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Complete one-loop corrections to the mass spectrum of charginos and neutralinos in the MSSM

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Abstract. The mass spectrum of the chargino–neutralino sector in the minimal supersymmetric standard model (MSSM) is calculated at the one-loop level, based on the complete set of one-loop diagrams. On-shell renormalization conditions are applied to determine the counterterms for the gaugino-mass-parameters M_1, M_2 and the Higgsino-mass parameter μ. The input is fixed in terms of three pole masses (two charginos and one neutralino); the other pole masses receive a shift with respect to the tree-level masses, which can amount to several GeV. The detailed evaluation shows that both the fermionic/sfermionic loop contributions and the non-(s)fermionic loop contributions are of the same order of magnitude and are thus relevant for precision studies at future colliders.

1 Introduction

Experiments at future high-energy colliders will be able to discover supersymmetric particles and to investigate their properties. Provided their masses are not too high, a linear electron-positron collider will be the best environment for precision studies of supersymmetric models [1], especially of the minimal supersymmetric standard model (MSSM). From precise measurements of masses, cross sections and asymmetries in chargino and neutralino production, the fundamental parameters can be reconstructed [2], to shed light on the mechanism of SUSY breaking.

In view of the experimental prospects it is inevitable to include higher-order terms in the calculation of the measureable quantities in order to achieve theoretical predictions matching the experimental accuracy. Former studies on chargino-pair production [3–5] and scalar-quark decays [6] have demonstrated that Born-level predictions can be influenced significantly by one-loop radiative corrections. In [5,6], with complete one-loop calculations performed in on-shell renormalization schemes, it was shown that besides the fermion- and sfermion-loop contributions also the virtual contributions from the supersymmetric gauge and Higgs sector are not negligible.

Since the masses of charginos and neutralinos are among the precision observables with lots of information on the SUSY-breaking structure, the relations between the particle masses and the SUSY parameters as well as the relations between the masses themselves are important theoretical objects for precision calculations. Previous studies were done in the $\overline{\mathrm{MS}}$ renormalization scheme [7,8] with running parameters. In [9] an on-shell scheme has been proposed to calculate the one-loop mass matrices X and Y of the chargino and neutralino sector, which after diagonalization yield the one-loop corrected mass eigenstates. The concrete evaluation of the mass spectrum in [9] has been performed with the subset of the diagrams involving only fermion and sfermion loops and is thus not yet complete at the one-loop level.

In this paper we present an on-shell calculation of the chargino and neutralino mass spectrum of the MSSM at the one-loop level, based on the entire set of one-loop diagrams. We specify the on-shell renormalization scheme by treating all particle masses as pole masses, and with field renormalization implemented in a way that allows to formulate the renormalized 2-point vertex functions as UV-finite matrices which become diagonal for external momenta on-shell. The masses of the two charginos and of one neutralino are used as input to fix the MSSM parameters μ, M_1, M_2. Since only the gaugino-mass parameters M_1, M_2 and the Higgsino-mass parameter μ can be renormalized independently in terms of three pole masses, with all other renormalization constants fixed in the gauge and Higgs sector, the residual eigenvalues of the tree-level mass matrices are no longer the pole positions of the corresponding dressed propagators; the pole masses hence receive a shift versus the tree-level masses, which is calculable in terms of the renormalized self-energies. As a byproduct, we obtain all the renormalization constants required to determine the various counterterms for the chargino–neutralino sector of the MSSM, being implemented in the MSSM version of `FeynArts` [10] for completion at the one-loop level.

After explaining the general structure of the renormalization of parameters and fields in Sects. 3 and 4, we specify the on-shell conditions in Sect. 5 and give the explicit solutions for the renormalization constants. The calculation of the predicted neutralino masses is outlined in Sect. 6, and a presentation and discussion of the numerical results is given in Sect. 7.

2 Notations and parameters

The bilinear part of the Lagrangian describing the chargino/neutralino sector of the MSSM,

$$\mathcal{L} = \mathcal{L}_{\rm kin} + \mathcal{L}_{\rm mass}\,, \tag{1}$$

is composed of the kinetic term

$$\begin{aligned}\mathcal{L}_{\rm kin} &= i\,\overline{\lambda}^a\,\overline{\sigma}^\mu\,(\partial_\mu\lambda)^a + i\,\overline{\lambda'}\,\overline{\sigma}^\mu\,(\partial_\mu\lambda')\\ &\quad + i\,\overline{\psi}_{H_1}\overline{\sigma}^\mu\partial_\mu\psi_{H_1} + i\,\overline{\psi}_{H_2}\overline{\sigma}^\mu\partial_\mu\psi_{H_2}\end{aligned} \tag{2}$$

and the mass term following from the expression

$$\begin{aligned}\mathcal{L}_{\rm mass} &= \sqrt{2}\,\Big[i\,H_1^\dagger\big(g\,\lambda^a\,T^a + \tfrac{1}{2}\,g'\,\lambda'\big)\psi_{H_1}\\ &\quad + i\,H_2^\dagger\big(g\,\lambda^a\,T^a + \tfrac{1}{2}\,g'\,\lambda'\big)\psi_{H_2} + \text{h.c.}\Big]\\ &\quad + \epsilon_{ij}\big(\mu\,\psi^i_{H_1}\psi^j_{H_2} + \text{h.c.}\big)\\ &\quad + \tfrac{1}{2}\big(M_1\lambda'\lambda' + M_2\lambda^a\lambda^a + \text{h.c.}\big)\end{aligned} \tag{3}$$

by substituting the vacuum configurations of the two Higgs-doublet fields $H_{1,2}$. The Lagrangian in two-component notation involves the Weyl spinors λ', λ^a $(a = 1, 2, 3)$ for the gauginos and $\psi^{1,2}_{H_i}$ for the Higgsino isospin components accompanying the components of the Higgs doublets, i.e.

$$H_i = \begin{pmatrix} H_i^1 \\ H_i^2 \end{pmatrix}, \quad \psi_{H_i} = \begin{pmatrix} \psi^1_{H_i} \\ \psi^2_{H_i} \end{pmatrix}. \tag{4}$$

Besides the gauge couplings g and g', the Lagrangian involves the μ parameter, the soft-breaking gaugino-mass parameters M_1 and M_2, and the Higgs vacua v_i, which are related to $\tan\beta = v_2/v_1$ and to the W mass $M_W = gv/2$ with $v = (v_1^2 + v_2^2)^{1/2}$.

3 Charginos

3.1 Lagrangian and mass eigenstates

Introducing a compact notation by collecting the chiral parts parts according to

$$\begin{aligned}\psi^R &\equiv \begin{pmatrix} \psi_1^R \\ \psi_2^R \end{pmatrix} = \begin{pmatrix} -i\,\lambda^- \\ \psi^2_{H_1} \end{pmatrix},\\ \psi^L &\equiv \begin{pmatrix} \psi_1^L \\ \psi_2^L \end{pmatrix} = \begin{pmatrix} -i\,\lambda^+ \\ \psi^1_{H_2} \end{pmatrix},\end{aligned} \tag{5}$$

with $\lambda^\pm = \frac{1}{\sqrt{2}}\big(\lambda^1 \mp i\lambda^2\big)$, leads to the conventional form of the bilinear terms of the Lagrangian (1)–(3) for the charginos,

$$\begin{aligned}\mathcal{L}_{\rm ch} &= i\left[\psi^{R\top}\sigma^\mu\partial_\mu\,\overline{\psi}^R + \overline{\psi}^{L\top}\overline{\sigma}^\mu\partial_\mu\,\psi^L\right]\\ &\quad - \left[\psi^{R\top}X\,\psi^L + \overline{\psi}^{L\top}X^\dagger\,\overline{\psi}^R\right].\end{aligned} \tag{6}$$

The mass matrix

$$X = \begin{pmatrix} M_2 & \sqrt{2}\,M_W\,\sin\beta \\ \sqrt{2}\,M_W\,\cos\beta & \mu \end{pmatrix} \tag{7}$$

can be diagonalized by two unitary matrices U and V, yielding the tree-level chargino mass eigenstates

$$\begin{aligned}\chi_j^R &= U_{jk}\,\psi_k^R,\\ \chi_j^L &= V_{jk}\,\psi_k^L,\\ U^*\,X\,V^\dagger &= \begin{pmatrix} m_{\tilde\chi_1^+} & 0 \\ 0 & m_{\tilde\chi_2^+} \end{pmatrix}.\end{aligned} \tag{8}$$

The tree-level definition of the corresponding chargino Dirac spinors $\tilde\chi_i^+$ $(i = 1, 2)$ is then given by

$$\tilde\chi_i^+ = \begin{pmatrix} \chi_i^L \\ \overline{\chi}_i^R \end{pmatrix}. \tag{9}$$

