equation (17) implies

$$V(K) - \delta^K V(K) = (1 - \delta^K) \frac{\alpha - c}{1 - \delta[\alpha + (1-\alpha)\delta^K]}. \tag{18}$$

Recall that if firm 1 delivers bad quality, then firm 2 sells at $\beta$ for the subsequent $K$ periods. The expected continuation payoff of firm 2, when it is inactive, is then

$$\begin{aligned} W(K) &= \delta\alpha W(K) + \delta(1-\alpha)\left[\beta \frac{1-\delta^K}{1-\delta} + \delta W(K)\right] \\ &> \frac{\delta(1-\alpha)}{1-\alpha\delta}\left[\beta \frac{1-\delta^K}{1-\delta}\right]. \end{aligned} \tag{19}$$

Clearly, the above consumer strategies are best responses to the firm strategies. Suppose consumers believe that a firm exerted low effort if it makes a deviating price offer. Then, the above strategies constitute an equilibrium if

$$\begin{aligned} V(K) - \delta^K V(K) &\geq \frac{c}{\delta(\alpha-\beta)}, \\ \delta^K V(K) &\geq \beta, \text{ and} \\ W(K) &\geq \beta. \end{aligned} \tag{20}$$

The first condition ensures firm 1 will exert high effort when it is active, and the last two conditions ensure that no firm will steal its competitor's turn. In view of (17) and (19) we see that for each finite $K$, $\delta^K V(K)$ and $W(K)$ can be made arbitrarily large by choosing a suitably large $\delta$. Hence consider the first condition in (20). Equation (18) gives that for each $K$

$$\begin{aligned} \lim_{\delta \longrightarrow 1} \left\{(1-\delta^K)V(K)\right\} &= (\alpha - c)\frac{K}{\alpha + (1-\alpha)(K+1)} \\ &= \frac{\alpha - c}{1 - \alpha + \frac{1}{K}}. \end{aligned}$$

It follows that for each sufficiently large $\delta$ there is a $K$ such that

$$V(K) - \delta^K V(K) \geq \frac{c}{\delta(\alpha-\beta)},$$

provided $\frac{\alpha-c}{1-\alpha} > \frac{c}{\alpha-\beta}$, which is the hypothesis, $v_M > 0$, of the claim. We have shown that the conditions in Eq. (20) will be satisfied for some $K$, given sufficiently patient firms. This completes the proof of the result since producer surplus in the equilibrium clearly dominates $\beta$, which is what is generated by the monopoly firm when $\alpha \leq \frac{c}{\alpha-\beta}$. □

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