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Peristaltic transport in a slip flow

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Abstract. Continuum-mechanic derivation of the entrainment of rarefied gases induced by a surface wave along walls (or peristaltic transport) in a confined parallel-plane microchannel is conducted by the perturbation method. Both no-slip and slip flow cases are investigated with the former ones matched with the previous approach by Fung and Yih. Critical reflux values due to first order slip-flow effects become trivial for the free pumping case, and decrease due to second order slip-flow effects after we compared them with no-slip cases.

PACS. 05.60.-k Transport processes: theory – 47.45.-n Rarefied gas dynamics

1 Introduction

Rarefied gases flowing in rigid channels with the dominant parameter being the Knudsen number ($K_n = mfp/l$, mfp is the mean free path of the gas, l is the width of the channel) have been considered since 1875 [1–3]. Recently researchers [4,5] have started to investigate the slip flow ($0.001 \le K_n < 0.15$) within (static) rigid corrugated-wall plane microchannels which are common in microdomains of MEMS (Micro-Electro-Mechanical Systems) [6] applications, and have found certain interesting physical behaviors due to the small wavy-roughness elements along the walls. The non-zero velocity-slip [7] normally comes from the incomplete momentum transfer along the gas-surface interacting (collision and reflection) boundary when the pressure in the channel is rather low [8,9]. Meanwhile, the microdomain will induce the slip flow because of the low pressure or the characteristic length scale of the cross-sectional geometry being in sub-microns.

We know, however, that most of the electrostatic force balancing sensors and resonant sensors in MEMS perform over a wide dynamic range and with a high sensitivity, respectively [10]. Besides, the bulk (cross-section) size of the microchannel is at most a few tens of $O(\mu\mathrm{m})$ and the wall of the microchannel is almost sub-micron. Microchannels built in the microstructure of MEMS are easily subjected to environment noises, such as oscillations or vibrations, externally excited traveling waves, etc. These kinds of dynamic effects could be neglected when rarefied gases are flowing in rigid (or thick-walled) macrochannels. But, considering the material of microfabricated walls, which is not so rigid as the traditional one (*e.g.* metals or alloys) and the typical micron-thickness of the wall (even the material is silicon-based), we need to take into account the non-steady effect due to non-static noises upon the walls as the rarefied gas is flowing within these rather-thin walls. That is to say, the flow rate of rarefied gases in microchannels might be tuned by these dynamic noises even though they are minor from the macroscopic point of view.

To extend our interests in the study of the flow of rarefied gases in microdomains [4,5], we shall investigate what happens when a sinusoidal wave is assumed to travel down the walls of a 2D microchannel of constant width and rather-long length where the velocity-slip effect is present. A similar study, which is related to the peristaltic pumping of the viscous liquid, has been conducted with the no-slip boundary condition along the wall [11]. Extensive references about the peristaltic pumping could be traced in the recent paper [12] which still treated no-slip (but multiphase) flow. Our study here, however, might be directly related to the gas flowing in bronchiole or micro-bronchia or noise induced free pumping [13,14]. The critical reflux conditions we shall present are completely different from previous no-slip cases for all ranges of Reynolds number and wave number [11]. In this study, we assume that the Mach number $M_a \ll 1$, and the governing equations are the incompressible Navier-Stokes equations which are associated with the relaxed velocity-slip boundary conditions along the walls.

2 Formulations

We firstly consider a 2D channel of uniform thickness filled with a homogeneous rarefied gas (Newtonian viscous fluid). The walls of the channel are not absolutely rigid, on which are imposed traveling sinusoidal waves of small amplitude a. The vertical displacements of the upper and lower walls ($y = d$ and $-d$) are thus presumed to be η and $-\eta$, respectively, where $\eta = a \cos \frac{2\pi}{\lambda}(x - ct)$, λ is the wave length, and c the wave speed. x and y are

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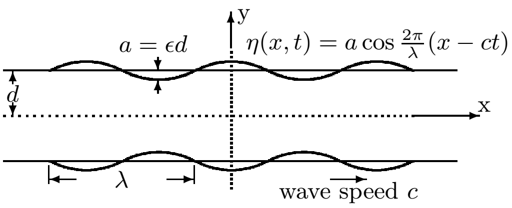

Fig. 1. Schematic diagram of the peristaltic motion of the walls.

