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Measurement of Principal-Stress Directions from Photoelastic Experiment Using Generalized Phase-Shift Method*

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A generalized phase-shift method is developed for automatic measurement of principal-stress directions in the whole field of a model from photoelastic experiments. This method uses a set of light intensities obtained from the rotation of polaroids in dark-field plane polariscopes at each point. To investigate the accuracy of the measurement, time-series light intensity curves with different noise and different numbers of data which constitute the curves are used in computer simulation. The error of the measurement of the principal-stress directions obtained by the method is not greatly affected by noise when the number of data exceeds 50. This method is applied to a circular disk subjected to a concentrated load. The principal-stress directions obtained by the method are found to be in good agreement with the theory.

Key words: experimental mechanics, photoelasticity, image processing, phase shift method, principal stress direction

1. Introduction

The photoelastic method is one of the most effective methods for whole-field stress analysis. Photoelastic fringe parameters, fringe orders and principal-stress directions in the whole field of a model are required in this analysis. There are many available methods for obtaining the principal-stress directions,[1-12] and with them the determination of the principal-stress directions has been markedly improved. Among them the phase-shift method,[3-7] which can be used to obtain the principal-stress directions from a combination of several images taken by the rotation of optical components, is promising for the determination of principal-stress directions in the whole field. This method assumes that the time-series light intensity at any point obtained by the rotation of optical components lies exactly on a sinusoidal curve. Actually, the light intensity does not lie exactly on such a curve because of noise caused by the variation in the intensity of a light source and by dust on optical components. Such noise has an adverse effect on the results obtained by the phase-shift method. A method based on Fourier transform[13] has been used to reduce the effect of such noise on the separation of isochromatics and isoclinics from images obtained by rotating the crossed polaroids in a plane polariscope.

In this study, a generalized phase-shift method is developed for suppressing such adverse effects and for automatically measuring principal-stress directions in the whole field, and is applied to the theoretically and experimentally obtained light intensities with noise. The effect of noise is discussed.

2. Generalized Phase-Shift Method

When the polarizer and analyzer in the dark field are simultaneously rotated by θ from a selected reference direction R, the light intensitiy I at any point emerging from the dark-field plane polariscopes with the monochromatic light source is

$$I=I_C\sin^2 2(\phi-\theta)+I_B\,, \tag{1}$$

where $I_C=a^2\sin^2\pi N$, a is the amplitude of polarized light transmitted through the polarizer, N is the fringe order, ϕ is the direction of principal stress σ_1, and I_B is the background light intensity.

The time-series intensity data I_m $(m=0, 1, 2, \ldots, n)$ shown in Eq. (2) are obtained by Eq. (1) according to the rotation of the polaroids at every angle $\Delta\theta(=\theta/n)$ from $\theta=0$ to $\theta=\pi/2$ in a dark-field plane polariscope at each point.[14,15]

$$\begin{aligned} I_0&=I_C\sin^2 2\phi+I_B,\\ I_1&=I_C\sin^2 2(\phi-\Delta\theta)+I_B,\\ I_2&=I_C\sin^2 2(\phi-2\Delta\theta)+I_B,\\ &\vdots\\ I_n&=I_C\sin^2 2(\phi-n\Delta\theta)+I_B, \end{aligned} \tag{2}$$

By applying the Fourier-series expansion to I_m, ϕ in Eq. (1) can be calculated as

$$\phi=\frac{1}{4}\left[\frac{\pi}{2}-\tan^{-1}\left(\frac{b_1}{a_1}\right)\right], \tag{3}$$

where

$$\begin{aligned} a_1&=\frac{2}{n}\sum_{m=0}^{n-1}I_m\cos\left(m\frac{2\pi}{n}\right),\\ b_1&=\frac{2}{n}\sum_{m=0}^{n-1}I_m\sin\left(m\frac{2\pi}{n}\right), \end{aligned} \tag{4}$$

and n is the number of points constituting the data.