The squares of the tree-level masses $m_{\tilde\chi_1^+}$ and $m_{\tilde\chi_2^+}$ arise as the eigenvalues of the hermitian matrix $XX^\dagger$,

$$\begin{aligned}m^2_{\tilde\chi_1^+,\tilde\chi_2^+} &= \frac{1}{2}\Big\{M_2^2 + |\mu|^2 + 2M_W^2\\ &\quad \mp\Big[(M_2^2 - |\mu|^2)^2 + 4M_W^4\cos^2 2\beta\\ &\quad + 4M_W^2(M_2^2 + |\mu|^2 + 2\,\text{Re}(\mu)\,M_2\,\sin 2\beta)\Big]^{\frac{1}{2}}\Big\}.\end{aligned} \tag{10}$$

3.2 Renormalization of the chargino sector

Starting from the chargino Lagrangian (6), we introduce renormalization constants for the mass matrix X and for the fields ψ^L, ψ^R by the transformation

$$\begin{aligned}X &\to X + \delta X\\ \psi^L &\to \Big(1 + \tfrac{\delta Z^L}{2}\Big)\,\psi^L\\ \psi^R &\to \Big(1 + \tfrac{\delta Z^R}{2}\Big)\,\psi^R\,.\end{aligned} \tag{11}$$

The matrix δX is made of the counterterms for the parameters in the mass matrix X in (7),

$$\delta X = \begin{pmatrix} \delta M_2 & \sqrt{2}\,\delta\big(M_W\,\sin\beta\big) \\ \sqrt{2}\,\delta\big(M_W\,\cos\beta\big) & \delta\mu \end{pmatrix}. \tag{12}$$

The matrix-valued field-renormalization constants δZ^L and δZ^R are chosen diagonal. This minimal set of renormalization constants is sufficient to render both S-matrix elements and Green functions for charginos finite [11].

For later convenience, we define in a next step the one-loop versions of the transformations (8) of the fields $\psi^{L,R}$ by

$$\begin{aligned}\chi^L &= R_L\,\psi^L\\ \chi^R &= R_R\,\psi^R\end{aligned} \tag{13}$$

with general complex, non-singular 2×2-matrices R_L and R_R, which are UV finite. In the one-loop expansion, these two matrices can be written as follows,

$$\begin{aligned}R_L &= \left(1+\tfrac{\delta Z^V}{2}\right)V\,,\\ R_R &= \left(1+\tfrac{\delta Z^U}{2}\right)U\,,\end{aligned} \tag{14}$$

where U and V are the unitary matrices from (8) and δZ^U, δZ^V are general complex 2×2-matrices of one-loop order. By combining the field transformations (11) and (13), (14) one finds

$$\begin{aligned}\psi^L &\to \left(1+\tfrac{\delta Z^L}{2}\right)V^\dagger\left(1-\tfrac{\delta Z^V}{2}\right)\chi^L\\ &= \left(V^\dagger+\tfrac{\delta Z^L}{2}V^\dagger-V^\dagger\tfrac{\delta Z^V}{2}\right)\chi^L\,,\\ \psi^R &\to \left(1+\tfrac{\delta Z^R}{2}\right)U^\dagger\left(1-\tfrac{\delta Z^U}{2}\right)\chi^R\\ &= \left(U^\dagger+\tfrac{\delta Z^R}{2}U^\dagger-U^\dagger\tfrac{\delta Z^U}{2}\right)\chi^R\,,\end{aligned} \tag{15}$$

which shows that in the renormalized MSSM Lagrangian δZ^L and δZ^V can only occur in the combination $\frac{\delta Z^L}{2}V^\dagger - V^\dagger\frac{\delta Z^V}{2}$, whilst δZ^R and δZ^U always combine to $\frac{\delta Z^R}{2}U^\dagger - U^\dagger\frac{\delta Z^U}{2}$. Hence, actually only 4 complex renormalization constants are available for each L and R part. In order to eliminate the redundant parameters we define new field-renormalization constants

$$\begin{aligned}\delta\tilde Z^L &= V\left[\delta Z^L V^\dagger - V^\dagger\delta Z^V\right] = V\delta Z^L V^\dagger - \delta Z^V\,,\\ \delta\tilde Z^R &= U\left[\delta Z^R U^\dagger - U^\dagger\delta Z^U\right] = U\delta Z^R U^\dagger - \delta Z^U\,,\end{aligned} \tag{16}$$

which are now general complex 2×2-matrices.

Applying the transformations (15) for the fields and (7) for the parameters to the Lagrangian (6) yields the Born and the counterterm parts. After a Fourier transformation they read, with 4-component spinors and the projectors $\omega_{L,R} = (1\mp\gamma_5)/2$:

$$\mathcal{L}_{\text{Born}} = \overline{\tilde\chi_i^+}\left[\not p\,\delta_{ij} - \omega_L\,(U^* X V^\dagger)_{ij} - \omega_R\,(V X^\dagger U^\top)_{ij}\right]\tilde\chi_i^+, \tag{17a}$$

$$\begin{aligned}\mathcal{L}_{\text{CT}} = \overline{\tilde\chi_i^+}\not p\Big[&\tfrac12\left(\delta\tilde Z^{R^*}+\delta\tilde Z^{R^\top}\right)_{ij}\omega_R\\ &+\tfrac12\left(\delta\tilde Z^L+\delta\tilde Z^{L^\dagger}\right)_{ij}\omega_L\Big]\tilde\chi_j^+ - \overline{\tilde\chi_i^+}\Bigg[\Big[U^*\delta X V^\dagger\\ &+\tfrac12\,\delta\tilde Z^{R^\top}U^* X V^\dagger + \tfrac12\,U^* X V^\dagger\delta\tilde Z^L\Big]_{ij}\omega_L\\ &+\Big[V\delta X^\dagger U^\top + \tfrac12\,V X^\dagger U^\top\delta\tilde Z^{R^*}\\ &+\tfrac12\,\delta\tilde Z^{L^\dagger} V X^\dagger U^\top\Big]_{ij}\omega_R\Bigg]\tilde\chi_j^+\,.\end{aligned} \tag{17b}$$

The renormalized self-energies $\hat\Sigma_{ij}(p)$ for the chargino system are given by the unrenormalized self-energies $\Sigma_{ij}(p)$ plus the corresponding counterterms, obtained as derivatives of the counterterm Lagrangian (17b) with respect to the fields $\overline{\tilde\chi_i^+}$ and $\tilde\chi_j^+$,

$$\hat\Sigma_{ij}(p) = \Sigma_{ij}(p) + \frac{\partial}{\partial\tilde\chi_j^+}\frac{\partial}{\partial\overline{\tilde\chi_i^+}}\,\mathcal{L}_{\text{CT}}\ . \tag{18}$$

Thus, by using the decomposition into Lorentz covariants

$$\begin{aligned}\Sigma_{ij}(p) &= \not p\,\omega_L\Sigma^L_{ij}(p^2) + \not p\,\omega_R\Sigma^R_{ij}(p^2)\\ &\quad+\omega_L\Sigma^{SL}_{ij}(p^2) + \omega_R\Sigma^{SR}_{ij}(p^2)\,,\end{aligned} \tag{19}$$

one immediately obtains

$$\hat\Sigma^R_{ij}(p^2) = \Sigma^R_{ij}(p^2) + \tfrac12\left(\delta\tilde Z^{R^*}+\delta\tilde Z^{R^\top}\right)_{ij} \tag{20a}$$

$$\hat\Sigma^L_{ij}(p^2) = \Sigma^L_{ij}(p^2) + \tfrac12\left(\delta\tilde Z^{L}+\delta\tilde Z^{L^\dagger}\right)_{ij} \tag{20b}$$

$$\begin{aligned}\hat\Sigma^{SR}_{ij}(p^2) = \Sigma^{SR}_{ij}(p^2) - \Big[&V\delta X^\dagger U^\top + \tfrac12\,V X^\dagger U^\top\delta\tilde Z^{R^*}\\ &+\tfrac12\,\delta\tilde Z^{L^\dagger}V X^\dagger U^\top\Big]_{ij}\end{aligned} \tag{20c}$$

$$\begin{aligned}\hat\Sigma^{SL}_{ij}(p^2) = \Sigma^{SL}_{ij}(p^2) - \Big[&U^*\delta X V^\dagger + \tfrac12\,\delta\tilde Z^{R^\top}U^* X V^\dagger\\ &+\tfrac12\,U^* X V^\dagger\delta\tilde Z^L\Big]_{ij}\end{aligned} \tag{20d}$$

for the scalar coefficients.