Cartesian coordinates, with x measured in the direction of wave propagation and y measured in the direction normal to the mean position of the walls (see Fig. 1). It would be expedient to simplify these equations by introducing nondimensional variables. We have a characteristic velocity c and three characteristic lengths a, λ, and d. The following variables based on c and d could thus be introduced:

$$x' = \frac{x}{d}, \quad y' = \frac{y}{d}, \qquad u' = \frac{u}{c}, \quad v' = \frac{v}{c},$$

$$\eta' = \frac{\eta}{d}, \quad \psi' = \frac{\psi}{c\,d}, \qquad t' = \frac{c\,t}{d}, \quad p' = \frac{p}{\rho c^2}\cdot$$

The amplitude ratio ϵ, the wave number α, and the Reynolds number Re are defined by

$$\epsilon = \frac{a}{d}, \qquad \alpha = \frac{2\pi d}{\lambda}, \qquad Re = \frac{c\,d}{\nu}\cdot$$

We shall seek a solution in the form of a series in the parameter ϵ:

$$\psi = \psi_0 + \epsilon\psi_1 + \epsilon^2\psi_2 + \cdots,$$
$$\frac{\partial p}{\partial x} = \left(\frac{\partial p}{\partial x}\right)_0 + \epsilon\left(\frac{\partial p}{\partial x}\right)_1 + \epsilon^2\left(\frac{\partial p}{\partial x}\right)_2 + \cdots,$$

with $u = \partial\psi/\partial y$, $v = -\partial\psi/\partial x$. The 2D ($x-$ and $y-$) momentum equations and the equation of continuity could be in terms of the stream function ψ if the pressure (p) term is eliminated. The final governing equation is

$$\frac{\partial}{\partial t}\nabla^2\psi + \psi_y\nabla^2\psi_x - \psi_x\nabla^2\psi_y = \frac{1}{Re}\nabla^4\psi,$$
$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \tag{1}$$

and subscripts indicate the partial differentiation. Thus, we have

$$\frac{\partial}{\partial t}\nabla^2\psi_0 + \psi_{0y}\nabla^2\psi_{0x} - \psi_{0x}\nabla^2\psi_{0y} = \frac{1}{Re}\nabla^4\psi_0, \tag{2}$$

$$\frac{\partial}{\partial t}\nabla^2\psi_1 + \psi_{0y}\nabla^2\psi_{1x} + \psi_{1y}\nabla^2\psi_{0x} - \psi_{0x}\nabla^2\psi_{1y} - \psi_{1x}\nabla^2\psi_{0y} = \frac{1}{Re}\nabla^4\psi_1, \tag{3}$$

$$\frac{\partial}{\partial t}\nabla^2\psi_2 + \psi_{0y}\nabla^2\psi_{2x} + \psi_{1y}\nabla^2\psi_{1x} + \psi_{2y}\nabla^2\psi_{0x} - \psi_{0x}\nabla^2\psi_{2y} - \psi_{1x}\nabla^2\psi_{1y} - \psi_{2x}\nabla^2\psi_{0y} = \frac{1}{Re}\nabla^4\psi_2, \tag{4}$$

and other higher order forms. The fluid is subjected to boundary conditions imposed by the symmetric motion of the walls and the non-zero velocity-slip: $u = \mp K_{\mathrm{n}}\mathrm{d}u/\mathrm{d}y$ [4,5,7], $v = \pm\partial\eta/\partial t$ at $y = \pm(1+\eta)$. The boundary conditions may be expanded in powers of η and then ϵ:

$$\psi_{0y}|_1 + \epsilon[\cos\alpha(x-t)\psi_{0yy}|_1 + \psi_{1y}|_1] + \epsilon^2\Big[\frac{\psi_{0yyy}|_1}{2}\cos^2\alpha(x-t) + \cos\alpha(x-t)\psi_{1yy}|_1 + \psi_{2y}|_1\Big] + \cdots = -K_{\mathrm{n}}\Big\{\psi_{0yy}|_1 + \epsilon[\cos\alpha(x-t)\psi_{0yyy}|_1 + \psi_{1yy}|_1] + \epsilon^2\Big[\frac{\psi_{0yyyy}|_1}{2}\cos^2\alpha(x-t) + \cos\alpha(x-t)\psi_{1yyy}|_1 + \psi_{2yy}|_1\Big] + \cdots\Big\} \tag{5}$$

$$\psi_{0x}|_1 + \epsilon[\cos\alpha(x-t)\psi_{0xy}|_1 + \psi_{1x}|_1] + \epsilon^2\Big[\frac{\psi_{0xyy}|_1}{2}\cos^2\alpha(x-t) + \cos\alpha(x-t)\psi_{1xy}|_1 + \psi_{2x}|_1\Big] + \cdots = -\epsilon\alpha\sin\alpha(x-t). \tag{6}$$

2.1 $K_{\mathrm{n}} \sim O(\epsilon)$ case

Considering the case of $K_{\mathrm{n}} \sim O(\epsilon)$ from the above boundary conditions, which means the mean free path of rarefied gases is of the same magnitude with the amplitude of the wall-surface wave, we have

$$\psi_{0y}(\pm 1) = 0, \qquad \psi_{0x}(\pm 1) = 0;$$

$$\cos\alpha(x-t)\psi_{0yy}|_{\pm 1} \pm \psi_{1y}|_{\pm 1} = \mp K_{\mathrm{n}}\psi_{0yy}(\pm 1),$$

$$\cos\alpha(x-t)\psi_{0xy}|_{\pm 1} + \psi_{1x}|_{\pm 1} = \mp\alpha\sin\alpha(x-t);$$

$$\frac{\psi_{0yyy}|_{\pm 1}}{2}\cos^2\alpha(x-t) \pm \cos\alpha(x-t)\psi_{1yy}|_{\pm 1} \pm \psi_{2y}|_{\pm 1} = \mp K_{\mathrm{n}}[\cos\alpha(x-t)\psi_{0yyy}|_{\pm 1} \pm \psi_{1yy}|_{\pm 1}],$$

$$\frac{\psi_{0xyy}|_{\pm 1}}{2}\cos^2\alpha(x-t) \pm \cos\alpha(x-t)\psi_{1xy}|_{\pm 1} \pm \psi_{2x}|_{\pm 1} = 0.$$

Equations above, together with the condition of symmetry and a uniform pressure gradient in the x-direction, $(\partial p/\partial x)_0 =$ constant, yield:

$$\psi_0 = K_0[y - \frac{y^3}{3}], \qquad K_0 = \frac{Re}{2}(-\frac{\partial p}{\partial x})_0, \tag{7}$$

$$\psi_1 = \frac{1}{2}\{\phi(y)\mathrm{e}^{\mathrm{i}\alpha(x-t)} + \phi^*(y)\mathrm{e}^{-\mathrm{i}\alpha(x-t)} + \phi_0(y)\}, \tag{8}$$

where the asterisk denotes the complex conjugate. A substitution of ψ_1 into equation (3) yields

$$\left\{\frac{\mathrm{d}^2}{\mathrm{d}y^2} - \alpha^2 + \mathrm{i}\alpha Re[1 - K_0(1 - y^2)]\right\}\left(\frac{\mathrm{d}^2}{\mathrm{d}y^2} - \alpha^2\right)\phi - 2\mathrm{i}\alpha K_0 Re\phi = 0, \quad \phi_{0_{yyyy}} = 0. \quad (9)$$

The boundary conditions are

$$\phi_y(\pm 1) = 2K_0, \quad \phi_{0_y}(\pm 1) = 4K_0K_\mathrm{n}, \quad \phi(\pm 1) = \pm 1. \quad (10)$$