3. Accuracy of Measurement

The main factors which affect the accuracy of the measurement of principal-stress directions obtained by the proposed method are the amount of noise and noise distribution in time-series light intensity values, and the number of points constituting the time-series intensity data. To investigate the effects of these factors on the

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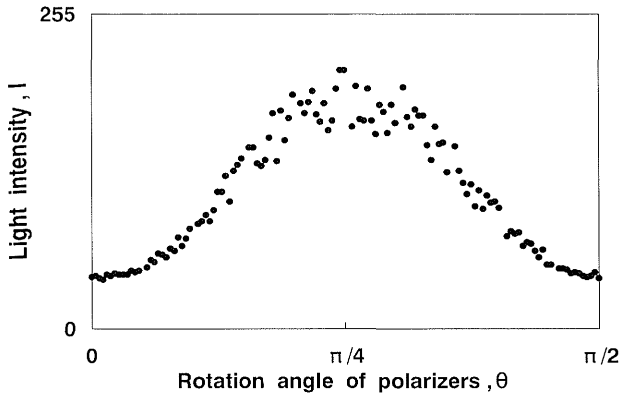

Fig. 1. Examples of actual time-series light intensity obtained by rotation of polaroids.

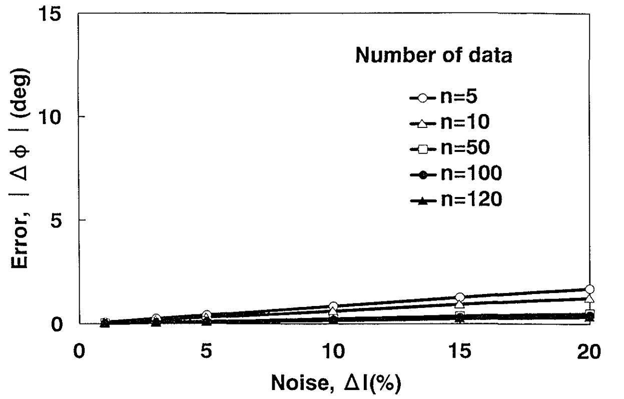

Fig. 3. Effects of noise ΔI on error of principal-stress directions $|\Delta\phi|$ for N=0.25 and ϕ=10 deg.

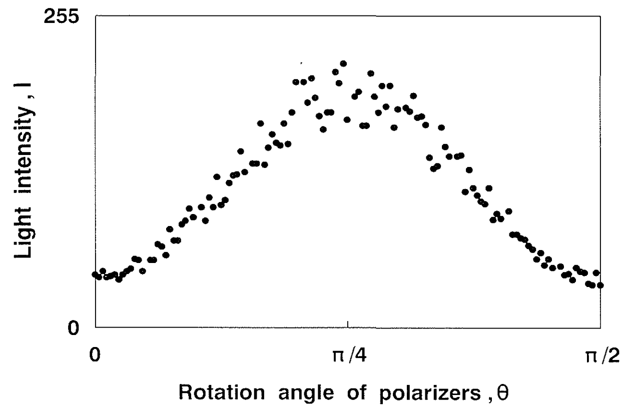

Fig. 2. Examples of artificial time-series light intensity used in simulation for $\Delta I=\pm15\%$, N=0.5 and ϕ=0 deg.

