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Large scale correlations for energy injection mechanisms in swirling turbulent flows

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Abstract. We report an experimental study of large scale correlations in the power injected in turbulent swirling flows generated in the gap between two coaxial rotating disks. We measure the pressure fluctuations on the blades of one disk, as well as the pressure drop between the leading and the trailing edges of the rotating blades, *i.e.* the local drag force. Measurements at different positions on one blade and on two successive blades display a correlation length much larger than the ones usually expected in turbulent flows. The time lag for which the correlation between two points is maximum, strongly depends on the global flow configuration. These results help us to understand the statistical properties of the injected power fluctuations in turbulent swirling flows.

PACS. 47.27.-i Turbulent flows, convection and heat transfer – 47.27.Jv High-Reynolds-number turbulence

1 Introduction

Despite their practical interest, only a few studies have been performed on the statistical properties of the power injected in turbulent flows. On the theoretical side, the usual phenomenological approach of turbulence does not take into account these fluctuations, and consider that all *global quantities*, *i.e.* quantities averaged on the whole flow volume (respectively on its boundaries), are constant. This approximation could be justified if, as usually assumed, any correlation length λ in turbulence were much smaller than the system size L. This allows to replace all space integrals by a sum of independent random variables. Then the central limit theorem implies Gaussian fluctuations for all global quantities. Moreover, their relative fluctuations then should decrease like $1/\sqrt{N}$, where $N = (L/\lambda)^d$ with $d = 3$, (respectively 2). One could thus expect that the rms power fluctuations relative to their mean value, $\sigma(P)/\langle P\rangle$, become negligible in the limit of large Reynolds number, Re, because λ is a decreasing function of Re [1].

However, it has been shown in a confined turbulent flow [2] that the power injected in order to maintain a constant Reynolds number displays large temporal fluctuations (rms of about 10%) and a non-Gaussian probability distribution function. In the above experiment, the flow motion was produced in the gap of two coaxial rotating disks set with blades to increase entrainment. We present here measurements of pressure fluctuations on the blades of one of the rotating disks maintaining the turbulent flow. The pressure drop between the leading and the trailing edges of a blade is the main contribution to the local drag force on the blade, and its rate of work, averaged on all the blades, is equal to the power injected in the flow. We show that both the pressure field on the blades surface and the local drag force display large scale correlations on the system size L. This helps to understand the statistical properties of the fluctuations of the power injected into the flow.

2 Experimental set-up and measurements

The experimental set-up has been described in reference [2]. Two coaxial disks of radius $R = 10$ cm, $H = 20$ cm apart, are fitted with vertical blades of height $h_\mathrm{b} = 2$ cm, perpendicular to the disk surface. They are driven by independent 450 Watt d.c. motors, the rotation frequencies of which are adjustable from 0 to 45 Hz and controlled by a feed-back loop. Air is the working fluid and the disks are enclosed in a cylindrical cylinder 23.2 cm in diameter.

Pressure fluctuations are measured by four piezoelectric transducers PCB 103A mounted flush with the leading or trailing vertical edges of a blade. Their active diameter is 2.1 mm, their low frequency cut-off at −5% is 0.05 Hz and their rise-time is 25 μs. The transducers are mounted

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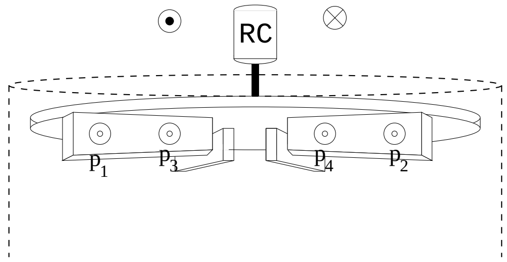

Fig. 1. Pressure sensors positions on the leading edge and on the trailing edge of two successive blades of the upper disk. The disk radius is 10 cm. The pressure sensor p_1 is at 9 cm from the disk center on the leading edge. The sensor p_2 is at the same distance on the trailing edge. p_3 is at 4 cm on the leading edge and p_4 is at 4 cm on the trailing edge.

either on the same blade, two on the leading edge, p_1 and p_3, 9 cm (respectively 4 cm) away from the disk axis, and the two others on the trailing edge, p_2 and p_4, opposite to p_1 and p_3, or on different blades of the upper disk (see Fig. 1). Electric signals from the rotating pressure transducers are retrieved using Air Precision T13HP slip rings.

