Tau lepton Michel parameters and new physics

A. Rougé[a]

LPNHE Ecole Polytechnique-IN2P3/CNRS, 91128 Palaiseau Cedex, France

Abstract. A complete discussion of the constraints on the Michel parameters and the ambiguities of their interpretation is presented. Estimators of new physics, optimized for a very wide class of hypotheses and models, are proposed.

1 Introduction

The leptonic decay of a charged lepton $l_1^- \to l_2^- \nu_1 \bar{\nu}_2$ is described by its Michel parameters [1–4]. Complementary parameters must be introduced [5] to describe the secondary lepton polarization. The measurements of all the parameters and the cross-section for the $\nu_1 l_2^- \to l_1^- \nu_2$ reaction allow to put upper limits on all the non-V-A amplitudes [5,6] and "prove experimentally" the V-A structure of the weak interaction.

In the case of the μ lepton, this program was followed and successfully completed fifteen years ago [6,7].

For the τ lepton, it is customary [7–9] to present the experimental results, in a similar way, as upper values of the possible coupling constants. However, the validity of the Standard Model is today so well established, that the interest of τ Michel parameter measurements is no more to prove the V-A structure but to look for small deviations from its predictions that would be evidence of new physics. Moreover, the additional measurements, which are necessary to complete the experimental proof of the V-A structure, will not be performed in a foreseeable future, while large statistics of τ pairs will soon be available at B-factories, allowing more precise measurements of the Michel parameters.

Therefore it seems worthwhile to look for the combinations of the measured parameters which are the most sensitive to non-standard effects. This is the aim of the present paper.

2 The general framework

If only the momentum of the final state charged lepton is measured, the decay of a polarized τ is entirely described, in its centre-of-mass, by the distribution

$$\frac{1}{x^2 \Gamma} \frac{d\Gamma}{d\Omega dx} = W_0(x) + P_\tau W_1(x) \cos\theta , \qquad (1)$$

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a e-mail: Andre.Rouge@in2p3.fr

Table 1. Contributions of neutral currents to the g_{ij}^γ coupling constants

Neutral current	Charged coupling constants					
V	g_{LL}^V	g_{RR}^V	g_{LL}^S	g_{RR}^S		
S	g_{LR}^V	g_{RL}^V	g_{LR}^S	g_{LR}^T	g_{RL}^S	g_{RL}^T
			$g_{LR}^S = 2g_{LR}^T$		$g_{RL}^S = 2g_{RL}^T$	
T			g_{LR}^S	g_{LR}^T	g_{RL}^S	g_{RL}^T
			$g_{LR}^S = -6g_{LR}^T$		$g_{RL}^S = -6g_{RL}^T$	

where P_τ is the τ polarization, θ the angle between the polarization and the charged lepton momentum and $x = E_l/E_l^{\max}$ the normalized lepton energy.
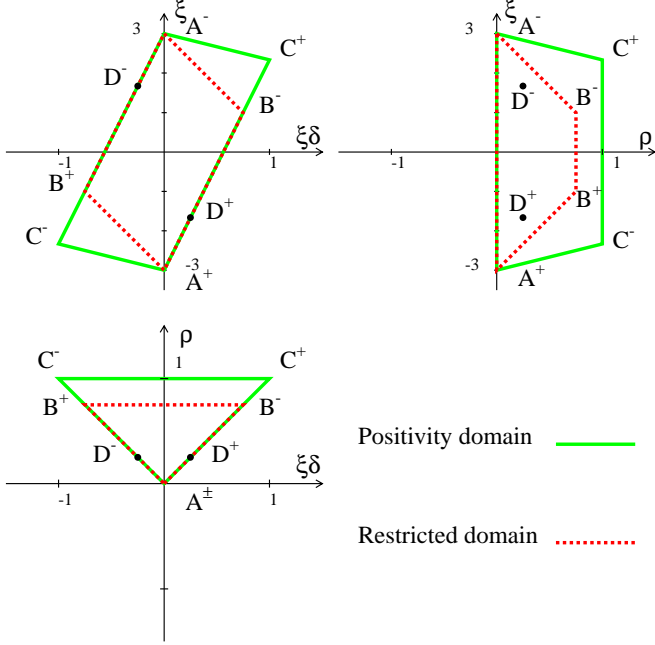
Taking advantage of the weak interaction short range, the decay can be represented by the most general four fermion contact interaction [1–4], written below in the helicity projection formalism [6, 10–12]:

$$\mathcal{M} = 4\frac{G_{\tau l}}{\sqrt{2}} \sum_{\substack{\gamma = S,V,T \\ i,j=R,L}} g_{ij}^\gamma \langle \bar{l}_i | \Gamma^\gamma | (\nu_l)_n \rangle \langle (\bar{\nu}_\tau)_m | \Gamma_\gamma | \tau_j \rangle,$$

$$l = e, \mu . \qquad (2)$$

$G_{\tau l}$ is the absolute coupling strength; the g_{ij}^γ are ten complex coupling constants describing the relative contribution of scalar ($\Gamma^S = 1$), vector ($\Gamma^V = \gamma^\mu$), and tensor ($\Gamma^T = \frac{1}{\sqrt{2}}\sigma^{\mu\nu}$) interactions, respectively, for given chiralities j, i of the τ and the charged decay lepton. The neutrino chiralities n and m are uniquely defined for a given set $\{\gamma, i, j\}$ by the chirality selections rules: conservation for vector coupling, reversal for scalar and tensor.

The same matrix element describes also the contributions of possible lepton-number-violating neutral currents [13]. The relationship between the neutral currents and the g_{ij}^γ constants, as given by the Fierz transformation, is summarized in Table 1.

Fig. 1. The allowed domain in the space of the three parameters, ρ, ξ, and $\xi\delta$

Table 2. Values of the coupling constants in the Standard Model and upper values compatible with the standard model prediction: $\xi = 1$, $\rho = \xi\delta = 3/4$

	$|g_{LL}^V|$	$|g_{LR}^V|$	$|g_{RL}^V|$	$|g_{RR}^V|$
maximum	1	1	1	1
SM	1	0	0	0
maximum $\rho = \xi\delta = 3/4$, $\xi = 1$	1	0	1/2	0

	$|g_{LL}^S|$	$|g_{LR}^S|$	$|g_{RL}^S|$	$|g_{RR}^S|$
maximum	2	2	2	2
SM	0	0	0	0
maximum $\rho = \xi\delta = 3/4$, $\xi = 1$	2	0	2	0

	-	$|g_{LR}^T|$	$|g_{RL}^T|$	-
maximum	-	$1/\sqrt{3}$	$1/\sqrt{3}$	-
SM	-	0	0	-
maximum $\rho = \xi\delta = 3/4$, $\xi = 1$	-	0	1/2	-

The matrix element (2) can be used to compute the functions W_0 and W_1 of 1. They are described by the four real Michel parameters: ρ and η for W_0, ξ and $\xi\delta$ for W_1.

A convenient way to express the relation between the Michel parameters and the g_{ij}^γ coupling constants is to introduce [14] the six positive parameters

$$\alpha^+ = |g_{RL}^V|^2 + |g_{RL}^S + 6g_{RL}^T|^2/16 ,$$
$$\alpha^- = |g_{LR}^V|^2 + |g_{LR}^S + 6g_{LR}^T|^2/16 , \quad (3)$$
$$\beta^+ = |g_{RR}^V|^2 + |g_{RR}^S|^2/4 ,$$
$$\beta^- = |g_{LL}^V|^2 + |g_{LL}^S|^2/4 , \quad (4)$$
$$\gamma^+ = (3/16)|g_{RL}^S - 2g_{RL}^T|^2 ,$$
$$\gamma^- = (3/16)|g_{LR}^S - 2g_{LR}^T|^2 . \quad (5)$$

They satisfy the relation

$$1 = \alpha^+ + \alpha^- + \beta^+ + \beta^- + \gamma^+ + \gamma^- \quad (6)$$
$$= |g_{RR}^V|^2 + |g_{LR}^V|^2 + |g_{RL}^V|^2 + |g_{LL}^V|^2$$
$$+ \frac{1}{4}(|g_{RR}^S|^2 + |g_{LR}^S|^2 + |g_{RL}^S|^2 + |g_{LL}^S|^2)$$
$$+ 3(|g_{LR}^T|^2 + |g_{RL}^T|^2) ,$$

which means that the normalization is absorbed in the definition of $G_{\tau l}$.