Similarly, with

$$\psi_2 = \frac{1}{2}\{D(y) + E(y)\mathrm{e}^{\mathrm{i}2\alpha(x-t)} + E^*(y)\mathrm{e}^{-\mathrm{i}2\alpha(x-t)} + G(y)\mathrm{e}^{\mathrm{i}\alpha(x-t)} + G^*(y)\mathrm{e}^{-\mathrm{i}\alpha(x-t)}\}, \quad (11)$$

we have

$$D_{yyyy} = -\frac{\mathrm{i}\alpha Re}{2}(\phi\phi^*_{yy} - \phi^*\phi_{yy})_y, \quad (12)$$

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}y^2} - \left(4\alpha^2 - 2\mathrm{i}\alpha Re\right)\right]\left(\frac{\mathrm{d}^2}{\mathrm{d}y^2} - 4\alpha^2\right)E = \mathrm{i}2\alpha ReK_0(1 - y^2 + 2K_\mathrm{n})\left(\frac{\mathrm{d}^2}{\mathrm{d}y^2} - 4\alpha^2\right)E + \mathrm{i}4\alpha K_0 ReE + \frac{\mathrm{i}\alpha Re}{2}(\phi_y\phi_{yy} - \phi\phi_{yyy}) = 0; \quad (13)$$

$$\left\{\frac{\mathrm{d}^2}{\mathrm{d}y^2} - \alpha^2 + \mathrm{i}\alpha Re[1 - K_0(1 - y^2)]\right\}\left(\frac{\mathrm{d}^2}{\mathrm{d}y^2} - \alpha^2\right)G - 2\mathrm{i}\alpha K_0 ReG = Re\frac{\mathrm{i}\alpha\phi_{0_y}}{2}(-\alpha^2\phi + \phi_{yy}), \quad (14)$$

and the boundary conditions

$$D_y(\pm 1) \pm \frac{1}{2}[\phi_{yy}(\pm 1) + \phi^*_{yy}(\pm 1)] - 2K_0 = 0, \quad G_y(\pm 1) = \mp K_\mathrm{n}(\phi_{yy}(\pm 1) \mp K_0), \quad (15)$$

$$E_y(\pm 1) \pm \frac{1}{2}\phi_{yy}(\pm 1) - \frac{K_0}{2} = 0, \quad E(\pm 1) \pm \frac{1}{4}\phi_y(\pm 1) = 0. \quad (16)$$

2.2 Free pumping case

To simplify the approach and obtain preliminary analytical solutions of the above complicated equations and boundary conditions, we only consider the case in which $(\partial p/\partial x)_0$ vanishes or $K_0 = \psi_0 = 0$. Hence equations (9, 10) become

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}y^2} - \alpha^2\right)\left(\frac{\mathrm{d}^2}{\mathrm{d}y^2} - \bar{\alpha}^2\right)\phi = 0,$$

where

$$\bar{\alpha}^2 = \alpha^2 - \mathrm{i}\alpha Re \quad \text{and} \quad \phi_{0_{yyyy}} = 0, \quad (17)$$

$$\phi_y(\pm 1) = 0, \quad \phi(\pm 1) = \pm 1, \quad \phi_{0_y}(\pm 1) = 0. \quad (18)$$

After lengthy algebraic manipulations, we obtain

$$\phi = c_0\mathrm{e}^{\alpha y} + c_1\mathrm{e}^{-\alpha y} + c_2\mathrm{e}^{\bar{\alpha}y} + c_3\mathrm{e}^{-\bar{\alpha}y},$$

where $c_0 = (A + A_0)/\det$, $c_1 = -(B + B_0)/\det$, $c_2 = (C + C_0)/\det$, $c_3 = -(T + T_0)/\det$;

$$\det = A\mathrm{e}^{\alpha} - B\mathrm{e}^{-\alpha} + C\mathrm{e}^{\bar{\alpha}} - T\mathrm{e}^{-\bar{\alpha}},$$

$$A = \mathrm{e}^{\alpha}\bar{\alpha}^2(\mathrm{e}^{-2\bar{\alpha}} - \mathrm{e}^{2\bar{\alpha}}) - 2\alpha\bar{\alpha}\mathrm{e}^{-\alpha} + \alpha\bar{\alpha}\mathrm{e}^{\alpha}(\mathrm{e}^{-2\bar{\alpha}} + \mathrm{e}^{2\bar{\alpha}}),$$

