OPTICAL REVIEW Vol. 15, No. 5 (2008) 241–250

Multi-Objective Design and Extended Optimization for Developing a Miniature Light Emitting Diode Pocket-sized Projection Display

Yi-Chin FANG, Yih-Fong TZENG*, and Si-Xian LI

Department of Mechanical and Automation Engineering, National Kaohsiung First University of Science and Technology, 2 Jhuoyue Road, Nanzih District, Kaohsiung 811, Taiwan, R.O.C.

(Received October 29, 2007; Accepted July 1, 2008)

This research proposes a new extended optimization method for a miniature light emitting diode (LED) pocket-sized projection display, introducing integration of the Taguchi method and principal component analysis in order to optimize the multiple quality characteristics of an LED pocket-sized projection display. With the aid of interactive optimization, control factors with three different levels are carefully selected in the complicated preliminary experiments. A set of optimal design parameters is well selected for best results on the combined effects of the total luminous flux, illumination uniformity, and the packing size of the system. The selected control factors are inclusive of major lens and system specifications, such as lens overall length, X-CUB semi-aperture, length of light integrator, width of integrator, total internal reflection (TIR) prism entering semi-diameter for the TIR prism, air-gap of the TIR prism, and digital micromirror device (DMD) position; an L18 orthogonal array is applied and implemented in the experiments. According to experimental results, the optimal design parameters for the projection display can be determined as A1 (lens specifications: type I), B3 (lens length: overall length), C1 (X-CUB semi-aperture: 8 mm), D3 (integrator length: 36.6 mm), E2 (integrator width: 3.5 mm), F2 (TIR prism entering semi-diameter: 11 mm), G1 (TIR prism air-gap: 1.0024 mm), and H1 (DMD location: −0.5 mm). In addition, analysis of variance (ANOVA) is also employed to identify the factor A (lens specifications), factor D (integrator length), factor F (TIR prism entering semi-diameter), and factor G (TIR prism air-gap) as key parameters, which account for 71.82% of the total variance. The other factors when compared are found to have relatively weaker impacts on the process design. Furthermore, a confirmation experiment of the optimal design parameters shows that the aforesaid multiple performance characteristics are optimized to achieve the best levels. It is concluded that Taguchi method and principal component analysis (PCA) combine to optimize and then minimize the LED pocket-sized projection display system, which not only yields a sufficient understanding of the effects of control factors, but also produces an optimized design to ensure that the LED pocket-sized projection display system exhibits the best multiple performance characteristics.

Key words: optical design, optical engine, LED light source, Taguchi method, principal component analysis, LED pocket-sized projection display, optimization, analysis of variance (ANOVA), miniature projector, interactive optimization

1. Introduction

Markets for portable miniature projection systems have recently grown rapidly, thanks to widely used wireless image transmission and the compactness of portable computers. It has become an issue that, when presentations are given, projector modules have to be accommodated in extra-thin portable computers with a reasonable projection view. By wireless transmission, images or information stored in computers could be easily accessed by thin monitors with high contrast, such as liquid crystal display (LCD) TV. But, for most informal cases, the projection from an extra thin portable computer could easily deliver the information and pictures in presentations.[1,2] In addition, for further three-dimensional (3D) display, projection plays its part in 3D presentation by a two-lens dissolve design.[3,4] It is very difficult for a traditional light source (lamp) to be assembled within an extra-thin portable computer due to the size of the optical engine and heat problems. To replace it, variable optical engine designs have been studied using high-power light emitting diode (LED) as a light source, an advance which is now possible thanks to advanced semiconductor development and processing. Following the growth of the LED light source with advanced circuit control which provides RGB light in a color-sequential mode,[5] optical engines for projectors could be significantly minimized in size and promise even more significant innovations.[6] Furthermore, with LED illuminators, white points and color gamuts can be dynamically improved, offering an optimization of the source material and viewing conditions.[7] Therefore, in the current business market, pocket-sized projectors are comprehensively discussed, even those which work and are compatible with products such as digital still cameras, personal digital assistant (PDA) or gaming devices.[8]

This paper proposes a newly developed optical engine design and lens design with a view to minimizing the optical system of a miniature projection system. RGB LED light sources are employed in this research, with a 0.55 in. digital micromirror device (DMD) panel for projection display. Projection brightness and uniformity play a role in this research as well. Generally speaking, any modifications of the components of a miniature projection system, such as

*E-mail address: franktzeng@ccms.nkfust.edu.tw

lens overall length, integrator length, TIR prism and X-CUB aperture or DMD position, may lead to significant consequences on the system performance such as luminous flex and uniformity.[9] In order to achieve maximum performance in the design process, the Taguchi method is employed in optimization procedures with orthogonal arrays of statistically designed experiments, in the hope of obtaining the best results with the fewest possible experiments.[10] According to industrial history, the Taguchi method has been successfully developed to optimize and analyze design processes with static and dynamic characteristics;[11–14] however, this method has difficulties in optimizing systems with inter-correlated multiple performance characteristics (MPC) such as the projection system in this research. Therefore, principal component analysis (PCA), as a useful statistical technique, was introduced in this study to examine the relationships between a given data set of MPC to improve the system's performance. A new set of uncorrelated data of MPC, called principal components, can be derived by PCA in descending order of their ability to explain the variance of the original dataset.[15] With the aid of PCA, the principal components and their explanatory power can be further integrated as a single quality characteristic for the MPC optimization of the LED pocket-sized projection display system. Experiments are simulated and results are evaluated using the optics software tool, ASAP, which is one of the main optical design tools for optical engines and non-image optics.