The ρ, ξ, and $\xi\delta$ parameters are given by

$$\rho = \frac{3}{4}(\beta^+ + \beta^-) + (\gamma^+ + \gamma^-) ,$$
$$\xi = 3(\alpha^- - \alpha^+) + (\beta^- - \beta^+) + \frac{7}{3}(\gamma^+ - \gamma^-) , \quad (7)$$
$$\xi\delta = \frac{3}{4}(\beta^- - \beta^+) + (\gamma^+ - \gamma^-) .$$

In geometrical terms, the point of coordinates ρ, ξ, and $\xi\delta$, in the space of the parameters, is the barycentre of six points, $A^\pm$, $B^\pm$, and $C^\pm$ with the weights $\alpha^\pm$, $\beta^\pm$, and $\gamma^\pm$ respectively. Since the point B^- lies on the A^+C^+ segment, and B^+ on A^-C^-, the allowed domain is just the tetrahedron $A^+A^-C^+C^-$ (Fig. 1).

The Standard Model prediction, $|g_{LL}^V| = 1$, implies $\rho = \xi\delta = 3/4$, $\xi = 1$ and is represented geometrically by the point B^-, but $\beta^- = 1$ does not imply $|g_{LL}^V| = 1$, and the location of B^- on the A^+C^+ segment introduces further ambiguities.

The value $\beta^- = 1$ is also obtained if $|g_{LL}^S| = 2$ and all the other constants equal to zero, and there is another possibility to reproduce the parameters predicted by the Standard Model, namely $\beta^- = \beta^+ = \alpha^- = \gamma^- = 0$, $\gamma^+ = 3/4$, and $\alpha^+ = 1/4$. So, with $\xi = 1$, $\rho = \xi\delta = 3/4$, $|g_{RL}^V|$ can reach an upper value of $1/2$ and it is also possible that all the constants vanish but g_{RL}^S and g_{RL}^T. In this last case, we get

$$|g_{RL}^S|^2 + 12|g_{RL}^T|^2 = 4 , \quad 2|g_{RL}^T| = -|g_{RL}^S|\cos\phi_{ST} , \quad (8)$$

where ϕ_{ST} is the relative phase of the two amplitudes. Accordingly, the upper possible values of $|g_{RL}^S|$ and $|g_{RL}^T|$ are 2 and $1/2$ respectively.

The upper values of the constants, compatible with the Standard Model prediction, are given in Table 2. More stringent experimental limits [15] are not the consequences of the data but of additional hypotheses or constraints. As the physically interesting region is the neighbourhood of $|g_{LL}^V| = 1$, no relevant bound on the non-standard τ left-handed couplings can be extracted from the measurement of ρ, ξ and $\xi\delta$ without additional hypotheses.

Since the Standard Model prediction is represented by B^- which is located on A^+C^+, a convenient set of param-
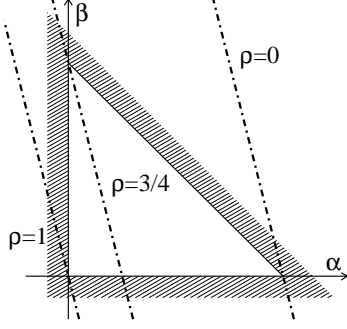
Fig. 2. The allowed domain in the α, β plane for a given value of ρ

eters is given by two equations of the A^+C^+ line and a third variable which specify the position on the segment. Using the equations of the faces $A^+C^+A^-$ and $A^+C^+C^-$ gives expressions which are positive for points inside the tetrahedron. They are more usefully combined into

$$\mathcal{P}_R^\tau = \frac{1}{2}\left[1 + \frac{\xi}{3} - \frac{16}{9}\xi\delta\right] = \beta^+ + \alpha^- + \gamma^- \qquad (9)$$

$$= |g_{RR}^V|^2 + |g_{LR}^V|^2 + \frac{1}{4}|g_{RR}^S|^2 + \frac{1}{4}|g_{LR}^S|^2 + 3|g_{LR}^T|^2 \,,$$

$$\mathcal{S}_R^\tau = \frac{2}{3}\left[\rho - \xi\delta\right] = \beta^+ + \frac{4}{3}\gamma^- \qquad (10)$$

$$= |g_{RR}^V|^2 + \frac{1}{4}|g_{RR}^S|^2 + \frac{1}{4}|g_{LR}^S - 2g_{LR}^T|^2 \,,$$

which, owing to their normalization, can be interpreted as fractional contributions of τ right-handed couplings to its leptonic partial width and used to bound the g_{iR}^γ amplitudes. For the third parameter, ξ or ρ can be chosen.