$$A_0 = \mathrm{e}^{-\alpha}\bar{\alpha}^2(\mathrm{e}^{-2\bar{\alpha}} - \mathrm{e}^{2\bar{\alpha}}) + 2\alpha\bar{\alpha}\mathrm{e}^{\alpha} - \alpha\bar{\alpha}\mathrm{e}^{-\alpha}(\mathrm{e}^{2\bar{\alpha}} + \mathrm{e}^{-2\bar{\alpha}}),$$

$$B = \mathrm{e}^{-\alpha}\bar{\alpha}^2(\mathrm{e}^{-2\bar{\alpha}} - \mathrm{e}^{2\bar{\alpha}}) + 2\alpha\bar{\alpha}\mathrm{e}^{\alpha} - \alpha\bar{\alpha}\mathrm{e}^{-\alpha}(\mathrm{e}^{-2\bar{\alpha}} + \mathrm{e}^{2\bar{\alpha}}),$$

$$B_0 = \mathrm{e}^{\alpha}\bar{\alpha}^2(\mathrm{e}^{-2\bar{\alpha}} - \mathrm{e}^{2\bar{\alpha}}) - 2\alpha\bar{\alpha}\mathrm{e}^{-\alpha} + \alpha\bar{\alpha}\mathrm{e}^{\alpha}(\mathrm{e}^{-2\bar{\alpha}} + \mathrm{e}^{2\bar{\alpha}}),$$

$$C = \mathrm{e}^{-\alpha}\alpha\bar{\alpha}(\mathrm{e}^{\bar{\alpha}-\alpha} - \mathrm{e}^{\alpha-\bar{\alpha}}) - \alpha\mathrm{e}^{2\alpha+\bar{\alpha}}(\alpha - \bar{\alpha}) + \alpha\mathrm{e}^{-\alpha}(\alpha\mathrm{e}^{\bar{\alpha}-\alpha} - \bar{\alpha}\mathrm{e}^{\alpha-\bar{\alpha}}), \quad (19)$$

$$C_0 = \mathrm{e}^{\alpha}\alpha\bar{\alpha}(\mathrm{e}^{\bar{\alpha}-\alpha} - \mathrm{e}^{\alpha-\bar{\alpha}}) - \alpha(\alpha\mathrm{e}^{2\alpha-\bar{\alpha}} - \bar{\alpha}\mathrm{e}^{\bar{\alpha}}) + \alpha(\alpha\mathrm{e}^{-(\bar{\alpha}+2\alpha)} - \bar{\alpha}\mathrm{e}^{-(2\alpha+\bar{\alpha})}), \quad (20)$$

$$T = \mathrm{e}^{-\alpha}\alpha\bar{\alpha}(\mathrm{e}^{\bar{\alpha}+\alpha} - \mathrm{e}^{-(\alpha+\bar{\alpha})}) - \alpha\bar{\alpha}(\mathrm{e}^{2\alpha-\bar{\alpha}} - \mathrm{e}^{\bar{\alpha}}) + \alpha^2\mathrm{e}^{-\alpha}(-\mathrm{e}^{2\alpha} + \mathrm{e}^{-2\alpha}), \quad (21)$$

$$T_0 = \mathrm{e}^{\alpha}\alpha\bar{\alpha}(\mathrm{e}^{\bar{\alpha}+\alpha} - \mathrm{e}^{-(\alpha+\bar{\alpha})}) - \alpha\bar{\alpha}(\mathrm{e}^{-\bar{\alpha}} - \mathrm{e}^{\bar{\alpha}-2\alpha}) + \alpha^2\mathrm{e}^{\alpha}(-\mathrm{e}^{2\alpha} + \mathrm{e}^{-2\alpha}). \quad (22)$$