1.1 Introduction to the DLP projector

Digital light processing (DLP) is a revolutionary new way to project and display information. Based on the DMD developed by Texas Instruments, DLP creates the final link in displaying digital visual information. DLP technology is being provided as subsystems to market leaders in the consumer, business, and professional segments of the projection display industry. In the same way as the compact disc revolutionized the audio industry, DLP will revolutionize video projection. DLP has three key advantages over existing projection technologies. The inherent digital nature of DLP enables noise-free, precise image quality with digital gray scale and color reproduction. Its digital nature also positions DLP as the final link in the digital video infrastructure. DLP is more efficient than its rival, transmissive LCD technology, because it is based on the reflective DMD and does not require polarized light. Finally, the close spacing of the micro mirrors causes video images to be projected as seamless pictures with higher perceived resolution. For movie projection, computer slide presentations or an interactive, multi-person, worldwide collaboration, DLP has become the only choice for digital visual communications, today and in the future.[16]

1.2 Digital light processing: How it works

Just as a central processing unit (CPU) is the heart of a computer, so a DMD is the cornerstone of DLP. One-, two-, and three-chip DLP systems have been built to serve different markets. A DLP-based projector system includes memory and signal processing to support a fully digital approach. Other elements of a DLP projector include a light source, a color filter system, a cooling system, and illumination and projection optics.

A DMD can be described simply as a semiconductor light switch. It is made up of thousands of tiny, square, 13.6 μm mirrors, fabricated on hinges above a static random access memory (SRAM). Each mirror is capable of switching a pixel of light. The hinges allow the mirrors to tilt between two states, $+10°$ for "on" and $-10°$ for "off". When the mirrors are not operating, they sit in a zero-degree state.[17]

2. Design of a Miniature LED Pocket-Sized Projection Display System

A DLP miniature projection system is composed of an optical engine system and light valves for illumination, and an imaging system.[9] In this experiment, LEDs replace the current high intensity discharge (HID) lamps; an LED light source makes minimization of the DLP projection system possible, thanks to the removal of a color wheel; the LED illuminator light is controlled by switching the different colored LEDs within one frame time of the video signal.[7]

The DLP optical engine with LED has been studied and developed over several years,[1,5,7] many experiments and prototypes have been constructed and even sold on the market. However, there is still room for improvement in such aspects as size, light efficiency and uniformity. In this experiment, a basic design for a LED pocket-sized projection display system is shown in Fig. 1. Red (R), green (G), blue (B) high-power LEDs in a Luxeon™ form-factor package are separately located in the optical engine. Table 1 shows the data of the LEDs. Injection-molded acrylic collimators in the front of the LED redistribute the light.[18] Then the RGB colors are recomposed by Dichroic X-CUB. A condenser lens is designed to focus the light on the entrance of a newly developed U-shaped integrator rod to keep the output light uniform; the aperture stop of the optical engine is set at the output gate of the U-shaped rod as the entrance pupil of the optical engine system; in this research, the U-shaped rod is used for the first time due to the minimization of the optical system. A relay lens set working with the TIR prism will keep the light directed on the 0.55″ DLP™ panel-DMD. The TIR prism is employed as an angular analyzer for light from the light veil and relay lens

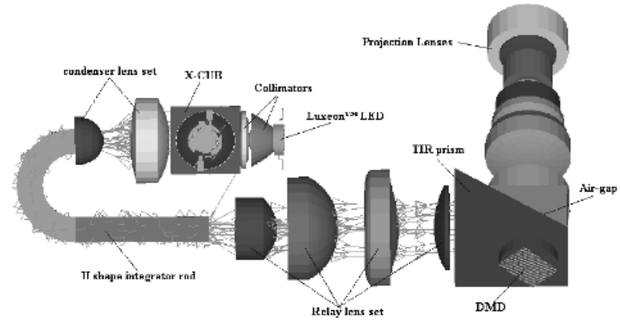


Fig. 1. Architecture of an LED pocket-sized projection display system.

Table 1. Luxeon™ LEDs of the light source of the projection display system.[19,20)]

Flux characteristics at 1000 mA, junction temperature, TJ = 25 °C			
Color	Luxeon emitter	Typical luminous flux (lm)	Radiation pattern
Red	LXHL-PD01	44	
Green	LXHL-PM09	80	Lambertian
Blue	LXHL-PB09	30	

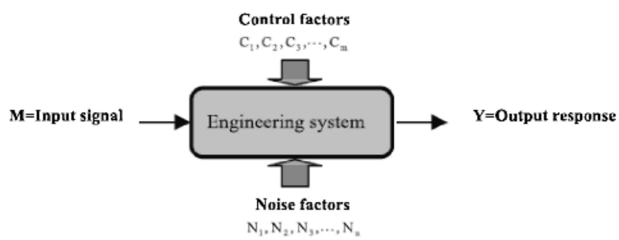


Fig. 2. The engineering system using the Taguchi method.

which separate the image and the illumination light at an angle.[9)] In this design, the image projected on the screen has a projected size of 17″ and its objective distance is approximately 600 mm.

3. Taguchi Method

3.1 Engineering system and parameter design

All man-made machines or set ups are classified as engineering systems according to THE Taguchi method. As shown in Fig. 2, an engineering system generally consists of four sections: the signal factor, control factor, noise factor, and output response. The signal factor is the input from the user to the system for a specified output response. If the system's output response changes with the input signal, the system is considered to possess dynamic characteristics according to the Taguchi method. Parameters which are easy or inexpensive to control are chosen as the control factors, while parameters which are difficult or expensive to control are designated as noise factors.