The last Michel parameter η can be written

$$\eta = 2\,\mathrm{Re}\,[\,g_{RL}^{V*}\,(g_{LR}^S + 6g_{LR}^T)/4\,]$$
$$+ 2\,\mathrm{Re}\,[\,g_{LR}^{V*}\,(g_{RL}^S + 6g_{RL}^T)/4\,] + \mathrm{Re}\,[\,g_{LL}^{V*}\,g_{RR}^S/2\,]$$
$$+ \mathrm{Re}\,[\,g_{RR}^{V*}\,g_{LL}^S/2\,] \,. \qquad (11)$$

This expression implies the inequality

$$|\eta| \le \alpha + \frac{\beta}{2} \le 1 - \rho + \frac{1}{4}\beta_{\max}(\rho) \,, \qquad (12)$$

where, $\alpha = \alpha^+ + \alpha^-$, $\beta = \beta^+ + \beta^-$, and $\beta_{\max}(\rho)$ is the greatest value of β, compatible with the value of ρ. Figure 2 shows that $\beta_{\max}$ is reached for $\gamma = 1 - \alpha - \beta = 0$ when $\rho < 3/4$ and for $\alpha = 0$ for $\rho > 3/4$, leading to the rather weak bound:

$$|\eta| \le 1 - \frac{2}{3}\rho\left(\rho \le \frac{3}{4}\right)\,, \quad |\eta| \le 2(1-\rho)\left(\rho \ge \frac{3}{4}\right) \quad (13)$$

3 The restricted domain

If only charged vector currents are present ($g_{ij}^S = 0, g_{ij}^T = 0$), we have $\gamma^+ = \gamma^- = 0$ and the allowed domain is the $A^+A^-B^+B^-$ tetrahedron that we call the restricted domain.

It is interesting to remark that, for a very wide class of models and hypotheses, the allowed domain for the Michel parameters is this restricted domain[1] and that getting a point outside it requires the conspiracy of scalar and tensor couplings [14].

We give below a list of general hypotheses leading to the restricted domain. We use for that the Lorentz covariance of the charged and neutral currents and the properties of the Fierz transformation only. More specific constraints associated with the hypothesis of a single boson exchange can be found in [12] and [16].

When the allowed domain of the parameters is the restricted one, the point B^-, which is the Standard Model prediction, is one of its vertices, therefore there is a third positive quantity whose vanishing is associated with this point. We use for it $1 - 4\rho/3$. Its precise physical meaning and the ambiguities in the interpretation of the measurements depend nevertheless on the hypothesis that lead to the restriction.

$g_{ij}^S = 0$

If all the scalar coupling constants vanish, the condition $\alpha^+ = 0$ implies $\gamma^+ = 0$, therefore the complete domain is not allowed. Defining the new parametrization

$$\alpha_{\mathrm{NS}}^+ = |g_{RL}^V|^2 \,, \qquad \gamma_{\mathrm{NS}}^+ = 3|g_{RL}^T|^2 \,, \qquad (14)$$
$$\alpha_{\mathrm{NS}}^- = |g_{LR}^V|^2 \,, \qquad \gamma_{\mathrm{NS}}^- = 3|g_{LR}^T|^2 \,,$$

we get the relations

$$1 = \alpha_{\mathrm{NS}}^+ + \alpha_{\mathrm{NS}}^- + \beta^+ + \beta^- + \gamma_{\mathrm{NS}}^+ + \gamma_{\mathrm{NS}}^- \,, \qquad (15)$$

$$\rho = \frac{3}{4}(\beta^+ + \beta^-) + \frac{1}{4}(\gamma_{\mathrm{NS}}^+ + \gamma_{\mathrm{NS}}^-) \,,$$

$$\xi = 3(\alpha_{\mathrm{NS}}^- - \alpha_{\mathrm{NS}}^+) + (\beta^- - \beta^+) - \frac{5}{3}(\gamma_{\mathrm{NS}}^+ - \gamma_{\mathrm{NS}}^-) \,, \quad (16)$$

$$\xi\delta = \frac{3}{4}(\beta^- - \beta^+) + \frac{1}{4}(\gamma_{\mathrm{NS}}^+ - \gamma_{\mathrm{NS}}^-) \,.$$