4 Neutralinos

4.1 Lagrangian and mass eigenstates

The bilinear, non-interacting, part of the neutralino Lagrangian derived from (2) and (3) can be written in the conventional compact form

$$\begin{aligned}\mathcal{L}_{\text{n}} = &\tfrac{i}{2}\left[\psi^{0^\top}\sigma^\mu\partial_\mu\,\overline\psi^0 + \overline\psi^{0^\top}\overline\sigma^\mu\partial_\mu\,\psi^0\right]\\ &-\tfrac12\left[\psi^{0^\top}Y\,\psi^0 + \overline\psi^{0^\top}Y^\dagger\,\overline\psi^0\right]\,,\end{aligned} \tag{21}$$

where the Weyl spinors of the neutral field components are arranged as quadruples

$$\begin{aligned}\psi^{0^\top} &= \left(-i\lambda',\,-i\lambda^3,\,\psi^1_{H_1},\,\psi^2_{H_2}\right)\,,\\ \overline\psi^{0^\top} &= \left(i\overline\lambda',\,i\overline\lambda^3,\,\overline\psi^1_{H_1},\,\overline\psi^2_{H_2}\right)\,.\end{aligned} \tag{22}$$

The symmetric mass matrix, with $s_W = \sin\theta_W$, $c_W = \cos\theta_W$ for the electroweak mixing angle, (see (23) on top

$$Y = \begin{pmatrix} M_1 & 0 & -M_Z\, s_W\, \cos\beta & M_Z\, s_W\, \sin\beta \\ 0 & M_2 & M_Z\, c_W\, \cos\beta & -M_Z\, c_W\, \sin\beta \\ -M_Z\, s_W\, \cos\beta & M_Z\, c_W\, \cos\beta & 0 & -\mu \\ M_Z\, s_W\, \sin\beta & -M_Z\, c_W\, \sin\beta & -\mu & 0 \end{pmatrix}, \tag{23}$$

of the page) can be diagonalized with the help of a unitary 4×4 matrix N,

$$N^* Y N^\dagger = M^D = \begin{pmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{pmatrix}, \tag{24}$$

yielding the neutralino mass eigenstates as linear combinations of the fields in (22):

$$\chi_i^0 = N_{ij}\, \psi_j^0\,, \qquad \tilde{\chi}_i^0 = \begin{pmatrix} \chi_i^0 \\ \overline{\chi_i^0} \end{pmatrix}. \tag{25}$$

In the 4-component notation, the neutralino Majorana spinors are denoted by $\tilde{\chi}_i^0$ (with $i = 1, \ldots, 4$).

4.2 Renormalization of the neutralino sector

Starting from the neutralino Lagrangian (21), the mass-matrix Y and the fields ψ^0 are – in analogy to the chargino case – transformed by the following parameter and field renormalization,

$$\begin{aligned} &Y \to Y + \delta Y \\ &\psi^0 \to \left(1 + \tfrac{\delta Z^0}{2}\right) \psi^0 \ , \end{aligned} \tag{26}$$

The counterterm matrix δY contains, besides those parameter counterterms already present in (12), the counterterms for the Z mass and for the electroweak mixing angle, respectively. The matrix-valued renormalization constant δZ^0 is chosen diagonal. As in the chargino case, this is sufficient for UV finiteness. In the next step, for later convenience, we define the one-loop version of χ^0 as a linear transformation of the renormalized fields ψ^0 via

$$\chi^0 = R\, \psi^0 \tag{27}$$

with a complex, non-singular and UV-finite 4×4-matrix R. For the one-loop expansion we can write

$$R = \left(1 + \tfrac{\delta Z^N}{2}\right) N\,, \tag{28}$$

where N is the unitary matrix from (24) whereas δZ^N is a general complex 4×4-matrix of one-loop order.

Performing the renormalization and redefinition of the neutralino fields according to (26), (27) and (28), one ends up with the following net substitution

$$\begin{aligned} \psi^0 &\to \left(1 + \tfrac{\delta Z^0}{2}\right) N^\dagger \left(1 - \tfrac{\delta Z^N}{2}\right) \chi^0 \\ &= \left(N^\dagger + \tfrac{\delta Z^0}{2} N^\dagger - N^\dagger \tfrac{\delta Z^N}{2}\right) \chi^0 \ . \end{aligned} \tag{29}$$

This makes obvious that the renormalization constants δZ^0 and δZ^N can only occur in the combination $\frac{\delta Z^0}{2} N^\dagger - N^\dagger \frac{\delta Z^N}{2}$ throughout the MSSM Lagrangian. In order to avoid redundances we define new field-renormalization constants

$$\delta\tilde{Z}^0 = N\big[\delta Z^0 N^\dagger - N^\dagger \delta Z^N\big] = N \delta Z^0 N^\dagger - \delta Z^N\,, \tag{30}$$

in analogy to those of the chargino case. $\delta\tilde{Z}^0$ is now a general complex 4×4-matrix.

Expressing the Lagrangian (21) in terms of the new fields χ^0 and substituting the mass matrix according to (26) yield the Born and the counterterm Lagrangian for the neutralinos. Using the 4-component Majorana spinors $\tilde{\chi}_j^0$ from (25) they read, after a Fourier transformation:

$$\begin{aligned} \mathcal{L}_{\text{Born}} = \tfrac{1}{2}\, \overline{\tilde{\chi}_i^0} \Big[\not{p}\, \delta_{ij} - \big(N^*\, Y\, N^\dagger\big)_{ij}\, \omega_L \\ - \big(N\, Y^\dagger\, N^\top\big)_{ij}\, \omega_R \Big]\, \tilde{\chi}_j^0\,, \end{aligned} \tag{31a}$$

$$\begin{aligned} \mathcal{L}_{\text{CT}} = \tfrac{1}{2}\, \overline{\tilde{\chi}_i^0}\, \not{p} \Big[\tfrac{1}{2} \left(\delta\tilde{Z}^{0*} + \delta\tilde{Z}^{0\top}\right)_{ij} \omega_R \\ + \tfrac{1}{2} \left(\delta\tilde{Z}^0 + \delta\tilde{Z}^{0\dagger}\right)_{ij} \omega_L \Big]\, \tilde{\chi}_j^0 - \tfrac{1}{2}\, \overline{\tilde{\chi}_i^0} \Big[\Big(N^* \delta Y N^\dagger \\ + \tfrac{\delta\tilde{Z}^{0\top} N^* Y N^\dagger + N^* Y N^\dagger \delta\tilde{Z}^0}{2} \Big)_{ij} \omega_L + \Big(N \delta Y^\dagger N^\top \\ + \tfrac{N Y^\dagger N^\top \delta\tilde{Z}^{0*} + \delta\tilde{Z}^{0\dagger} N Y^\dagger N^\top}{2} \Big)_{ij} \omega_R \Big]\, \tilde{\chi}_j^0\,. \end{aligned} \tag{31b}$$

The neutralino self-energies are decomposed into Lorentz covariants as given in (19). The renormalized self-energies are obtained by adding the appropriate counterterms following from (31b) with the help of (18), yielding

$$\hat{\Sigma}_{ij}^R(p^2) = \Sigma_{ij}^R(p^2) + \tfrac{1}{2} \left(\delta\tilde{Z}^{0*} + \delta\tilde{Z}^{0\top}\right)_{ij} \tag{32a}$$

$$\hat{\Sigma}_{ij}^L(p^2) = \Sigma_{ij}^L(p^2) + \tfrac{1}{2} \left(\delta\tilde{Z}^0 + \delta\tilde{Z}^{0\dagger}\right)_{ij} \tag{32b}$$

$$\begin{aligned} \hat{\Sigma}_{ij}^{SR}(p^2) = \Sigma_{ij}^{SR}(p^2) - \Big(N \delta Y^\dagger N^\top \\ + \tfrac{N Y^\dagger N^\top \delta\tilde{Z}^{0*} + \delta\tilde{Z}^{0\dagger} N Y^\dagger N^\top}{2} \Big)_{ij} \end{aligned} \tag{32c}$$

$$\begin{aligned} \hat{\Sigma}_{ij}^{SL}(p^2) = \Sigma_{ij}^{SL}(p^2) - \Big(N^* \delta Y N^\dagger \\ + \tfrac{\delta\tilde{Z}^{0\top} N^* Y N^\dagger + N^* Y N^\dagger \delta\tilde{Z}^0}{2} \Big)_{ij} \ . \end{aligned} \tag{32d}$$

Since neutralinos are Majorana fermions, the appropriate renormalized self-energies have to obey the relations

$$\hat{\Sigma}_{ij}^L(p^2) = \hat{\Sigma}_{ji}^R(p^2)\,,$$

$$
\begin{aligned}
\hat{\Sigma}^{SR}_{ij}(p^2) &= \hat{\Sigma}^{SR}_{ji}(p^2)\,,\\
\hat{\Sigma}^{SL}_{ij}(p^2) &= \hat{\Sigma}^{SL}_{ji}(p^2)\,,
\end{aligned} \tag{33}
$$

which are in accordance with the structure of the counterterms in (32).