The Taguchi method uses an orthogonal array to execute experiments and to analyze results. Using an orthogonal array can substantially reduce the time and cost of developing a new product or technique and thereby increase the competitiveness of the product in the open market. Taking the L12 (2^{11}) orthogonal array as an example, the initially required $2^{11} = 2{,}048$ sets of experiments can be significantly reduced to 12 sets, saving time and cost and, in addition, achieving similar results to a full factorial experimental set up. Moreover, interaction amongst the factors could be evenly distributed to each column, ensuring that the effect of interaction is minimized. Orthogonal arrays consist of inner and outer columns, with the former designated the control factors while the latter are named the input signal and noise factors. The principle of the Taguchi method is to allow the design factors to be subjected to the tests of the noise factors located at the outer columns of the array such that the optimized control factors will be effective in combating the influence of the noise factors acting on the product quality.

3.2 Control factors and their levels

From the viewpoint of optical design and engineering, control factors play a significant role in system optimization. Choice of these factors depends upon the system and its design philosophy, which might vary with specifications and designer's characteristics. During the optical design process, "Interactive Optimization" in the optical design software might be employed in order to evaluate manually the degradation of system performance. According to the degradation, three levels of each control factors could be well-defined.

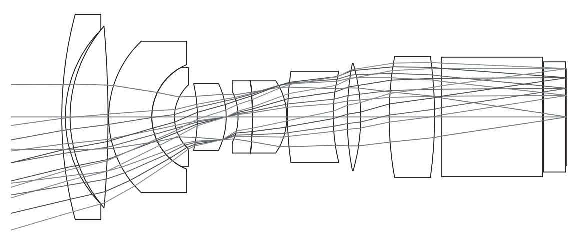
When the design of an LED projector is begun in this case, variable factors are tested and evaluated; for example, lens aperture diameter, rear lens diameter, front lens diameter and back focal length are considered as variable factors which might have significant contributions to the system performance. At the next step, numerous experiments will be done in order to determine the three levels of each control factor following the system performance degradation.

Eight dominant design parameters for developing the projection display system are identified as the control factors and are listed in Table 2 together with their alternative levels. It is noted that most of the factors have three levels, except for factor A, which has two levels, the projection lenses of specifications types I and II, the main difference between the two types being the offset of the projection levels.[9)] All data on the projection lens plots and lens specifications design are shown in Fig. 3 and Table 3. For factor B, which is the projection lenses' overall length dimensions, level 3 is the lenses' overall length, while level 2 and level 1, being the lenses overall length, are reduced by 3 and 6 mm. Moreover, the levels of the factors C, D, E, F, G are all the different dimensions of the optical components. Finally, for factor H, level 2, we set the DLP™ panel DMD location position of the design of the initial condition parameters of the projection display system as the origin "0", and shifted the location positions of the DMD along the z coordinate for -5 and 5 mm as level 1 and level 3. This will influence the luminous flux and illuminate the uniformity performance of the projection display system. All levels are carefully determined. Noise factors are neglected for the sake of simplifying the system design.

The main design and optimization goal is uniformity and brightness rather than modulation transfer function (MTF). Optics plays a role in resolution issue such as MTF and aberrations like distortion and flare. If optimization goals apply for uniformity and brightness, the critical issues from the optics are the following: F-number, exit pupil, and relative illuminance (RI). Firstly, focal length is fixed

Table 2. Control factors and their levels.

	Control factors	Levels		
		1	2	3
A	Lens specifications	I (type)	II (type)	
B	Lens length (mm)	Overall length minus 6	Overall length minus 3	Overall length
C	X-CUB semi-aperture (mm)	8	9	10
D	Integrator length (mm)	30.6	33.6	36.6
E	Integrator width (mm)	3	3.5	4
F	TIR prism entering semi-diameter (mm)	10	11	12
G	TIR prism air-gap (mm)	1.0024	1.1024	1.2024
H	DMD location (mm)	−0.5	0	0.5



(a)

(b)

Fig. 3. Projection lenses plots of the lens specifications (a) type I and (b) type II.

Table 3. Lens specifications design data of types I and II projection lenses.

Lens specifications			P.S.
Type	I	II	
Image height (mm)	6.3	7.7	
Screen size (mm)	17″	17″	
F-number	2.4	2.4	
Objective distance (mm)	600	600	
MTF (%)	≧88	≧81	36 lp/mm
Optical distortion (%)	<2	<2	
Relative illumination (%)	≧70	≧70	
Exit pupil position (mm)	<−300	<−300	
Focal length (mm)	22.42	22.42	
Overall length (mm)	102.33	95.57	
Offset (mm)	1 : 1	1.2 : 1	

normally due to the specification-defined field of view (FOV). For DLP projection system, the F-number must be fixed with cat eyes correction thanks to a DMD 12 degree shift. The cat eyes correction is employed to block the revealing stray light. With regard to RI, 70% is the minimum required for projection lens design. From point view of optical design, it will be very difficult for short zoom ratio projector lens to surpass 70% RI or designer has to minimize it to 70% because RI and MTF are traded-off in most cases. Therefore, it is not a proper solution for RI to be selected as a main factor.