The point of coordinates ρ, ξ, and $\xi\delta$ is now the barycentre of $A^\pm$, $B^\pm$, and two new points $C_{\mathrm{NS}}^\pm \equiv D^\pm$ whose coordinates are $(1/4, \mp 5/3, \pm 1/4)$ but, since D^+ is located on the A^+B^- segment and D^- on A^-B^+ (Fig. 1), the allowed region is the restricted domain. The third constraint can be written, in terms of the g_{ij}^γ's, as

$$1 - \frac{4}{3}\rho = |g_{RL}^V|^2 + |g_{LR}^V|^2 + 2\,(|g_{RL}^T|^2 + |g_{LR}^T|^2) \,. \quad (17)$$

Measurements represented by B^- imply a V-A structure.

The combination of charged, vector and tensor currents belongs to this class.

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[1] This result is almost obvious from the "charge retention" formalism, where γ^+ and γ^- are related to (neutral) tensor currents [4], and Table 1

$g_{ij}^T = 0$

If $g_{ij}^T = 0$, we define

$$\alpha_{\mathrm{NT}}^+ = |g_{RL}^V|^2 \,, \qquad \gamma_{\mathrm{NT}}^+ = \frac{1}{4}\, |g_{RL}^S|^2 \,, \qquad (18)$$

$$\alpha_{\mathrm{NT}}^- = |g_{LR}^V|^2 \,, \qquad \gamma_{\mathrm{NT}}^- = \frac{1}{4}\, |g_{LR}^S|^2 \,,$$

and obtain

$$1 = \alpha_{\mathrm{NT}}^+ + \alpha_{\mathrm{NT}}^- + \beta^+ + \beta^- + \gamma_{\mathrm{NT}}^+ + \gamma_{\mathrm{NT}}^- \,, \qquad (19)$$

$$\rho = \frac{3}{4}\, (\beta^+ + \beta^- + \gamma_{\mathrm{NT}}^+ + \gamma_{\mathrm{NT}}^-) \,,$$

$$\xi = 3\,(\alpha_{\mathrm{NT}}^- - \alpha_{\mathrm{NT}}^+) + (\beta^- - \beta^+) + (\gamma_{\mathrm{NT}}^+ - \gamma_{\mathrm{NT}}^-) \,, \quad (20)$$

$$\xi\delta = \frac{3}{4}\, (\beta^- - \beta^+ + \gamma_{\mathrm{NT}}^+ - \gamma_{\mathrm{NT}}^-) \,.$$

Here the points C_{NT}^+ and C_{NT}^- coincide with the points B^- and B^+ respectively. The allowed domain is then the restricted one. The third constraint is

$$1 - \frac{4}{3}\, \rho = |g_{RL}^V|^2 + |g_{LR}^V|^2 \,. \qquad (21)$$

The vector couplings only are bounded by the measurement of ρ, due to the ambiguity between $B^\pm$ and $C_{\mathrm{NT}}^\mp$.

The combination of charged, vector and scalar currents and neutral, vector currents belongs to this class.

$g_{RL}^S = 2g_{RL}^T$, $g_{LR}^S = 2g_{LR}^T$

The restriction of the domain is evident from 5. The third constraint can be written

$$1 - \frac{4}{3}\, \rho = |g_{RL}^V|^2 + |g_{LR}^V|^2 + \frac{1}{4}\, (|g_{LR}^S|^2 + |g_{RL}^S|^2)$$
$$+ 3\,(|g_{LR}^T|^2 + |g_{RL}^T|^2) \,. \quad (22)$$

Measurements represented by B^- imply a V-A structure.

The combination of charged, vector currents and neutral, vector and scalar currents belongs to this class.

"V-A plus something"[2]

In an especially interesting family of models [16] the decay is described by the addition of a single, non-standard contribution to the Standard Model amplitude. Their predictions are perfectly transparent in the present geometrical presentation. They follow at once from the definitions of the parameters and the properties displayed in Table 1.

- If the non-standard contribution is a charged vector current, all the restricted domain is allowed. The only new prediction is $\eta = 0$.