The fluid motion is mainly created by the motion of the blades. We can compute the injected power, P, by multiplying the Navier–Stokes equation by the velocity and taking the average over all the flow volume. To leading order, we get

$$P \approx -\Omega \int_{S_{\rm b}} \Delta p(r,t)\, r\mathrm{d}S, \tag{1}$$

where Ω is the angular rotation rate, $\Delta p(r,t)$ is the pressure drop between the leading and trailing edges of a blade at a distance r from the rotation axis, and $S_{\rm b}$ is the total surface of the blades leading vertical edges. $\Delta p(r,t)$ times the active surface of a pressure sensor is roughly the local drag force applied on the blade at distance r from the rotation axis, and its work, integrated on the blades leading vertical edges, gives the power injected into the flow. There is actually another contribution to the injected power which is due to the work of the viscous friction forces on the blades and on the disks, but it is known to be negligible compared to the pressure drag at large Re [3]. In the absence of blades and with smooth disks, only the viscous forces contribute to the injected power, and qualitative differences in the mean rate of energy injection have been observed [4,5].

The main interest of our local pressure or drag measurements is to study the spatiotemporal correlations in the energy injection mechanisms in the turbulent flow, *i.e.* to determine if the right hand side of equation (1) can or cannot be replaced by a sum of independent random contributions in the limit of large Re. These measurements are not performed to get a quantitative evaluation of the injected power P using equation (1). A lot more pressure sensors should be used for that purpose and there exist other simpler measurement methods, for instance the electric power consumption [2] or torque measurements [4]. Note however that the determination of the injected power using pressure measurements present several advantages compared to these global measurements: no high frequency filtering due to the inertia of the disks and rotors, no bias due to Joule dissipation in the motors, etc. It may thus be an efficient method to measure drag fluctuations on bluff bodies in high Re flows.

Most of the experimental results presented below have been obtained with counter-rotating disks at the same rotation rate but in an asymmetric configuration: we used an upper disk with four blades on which pressure was measured but a lower disk with eight blades. With these non symmetric conditions, the shearing zone between the disks moves toward the upper disk so that the measurements are done in a strongly turbulent flow. We have compared this situation with the symmetric one with four blades on each disks. We have also studied the open flow geometry as well as the flow generated by co-rotating disks. These systematic studies have been reported in reference [6]. We will recall them here as needed, to underline the differences with our reference flow.

Note that a convenient definition of the Reynolds number, Re, that takes into account the number of blades, $N_{\rm b}$, and their height, $h_{\rm b}$, is $Re = 2N_{\rm b}h_{\rm b}R\Omega/\nu$, where ν the kinematic viscosity. The flow is indeed created by blades of characteristic size $h_{\rm b}$ moving at typical velocity $R\Omega$ in the fluid. The rotation rate being in the range $[4, 44\,\mathrm{Hz}]$, we have one decade in Reynolds number, $9 \times 10^4 < Re < 9 \times 10^5$.

3 Results

Power density spectra (PDS) of pressure fluctuations on the leading edge, p_1, (respectively the trailing edge, p_2), of a rotating blade are displayed in Figure 2, and compared to the pressure fluctuations on the cylindrical boundary in the mid-plane between the disks.

We first observe that the pressure PDS on the rotating blades involves sharp peaks at the rotation frequency and its harmonics. In the symmetric counter-rotating configuration (two disks with four blades), only the fourth harmonic clearly emerges from the background. In the open flow configuration, the pressure PDS on the leading edge is similar to the one of Figure 2a, but with smaller peaks, whereas they almost vanish in the pressure PDS on the trailing edge. Thus, the pressure field on the rotating blades involves a large coherent part which is smaller in the bulk of the flow, but strongly depends on the global flow geometry. The pressure field on one disk, and thus the injected power by that disk, depends on the characteristics of the other disk and on the open or shrouded flow configuration.