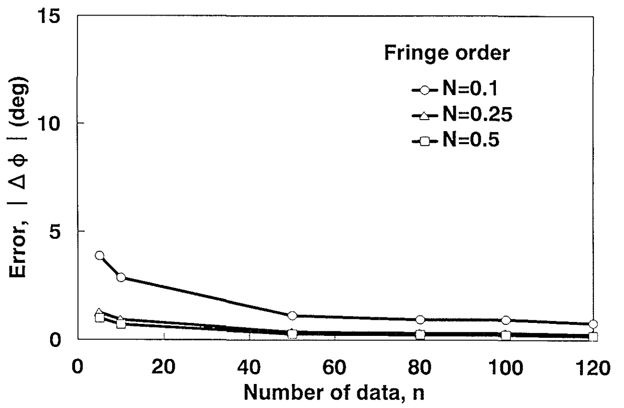

Fig. 4. Effects of number of points constituting time-series intensity data n on error of principal-stress directions $|\Delta\phi|$ for $\Delta I=\pm15\%$ and ϕ=10 deg.

accuracy, a computer simulation is used for the time-series intensity data with noise obtained from Eq. (1) for given N, ϕ, and ϕ in the range of 0 to $\pi/2$.

The amount of noise ΔI is determined by random numbers in the range of $\pm1\%$ to $\pm20\%$ of the light intensity at each point on the time-series intensity data. The range is determined with reference to an actual intensity data at $N \fallingdotseq 0.5$ and $\phi \fallingdotseq 0$ in a circular disk subjected to a diametrically compressive load in a plane polariscope with a mercury lamp as shown in Fig. 1. An amplitude a^2 of 160 and background intensity I_B of 40 are used, which are commonly encountered. Numbers of points used to constitute the data n are 5 to 120. Figure 2 shows examples of the time-series intensity data obtained by the above procedure for n=120, $\Delta I=\pm15\%$, N=0.5 and ϕ=0 deg.

That the time-series intensity data obtained by random numbers are effective for computer simulation is shown by the fact that the intensity data curves obtained by simulation (Fig. 2) are similar to the actual ones (Fig. 1). The intensity data curves shown in Fig. 1 are, however, examples of the actual ones. Many noise distributions in time-series light intensity values must be used to investigate the influence of noise. Hence the seed numbers, which generate different sets of random numbers from 1 to 100 are used to simulate many kinds of actual time-series intensity data. One hundred time-series intensity data are obtained for each combination of ΔI, N, and ϕ. The average value of errors obtained from the 100 data is given in the following simulated results.

Figure 3 shows the effects of the amount of noise ΔI and the number of points constituting the data n on the error of the principal-stress directions $|\Delta\phi|$ obtained for N=0.25 and ϕ=10 deg. Although the errors are affected by n, errors for the time-series intensity data which are composed of light intensity values of about 50 take almost constant values, which are almost independent of ΔI. Similar results are also found for other N and ϕ.

Figure 4 is an example of the effects of n and the fringe order at a measured point N on $|\Delta\phi|$ obtained for $\Delta I=\pm$ 15% and ϕ=10 deg. Although the errors are affected by n and N, errors due to the time-series intensity data which are composed of light intensity values of about 50 take constant values, which depend on N. Similar results are also found for other ΔI and ϕ.

Figure 5 shows clearly the effects of the fringe order at a measured point N on $|\Delta\phi|$ obtained for $\Delta I=\pm15\%$ and ϕ=10 deg. The errors increase near N=0 and therefore

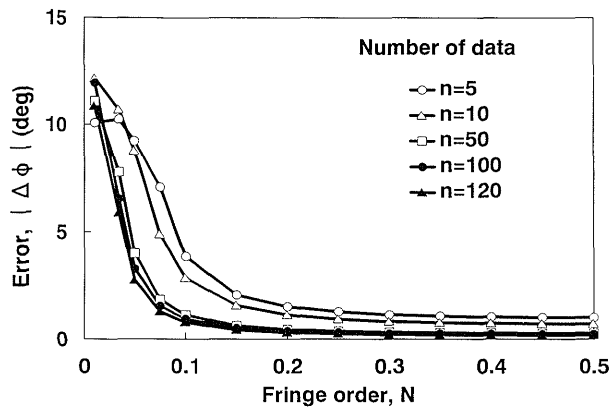

Fig. 5. Effects of fringe order at measured points N on error of principal-stress directions $|\Delta\phi|$ for $\Delta I=\pm15\%$ and $\phi=10$ deg.