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[2] The title of this paragraph is borrowed from C. Nelson [17]

- The hypothesis of an additional charged scalar current belongs to the second of the above defined classes with the further conditions $\alpha_{\mathrm{NT}}^\pm = 0$. The allowed domain is then the B^+B^- segment ($\rho = \delta = 3/4$), since the points $C_{\mathrm{NT}}^\pm$ and $B^\mp$ are identical.
- The contribution of a neutral vector is included in the same class with $\alpha_{\mathrm{NT}}^\pm = \gamma_{\mathrm{NT}}^\pm = 0$. The allowed domain is again the B^+B^- segment.
- The hypothesis of an additional neutral scalar leads to $\gamma^+ = \gamma^- = 0$ (third class) and $\beta^+ = 0$. The allowed domain is the two-dimensional one, spanned by the points A^+, A^-, and B^-. The corresponding condition is $\rho = \xi\delta$ and the bound on η is stricter: $|\eta| \leq 1 - 4\rho/3$.

4 Looking for new physics

In the standard approach described in Sect. 1, the indicators of new physics constructed with the Michel parameters are, besides the η parameter itself, the two positive quantities $\mathcal{P}_R^\tau$ and $\mathcal{S}_R^\tau$, defined by 9 and 10, which bound the coupling constants and can be interpreted as (non-independent) contributions of new physics to the τ decay.

Using the world-average values of the parameters [7], under the hypothesis of e-μ universality, yields

$$\mathcal{P}_R^\tau = 0.006 \pm 0.028 \,, \qquad \mathcal{S}_R^\tau = 0.001 \pm 0.016 \,.$$

If hypotheses are made that reduce the dimensionality of the domain, the determination of the parameters can be improved by a constrained fit. For instance, if the domain is the B^+B^- segment, there is only one free parameter. Neglecting the correlations between the measurements and taking advantage of the near equality of the errors on ξ and $4\xi\delta/3$, this parameter is merely their average $\bar{\xi}$ and the quantity $\mathcal{P}_R^\tau$ reduces to

$$\frac{1}{2}\,[1 - \bar{\xi}] = \frac{1}{2}\,\left[1 - \frac{\xi}{2} - \frac{2}{3}\xi\delta\right] \,. \qquad (23)$$

It is noteworthy that the same strategy can be followed under the much weaker hypotheses that imply the restricted domain. This is due to the fact that the measured value of ρ forces the point which represents the measurements in the parameter space to lie on the edge of the domain.

Quantitatively, the difference of ξ and $4\xi\delta/3$ is bounded by the inequality

$$\left|\xi - \frac{4}{3}\xi\delta\right| \leq 3\left(1 - \frac{4}{3}\rho\right). \qquad (24)$$

From the values of the errors [7] $\sigma(\xi - 4\xi\delta/3) = 0.044$ and $4\sigma(\rho) = 0.036$, it is clear that there is no physically relevant information in $\xi - 4\xi\delta/3$ and that even the τ-l universality prediction [12] $\delta = 3/4$ is better tested by the measurement of ρ than by comparing ξ and $\xi\delta$.

Assuming only the restricted domain, the indicators of new physics are

$$1 - \frac{4}{3}\rho = 0.004 \pm 0.012 \,, \qquad \frac{1}{2}\,[1 - \bar{\xi}] = 0.002 \pm 0.011 \,.$$

They bring a clear improvement of the sensitivity with respect to $\mathcal{P}_R^\tau$ and $\mathcal{S}_R^\tau$.

It must be noted that $1 - \bar{\xi}$ is not strictly positive but that excursions of $\bar{\xi}$ beyond 1 are severely limited by the inequality

$$\bar{\xi} - 1 \le \frac{1}{2}\left(1 - \frac{4}{3}\rho\right), \qquad (25)$$

since $\sigma(\bar{\xi}) = 0.022$ and $2\sigma(\rho)/3 = 0.006$.

5 Conclusion

A complete study of the constraints on the Michel parameters and the ambiguities of their interpretation has been presented.

It has been shown that, for a very wide class of hypotheses and models, which cause the same restriction of the parameter domain, the best indicators of new physics are the combinations, $1 - 4\rho/3$ and $(1 - \bar{\xi})/2 = (1/2 - \xi/4 - \xi\delta/3)$.

Compared to the customary estimators, their sensitivities are roughly twice better. The third available parameter, $\xi - 4\xi\delta/3$, is better determined by the geometry of the domain and the value of ρ than by its measurement.