The pressure PDS in Figure 2 are all rather flat at low frequency but their high frequency cut-off are different. The cut-off is at much larger frequency in the PDS of pressure on rotating blades, in particular on the leading edge. This is easily understood if one considers that the frequency cut-off is proportional to the advection velocity

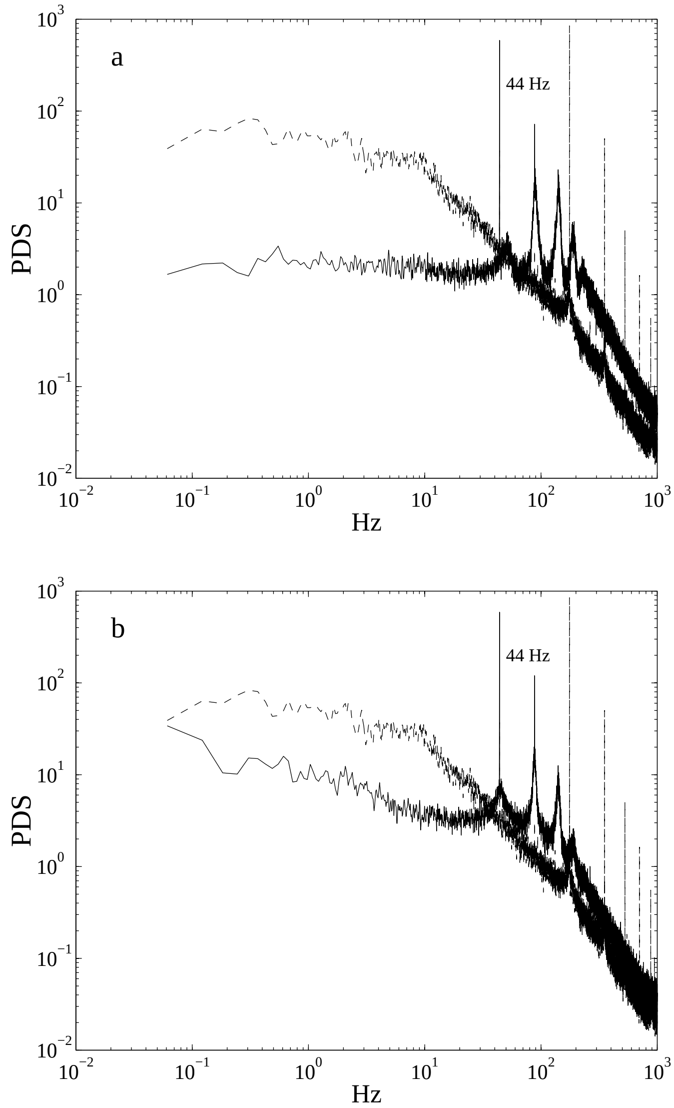

Fig. 2. (a) PDS of the pressure signal on the leading edge p_1 (full line) compared with the one of a pressure signal at the lateral boundary (dashed line). (b) PDS of the pressure signal on the trailing edge p_2 (full line). $\Omega/2\pi = 44$ Hz, $Re = 905\,000$.

of the spatial pressure disturbances on the pressure sensor. Then, according to Figure 2, the large scale velocity is roughly 10 times larger on the blades than in the bulk of the flow.

Finally, the slopes of the high frequency decay of the pressure PDS are roughly equal and in agreement with the $-7/3$ power law exponent predicted for isotropic turbulence [7]. The slight differences observed for the pressure PDS on rotating blades can be explained if one takes into account the effect of the Coriolis force in the equation for the pressure obtained by taking the divergence of the Navier-Stokes equation in the rotating frame of the blades [6].

