Short note

Zpif's law in the liquid gas phase transition of nuclei

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Abstract. Zpif's law in the field of linguistics is tested in the nuclear disassembly within the framework of isospin dependent lattice gas model. It is found that the average cluster charge (or mass) of rank n in the charge (or mass) list shows exactly inversely to its rank, i.e., there exists Zpif's law, at the phase transition temperature. This novel criterion shall be helpful to search the nuclear liquid gas phase transition experimentally and theoretically. In addition, the finite size scaling of the effective phase transition temperature at which the Zpif's law appears is studied for several systems with different mass and the critical exponents of ν and β are tentatively extracted.

PACS. 25.70.Pq Multifragment emission and correlations – 05.70.Jk Critical point phenomena – 24.10.Pa Thermal and statistical models

In intermediate energy heavy ion collisions (HIC) the hot nuclei with moderate temperature can be formed and finally de-excite by different decay modes, such as multifragmentation. This kind of multifragment emission shows a rise and fall with beam energy (excitation energy, or nuclear temperature, or impact parameter ...) [1], which probably relates to the critical point behavior in nuclear matter. The onset of multifragmentation, i.e., when the multiplicity N_{imf} of intermediate mass fragment (IMF) attains to the maximum, probably indicates of the coexistence of liquid and gas phases. Relations to the IMF distribution, theory predicted that there exists a minimum of power law parameter, τ_{min}, for the IMF cluster distribution when the liquid gas phase transition takes place [2].

On the other hand, the study on nuclear caloric curve is an attractive subject due to the fact that a temperature plateau exists when the first order liquid gas phase transition occurs. Experimentally He-Li isotopic temperature from Albergo's thermometer [3] for projectile-like Au spectators seems to have a temperature plateau in the excitation energy range of 3 to 10 MeV/u by ALADIN Collaborations [4], which was claimed that the first order nuclear liquid gas phase transition was observed experimentally. Nuclear caloric curves were also surveyed by several groups, such as INDRA [5], MSU [6], TAMU [7], EOS [8] and GSI [9] etc.

In addition, the possibility to extract critical exponents and critical behavior from finite-size systems has been attempted experimentally in [10].

However, even though the above trials to search for the liquid gas phase transition are multi-variant, these signatures are still in controversial debating [11–14]. In this context, it will be helpful and meaningful to present novel signature to characterize the nuclear liquid gas phase transition to guide the experimental analysis and theoretical predictions.

In this paper we will introduce, for the first time, Zpif's law into the study of nuclear liquid gas phase transition. Zpif's law [15] has been known as a statistical phenomenon concerning the relation between English words and their frequency in literature in the field of linguistics. In this Zpif's analysis, one calculates the histogram that gives the total number of occurrences of each word in a text. If all the words in the text are arranged in rank n order, from the most frequent to the least frequent, then such a histogram is found to be linear on double logarithmic paper, with a slope $-\lambda$, with $\lambda \approx 1$ for all languages. Attempts to understand the origin of the Zpif's law are connected to the hierarchical structure of language [16]. This relation was found not only in linguistics but also in other fields of sciences. For examples, the law appeared in distributions of populations in cities, distributions of areas of lakes, DNA base pair sequences [17] and cluster-

size distribution in percolation process [18]. In this paper, Zpif's law will be tested for the charge (or mass) distribution of nuclear clusters and evidenced to be a factor to characterize the phase transition.

Isospin dependent lattice gas model (I-LGM) was used to investigate the Zpif's law in nuclear cluster distribution. The lattice gas model was developed to describe the liquid-gas phase transition for atomic system by Lee and Yang [19] first. The same model has already been applied to nuclear physics for isospin symmetrical systems in the grancanonical ensemble [20] with a sampling of the canonical ensemble [11,21–26], and also for isospin asymmetrical nuclear matter in the mean field approximation [27]. Here we will make a brief description for the I-LGM model.