5 On-shell conditions

The propagators of the charginos and neutralinos explicitly depend on the mass parameters μ, M_2 and M_1 of the MSSM Lagrangian. We now define the on-shell values of these parameters through the pole positions of the propagators, which correspond to the physical masses of the charginos and neutralinos.

In addition we require for both charginos and neutralinos that the matrix of the renormalized one-particle-irreducible two-point vertex functions $\hat{\Gamma}^{(2)}_{ij}$ becomes diagonal for on-shell external momenta. This fixes the non-diagonal entries of the field-renormalization matrices; their diagonal entries are determined by normalizing the residues of the propagators.

The formulae of the previous sections are general enough to accommodate also complex MSSM parameters giving rise to intrinsic CP violation. In the following discussion we restrict ourselves to the simpler case of the CP-conserving MSSM with real parameters.

5.1 Charginos

In the case of CP conservation the on-shell renormalization conditions for the chargino sector correspond to the following relations between the renormalized self-energies (20), with $i,j = 1,2$ [the operation $\widetilde{\mathrm{Re}}$ replaces the momentum integral in the following term by its real part but does not change other complex coefficients]:

$$
U^* X V^\dagger = \mathrm{diag}(m_{\tilde{\chi}^+_1}, m_{\tilde{\chi}^+_2}) \tag{34a}
$$

$$
\begin{aligned}
& m_{\tilde{\chi}^+_j} \widetilde{\mathrm{Re}}\, \hat{\Sigma}^R_{ij}(m^2_{\tilde{\chi}^+_j}) + \widetilde{\mathrm{Re}}\, \hat{\Sigma}^{SL}_{ij}(m^2_{\tilde{\chi}^+_j}) = 0\\
& m_{\tilde{\chi}^+_j} \widetilde{\mathrm{Re}}\, \hat{\Sigma}^L_{ij}(m^2_{\tilde{\chi}^+_j}) + \widetilde{\mathrm{Re}}\, \hat{\Sigma}^{SR}_{ij}(m^2_{\tilde{\chi}^+_j}) = 0
\end{aligned} \tag{34b}
$$

$$
\begin{aligned}
& \widetilde{\mathrm{Re}}\, \hat{\Sigma}^L_{ii}(m^2_{\tilde{\chi}^+_i}) + 2 m_{\tilde{\chi}^+_i} \widetilde{\mathrm{Re}}\, \hat{\Sigma}^{SL\prime}_{ii}(m^2_{\tilde{\chi}^+_i})\\
& \quad + m^2_{\tilde{\chi}^+_i} \left(\widetilde{\mathrm{Re}}\, \hat{\Sigma}^{L\prime}_{ii}(m^2_{\tilde{\chi}^+_i}) + \widetilde{\mathrm{Re}}\, \hat{\Sigma}^{R\prime}_{ii}(m^2_{\tilde{\chi}^+_i}) \right) = 0.
\end{aligned} \tag{34c}
$$

The diagonal ($i = j$) equations in (34b) ensure that the positions of the propagator poles are not shifted by the renormalized self-energies. This means that the relations between the chargino pole masses $m_{\tilde{\chi}^+_i}$ and the MSSM parameters have the same form as in lowest order, also at the one-loop level.

Inserting the renormalized chargino self-energies (20) into the ten equations above and solving for the renormalization constants one obtains the explicit expressions

$$
\begin{aligned}
\delta M_2 = \Bigg[& U_{22} V_{22} \left(m_{\tilde{\chi}^+_1} \left[\widetilde{\mathrm{Re}}\, \Sigma^L_{11}\left(m^2_{\tilde{\chi}^+_1}\right) + \widetilde{\mathrm{Re}}\, \Sigma^R_{11}\left(m^2_{\tilde{\chi}^+_1}\right) \right] \right.\\
& \left. + 2 \widetilde{\mathrm{Re}}\, \Sigma^{SL}_{11}\left(m^2_{\tilde{\chi}^+_1}\right) \right)\\
& - U_{12} V_{12} \left(m_{\tilde{\chi}^+_2} \left[\widetilde{\mathrm{Re}}\, \Sigma^L_{22}\left(m^2_{\tilde{\chi}^+_2}\right) + \widetilde{\mathrm{Re}}\, \Sigma^R_{22}\left(m^2_{\tilde{\chi}^+_2}\right) \right] \right.\\
& \left. + 2 \widetilde{\mathrm{Re}}\, \Sigma^{SL}_{22}\left(m^2_{\tilde{\chi}^+_2}\right) \right) + 2 (U_{12} U_{21} - U_{11} U_{22}) V_{12} V_{22}\\
& \times \delta(\sqrt{2} M_W \sin\beta) + 2\, U_{12} U_{22} (V_{12} V_{21} - V_{11} V_{22})\\
& \times \delta(\sqrt{2} M_W \cos\beta) \Bigg] \Big/ \Delta\,,
\end{aligned} \tag{35a}
$$

$$
\begin{aligned}
\delta\mu = \Bigg[& U_{11} V_{11} \left(m_{\tilde{\chi}^+_2} \left[\widetilde{\mathrm{Re}}\, \Sigma^L_{22}\left(m^2_{\tilde{\chi}^+_2}\right) + \widetilde{\mathrm{Re}}\, \Sigma^R_{22}\left(m^2_{\tilde{\chi}^+_2}\right) \right] \right.\\
& \left. + 2 \widetilde{\mathrm{Re}}\, \Sigma^{SL}_{22}\left(m^2_{\tilde{\chi}^+_2}\right) \right)\\
& - U_{21} V_{21} \left(m_{\tilde{\chi}^+_1} \left[\widetilde{\mathrm{Re}}\, \Sigma^L_{11}\left(m^2_{\tilde{\chi}^+_1}\right) + \widetilde{\mathrm{Re}}\, \Sigma^R_{11}\left(m^2_{\tilde{\chi}^+_1}\right) \right] \right.\\
& \left. + 2 \widetilde{\mathrm{Re}}\, \Sigma^{SL}_{11}\left(m^2_{\tilde{\chi}^+_1}\right) \right) + 2\, U_{11} U_{21} (V_{12} V_{21} - V_{11} V_{22})\\
& \times \delta(\sqrt{2} M_W \sin\beta) + 2\, (U_{12} U_{21} - U_{11} U_{22}) V_{11} V_{21}\\
& \times \delta(\sqrt{2} M_W \cos\beta) \Bigg] \Big/ \Delta\,,
\end{aligned} \tag{35b}
$$

with $\quad \Delta = 2(U_{11} U_{22} V_{11} V_{22} - U_{12} U_{21} V_{12} V_{21})$;

$$
\begin{aligned}
\delta \tilde{Z}^L_{ii} = & - \widetilde{\mathrm{Re}}\, \Sigma^L_{ii}\left(m^2_{\tilde{\chi}^+_i}\right) - m^2_{\tilde{\chi}^+_i} \left[\widetilde{\mathrm{Re}}\, \Sigma^{L\prime}_{ii}\left(m^2_{\tilde{\chi}^+_i}\right) \right.\\
& \left. + \widetilde{\mathrm{Re}}\, \Sigma^{R\prime}_{ii}\left(m^2_{\tilde{\chi}^+_i}\right) \right] - 2\, m_{\tilde{\chi}^+_i} \widetilde{\mathrm{Re}}\, \Sigma^{SL\prime}_{ii}\left(m^2_{\tilde{\chi}^+_i}\right)\,,\\
\delta \tilde{Z}^L_{ij} = & \frac{2}{m^2_{\tilde{\chi}^+_i} - m^2_{\tilde{\chi}^+_j}} \cdot \Bigg[m^2_{\tilde{\chi}^+_j} \widetilde{\mathrm{Re}}\, \Sigma^L_{ij}\left(m^2_{\tilde{\chi}^+_j}\right)\\
& + m_{\tilde{\chi}^+_i} m_{\tilde{\chi}^+_j} \widetilde{\mathrm{Re}}\, \Sigma^R_{ij}\left(m^2_{\tilde{\chi}^+_j}\right) + m_{\tilde{\chi}^+_i} \widetilde{\mathrm{Re}}\, \Sigma^{SL}_{ij}\left(m^2_{\tilde{\chi}^+_j}\right)\\
& + m_{\tilde{\chi}^+_j} \widetilde{\mathrm{Re}}\, \Sigma^{SL}_{ji}(m^2_{\tilde{\chi}^+_j}) - m_{\tilde{\chi}^+_i} (U \delta X V^\top)_{ij}\\
& - m_{\tilde{\chi}^+_j} (U \delta X V^\top)_{ji} \Bigg]\,,
\end{aligned} \tag{35c}
$$