A Appendix: Measurement of the parameters

In the numerical exercise above, the error correlations were neglected and the e-μ universality was assumed. We will discuss briefly this two approximations.

At low energy, where the τ's are unpolarized, the ρ and η parameters are determined by the single-lepton laboratory energy distributions and the ξ and $\xi\delta$ parameters by the spin-correlated $\tau^+\tau^-$ decay distributions. Estimates of the covariance matrices for measurements at 4 GeV and 10 GeV can be found in [8].

At the Z peak, the τ polarization makes ambiguous the interpretation of a single tau leptonic-decay distribution and, since the transverse spin correlations depend on the Z couplings, only the helicity correlation of τ^+ and τ^-, which is equal to -1, is used. If the decay distribution of a τ^- in the channel a, is written,

$$W_a(x^-) = f_a(x^-) + P_\tau\, g_a(x^-)\ , \qquad (26)$$

the correlated distribution for the a and b channels reads then

$$\begin{aligned} W_{ab}(x^-,x^+) = {}& f_a(x^-)f_b(x^+) + g_a(x^-)g_b(x^+) \qquad (27)\\ &+ P_\tau\left[\,f_a(x^-)g_b(x^+) + f_b(x^+)g_a(x^-)\,\right], \end{aligned}$$

where P_τ is the τ^- polarization.

For a hadronic decay, with the notations of [19], $x^\pm$ is the optimal variable ω and the decay distribution is

$$W(\omega) = \hat{f}(\omega)[1 + \xi_h\, P_\tau\, \omega]\ , \qquad (28)$$

where the ξ_h parameter is equal to 1 in the standard model[3].

For a leptonic decay, $x^\pm$ is the normalized energy of the charged lepton, $y = (E_l/E_l^{\max})_{\mathrm{LAB}}$. Defining the parameters,

$$\tilde{\rho} = 1 - \frac{4}{3}\rho, \quad \tilde{\delta} = \xi - \frac{4}{3}\xi\delta, \quad \tilde{\eta} = \frac{m_l}{m_\tau}\eta, \qquad (29)$$

which vanish in the Standard model, and the functions

$$\begin{aligned} h_0(y) &= \frac{1}{3}(5 - 9y^2 + 4y^3)\,, \qquad (30)\\ h_1(y) &= \frac{1}{3}(1 - 9y^2 + 8y^3)\,, \\ h_2(y) &= \frac{1}{3}(1 - 12y + 27y^2 - 16y^3)\,, \\ h_3(y) &= 12(1 - y)^2\,, \end{aligned}$$

the decay distribution reads

$$\begin{aligned} W(y) = {}& f_{\rho,\eta}(y) + P_\tau g_{\xi,\xi\delta}(y) = \qquad (31)\\ & \frac{1}{1+4\tilde{\eta}}\left\{h_0(y) + [\tilde{\rho} + P_\tau\xi]h_1(y) + \tilde{\delta}P_\tau h_2(y) + \tilde{\eta}h_3(y)\right\}. \end{aligned}$$

The presence of the polarization allows the measurement of a new parameter, $\tilde{\delta}P_\tau$, but, as previously mentioned, it also introduces an ambiguity in the interpretation of the first one which is now $\tilde{\rho} + P_\tau\xi$.

The same kind of ambiguity also arises in the e-μ correlation. For $P_\tau = 0$, only the parameters, δ_μ, δ_e, and the product $\xi_\mu\xi_e$ are measurable [8]. For $P_\tau \ne 0$, neglecting $\tilde{\eta}$ for the sake of clarity, and keeping only terms of the first order in the Standard Model violating parameters, $\tilde{\rho}$, $\tilde{\delta}$ and $1 - \xi$, the correlated distribution can be written

$$\begin{aligned} W(y_e,y_\mu) \sim {}& h_0(y_\mu)h_0(y_e) - h_1(y_\mu)h_1(y_e)\\ &+ [\tilde{\rho}_e + P_\tau\xi_e]h_0(y_\mu)h_1(y_e) + [\tilde{\rho}_\mu + P_\tau\xi_\mu]h_0(y_e)h_1(y_\mu)\\ &+ [\xi_\mu + \xi_e + P_\tau(\tilde{\rho}_e + \tilde{\rho}_\mu)]h_1(y_\mu)h_1(y_e)\\ &+ \tilde{\delta}_e h_2(y_e)[h_1(y_\mu) + P_\tau h_0(y_\mu)] \qquad (32)\\ &+ \tilde{\delta}_\mu h_2(y_\mu)[h_1(y_e) + P_\tau h_0(y_e)]\,. \end{aligned}$$

Even if P_τ is known, there is only three measurements to determine the four parameters ρ_μ, ρ_e, ξ_μ and ξ_e. The ambiguity is displaced but not suppressed[4].