In the isospin dependent lattice gas model, A (= $N + Z$) nucleons with an occupation number s which is defined $s = 1$ (-1) for a proton (neutron) or $s = 0$ for a vacancy, are placed on the L sites of lattice. Nucleons in the nearest neighboring sites interact with an energy $\epsilon_{s_i s_j}$. The hamiltonian is written as

$$E = \sum_{i=1}^{A} \frac{P_i^2}{2m} - \sum_{i<j} \epsilon_{s_i s_j} s_i s_j. \qquad (1)$$

The interaction constant $\epsilon_{s_i s_j}$ is chosen to be isospin dependent and be fixed to reproduce the binding energy of the nuclei [25]:

$$\epsilon_{nn} = \epsilon_{pp} = 0.MeV,$$
$$\epsilon_{pn} = -5.33MeV, \qquad (2)$$

which indicates the repulsion between the nearest neighboring like-nucleons and attraction between the nearest neighboring unlike-nucleons. This kind of isospin dependent interaction incorporates, to a certain extent, Pauli exclusion principle and effectively avoids producing unreasonable clusters, such as di-proton and di-neutron etc. Three-dimension cubic lattice with L^3 sites is used which results in $\rho_f = \frac{A}{L^3}\rho_0$ of an assumed freeze-out density of disassembling system, in which ρ_0 is the normal nuclear density. The disassembly of the system is to be calculated at ρ_f, beyond which nucleons are too far apart to interact. Nucleons are put into lattice by Monte Carlo Metropolis sampling. Once the nucleons have been placed we also ascribe to each of them a momentum by Monte Carlo samplings of Maxwell-Boltzmann distribution.

Once this is done the I-LGM immediately gives the cluster distribution using the rule that two nucleons are part of the same cluster if

$$P_r^2/2\mu - \epsilon_{s_i s_j} s_i s_j < 0. \qquad (3)$$

This prescription is evidenced to be similar to the Coniglio-Klein's prescription [28] in condensed matter physics and shown to be valid in lattice gas type calculations [11,21,23,26].

We first chose the medium size nuclei ^{129}Xe to analyze the phase transition point behavior in nuclear disassembly and its Zpif's law from the cluster distribution. The freeze-out density ρ_f is chosen to be 0.38 ρ_0 due to the data were
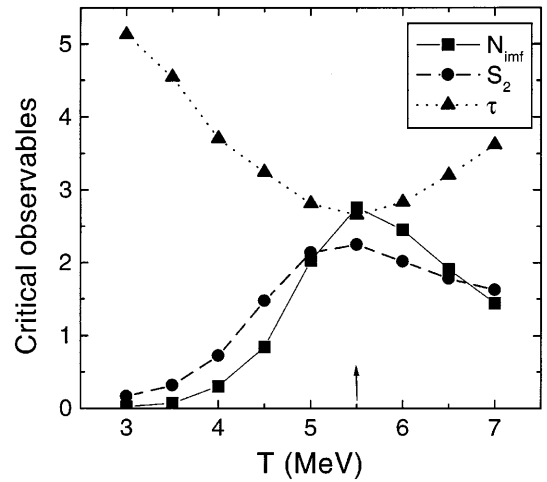

Fig. 1. The effective power-law parameter, τ, the second moment of the cluster distribution, S_2, and the multiplicity of intermediate mass fragments, N_{imf} as a function of temperature for the disassembly of ^{129}Xe. The arrow represents the position of phase transition temperature

best fitted by a ρ_f between $0.3\rho_0$ and $0.4\rho_0$ in the previous LGM calculations [23,29], which corresponds to the cubic lattice with size $L = 7$ for ^{129}Xe. 1000 events are simulated for each calculation.

In order to check the phase transition behavior in the I-LGM, we will firstly show the results of some physical observables, namely the effective power-law parameter, τ, the second moment of the cluster distribution, S_2 [30], and the multiplicity of intermediate mass fragments, N_{imf} for the disassembly of ^{129}Xe in Fig. 1. These observables have been evidenced useful in previous works to judge the liquid gas phase transition, as shown in [25,31,32]. The valley of τ, the peaks of N_{imf} and S_2 happens around $T \sim 5.5$ MeV which is the signature of onset of phase transition. However, the aim of this paper is searching a novel signature of liquid gas phase transition besides the above observables. The above phase transition temperature will be only used as a reference of the novel signature, as stated below.