To obtain a simple solution which relates to the mean flow so long as only terms of $O(\epsilon^2)$ are concerned, we see that if every term in the x-momentum equation is averaged over an interval of time equal to the period of oscillation [4,5,11], we obtain for our solution as given by the above equations the mean pressure gradient

$$\overline{\frac{\partial p}{\partial x}} = \epsilon\overline{\left(\frac{\partial p}{\partial x}\right)_1} + \epsilon^2\overline{\left(\frac{\partial p}{\partial x}\right)_2} = \epsilon\frac{\phi_{0_{yyy}}}{2Re} + \epsilon^2\left[\frac{D_{yyy}}{2Re} + \frac{\mathrm{i}Re}{4}(\phi\phi^*_{yy} - \phi^*\phi_{yy})\right] + O(\epsilon^3) = \epsilon\frac{r_0}{Re} + \epsilon^2\frac{a_0}{Re} + O(\epsilon^3), \quad (23)$$

where r_0 and a_0 are integration constants for the integration of equations (9, 12). $\phi_{0_y} = -r_0(1 - y^2)$.
Now, from equation (14), we have

$$D_y(\pm 1) = -\frac{1}{2}[\phi_{yy}(\pm 1) + \phi^*_{yy}(\pm 1)]\}, \quad (24)$$

where $D_y(y) = a_0 y^2 + a_1 y + a_2 + \mathcal{C}(y)$, and from equation (12),

$$\mathcal{C}(y) = \frac{\alpha^2 Re^2}{2}\left[\frac{c_0 c_2^*}{g_1^2}\mathrm{e}^{(\alpha+\bar{\alpha}^*)y} + \frac{c_0^* c_2}{g_2^2}\mathrm{e}^{(\alpha+\bar{\alpha})y} + \frac{c_0 c_3^*}{g_3^2}\mathrm{e}^{(\alpha-\bar{\alpha}^*)y} + \frac{c_0^* c_3}{g_4^2}\mathrm{e}^{(\alpha-\bar{\alpha})y} + \frac{c_1 c_2^*}{g_3^2}\mathrm{e}^{(\bar{\alpha}^*-\alpha)y} + \frac{c_1^* c_2}{g_4^2}\mathrm{e}^{(\bar{\alpha}-\alpha)y} + \frac{c_1 c_3^*}{g_1^2}\mathrm{e}^{-(\bar{\alpha}^*+\alpha)y} + \frac{c_1^* c_3}{g_2^2}\mathrm{e}^{-(\bar{\alpha}+\alpha)y} + \frac{c_2 c_3^*}{g_5^2}\mathrm{e}^{(\bar{\alpha}-\bar{\alpha}^*)y} + \frac{c_2^* c_3}{g_5^2}\mathrm{e}^{(\bar{\alpha}^*-\bar{\alpha})y} + 2\frac{c_2 c_2^*}{g_6^2}\mathrm{e}^{(\bar{\alpha}^*+\bar{\alpha})y} + 2\frac{c_3 c_3^*}{g_6^2}\mathrm{e}^{-(\bar{\alpha}^*+\bar{\alpha})y}\right],$$

with $g_1 = \alpha + \bar{\alpha}^*$, $g_2 = \alpha + \bar{\alpha}$, $g_3 = \alpha - \bar{\alpha}^*$, $g_4 = \alpha - \bar{\alpha}$, $g_5 = \bar{\alpha} - \bar{\alpha}^*$, $g_6 = \bar{\alpha} + \bar{\alpha}^*$. In practical applications we must determine r_0, a_0 from considerations of conditions at the ends of the channel. a_1 equals to zero because of the symmetry of boundary conditions.
Once r_0, a_0 is specified [11], our solution for the mean speed (averaged over time) of flow is

$$\bar{U} = \epsilon\frac{\phi_{0_y}}{2} + \epsilon^2\frac{D_y}{2} = \epsilon\frac{-r_0(1-y^2)}{2} + \frac{\epsilon^2}{2}\{\mathcal{C}(y) - \mathcal{C}(1) + R_0 + a_0(y^2-1)\} \tag{25}$$

where $R_0 = -[\phi_{yy}(1) + \phi_{yy}^*(1)]/2$, which has a numerical value of about 3 for a wide range of α and Re [10].