$$
\begin{aligned}
\delta \tilde{Z}^R_{ii} = & - \widetilde{\mathrm{Re}}\, \Sigma^R_{ii}\left(m^2_{\tilde{\chi}^+_i}\right) - m^2_{\tilde{\chi}^+_i} \left[\widetilde{\mathrm{Re}}\, \Sigma^{L\prime}_{ii}\left(m^2_{\tilde{\chi}^+_i}\right) \right.\\
& \left. + \widetilde{\mathrm{Re}}\, \Sigma^{R\prime}_{ii}\left(m^2_{\tilde{\chi}^+_i}\right) \right] - 2\, m_{\tilde{\chi}^+_i} \widetilde{\mathrm{Re}}\, \Sigma^{SL\prime}_{ii}\left(m^2_{\tilde{\chi}^+_i}\right)\,,\\
\delta \tilde{Z}^R_{ij} = & \frac{2}{m^2_{\tilde{\chi}^+_i} - m^2_{\tilde{\chi}^+_j}} \cdot \Bigg[m^2_{\tilde{\chi}^+_j} \widetilde{\mathrm{Re}}\, \Sigma^R_{ij}\left(m^2_{\tilde{\chi}^+_j}\right)\\
& + m_{\tilde{\chi}^+_i} m_{\tilde{\chi}^+_j} \widetilde{\mathrm{Re}}\, \Sigma^L_{ij}\left(m^2_{\tilde{\chi}^+_j}\right) + m_{\tilde{\chi}^+_j} \widetilde{\mathrm{Re}}\, \Sigma^{SL}_{ij}\left(m^2_{\tilde{\chi}^+_j}\right)\\
& + m_{\tilde{\chi}^+_i} \widetilde{\mathrm{Re}}\, \Sigma^{SL}_{ji}\left(m^2_{\tilde{\chi}^+_j}\right) - m_{\tilde{\chi}^+_j} (U \delta X V^\top)_{ij}\\
& - m_{\tilde{\chi}^+_i} (U \delta X V^\top)_{ji} \Bigg]\,.
\end{aligned} \tag{35d}
$$

So far, the renormalization of two of our MSSM parameters (M_2 and μ) has been fixed by the chargino mass renormalization.

5.2 Neutralinos

The left-over mass-parameter of the MSSM Lagrangian yet to be determined is the Bino mass M_1 of the neutralino sector. In the on-shell strategy, it can be fixed, together with its counterterm, by the on-shell mass renormalization of one of the four neutralino states, which we choose to be $m_{\tilde{\chi}_1^0}$.

Moreover, the additional matrix (27) in the field-renormalization constants allows one to impose the condition of having diagonal renormalized 2-point vertex functions for each of the neutralinos on-shell, i.e. $(i \neq j)$

$$\hat{\Gamma}_{ij}^{(2)}(p) = 0 \quad \text{for either} \quad p^2 = m_{\tilde{\chi}_i^0}^2 \quad \text{or} \quad p^2 = m_{\tilde{\chi}_j^0}^2 .$$

This fixes the 12 non-diagonal entries of $\delta\tilde{Z}^0$. The remaining four diagonal entries are determined by requiring unity for the residues of the neutralino propagators.

In the case of CP-conservation this leads to the following conditions:

$$\begin{aligned} &\left(NYN^\top\right)_{11} = m_{\tilde{\chi}_1^0} \\ &NYN^\top = \mathrm{diag}(m_1, m_2, m_3, m_4) \equiv M^D \end{aligned} \tag{36a}$$

$$\begin{aligned} &m_{\tilde{\chi}_j^0} \widetilde{\mathrm{Re}}\, \hat{\Sigma}_{ij}^L(m_{\tilde{\chi}_j^0}^2) + \widetilde{\mathrm{Re}}\, \hat{\Sigma}_{ij}^{SL}(m_{\tilde{\chi}_j^0}^2) = 0 \\ &\qquad \mathrm{for}(i \neq j) \vee (i = j = 1) \end{aligned} \tag{36b}$$

$$\begin{aligned} &\widetilde{\mathrm{Re}}\, \hat{\Sigma}_{ii}^L(m_{\tilde{\chi}_i^0}^2) + 2m_{\tilde{\chi}_i^0}^2\, \widetilde{\mathrm{Re}}\, \hat{\Sigma}_{ii}^{L\prime}(m_{\tilde{\chi}_i^0}^2) \\ &\qquad +2m_{\tilde{\chi}_i^0} \widetilde{\mathrm{Re}}\, \hat{\Sigma}_{ii}^{SL\prime}(m_{\tilde{\chi}_i^0}^2) = 0 \quad . \end{aligned} \tag{36c}$$

The value of M_1 is related to the mass $m_{\tilde{\chi}_1^0}$ of $\tilde{\chi}_1^0$ by means of (36a). The condition (36b) for $i = j = 1$ ensures that this is also the pole mass at the one-loop level. After this step, all eigenvalues m_j $(j = 1, \ldots, 4)$ of the matrix Y are known. However, only $m_1 \equiv m_{\tilde{\chi}_1^0}$ is equal to the pole mass. The other eigenvalues $m_{2,3,4}$ are the Born approximations of the corresponding physical neutralino masses. They get corrections at the one-loop level, as discussed in the next section.

Inserting the renormalized neutralino self-energies (32) and solving (36) for the renormalization constants one finds the explicit expressions

$$\begin{aligned} \delta M_1 = \frac{1}{N_{11}^2} \Bigg[& 2N_{11} \Big[N_{13}\, \delta(M_Z \sin\theta_W \cos\beta) \\ & -N_{14}\, \delta(M_Z \sin\theta_W \sin\beta) \Big] \\ & -N_{12} \Big[2N_{13}\, \delta(M_Z \cos\theta_W \cos\beta) \\ & -2N_{14}\, \delta(M_Z \cos\theta_W \sin\beta) + N_{12}\, \delta M_2 \Big] \\ & +2N_{13}N_{14}\, \delta\mu + m_{\tilde{\chi}_1^0} \widetilde{\mathrm{Re}}\, \Sigma_{11}^L(m_{\tilde{\chi}_1^0}^2) \\ & +\widetilde{\mathrm{Re}}\, \Sigma_{11}^{SL}(m_{\tilde{\chi}_1^0}^2) \Bigg] , \end{aligned} \tag{37a}$$

$$\begin{aligned} \delta\tilde{Z}_{ii}^0 = -\widetilde{\mathrm{Re}}\, \Sigma_{ii}^L\left(m_{\tilde{\chi}_i^0}^2\right) - 2m_{\tilde{\chi}_i^0} \Big[& m_{\tilde{\chi}_i^0}\, \widetilde{\mathrm{Re}}\, \Sigma_{ii}^{L\prime}\left(m_{\tilde{\chi}_i^0}^2\right) \\ & +\widetilde{\mathrm{Re}}\, \Sigma_{ii}^{SL\prime}\left(m_{\tilde{\chi}_i^0}^2\right) \Big] , \end{aligned} \tag{37b}$$

$$\begin{aligned} \delta\tilde{Z}_{ij}^0 = \Bigg\{ \Bigg(& 2\Big[m_{\tilde{\chi}_j^0}\, \widetilde{\mathrm{Re}}\, \Sigma_{ij}^L\left(m_{\tilde{\chi}_j^0}^2\right) + \widetilde{\mathrm{Re}}\, \Sigma_{ij}^{SL}\left(m_{\tilde{\chi}_j^0}^2\right) \\ & - \left(N\delta YN^\top\right)_{ij} \Big] \Bigg) \Bigg/ \Bigg(m_{\tilde{\chi}_i^0} - m_{\tilde{\chi}_j^0} \Bigg) \Bigg\} . \end{aligned} \tag{37c}$$

Together with the renormalization constants from the gauge and Higgs sector, outlined in the following subsection, the renormalization of the 2-point functions of the chargino/neutralino sector of the MSSM is complete. Moreover, all renormalization constants are now available to determine all the counterterms required for one-loop calculations in the neutralino–chargino sector of the MSSM.

5.3 Renormalization constants from other sectors

The two sets of (35) and (37) for the renormalization constants contain explicitly the counterterms for the quantities M_W, M_Z, θ_W of the gauge sector and for β, which is a parameter of the Higgs sector.