Now we present the results for testing Zpif's law in the charge distribution of clusters. The law states that the relation between the sizes and their ranks is described by $Z_n = c/n$ (n=1, 2, 3, ...), where c is a constant and Z_n is the average charge (or mass) of rank n in a charge (or mass) list when we arrange the clusters in the order of decreasing size. For instance the charge Z_2 of the second largest cluster with rank $n = 2$ is one-half of the charge Z_1 of the largest cluster, the charge Z_3 of the third largest cluster with rank $n = 3$ is one-third of the charge Z_1 of the largest cluster, and so on. In the simulations of this work, we averaged the charges for each rank in charge lists of the events: we averaged the charges for the largest clusters in each event, averaged them for the second largest clusters, averaged them for the third largest clusters, and so on. From the charges averaged, we examined the relation between the charges Z_n and their ranks n. Figure 2 shows such relations of Z_n and n for Xe in different tem-
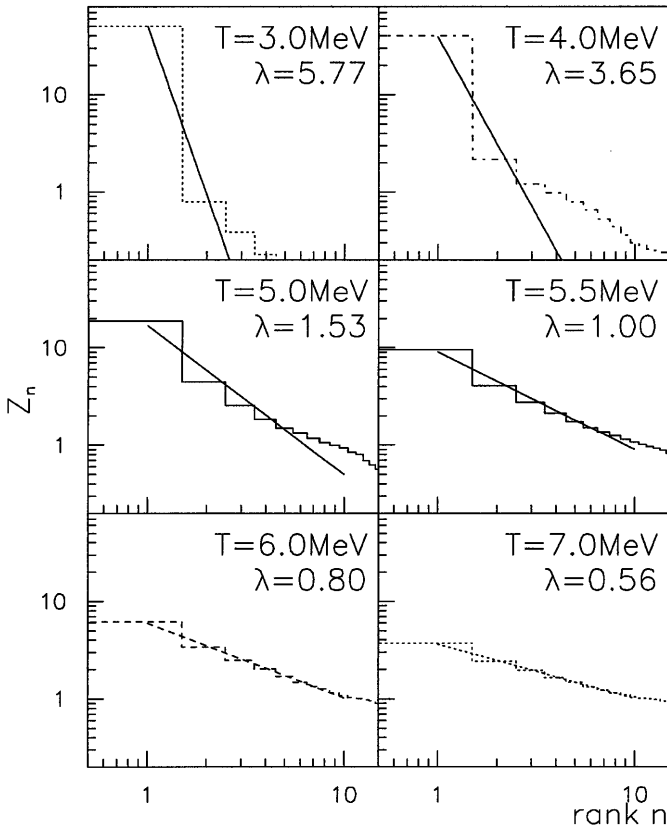
Fig. 2. The average charge Z_n with rank n as a function of n for ^{129}Xe. The histograms are the calculation results and the straight lines are their fits with $Z_n \propto n^{-\lambda}$

Fig. 3. The slope parameter λ of Z_n to n (a) and the χ^2 test for Zpif's law (b) as a function of temperature for ^{129}Xe. The arrow represents the position of phase transition temperature

perature. The histogram is the simulated results and the straight lines represent the fit with $Z_n \propto n^{-\lambda}$ in the range of $1 \leq n \leq 10$, where λ is the slope parameter. λ is 5.77 at $T = 3$ MeV. Then we increased the temperature and examined the same relation and obtained $\lambda = 3.65$ and 1.53 at $T = 4$ and 5 MeV, respectively. Up to $T = 5.5$ MeV, $\lambda = 1.00$, i.e., at this temperature the relation is satisfied to the Zpif's law: $Z_n \propto n^{-1}$. When temperature continues to increases, λ continues to decreases, for instance, $\lambda = 0.80$ at $T = 6$ MeV and $\lambda = 0.56$ at $T = 7$. This temperature having the Zpif's law, denoted as T_A, is consistent with the phase transition temperature obtained in Fig. 1, illustrating that the Zpif's law is also a good judgement to phase transition. From the statistical point of view, the Zpif's law is related to the critical phenomenon [2,33]. Figure 3a summarizes the parameter λ as a function of temperature. Clearly the Zpif's law ($\lambda = 1$) reveals at phase transition point.