Another example is that we found the semi-diameter of TIR prism play a role in system performance for light efficiency and uniformity rather than lens RI after a number of interactive optimization experiments at ASAP. Therefore, semi-diameter of the TIR prism has become a main control factor.

With regard to the main factors for optics, lens 1 and lens 2 with different exit pupil aberrations are usually pointed out. Normally, the exit pupil behaves like a perfect circle at on-axis and like cat eyes at off-axis. However, the pupil might have a 3D shape due to transverse aberrations; for example, distortion at transverse aberration is the same as pupil coma aberration. When optics and an optical engine are combined with, their 3D distortion pupil will be interactively matched, but some rays will be missing if the matching is not perfect for each pupil from optics and the optical engine. Every optical design has its unique aberrations due to a different optical layout; therefore, their 3D pupil aberration is different with their unique layout. Even a system with the same optics with various tolerances might have slightly different light efficiency and uniformity due to non-perfection of the 3D pupil matching at a different rotating angle. Therefore, the optics and its layout are the main factor due to pupil aberration, and different optical layout with different aberrations and its characteristics are considered to be the main factor for refined extended optimization.

3.3 *System performance evaluation using signal-to-noise ratio*

The signal-to-noise (S/N) ratio originates in the communication field. The Taguchi method expands its function into the quality engineering area. For any engineered system, the input signal, control factors and noise factors come together to perform its designed function. Some measurable output responses, generally referred to as the performance characteristics, are used to express how well the system performs the functions. The so-called S/N ratio is used to evaluate the output response. As stated above, the performance characteristics to be measured are the luminous flux, illumination uniformity performance, and the packing size of the system. In a study where there are greater amounts of the luminous flux, the illumination uniformity ratio is better. The larger-the-better (LTB) S/N ratio formula is

$$\text{LTB S/N} = -10\log\left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i^2}\right) \quad (1)$$

Moreover, smaller amounts of the packing size of a system are better; the smaller-the-better (STB) S/N ratio formula is shown as follows:

$$\text{STB S/N} = -10\log\left(\frac{1}{n}\sum_{i=1}^{n}y_i^2\right), \quad (2)$$

where y_i denotes the values of the luminous flux, illumination uniformity ratio and the packing size of the system.

The S/N ratio measures the level of system performance and the effects of noise factors on performance. The higher this ratio, the more the system is doing what it is intended to do, regardless of noise factors; the system is more robust against noise.

4. Principal Component Analysis Approach

4.1 Theory and formulas

PCA is a technique which provides a way of exploring multivariate data. The original initial variables are transformed into another dimension set of uncorrelated variables, e.g., the principal components, by calculating the eigenvectors of the covariance matrix of the original inputs. The transformed variables are ranked according to their variance, thereby reflecting decreasing levels of ability to capture the whole information content of the original dataset.

Although P components are required to reproduce the total system variability, often much of this variability can be accounted for by a small k of the principal components. The k principal components can then replace the initial p variables, and the original dataset, consisting of n measurements on p variables, is reduced to a dataset consisting of n measurements on k principal components.

Let $X = (X_1, \ldots, X_p)^{\mathrm{T}}$ have covariance matrix $\sum$, with eigenvalue–eigenvector pairs $(\lambda_1, V_1), (\lambda_2, V_2), \ldots, (\lambda_p, V_p)$, where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p \geq 0$. The ith principal component is given by

$$Y_i = V_{i1}X_1 + V_{i2}X_2 + \cdots + V_{ip}X_p \quad (i = 1, 2, \ldots, p) \quad (3)$$

with $\mathrm{Var}(Y_i) = V_i^{\mathrm{T}} \sum V_i = \lambda_i$, and $\mathrm{Cov}(Y_i, Y_k) = V_i^{\mathrm{T}} \sum V_i = 0$, $I \neq k$. The total system variance is given by

$$\sum_{i=1}^{p}\mathrm{Var}(X_i) = \sum_{i=1}^{p}\mathrm{Var}(Y_i) = \sum_{i=1}^{p}\lambda_i. \quad (4)$$

The proportion of total variance explained by the ith principal component is defined as its explanatory power. The explanatory power of the ith principal component equals to

$$\frac{\lambda_i}{\sum_{i=1}^{p}\lambda_i} \quad (5)$$

4.2 Data pre-processing

The dataset in PCA consists of a number of observations, n, where each observation contains values for a set of p variables. Thus, the dataset can be represented by a $n \times p$ matrix. Often, to keep some observations or variables from discriminating the calculations, the data are normalized prior to finding PCs. Such data preprocessing can avoid being influenced by the units used for evaluating the multiple performance characteristics of the machining process under investigation. Normalization of the data may provide fair information for determining the optimal levels of process parameters.

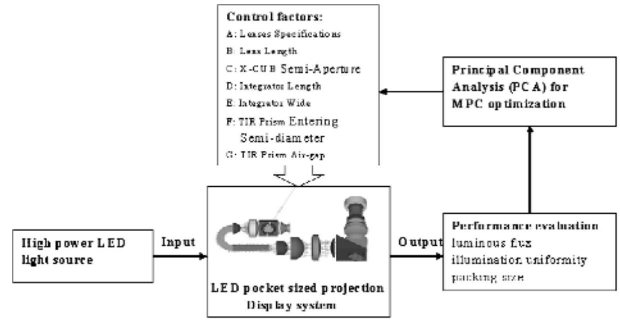


Fig. 4. Flow structure chart for application of the Taguchi method.