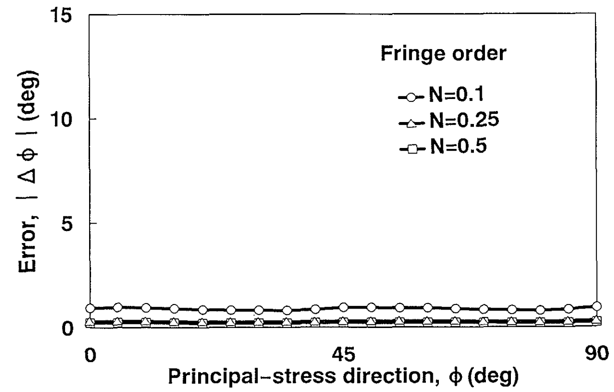

Fig. 6. Effects of principal-stress direction at measured points ϕ on error of principal-stress directions $|\Delta\phi|$ for $n=100$ and $\Delta I=\pm15\%$.

near integer-order fringes, and decrease with increase of n. The number n above 50 gives almost constant error for the fringe order at each measured point. The errors $|\Delta\phi|$ for $N=0.1$, 0.25 and 0.5 are 0.886, 0.290 and 0.221 deg for $n=100$, respectively.

Figure 6 shows the effects of the direction of principal stresses ϕ and fringe order N at a measured point on $|\Delta\phi|$ obtained for $n=100$ and $\Delta I=\pm15\%$. The almost constant error, which is dependent on N, is obtained almost independent of ϕ. Similar results are also found for other ΔI.

The proposed method is thus found to accurately measure the principal-stress directions for the time-series intensity data which are composed of almost 50 light intensity values, and is almost independent of the amount of noise.

4. Experimental Setup

The principal-stress directions in a model are measured using an automatic polariscope system as shown in Fig. 7. The polarizer and analyzer are simultaneously rotated using the stepper motor driven by the number of pulses transmitted from a personal computer. Images which include both isochromatics and isoclinics are taken by the

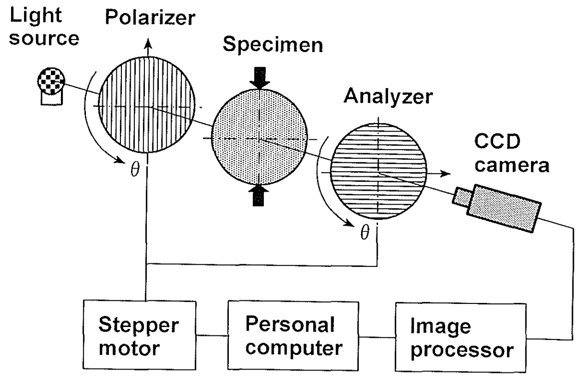

Fig. 7. Experimental setup for measuring principal-stress directions in the whole field.

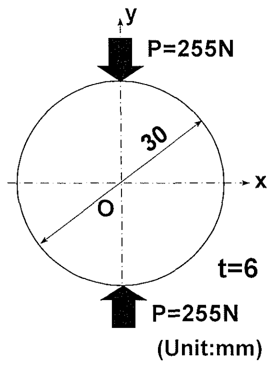

Fig. 8. Circular disk subjected to diametrically concentrated load.

charge coupled device camera, digitized into 8 bits (256 levels) by the image processor, and stored as a 256×256 pixel array in a hard disk for each step of the rotation. In this study, the polarizer and analyzer are rotated with an interval of about 0.7 deg, which is obtained by dividing 90 deg into 128 equal angles. As a result, 129 images in the dark and light fields are taken between 0 and 90 deg. A white lamp in a plane polariscope is used for obtaining principal-stress directions.

After the 129 images are taken in each field, the time-series intensity data are made at each pixel using these images, and stored in a hard disk. The time-series intensity data are used to obtain principal-stress directions at each pixel using the Fourier-series expansion.