In order to further illustrate that the Zpif's law exists most probably in phase transition point, we directly reproduce the histograms with Zpif's law: $Z_n = c/n$. In this case, c is sole parameter, but what we are interesting in is its truth of the hypothesis of Zpif's law: the χ^2 test. Figure 3b demonstrates the χ^2/ndf for the Z_n - n relations at different T. As expected, there is the minimum χ^2/ndf around the phase transition temperature, which
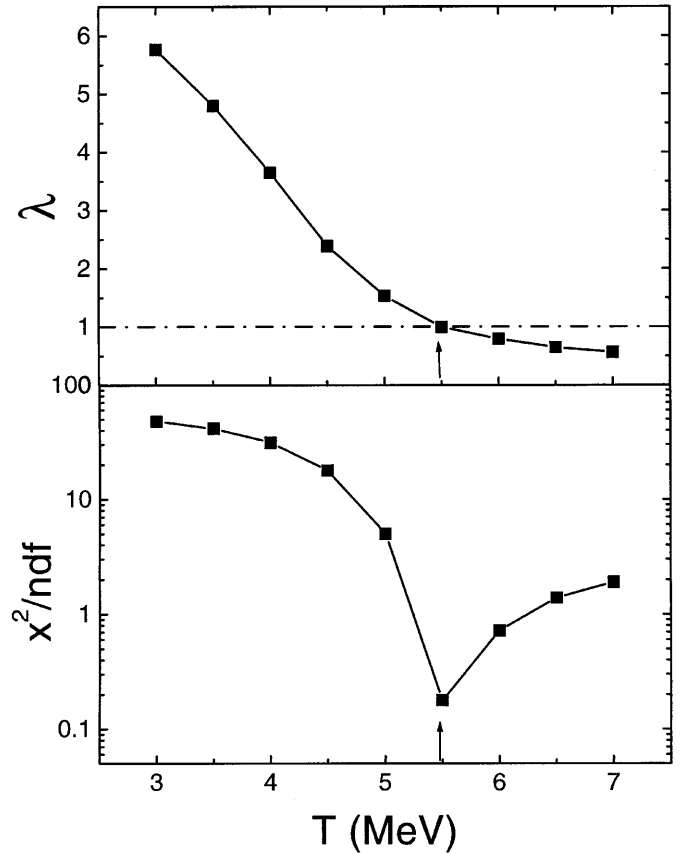
further support that Zpif's law of the fragment distribution reveals when the liquid gas phase transition occurs.

In the above examinations, the system size is finite, so the effective phase transition temperature, T_A, deduced from the Zpif's law shall be only valid for a specific finite size nuclei. As well known, the finite size scaling is an important factor in studying the phase transition of finite matter. In this context, we will investigate such an effect for T_A with the help of the Zpif-type analysis for nuclear clusters and predict the value of critical temperature, T_C, at which the Zpif's law will appear for the infinite nuclear matter with the same N/Z and freeze-out density as ^{129}Xe in the framework of isospin dependent lattice gas model. When we examine the values T_C of thresholds of phase transition, we usually extrapolate the effective phase transition temperature T_A as $L \to \infty$ [34]. This is based on the extrapolation rule