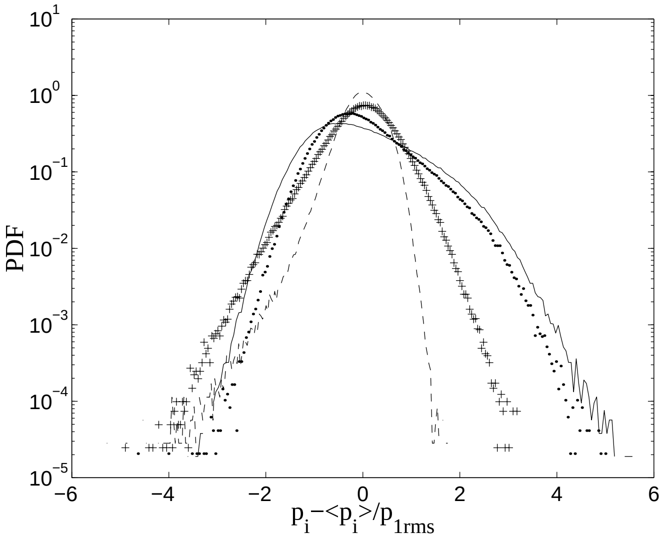

Fig. 3. PDFs of pressure fluctuations at the previously defined positions on a rotating blade: p_1 (full line), p_2 (dashed line), p_3 (dotted line) and p_4 (+++), reduced by the standard deviation of the sensor p_1. $Re = 905\,000$.

The probability density functions of the pressure fluctuations (PDF) on a moving blade are presented in Figure 3 for $Re = 9\times10^5$. All the PDF are related to the standard deviation of the pressure fluctuations on the leading edge, p_1. Pressure PDF on the trailing edge, p_2 and p_4, display a negative skewness (respectively equal to -0.53 and -0.33). As already observed for pressure at a fixed boundary in such flows [8,9], the tail of low pressure events corresponds to vorticity concentrations, here in the wake of the blades. On the contrary, a large positive skewness is observed for pressure PDF on the leading edge of the blades (respectively 0.61 and 0.78 for p_1 and p_3). High pressure fluctuations are connected to regions with strong strain which are expected in the vicinity of the leading edge of the blades. The asymmetry of the pressure PDFs thus reflects the different flow geometries in the vicinity of the leading (respectively trailing) edges of the moving blades.

We have thus a simple explanation for the negative skewness of the injected power observed in confined turbulent swirling flows [2]: equation (1) indeed shows that the injected power is a sum of terms involving the pressure on the trailing edge minus the pressure at the leading edge at each radius of the blade. If all these contributions add in a coherent way, the PDF of the injected power should have a tail of low power events, *i.e.* a negative skewness. This mechanism of course operates only if the pressure field on the blades has a large enough spatial coherence. Otherwise, as said above, if the sum consists of random independent contributions, one expects a Gaussian PDF. We thus need to estimate the spatial correlation length of pressure fluctuations on the blades.

We have first studied the correlation between two pressure transducers located on the leading edge of the same

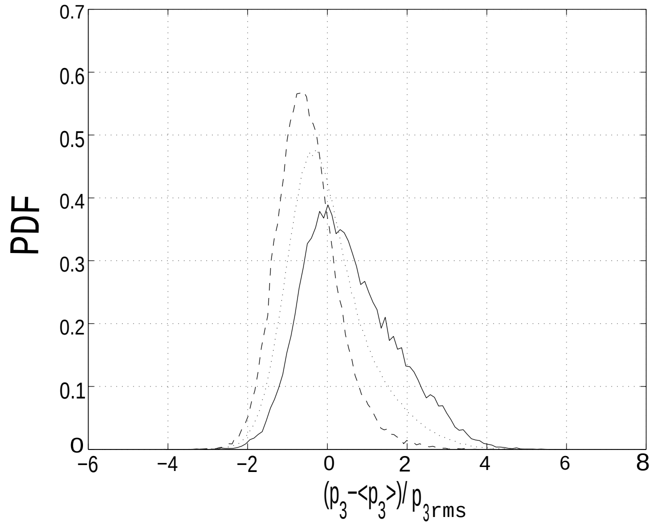

Fig. 4. PDF of pressure fluctuations p_3 conditioned by intense pressure events (larger than twice the standard deviation) recorded for p_1 at the same time (dashed line) or after a time lag $\tau_c = 4$ms (full line); non conditioned PDF (dotted line). $Re = 520\ 000$.