Using several known values of the polarization, all the parameters can be determined from single-decay distributions. At the Z peak, and/or with polarized beam, the τ polarization is a function of the production angle θ, hence the Michel parameters can be measured by the θ-y correlation [21].

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[3] Since $P_{\tau^+} = -P_{\tau^-}$ and $\xi_h^{\tau^+} = -\xi_h^{\tau^-}$, the decay distribution is independent of the τ charge. In the case of $\tau \to \nu 3\pi$, this last property is true only if no pseudoscalar variable constructed from the π momenta is used in the definition of ω. A more general analysis is presented in [18]

[4] Therefore, the analysis [20] necessarily uses additional hypotheses

Table 3. Ideal statistical errors on the Michel parameters (in%) and their correlation coefficients for 2×10^5 $\tau^+\tau^-$ pairs at the Z peak

	$\sigma(\rho)$	$\sigma(\xi)$	$\sigma(\xi\delta)$	$\sigma(\xi - \frac{4}{3}\xi\delta)$
μ	2.3	6.0	4.0	7.3
e	1.3	5.5	3.9	7.2

	$C(\rho,\xi)$	$C(\rho,\xi\delta)$	$C(\xi,\xi\delta)$
μ	0.24	0.15	0.18
e	-0.21	-0.06	0.10

The other measurements [15, 22–24] use all the hadron-hadron, lepton-lepton and hadron-lepton final states to obtain the complete set of parameters up to a global sign ambiguity which is solved, for instance, by the result of [21].

To calculate the covariance matrix V of the Michel parameter measurements, we assume that, in an ideal experiment at the Z peak, similar to [15, 22–24], all the decays into μ, e, π, ρ and a_1 and their correlations are used to determine the Michel parameters, the hadronic, ξ_h parameters and the τ polarization. Asymptotically, V is given by

$$\left(V^{-1}\right)_{ij} \sim -\sum_s N_s \iint W_s \frac{\partial^2 \log W_s}{\partial \alpha_i \partial \alpha_j} dx^+ dx^- , \quad (33)$$

where s labels one of the twenty classes of events, e-μ, e-e, e-π, etc, and N_s is the number of events in the class. The computation is made straightforward by the fact that the distributions are quadratic functions of all the parameters, except η which appears in the normalizations.

The computed covariance matrix is similar to its estimations [8] for measurements at lower energy. The largest correlation coefficients are $C(\rho_\mu, \eta_\mu) = 0.82$ and $C(\xi_\mu, \eta_\mu) = 0.42$. The numerical values relevant for the analysis of Sect. 4 are given in Table 3.

For the $\tau \to e\nu\bar{\nu}$ channel, the inequality $\sigma(\tilde{\delta}) > 3\sigma(\tilde{\rho})$ is satisfied and the weights of ξ and $4\xi\delta/3$ in their optimal combination are 0.47 and 0.53 respectively. All the hypotheses made in Sect. 4 are verified.

For the $\tau \to \mu\nu\bar{\nu}$ channel, the weights are 0.43 and 0.57 but, owing to the correlations with η_μ, the ideal error on $\tilde{\delta}$ is slightly smaller than $3\sigma(\tilde{\rho})$. However, in a more realistic estimation, the inefficiencies in the identification of the various decay channels reduce the statistics for the classes of events with two analysed decays and increase it for the events with only one identified decay which contribute mainly to the measurement of ρ. Therefore, the same analysis scheme remains basically valid.

From an experimental point of view, the universality hypothesis allows to constrain the value of η_μ by the measurement of the $\tau \to e\nu\bar{\nu}$ and $\tau \to \mu\nu\bar{\nu}$ branching ratios [15, 20].

If the variation of the parameters is limited to the above defined restricted domain, the deviations of the parameters from their Standard Model values must have the same sign in the e and μ channels. Therefore the universality hypothesis can perhaps reduce the sensitivity to these deviations but complete cancelations are not possible.

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