$$|T_C - T_A| \propto L^{-1/\nu}, \qquad (4)$$

where ν is the critical exponent for correlation length. For three dimension (3D) percolation class, $\nu = 0.9$; for the 3D Ising class or liquid gas class, $\nu = 0.6$ [35]. Below we will examine if the finite size scaling law is valid for the phase transition temperature at which the Zpif's law occurs in the nuclear system and tentatively extract the critical exponents ν and β.
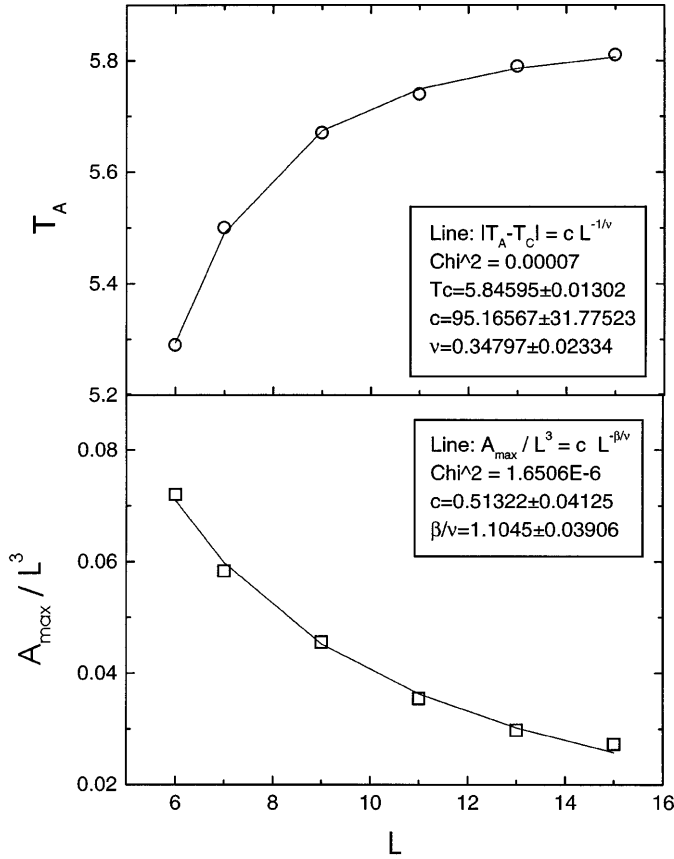
Fig. 4. The effective phase transition temperature (a) and the normalized largest cluster size A_{max}/L^3 (b) is plotted as a function of size of lattice. From (a) the critical temperature T_C for infinite system and the critical exponent ν can be extracted; From (b) the critical exponent β/ν can be obtained

We simulated the nuclear disassembly for systems with the same N/Z as ^{129}Xe but A = 80, 274, 500, 830 and 1270 within the cubic sites of 6^3, 9^3, 11^3, 13^3 and 15^3, respectively, which will result in the same freeze-out density as ^{129}Xe for comparison. The effective phase transition temperatures, T_A, at which Zpif's law ($\lambda = 1$) appears are extracted for each system. The Fig. 4a summarizes such T_A as a function of L, from which we can get the two fit parameters of T_C and ν with the (4). The best fit parameter is shown in the figure, i.e., $T_C = 5.846 \pm 0.013$ MeV and $\nu = 0.348 \pm 0.023$. It tells us that the scaling law (4) is rather valid even for the effective phase transition temperature deduced from the Zpif's law. Viewing from the fit values of ν, it seems to approach to the value of 3D Ising or liquid gas class than the 3D percolation class, which probably indicate that the nuclear disassembly belongs to 3D liquid gas universality class, while the corresponding extrapolated critical temperature of $T_C \simeq 5.846$ MeV for the infinite nuclear matter with $N/Z = 1.39$ and $0.38\rho_0$ as $A \to \infty$. In addition, the finite size scaling can be further investigated by the source size dependent largest cluster at the phase transition point where the Zpif's law appears

and hence to extract the critical exponent β by the scaling law:
$$A_{max}/L^3 \propto L^{-\beta/\nu}, \qquad (5)$$
where A_{max} is the size of the largest cluster at the phase transition point where $\lambda = 1$ and L is the lattice size of studied system. For three dimension (3D) percolation class, $\beta/\nu = 0.44$; for the Ising class or liquid gas class, $\beta/\nu = 0.51$ [35]. Figure 4b shows the scaling law of (5) for A = 80, 129, 274, 500, 830 and 1270. This method leads to $\beta/\nu = 1.105 \pm 0.039$ which is close to the classical value. However, we would like to emphasize that one should not be nutty to the value itself of the exponents because they are very sensitive to the reduced T_A and A_{max}. In addition, the size of lattice should be as large as possible if one really wants to extract the precise exponents. What we want to say here is that the finite size scaling is valid for the Zpif-type analysis and it is also possible, in principal, to study the critical exponents from there.

In conclusion, Zpif-type analysis in the field of linguistics is applied to the cluster charge (mass) distribution which has been generated with an isospin dependent lattice gas model. The cluster distributions at the phase transition point show exactly inversely to its rank, i.e., there exists Zpif's law. This criterion shall be useful in searching the nuclear liquid gas phase transition experimentally and theoretically in combination with the other phase transition observables, such as τ, S_2 and N_{imf} etc. In addition, the finite size effect of the effective phase transition temperature at which the Zpif's law appears is also addressed and the critical exponents of ν and β are tentatively extracted. It seems that the nuclear disassembly approaches to the 3D liquid gas universality class in the framework of the isospin dependent lattice gas model.

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