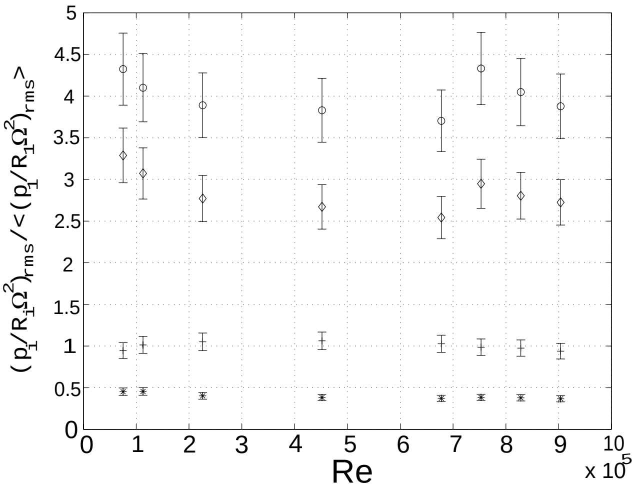

Fig. 5. rms fluctuations of the pressure p_i divided by the square of the local velocity, $(R_i\Omega)^2$, as functions of the Reynolds number. All measurements are divided by the average value computed for p_1 on the full range of Reynolds numbers.

blade, one in the vicinity of the outer radius of the blade, p_1, and the other in the vicinity of the inner radius, p_3 (see Fig. 1). To wit, we have plotted in Figure 4 the PDF of p_3 conditioned or not to the presence of intense pressure fluctuations for p_1, observed at the same time or after a time lag τ_c. If p_1 and p_3 are independent then the conditioned and non conditioned PDFs should be the same. Otherwise, the probability of the rare events for p_3 should be strongly affected by the way its PDF is conditioned. The optimum time lag, τ_c, can be estimated by looking at the cross-correlation between p_1 and p_3 which shows a maximum for $\tau_c = 4$ ms. The PDFs in Figure 4 show that the probability of rare events for p_3 (larger than twice the standard deviation) is reduced by a factor 10 when an intense event is recorded at the same time for p_1, but increased by a factor 3 if an intense event is recorded for p_1 after a time lag $\tau_c = 4$ ms. These two transducers being 5 cm apart, this shows that spatially localized pressure fluctuations are advected along the blade with a characteristic velocity larger than 10 m/s. Large scale correlations in the pressure field on the blades are thus clearly observed. However, the way these pressure or local drag fluctuations add in equation (1) to give the shape of the PDF of the injected power fluctuations cannot be simply predicted because of the time lags which strongly depend on the global flow geometry. In the symmetric case or in the open flow geometry, the individual pressure PDFs are similar to the ones shown in Figure 3, but the correlation time τ_c between pressure measurements on the same blade vanishes.

We have similarly studied the correlation of the pressure field on two successive blades. We have also observed a strong correlation with a time lag $1/4\Omega$, *i.e.* the time requisited to find the two successive blades at the same place in the flow volume.

4 Concluding remarks

We have shown the existence of large scale correlations in the pressure field or the local drag force on the rotating blades generating a turbulent swirling flow. We thus do not expect the fluctuations of the injected power to display Gaussian statistics. These features are consistent with the strong correlation of the power injection at each disk as reported in [2,11]; in addition they are not not specific to swirling flows and have been also observed in turbulent convection [10].

However, the manner the local fluctuations of drag add to give the shape of the injected power PDF cannot be simply predicted; depending on the time lag that exists between different locations, these local fluctuations may be instantaneously correlated or anti-correlated. If they mostly add coherently (configuration symmetric with respect to the midplane), then the shape of the power PDF with a tail of low power rare events, is in agreement with our measurements of the local drag fluctuations. However, this shape is likely to be modified in the presence of a mean flow resulting from any asymmetry with respect to the horizontal midplane between the disks.

Similarly, it is not simple to predict the behavior of the *rms* power fluctuations related to their mean value as the Reynolds number is increased, using local drag measurements. On our Reynolds number range, we have checked that the local pressure or drag fluctuations on the blades scale like the square of the rotation rate (see Fig. 5). If they add coherently in equation (1), this would lead to rms power fluctuations scaling like the cube of the rotation rate, *i.e.* constant power fluctuations related to their mean value. This is in agreement with the global measurements of reference [2] performed on the same range of Re, but similar experiments performed on a larger range show a slow decrease of the relative power fluctuations as Re is increased [11]. This deserves further studies on larger range of Re.

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