3 Results and discussion

Numerical calculations confirm that the mean streamwise velocity distribution (averaged over time) due to the peristaltic motion in the case of free pumping is dominated by R_0 (or K_n) and the parabolic distribution $-r_0(1-y^2)$, $-a_0(1-y^2)$. R_0 which defines the boundary value of D_y has its origin in the y-gradient of the first-order streamwise velocity distribution, as can be seen in equation (14).

In addition to the terms mentioned above, there is a perturbation term (within the 2nd order) which varies across the channel: $\mathcal{C}(y) - \mathcal{C}(1)$ [10]. Let us define it to be

$$F(y) = \frac{-200}{\alpha^2 Re^2}[\mathcal{C}(y) - \mathcal{C}(1)]. \tag{26}$$

To compare with previous no-slip ($K_\mathrm{n} = 0$) results [11], we plot two cases, $\alpha = 0.1$, $Re = 1$ and $\alpha = 0.4$, $Re = 1$, of our results: $K_\mathrm{n} = 0$ with Fung and Yih's [11] into Figure 2. This figure confirms our approaches since we can recover previous results by using $K_\mathrm{n} = 0$.

Now, let us define a critical reflux condition as one for which the mean velocity $\bar{U}(y)$ goes to to zero at the center-line $y = 0$. With equations (18, 20, 21), we could only have, for the second order term

$$a_0 = Re\overline{\left(\frac{\partial p}{\partial x}\right)_2} = -\left[\frac{\alpha^2 Re^2}{200}F(0) - R_0\right], \tag{27}$$

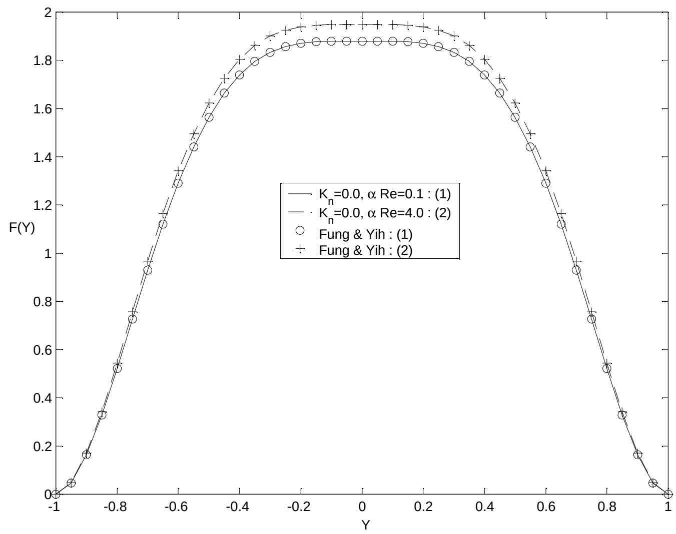

Fig. 2. Comparison of the mean-velocity perturbation function $F(y)$.

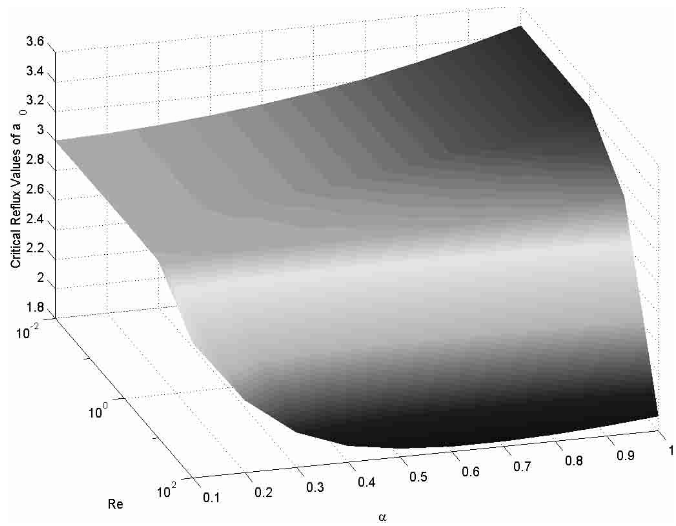

Fig. 3. Comparison of $a_0 = Re\,\overline{\mathrm{d}p/\mathrm{d}x_2}$ at the critical-reflux condition.

which means the critical reflux condition is reached when a_0 has the above value. Pumping against a positive pressure-gradient greater than the critical value would result in a backward flow (reflux) in the central region of the stream. This critical value depends on α, Re, and K_n. There will be no reflux if the pressure gradient is smaller than this a_0.