The original data are converted into a range between 0 and 1, in which 0 indicates the worst performance and 1 the best. The formula employed to normalize the observed values of the luminous flux, illumination uniformity performance, and the packing size of system is

$$\hat{x}_i(k) = \frac{x_i(k) - \min\limits_k x_i(k)}{[\max\limits_k x_i(k)] - \min\limits_k x_i(k)} \quad i = 1, 3, 4 \quad (6)$$

5. Strategy for Optimizing the Multiple Performance Characteristics

In this study a hybrid approach of integrating the Taguchi method with the PCA is employed, to optimize the aforesaid system performances of the LED pocket-sized projection display system. Figure 4 is the flow structure chart.

6. Experimental Results and Discussion

6.1 System performance evaluations and normalization

The multiple performance characteristics of the LED pocket-sized projection display selected in the study are the luminous flux, illumination uniformity, and the packing size of the system. Table 4 lists a complete experimental layout of L18, and the performance evaluation of the three quality characteristics. The luminous flux and illumination uniformity are evaluated using the LTB S/N ratio and the packing size of the system is assessed using the STB S/N ratio. The results are displayed in Table 5.

6.2 Principal component analysis

Before the PCA calculation, data pre-processing was undertaken. Table 6 lists the normalization of the performance evaluation of the MPC listed in Table 5 to a range between 0 and 1. Table 7 shows the correlation coefficients

Table 4. Performance evaluation of all the experimental arrangements in L18.

Test no.	Control factors								Performance evaluation of quality characteristics		
	A	B	C	D	E	F	G	H	Luminous flux (lm)	Illumination uniformity (%)	Packing size (cm^3)
1	1	1	1	1	1	1	1	1	25.29	63.74	711.13
2	1	1	2	2	2	2	2	2	18.48	60.88	769.35
3	1	1	3	3	3	3	3	3	12.96	51.90	828.71
4	1	2	1	1	2	2	3	3	20.39	61.04	771.89
5	1	2	2	2	3	3	1	1	15.98	50.09	852.41
6	1	2	3	3	1	1	2	2	31.2	47.39	824.4
7	1	3	1	2	1	3	2	3	17.98	39.27	840.34
8	1	3	2	3	2	1	3	1	17.92	50.87	866.13
9	1	3	3	1	3	2	1	2	14.8	49.34	811.53
10	2	1	1	3	3	2	2	1	6.81	57.91	766.96
11	2	1	2	1	1	3	3	2	11.35	37.47	688.9
12	2	1	3	2	2	1	1	3	9.75	58.48	696.05
13	2	2	1	2	3	1	3	2	6.61	50.74	728.93
14	2	2	2	3	1	2	1	3	11.59	46.2	753.5
15	2	2	3	1	2	3	2	1	8.34	35.15	721.52
16	2	3	1	3	2	3	1	2	6.29	72.59	833.25
17	2	3	2	1	3	1	2	3	5.99	45.64	730.92
18	2	3	3	2	1	2	3	1	9.99	65.5	774.45

Table 5. S/N ratio data of all experimental arrangements.

Test no.	Control factors								Performance evaluation of quality characteristics		
	A	B	C	D	E	F	G	H	Luminous flux (lm) S/N (db)	Illumination uniformity S/N (db)	Packing size S/N (db)
1	1	1	1	1	1	1	1	1	28.058	36.088	−57.039
2	1	1	2	2	2	2	2	2	25.334	35.689	−57.722
3	1	1	3	3	3	3	3	3	22.252	34.303	−58.368
4	1	2	1	1	2	2	3	3	26.188	35.712	−57.751
5	1	2	2	2	3	3	1	1	24.071	33.995	−58.613
6	1	2	3	3	1	1	2	2	29.883	33.513	−58.322
7	1	3	1	2	1	3	2	3	25.095	31.881	−58.489
8	1	3	2	3	2	1	3	1	25.066	34.129	−58.751
9	1	3	3	1	3	2	1	2	23.405	33.863	−58.186
10	2	1	1	3	3	2	2	1	16.662	35.255	−57.695
11	2	1	2	1	1	3	3	2	21.099	31.473	−56.763
12	2	1	3	2	2	1	1	3	19.780	35.340	−56.852
13	2	2	1	2	3	1	3	2	16.404	34.107	−57.253
14	2	2	2	3	1	2	1	3	21.281	33.292	−57.541
15	2	2	3	1	2	3	2	1	18.423	30.918	−57.165
16	2	3	1	3	2	3	1	2	15.973	37.217	−58.415
17	2	3	2	1	3	1	2	3	15.548	33.186	−57.277
18	2	3	3	2	1	2	3	1	19.991	36.324	−57.779

among the MPCs. It is understood that the luminous flux of the display system has a relatively strong interrelationship with the packing size of the system, but of an inverse kind. However, there is a very weak interrelationship between luminous flux and illumination uniformity. Illumination uniformity has little interrelationship with the packing size. This may indicate that a better design of luminous flux and packing size are needed in order to achieve good luminous performances for an LED pocket-sized projection display system.

Table 8 shows the PCA of the MPC observed in the L18 experiments leading to three PCs with eigenvalues of 1.358, 0.998, and 0.644. The first PC has an explanatory power of 45.278% for the total variance of the data set of the multiple performance characteristics. The explanatory powers for the second and third PCs are 33.259, and 21.464%, respectively, according to their ability to account for the total variance. The observation agrees well with the interrelationships reflected in the correlation coefficient matrix of the MPC. These eigenvectors corresponding to the three eigenvalues of the correlation matrix are shown in Table 9. The eigenvectors can be treated as the weighting number such that the matrix of the three PCs is expressed as:

Table 6. Normalization of the performance evaluation in Table 5.