5. Application

The proposed method is applied to experimental images obtained from a circular disk of 30 mm diameter subjected to diametrically compressive loads P of 255 N for obtaining principal-stress directions as shown in Fig. 8. This model is made of an epoxy resin plate of 6 mm thickness t. Figure 9 shows the isochromatics obtained in a dark-field circular polariscope.

Figures 10 and 11 show the principal-stress directions in the whole field of the disk and along a horizontal line of $y/a=0.5$, respectively. In Fig. 11, the integer-order fringes are shown by $N=0$, 1 and 2. The proposed method gives

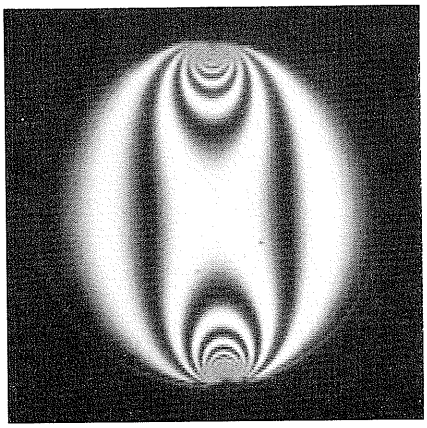

Fig. 9. Isochromatics from the specimen shown in Fig. 8.

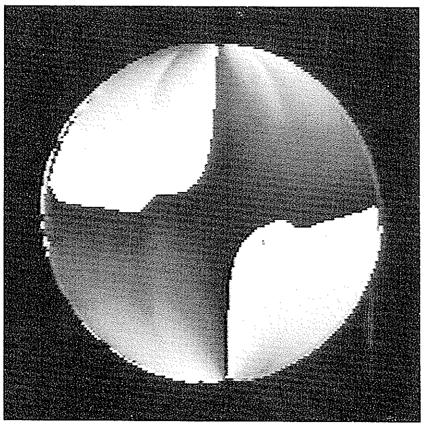

Fig. 10. Principal-stress directions in the whole field.

noticeable error near zero-order fringes, which exist at the edge of the disk. This results mainly from initial stress called edge stress. The edge stress is seen as the white region at the left edge in Fig. 9. In addition, the results obtained by the proposed method do not agree with those of theory[16] near first-order fringes, which exist inside the zero-order fringes. In general, the results are in good agreement with those of theory except for the region near the zero- and first-order fringes.

6. Conclusions

A generalized phase-shift method is developed for reducing the influence of noise caused by the variation in light intensity of the light source and dust on optical components, and for automatically measuring principal-stress directions in the whole field from photoelastic experiments. To investigate the accuracy of the measurement of principal-stress directions, a computer simulation is used.

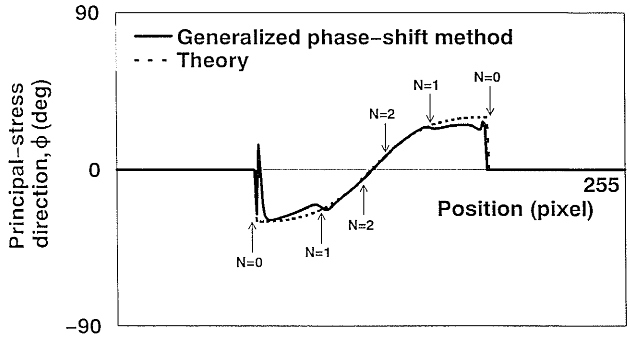

Fig. 11. Distributions of principal-stress directions along the horizontal line of $y/a=0.5$ of a circular disk with radius a.

The method is found to accurately measure the principal-stress directions for the time-series intensity data which are composed of almost 50 light intensity values, and is almost independent of the amount of noise. The method was further applied to the experimental images obtained from a circular disk subjected to a diametrically compressive load. The result was in good agreement with the theory.

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