We plot the 3D view of $a_0(\alpha, Re; K_\mathrm{n} = 0)$ in Figure 3 where α has the range between 0.1 and 1.0; $Re = 0.01, 0.1, 1, 10, 100$. Similarly, those values of a_0 for the case of $K_\mathrm{n} = 0$ recover previous results exactly [11].

For the first order effect, the critical reflux condition can occur as the trivial case as shown in equation (19) for $r_0 = 0$. We then compare second order slip-effect results with no-slip ones and put them into Table 1. The critical reflux values decrease for the slip cases: $K_\mathrm{n} = 0.01$ for all ranges of (Re, α). This result is different from the previous

Table 1. Comparison of critical reflux (a_0) values.

		Fung and Yih	Present: No-slip	Slip ($K_\mathrm{n} = 0.01$)
Re	α	Re $(\overline{\mathrm{d}p/\mathrm{d}x})_2\vert_\mathrm{cr}$	Re $(\overline{\mathrm{d}p/\mathrm{d}x})_2\vert_\mathrm{cr}$	Re $(\overline{\mathrm{d}p/\mathrm{d}x})_2\vert_\mathrm{cr}$
0.01	0.1	3.0035	3.0040	2.8881
	0.2	3.0157	3.0161	2.9002
	0.3	3.0365	3.0365	2.9207
	0.4	3.0656	3.0656	2.9497
	0.5	3.1038	3.1039	2.9877
	0.6	3.1519	3.1519	3.0355
	0.7	3.2105	3.2106	3.0936
	0.8	3.2806	3.2806	3.1629
	0.9	3.3630	3.3630	3.2444
	1.0	3.4587	3.4587	3.3390
0.10	0.1	3.0040	3.0040	2.8881
	0.2	3.0161	3.0161	2.9002
	0.3	3.0365	3.0365	2.9207
	0.4	3.0656	3.0656	2.9497
	0.5	3.1039	3.1039	2.9877
	0.6	3.1519	3.1519	3.0355
	0.7	3.2106	3.2106	3.0936
	0.8	3.2806	3.2806	3.1629
	0.9	3.3630	3.3630	3.2444
	1.0	3.4587	3.4587	3.3389
1.0	0.1	3.0040	3.0040	2.8881
	0.2	3.0160	3.0160	2.9001
	0.3	3.0362	3.0362	2.9203
	0.4	3.0650	3.0650	2.9491
	0.5	3.1029	3.1029	2.9868
	0.6	3.1505	3.1505	3.0341
	0.7	3.2086	3.2086	3.0917
	0.8	3.2780	3.2780	3.1603
	0.9	3.3597	3.3597	3.2410
	1.0	3.4545	3.4545	3.3347
10.0	0.1	3.0003	3.0003	2.8844
	0.2	3.0015	3.0015	2.8856
	0.3	3.0037	3.0037	2.8879
	0.4	3.0075	3.0075	2.8916
	0.5	3.0135	3.0135	2.8975
	0.6	3.0228	3.0228	2.9065
	0.7	3.0363	3.0363	2.9198
	0.8	3.0556	3.0556	2.9386
	0.9	3.0820	3.0820	2.9645
	1.0	3.1173	3.1173	2.9992

one [11] which didn't consider velocity-slip effects. The direct interpretation about this behavior is that once there are velocity-slips along both walls, the backward gases flow is much more easily triggered than no-slip ones inside the channel so that the pumping power should be increased at the beginning. Meanwhile, as there is early backward flow due to slip-flow effects, the gas flow might become more unstable compared to the previous no-slip cases. Thus the detailed study of gases flowing in bronchiole or micro-bronchia should be performed as early as possible for our health's sake.

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