In the on-shell scheme, the renormalization of the electroweak mixing angle with $c_W = M_W/M_Z$ is deduced from the renormalization of the W- and Z-boson masses, at the one-loop level via the relation

$$\delta s_W^2 \;=\; c_W^2 \left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right) .$$

The on-shell counterterms for M_W and M_Z are given by the transverse parts of the respective vector-boson self-energies evaluated on their mass shell,

$$\begin{aligned} \delta M_W^2 &= \mathrm{Re}\, \Sigma_{WW}^{\mathrm{trans}}(M_W^2) , \\ \delta M_Z^2 &= \mathrm{Re}\, \Sigma_{ZZ}^{\mathrm{trans}}(M_Z^2) . \end{aligned} \tag{38}$$

Following [12] we fix the renormalization constant for $\tan\beta$ by the condition

$$\frac{\delta\tan\beta}{\tan\beta} = \frac{1}{2M_Z \sin\beta\, \cos\beta} \cdot \mathrm{Im}\left[\widetilde{\mathrm{Re}}\, \Sigma_{A^0Z}(M_A^2) \right] . \tag{39}$$

Another option, which has been applied in the recent version of *FeynHiggs* [13], would be a $\overline{\mathrm{MS}}$ renormalization of $\tan\beta$.

6 Neutralino masses

The renormalization procedure presented in the last section assures the neutralino fields not to mix with each other on their specific mass-shell. Thus the one-loop corrected masses of the remaining three neutralinos can simply be determined by finding those momenta $p_i^2 = m_{\tilde{\chi}_i^0}^2$ which obey the relation

$$\widetilde{\mathrm{Re}}\, \left[\hat{\Gamma}_{ii}^{(2)}(p_i) \right] u(p_i) = 0 \quad , \tag{40}$$

where

$$\hat{\Gamma}^{(2)}_{ij}(p) = (\not{p} - m_i)\,\delta_{ij} + \hat{\Sigma}_{ij}(p)\,,$$

$$\hat{\Sigma}_{ij}(p) = \not{p}\,\omega_L \hat{\Sigma}^L_{ij}(p^2) + \not{p}\,\omega_R \hat{\Sigma}^R_{ij}(p^2) + \omega_L \hat{\Sigma}^{SL}_{ij}(p^2) + \omega_R \hat{\Sigma}^{SR}_{ij}(p^2)\,, \qquad (41)$$

are the renormalized neutralino two-point vertex-functions with the self-energies (32) and $u(p_i)$ the i-neutralino wave function in momentum space. At one-loop order, the condition (40) has the solution

$$m_{\tilde{\chi}^0_i} = m_i \left[1 - \widetilde{\mathrm{Re}}\,\hat{\Sigma}^L_{ii}(m_i^2)\right] - \widetilde{\mathrm{Re}}\,\hat{\Sigma}^{SL}_{ii}(m_i^2) \qquad (42)$$

for the neutralino pole masses. Inserting (32) for the renormalized self-energies finally yields the neutralino masses in terms of the unrenormalized neutralino self-energies and the renormalization constants,

$$m_{\tilde{\chi}^0_i} = m_i\left[1 - \widetilde{\mathrm{Re}}\,\Sigma^L_{ii}(m_i^2)\right] - \widetilde{\mathrm{Re}}\,\Sigma^{SL}_{ii}(m_i^2) + \left(N\delta Y N^\top\right)_{ii} \quad . \qquad (43)$$

These masses are thus predictions arising from $m_{\tilde{\chi}^+_1}, m_{\tilde{\chi}^+_2}$, $m_{\tilde{\chi}^0_1}$ and depend in addition on the residual MSSM parameters that enter the self-energies and the counterterms at the one-loop level.

7 Numerical evaluation

7.1 Specification of μ, M_2 and M_1

In our on-shell approach, the pole masses of the two charginos, $m_{\tilde{\chi}^+_1}, m_{\tilde{\chi}^+_2}$, and of one neutralino, $m_{\tilde{\chi}^0_1}$, are considered as input parameters, to specify the chargino/neutralino Lagrangian in terms of physical quantities. This is equivalent to the specification of the parameters μ, M_1, M_2, which are related to the input masses in the same way as in lowest order, as a consequence of the on-shell renormalization conditions.

For given pole masses of the charginos, the values of M_2 and μ can be determined using (10). Inverting those relations one gets four solutions corresponding to different physical scenarios,

$$M_2 = a_\pm\,, \qquad \mu = \frac{a_\pm \cdot a_\mp^2}{2M_W^2 \sin\beta\,\cos\beta + m_{\tilde{\chi}^+_1}\, m_{\tilde{\chi}^+_2}}\,;$$
$$M_2 = b_\pm\,, \qquad \mu = \frac{b_\pm \cdot b_\mp^2}{2M_W^2 \sin\beta\,\cos\beta - m_{\tilde{\chi}^+_1}\, m_{\tilde{\chi}^+_2}}\,; \qquad (44)$$

with the abbreviations

$$a_\pm = \tfrac{1}{\sqrt{2}}\sqrt{m^2_{\tilde{\chi}^+_1} + m^2_{\tilde{\chi}^+_2} - 2M_W^2 \pm c_+}$$

$$b_\pm = \tfrac{1}{\sqrt{2}}\sqrt{m^2_{\tilde{\chi}^+_1} + m^2_{\tilde{\chi}^+_2} - 2M_W^2 \pm c_-} \qquad (45)$$

$$c_\pm = \sqrt{\left(m^2_{\tilde{\chi}^+_1} + m^2_{\tilde{\chi}^+_2} - 2M_W^2\right)^2 - 4\left(m_{\tilde{\chi}^+_1} m_{\tilde{\chi}^+_2} \pm 2M_W^2 \sin\beta\,\cos\beta\right)^2}.$$

After selecting a specific M_2–μ configuration all entries in Y from (23) are determined, except for $Y_{11} \equiv M_1$. The value of M_1 is obtained by the condition that the eigenvalue m_1 of Y coincides with the pole mass $m_{\tilde{\chi}^0_1}$. For the case of real parameters the appropriate eigenvalue equation leads to the unique solution

$$M_1 = \Big[-M_2\mu M_Z^2 \sin 2\beta + \big[\mu M_Z^2 \sin 2\beta - M_2\big(\mu^2 + M_Z^2 s_W^2\big)\big] m_{\tilde{\chi}^0_1} + \big[\mu^2 + M_Z^2\big] m^2_{\tilde{\chi}^0_1} + M_2 m^3_{\tilde{\chi}^0_1} - m^4_{\tilde{\chi}^0_1}\Big] \cdot \Big[\mu M_Z^2 c_W^2 \sin 2\beta - M_2\mu^2 + \big[\mu^2 + M_Z^2 c_W^2\big] m_{\tilde{\chi}^0_1} + M_2 m^2_{\tilde{\chi}^0_1} - m^3_{\tilde{\chi}^0_1}\Big]^{-1}. \qquad (46)$$

After this specification of the mass parameters, the Born-level diagonalization matrices U, V and N can be calculated.

7.2 Results and discussion

The self-energies appearing in the counter terms $\delta\mu$, δM_2, δM_1, δM_W^2, δM_Z^2 and $\delta\tan\beta$ as well as the neutralino self-energies in (43) have been calculated with the help of the program packages `FeynArts`, `FormCalc` and `LoopTools` [14]. For regularization we used the method of 'Constrained Differential Renormalization' [15]. At the one-loop level this prescription has been proven to be equivalent to 'Dimensional Reduction' [16], which is compatible with supersymmetry.

In the numerical evaluation, mixing between the fermion families has been neglected. Moreover, we assume a common sfermion-mass scale $m^2_{\{\tilde{q},\tilde{u},\tilde{d},\tilde{l},\tilde{e}\}} \equiv M^2_{\mathrm{susy}}$ for simplification. The gaugino parameters M_1 and M_2 are treated as independent.

All values for the masses in Figs. 1–3 have to be understood in units of GeV. The input values taken for the gauge-boson masses are $M_W = 80.419$ GeV, $M_Z = 91.1882$ GeV. For the MSSM parameters, unless stated differently, the following values have been used for the examples in the numerical presentation. Thereby, the trilinear $A_{u,d,e}$ parameters are assumed universal for the three generations.

$M_A = 150$ GeV	$M_{\mathrm{susy}} = 300$ GeV	$\tan\beta = 10$
$A_u = 100$ GeV	$A_d = 900$ GeV	$A_e = 900$ GeV

Before entering the presentation of our results we want to add a few comments regarding other treatments of on-shell renormalization. The scheme used in [6] for the calculation of sfermion decays into fermions is equivalent to the one specified here up to the treatment of field renormalization. In our case, the effective field-renormalization constants involve a finite one-loop redefinition of the diagonalization matrices U, V and N [see (16) and (30)] which allows a complete diagonalization of the self-energy matrices on-shell, whereas in [6] the U, V, N are not modified. Differences for the predicted neutralino masses in terms of the input masses, however, are only of higher order and thus accordingly very small. This was confirmed also numerically by an explicit comparison [17][1].