Test no.	Control factors								Performance evaluation of quality characteristics		
	A	B	C	D	E	F	G	H	Luminous flux	Illumination uniformity	Packing size
1	1	1	1	1	1	1	1	1	0.872746992	0.820719	0.861277
2	1	1	2	2	2	2	2	2	0.682651299	0.757416	0.517557
3	1	1	3	3	3	3	3	3	0.467650609	0.537359	0.192911
4	1	2	1	1	2	2	3	3	0.742248905	0.761035	0.50316
5	1	2	2	2	3	3	1	1	0.594577146	0.488411	0.069745
6	1	2	3	3	1	1	2	2	1	0.412004	0.215687
7	1	3	1	2	1	3	2	3	0.666030931	0.152835	0.132037
8	1	3	2	3	2	1	3	1	0.664005501	0.509718	0
9	1	3	3	1	3	2	1	2	0.548094979	0.467608	0.284415
10	2	1	1	3	3	2	2	1	0.077742613	0.68845	0.531148
11	2	1	2	1	1	3	3	2	0.387272616	0.088135	1
12	2	1	3	2	2	1	1	3	0.295199658	0.701956	0.954899
13	2	2	1	2	3	1	3	2	0.059680452	0.50619	0.75329
14	2	2	2	3	1	2	1	3	0.399951871	0.376936	0.608485
15	2	2	3	1	2	3	2	1	0.200549266	0	0.79792
16	2	3	1	3	2	3	1	2	0.029612112	1	0.169046
17	2	3	2	1	3	1	2	3	0	0.36012	0.741381
18	2	3	3	2	1	2	3	1	0.309934503	0.858278	0.488698

Table 7. Correlation coefficient matrix of the multiple performance characteristics.

Correlation coefficients	Luminous flux	Illumination uniformity	Packing size
Luminous flux	1.000	0.003	−0.331
Illumination uniformity	0.003	1.000	−0.134
Packing size	−0.331	−0.134	1.000

Table 8. Principal component analysis: eigenvalues, proportion explained, and cumulative total.

Principal components	Eigenvalues	Proportion explained (%)	Cumulative total (%)
1	1.358	45.278	45.278
2	0.998	33.259	78.536
3	0.644	21.464	100

Table 9. Eigenvectors corresponding to the eigenvalues.

Multiple performance characteristics	Principal components		
	PC_1	PC_2	PC_3
Luminous flux	0.654748712	−0.376376565	0.655454894
Illumination uniformity	0.271167	0.925926	0.261684
Packing size	−0.70624	0.006006	0.707792

$$[Y] = [M][X] \text{ or}$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} 0.654748712 & 0.271167 & -0.70624 \\ -0.376376565 & 0.925926 & 0.006006 \\ 0.655454894 & 0.261684 & 0.707792 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} \tag{7}$$

The score table for the PC is shown in Table 10. It is also clear that the luminous flux and the packing size of the system are more important than the illumination uniformity, due to their higher weightings in PC_1, which has the greatest explanatory power. There is a similar trend reflected in Y_3. Conversely, the illumination uniformity has the strongest power in Y_2 due to the largest weighting number.

In order to facilitate the MPC optimization in the study, the scores of the three PCs can be further integrated as a total PC score for the MPCI indices (MPCI) using the linear combination method according to their explanatory powers for the total variance. The matrix form for the total PC scores is thus formulated as

$$[\text{MPCI}] = [0.453 \quad 0.333 \quad 0.215] \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} \tag{8}$$

The total PC score results are also shown in Table 10. It is seen that the design conditions for test number 1 produces the best results on the MPCI in the L18.

6.3 *Effects of control factors on MPCI*

Those MPCIs in Table 10 can be further translated into the effect which each control factor has on the MPCI by computing their average values, as listed in Table 11. Figure 5 is its response graph. Table 11 suggests that the

Table 10. Principal component scores and the integrated total PC score as the multiple performance characteristics indices (MPCIs).

Test no.	PC_1	PC_2	PC_3	Total PC
	Y_1	Y_2	Y_3	MPCI
1	0.18572	0.43662	1.3964	0.52903
2	0.28683	0.44749	1.012	0.49592
3	0.31567	0.3227	0.58368	0.37554
4	0.337	0.42832	1.0418	0.51865
5	0.47248	0.22887	0.56689	0.41173
6	0.61414	0.0064046	0.91593	0.4768
7	0.38428	−0.10837	0.57	0.2603
8	0.57298	0.22205	0.56861	0.45533
9	0.2848	0.22839	0.68292	0.35149
10	−0.13753	0.61138	0.60705	0.27137
11	−0.42877	−0.058147	0.98469	−0.0021237
12	−0.29076	0.54459	1.053	0.27549
13	−0.35566	0.45076	0.70475	0.14015
14	−0.065654	0.20214	0.79147	0.20738
15	−0.43221	−0.07069	0.69621	−0.069772
16	0.17117	0.9158	0.40074	0.4681
17	−0.42594	0.3379	0.61898	0.052383
18	0.09053	0.68099	0.77364	0.43353

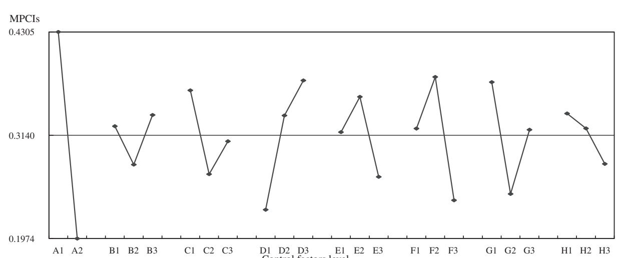


Fig. 5. Response graph of MPCI values.

best levels for each control factor are A_1, B_3, C_1, D_3, E_2, F_2, G_1, and H_1 due to their maximum MPCI values. The max–min value is equal to the range of MPCI variance, due to the change in the level setting. The larger the range, the more powerful impact the control factor has on the MPCI. The ranking in Table 11 demonstrates that factor A (lens specifications), factor D (integrator length), factor F (TIR prism entering semi-diameter) and factor G (TIR prism air-gap) have stronger impacts on the MPCI, while B, C, E, and H have relatively weak impacts. Factors A, D, F, and G should be strictly controlled for high MPCI on the LED pocket-sized projection display system.

Table 12. Analysis of variance on MPCI.

Factor	S	F	Variance	S′	F-ratio	Contribution (%)
A	0.2446	1.0000	0.2446	0.2446	5.3068	41.8903
B	0.0104	2.0000	0.0052	0.0104	0.1124	1.7748
C	0.0272	2.0000	0.0136	0.0272	0.2951	4.6588
D	0.0682	2.0000	0.0341	0.0682	0.7401	11.6848
E	0.0245	2.0000	0.0123	0.0245	0.2658	4.1971
F	0.0586	2.0000	0.0293	0.0586	0.6352	10.0286
G	0.0480	2.0000	0.0240	0.0480	0.5207	8.2211
H	0.0103	2.0000	0.0051	0.0103	0.1113	1.7570
Error	0.0922	2.0000	0.0461	0.0922	1.0000	15.7875
Total	0.5839	17.0000	0.0343	0.5839		100.0000

6.4 Analysis of variance

The analysis of variance is fundamentally similar to the analysis of the "max–min" range in the response table. The main difference is that the former can effectively separate the variation caused by experimental error and thereby identify the variations of the individual control factor. This enables a thorough examination to be made of the contribution of each control factor and also of the experimental error of the engineering system. The "max–min" range method, for its part, exhibits the effect of the entire range. From the analysis of the variation in Table 12, the effect of each control factor on the multiple performance characteristics becomes apparent. It is clear that factors A, D, F, and G are the key control parameters due to their higher contributions to the total variance. These four factors account for nearly 71.82% of the total variance on MPCI. This observation is very similar to the trend reflected in Table 11 and Fig. 5.

6.5 MPC optimization of system design

The objective for the project is to have MPC optimization in the LED pocket-sized projection display system. In general, many strategies and techniques are used to increase the value of the MPCI. According to the Taguchi method, the optimal conditions can be determined by resorting to the response table and graph of the MPCI. As shown in Table 11 and Fig. 5, the level of individual control factors which results in the largest MPCI are A1 (lens specifications: type I), B3 (lens length: overall length), C1 (X-CUB semi-aperture: 8 mm), D3 (integrator length: 36.6 mm), E2 (integrator width: 3.5 mm), F2 (TIR prism entering semi-diameter: 11 mm), G1 (TIR prism air-gap: 1.0024 mm), and H1 (DMD location: −0.5 mm).

Table 11. Response table of MPCI values.

	A	B	C	D	E	F	G	H
Level 1	**0.4305**	0.3242	**0.3646**	0.2299	0.3175	0.3215	**0.3739**	**0.3385**
Level 2	0.1974	0.2808	0.2701	0.3362	**0.3573**	**0.3797**	0.2478	0.3217
Level 3		**0.3369**	0.3072	**0.3758**	0.2671	0.2406	0.3202	0.2816
Max–min	0.2331	0.0560	0.0945	0.1458	0.0902	0.1391	0.1260	0.0569
Rank	1	8	5	2	6	3	4	7

Table 13. Comparison of the confirmation run between the initial and the optimal conditions.

Performance characteristics	Initial condition $A_2B_3C_2D_2E_2F_2G_2H_2$	Optimal condition $A_1B_3C_1D_3E_2F_2G_1H_1$	Gain
MPCI prediction	0.2491	0.7594	0.5103
MPCI confirmed	0.1892	0.7903	0.601
Luminous flux (db)	16.65	28.107	11.46
Illumination uniformity (db)	33.078	36.217	3.14
Packing size (db)	−58.116	−57.14	0.976

6.6 *MPCI prediction of optimal parameter design*

The optimum design for the LED pocket-sized projection display system under study consists of factor levels, which maximize the MPCI value. The best MPCI value for the three performances can be predicted using the additive law, as follows:[10)]

For optimal conditions: $A_1B_3C_1D_3E_2F_2G_1H_1$

$$\begin{aligned}\mathrm{MCPI}_{\mathrm{opt}} &= A_1 + B_3 + C_1 + D_3 + E_2 + F_2 \\ &\quad + G_1 + H_1 - 7 \times \mathrm{MCPI}_{\mathrm{average}} \\ &= 0.4305 + 0.3369 + 0.3646 + 0.3758 + 0.3573 \\ &\quad + 0.3797 + 0.3739 + 0.3385 - 7 \times 0.3140 \\ &= 0.7594.\end{aligned}$$

As for the initial conditions, $A_2B_3C_2D_2E_2F_2G_2H_2$, the MPCI is predicted in the same way, as

$$\begin{aligned}\mathrm{MCPI}_{\mathrm{initial}} &= A_2 + B_3 + C_2 + D_2 + E_2 + F_2 \\ &\quad + G_2 + H_2 - 7 \times \mathrm{MCPI}_{\mathrm{average}} \\ &= 0.1974 + 0.3369 + 0.2701 \\ &\quad + 0.3362 + 0.3573 + 0.3797 \\ &\quad + 0.2478 + 0.3217 - 7 \times 0.3140 \\ &= 0.2491,\end{aligned}$$

where $\mathrm{MPCI}_{\mathrm{average}}$ represents the average effects of the overall control factors. It can be seen that the MPCI value of the optimal conditions has an increase of approximately 0.5103 over the initial conditions.