In [9] the field-renormalization constants are introduced also in combination with one-loop redefinitions of the matrices U, V, N. The Z factors, however, have been fixed by an independent prescription adopted from [18]. In order to make this compatible with the on-shell renormalization conditions, this requires a re-adjustment of the entries in the chargino and neutralino mass matrices by finite shifts; otherwise M_W and M_Z in (7) and (23) would not be the on-shell (pole) masses of W and Z. As a consequence, the MSSM parameters in the mass matrices are in general different at tree level and one-loop order. In particular, the relations between M_2, μ and the chargino pole masses as well as between M_1 and the $\tilde{\chi}_1^0$ pole mass are not the tree-level relations but do contain additional terms of one-loop order. For comparisons one therefore has to keep in mind that the values of the formal parameters μ, M_1, M_2 in [9] are different from ours for the same physical situation, i.e. the physical masses $m_{\tilde{\chi}_1^+}$, $m_{\tilde{\chi}_2^+}$, $m_{\tilde{\chi}_1^0}$. The masses of the three other neutralinos, however, when calculated as observables from the same physical input, should be the same, up to small terms of formally higher orders. The evaluation in [9] was performed for the subclass of fermion/sfermion-loop contributions only; hence, for a numerical comparison, we had to turn off the non-(s)fermionic loop contributions of our approach and found indeed good agreement in the calculated neutralino masses for the examples given in [9][2].

7.2.1 Dependence of the neutralino masses on $m_{\tilde{\chi}_1^+}$ and $m_{\tilde{\chi}_2^+}$

Figure 1 shows the dependence of the neutralino masses on the input mass $m_{\tilde{\chi}_2^+}$ of the heavy chargino at three different values for the mass $m_{\tilde{\chi}_1^+}$ of the light chargino. For definiteness we choose the a_+ solution of the first line in (44) as a representative example (the other solutions show similar behaviour). The input mass from the neutralino sector is assumed to be $m_{\tilde{\chi}_1^0} = 110$ GeV throughout all the subdiagrams of Fig. 1. Depicted are: the tree-level approximation of the calculated neutralino masses, their values after including the corrections from the (s)fermionic loops only, and finally the complete one-loop corrected masses with all MSSM particles in the virtual states. For the heaviest neutralino mass $m_{\tilde{\chi}_4^0}$ the shift is small, not more than 175 MeV throughout the whole scanned parameter space, and nearly invisible in the graphical illustration. Accordingly, the relative corrections are less than 0.05% everywhere, and hence we do not give more than one graph in the figure.

The impression prima facie of a qualitatively different behaviour of the masses $m_{\tilde{\chi}_2^0}$ and $m_{\tilde{\chi}_3^0}$ in the different mass regions of $m_{\tilde{\chi}_1^+}$ in Fig. 1 can be explained as follows. Starting from $m_{\tilde{\chi}_1^+} = 170$ GeV and going to $m_{\tilde{\chi}_1^+} = 135$ GeV there is a point where the two neutralino-mass curves under consideration cross each other. At this specific value for $m_{\tilde{\chi}_1^+}$, the two particles are renumbered reflecting the changed order of their masses. The different form of this two mass curves for $m_{\tilde{\chi}_1^+} = 135$ GeV and $m_{\tilde{\chi}_1^+} = 100$ GeV stems from the fact that there is a formal singularity of M_1 at a value of $m_{\tilde{\chi}_1^+}$ close to 115 GeV in (46).

The one-loop corrections for the mass $m_{\tilde{\chi}_2^0}$ can reach 20% in the case that the mass splitting between the two charginos is large. For the second and third neutralino, the typical size of the loop corrections to the masses amount to about one per cent of the Born values.

A synopsis of all diagrams clearly shows that the one-loop contributions of the gauge and Higgs sector to the neutralino mass shifts are basically of the same order of magnitude as those resulting from the subclass of (s)fermionic loops.

7.2.2 Dependence of the neutralino masses on $\tan\beta$

In Fig. 2 and 3 the $\tan\beta$-dependence of the calculated neutralino masses is visualized for two different examples of the light-chargino mass $m_{\tilde{\chi}_1^+}$. The values for $\tan\beta$ are varied from 2 to 60. For the mass of the heavy chargino we choose $m_{\tilde{\chi}_2^+} = 350$ GeV, and the input neutralino mass is set to $m_{\tilde{\chi}_1^0} = 160$ GeV.

The left columns contain the predicted neutralino masses, again in Born approximation and at the one-loop level taking into account all MSSM particles in the loops. For comparison, the one-loop neutralino masses based on the approximation with (s)fermionic loops only are also shown. In the right columns the loop-induced mass shifts (in GeV) are displayed.

The variation of the one-loop shifts of the neutralino masses over the whole range of $\tan\beta$ is highest for the case of $\tilde{\chi}_2^0$. At low values for $\tan\beta$ the mass correction is about 2% of the Born value, decreasing steadily with raising $\tan\beta$ to approximately 0.7%.

In the parameter space being considered here the mass correction for the next-heaviest neutralino ranges from 1.5% to 2.5% of the Born value in the case of $m_{\tilde{\chi}_1^+} = 100$ GeV. It decreases for heavier charginos, as e.g. in the example of $m_{\tilde{\chi}_1^+} = 180$ GeV where it varies between 0.4% and 0.9%.

The one-loop corrections of the mass of the heaviest neutralino are small, below 0.1% of the respective Born values throughout the entire range of $\tan\beta$ under investigation.

A final remark addresses the option of performing a $\overline{\text{MS}}$ renormalization of $\tan\beta$, where only the $\overline{\text{MS}}$ UV-singularity of the r.h.s. in (39) is defined as the countert-

[1] We thank J. Guasch for the numerical comparison with the results based on the scheme of [6]

[2] We thank H. Eberl for providing us with the detailed numbers for the examples given in [9]

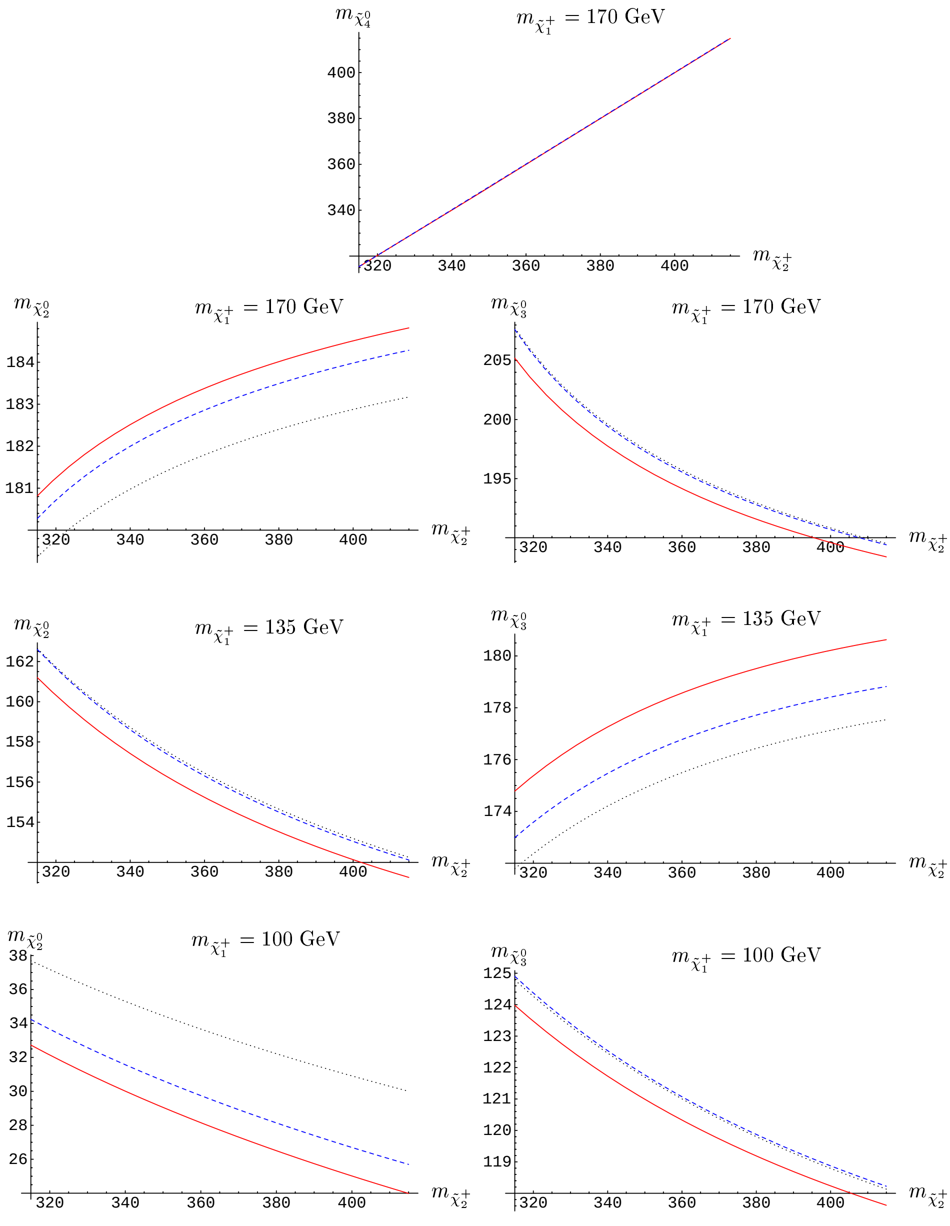

Fig. 1. Dependence of the calculated neutralino masses (in GeV) on the chargino masses $m_{\tilde{\chi}_1^+}$ and $m_{\tilde{\chi}_2^+}$, in Born approximation (*dotted*), including loop corrections with (s)fermions only (*dashed*), and with the complete one-loop contributions (*solid*). The input neutralino mass is chosen as $m_{\tilde{\chi}_1^0} = 110$ GeV throughout all diagrams. The plots for $m_{\tilde{\chi}_4^0}$ neutralino would look very much alike the one shown for all three different values of $m_{\tilde{\chi}_1^+}$

erm. This option has been installed in the version *FeynHiggs*1.2 of the *FeynHiggs* code to calculate the neutral MSSM Higgs-boson masses and couplings [13]. As a comparison of the respectively calculated neutralino masses with the neutralino-mass results based on (39) has shown [19], the differences are quite small, at most about 40 MeV, for the same input values in both schemes[3].

8 Conclusions

We have presented an on-shell renormalization of the chargino and neutralino mass spectrum of the MSSM at the one-loop level, based on the entire set of one-loop diagrams. An on-shell renormalization scheme has been specified treating all particle masses as pole masses, with renormalization constants implemented in a way that allows one to formulate the renormalized self-energies of the charginos and neutralinos as UV-finite matrices which are diagonal for external momenta on-shell. With the masses of both charginos and of one neutralino as input, the MSSM parameters μ, M_2, M_1 formally obey the lowest-order relations to these masses. The masses of the residual three neutralinos are calculated from the input, yielding mass shifts up to several GeV as compared to the tree-level approximation. The numerical investigation shows that the virtual contributions beyond those from the subset of diagrams with fermion/sfermion loops are in general of similar size as the purely (s)fermionic contributions. A proper treatment will therefore become necessary for precision studies within the MSSM at future colliders.

[3] We thank A. Freitas for the numerical comparison

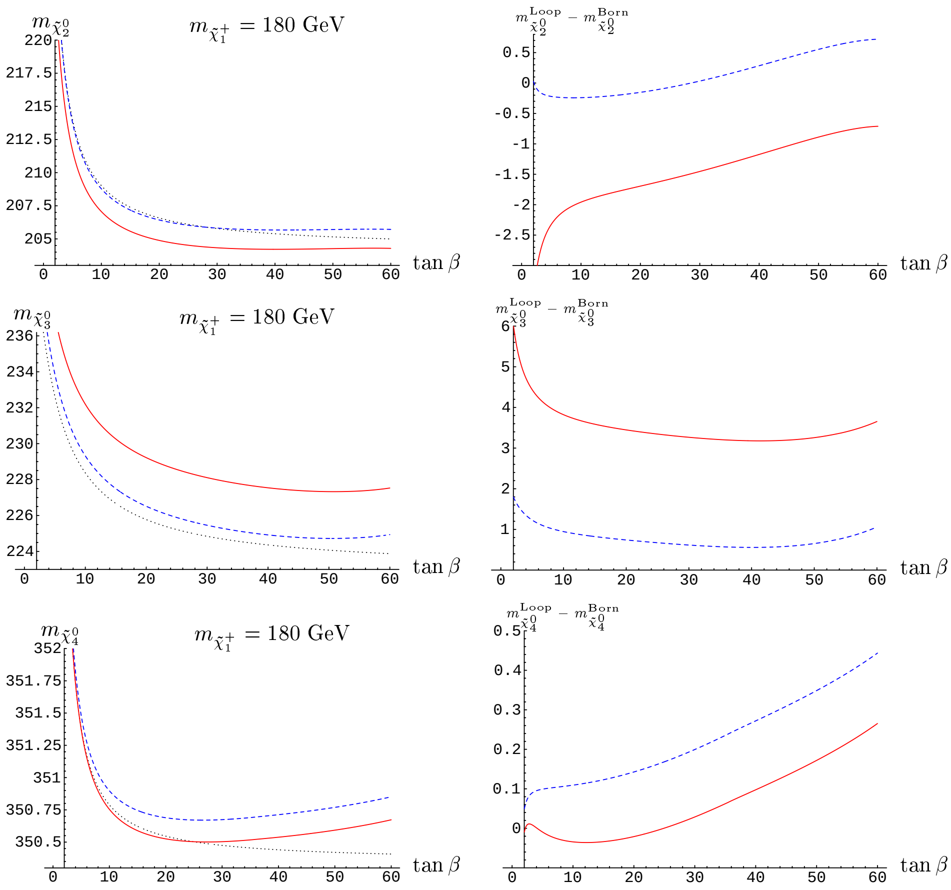

Fig. 2. Dependence of the neutralino masses on $\tan\beta$, in Born approximation (*dotted*), including loop corrections with (s)fermions only (*dashed*), and with the complete one-loop contributions (*solid*). The mass of the heavy chargino is set to $m_{\tilde{\chi}_2^+} = 350$ GeV and the input neutralino mass is chosen as $m_{\tilde{\chi}_1^0} = 160$ GeV throughout all the plots. Left columns: absolute mass values; right columns: mass shifts (in GeV)

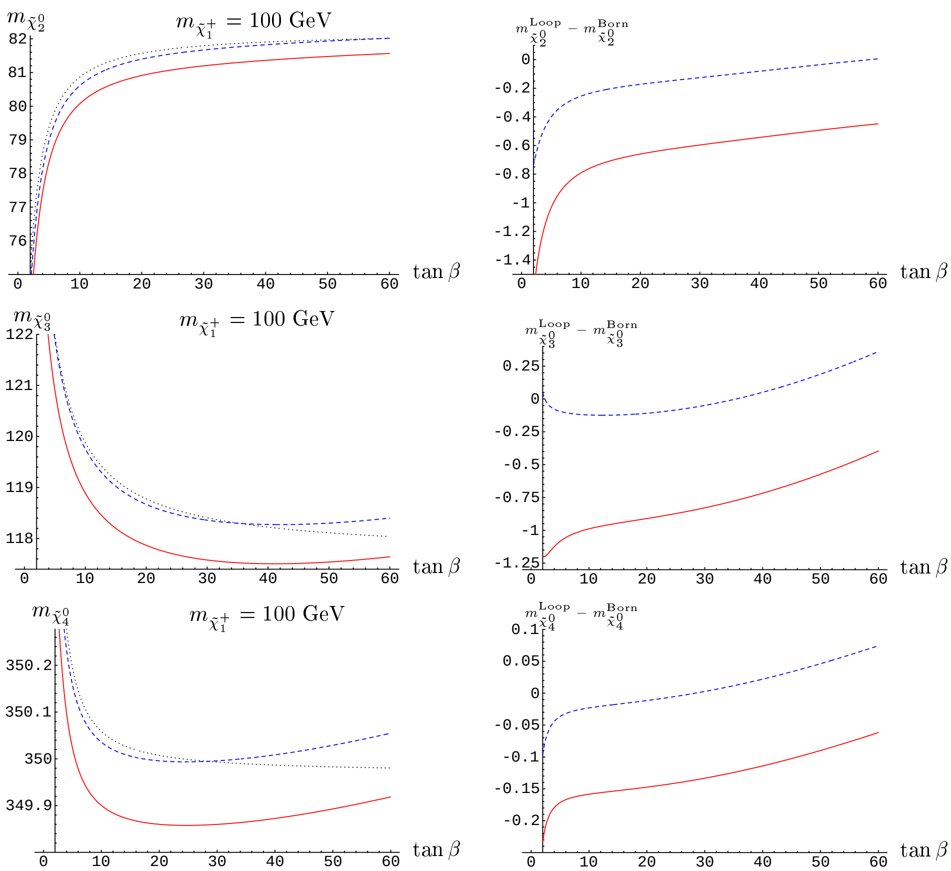

Fig. 3. Dependence of the neutralino masses on $\tan\beta$ in Born approximation (*dotted*), including loop-corrections with (s)fermions only (*dashed*) and with the complete one-loop radiative corrections (*solid*). The mass of the heavy chargino is set to $m_{\tilde{\chi}_2^+} = 350$ GeV and the input neutralino mass is chosen as $m_{\tilde{\chi}_1^0} = 160$ GeV throughout all diagrams. Left columns: absolute mass values; right columns: mass shifts (in GeV)

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