6.7 *Confirmation run*

Table 13 is a comparison of the confirmation run between the initial and the optimal conditions. It is observed that the confirmed MPCI 0.7903 for the optimal condition is still the best when compared to all the results in Table 10. Moreover, a similar observation is made of the three individual performance evaluations. The actual gain is 0.601 in MPCI; this is close to the predicted 0.5103, and this indicates that the best combination of the control factors level setting is robust enough against the noise effects and results in high reproducibility.

7. Conclusions

The research concludes that application of the Taguchi method, coupled with principal component analysis, is effective and efficient, and has helped to develop an optimal LED pocket-sized projection display system in size, uniformity and light efficiency, where the performance characteristics have been successfully optimized to meet our expectations. According to the results, the following summaries can be put forward and some conclusions drawn.

1. The optimized control factor settings are correctly defined: A1 (lens specifications: type I), B3 (lens length: overall length: from first element to DMD), C1 (X-CUB semi-aperture: 8 mm), D3 (integrator length: 36.6 mm), E2 (integrator width: 3.5 mm), F2 (TIR prism: entering semi-diameter: 11 mm), G1 (TIR prism air-gap: 1.0024 mm), and H1 (DMD location: −0.5 mm).
2. The most important contributors to the MPCI identified by both the response table/graph and the ANOVA analysis are: factor A (lens specifications), factor D (integrator length), factor F (TIR Prism Entering semi-diameter), and factor G (TIR Prism Air-gap). They account for about 71.82% of the total variance.
3. Compared to the initial design, the optimized parameter design is able to improve the luminous flux by 11.46 dB, the illumination uniformity by 3.14 dB, and the packing size by 0.976 dB.
4. The optimized design parameters lead to an actual gain of 0.7903 in MPCI which is very close to the predicted 0.7594. This shows very good reproducibility and confirms the success of the experiment.
5. According to the explanatory powers through PCA, the MPC of the LED pocket-sized projection display system parameters can be evaluated by an integrated total PC score, which can be mathematically expressed as $\mathrm{MPCI} = 0.453Y_1 + 0.333Y_2 + 0.215Y_3$.

References

1) M. Keuper, G. Harbers, and S. Paolini: SID Microdisplay Dig. Pap., Westminster, Colorado, 2003.
2) H. Kanayama, M. Maeda, T. Miwa, T. Ikeda, H. Murata, and K. Chihara: Consumer Electronics, ICCE Dig. Tech. Pap., 2006, p. 127.
3) T. Annen, W. Matusik, H. Pfister, H. P. Seidel, and M. Zwicker: Proc. XIII SPIE Conf. Stereoscopic Displays and Virtual Reality Systems, 2006, p. 231.
4) J. Konrad and K. Agniel: IEEE Trans. Image Process. **15** (2006) 128.
5) M. Keuper, G. Harbers, and S. Paolini: Proc. IDW, 2002, p. 5-1-504.
6) P. Beardsley, C. Forlines, R. Raskar, and J. VanBaar: IEEE Computer Society Conf. Computer Vision and Pattern Recognition (CVPR), 2005, Vol. 3, p. 112.
7) M. Keuper, G. Harbers, and S. Paolini: SID Microdisplay Dig.

Pap., Westminster, Colorado, 2002.
8) M. H. Keuper, S. Paolini, G. Harbers, and P. Tsang: SID Int. Symp. Dig. Tech. Pap. **34** (2003) 713.
9) H. Edwards and S. Matthew: *Projection Displays* (Wiley, New York, 1999) 2nd ed., p. 71.
10) Y. Wu and A. Wu: *Taguchi Methods for Robust Design* (ASME Press, New York, 2000) 1st ed., p. 125.
11) A. Alsaran, A. Celik, and C. Celik: Surf. Coat. Technol. **106** (2002) 219.
12) L. J. Yang: J. Mater. Process. Technol. **113** (2001) 521.
13) E. A. Elsayed and A. Chen: Int. J. Prod. Res. **31** (1993) 1117.
14) L. I. Tong, C. T. Su, and C. H. Wang: Int. J. Qual. Reliab. Manage. **14** (1997) 367.
15) R. K. Tomita, S. W. Park, and O. Sotomayor: Chem. Eng. J. **90** (2002) 283.
16) L. Yoder: Texas Instruments, 2002, http://www.dlp.com
17) Y. C. Fang, W. T. Lin, and H. L. Tsai: Proc. SPIE **6342** (2006) 63420Z.
18) Lumileds: Secondary Optics Design Considerations for Super-Flux LEDs, 2002.
19) Lumileds: Power light source Luxeon™ III Emitter, 2003.
20) Lumileds: Power light source Luxeon™ Emitter, 2006.