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Synchronization of Chaotic Oscillations in Mutually Coupled Semiconductor Lasers

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Synchronization of chaotic oscillations in mutually coupled semiconductor lasers is experimentally investigated. Synchronization of chaotic outputs from mutually injected lasers is observed not only in low frequency fluctuation regimes but also in high frequency fluctuation regions on the nano-second time scale. It is shown that the synchronization of our results is based not on complete chaos synchronization but on injection phenomena in laser systems, so called generalized chaos synchronization.

Key words: chaos synchronization, semiconductor lasers, optical injection, low frequency fluctuations

1. Introduction

Synchronization of chaotic oscillations in coupled nonlinear systems is a hot issue in chaos research.[1)] Since the prediction by Pecora and Carroll in 1990,[2)] synchronization of chaotic oscillations between two nonlinear systems has been reported in various fields of engineering. Experimental synchronization between two chaotic laser systems has already been demonstrated in solid state lasers[3,4)] and CO_2 lasers.[5)] Chaos synchronization in lasers is very important from the viewpoint of practical applications for secure optical communications and phase locking in semiconductor laser arrays. A configuration of unidirectional coupling between two lasers is employed for secure communication systems. In secure communications based on chaos, a message with a chaotic carrier is sent to a receiver. In the receiver, only the chaotic carrier from the transmitter is duplicated by chaos synchronization and, thus, the message can be easily decoded. Chaotic secure communications have been reported in several laser systems.[6–8)] On the other hand, the study of mutually coupled laser systems is an important issue for phase locking and control of laser arrays.[9)]

In semiconductor lasers, few experimental studies for synchronization of chaotic oscillations have been conducted,[10,11)] although there have been a number of numerical simulations.[12,13)] Synchronization of chaotic oscillations in two external cavity semiconductor lasers has been reported in low frequency fluctuation (LFF) regions less than several tens of MHz (LFF regimes)[11)] and also in high frequency ranges over the order of GHz (ordinary chaotic oscillations related to laser relaxation oscillation).[14,15)] On the other hand, chaotic fluctuations in a semiconductor laser are easily observed by the introduction of optical injection into the laser cavity. Instabilities and chaos induced by optical injection into semiconductor lasers have also been studied by many researchers.[16,17)] In most of those works, a laser coupled unidirectionally to the other laser, so that one of two lasers was always the master and the other was the slave. Hohl *et al.*[18,19)] discussed synchronization of chaotic oscillations in mutually coupled semiconductor lasers. They have done numerical simulations and some experiments, but they did not directly observe synchronized chaotic waveforms. Recently, Garcia-Ojalvo *et al.*[9)] theoretically investigated the phase locking nature in mutually coupled semiconductor laser arrays and discussed the importance of that study.

In this paper, we experimentally demonstrate synchronization of chaotic oscillations in mutually coupled semiconductor lasers both in LFF regimes and high frequency ranges over nano-second time variations. LFF in a semiconductor laser is one of the laser instabilities and chaotic routes induced by external optical feedback or optical injection. Since the frequency of LFFs is low enough to be easily observed by currently available electronic equipment, they are ideally suited to investigation of the dynamic properties of chaotic oscillations and synchronization. At first, we study synchronization of chaotic oscillations in LFF regimes. The synchronization of chaotic oscillations in two mutually coupled systems occurs under a master-slave configuration and is considered to be a generalized chaos synchronization scheme,[20)] since the delay time between the waveforms of the two laser outputs is equal to the time corresponding to a one-way trip from one of the lasers to the other. We also observe switching of the delay between the two output waveforms depending on the laser parameters. Finally, we study synchronization of chaotic oscillations for mutually coupled semiconductor lasers in higher frequency fluctuations over the range of GHz. For both regimes, chaotic outputs from the lasers are synchronized under appropriate parameter conditions.

2. Experiments and Results

The experimental setup is shown in Fig. 1. The semiconductor lasers used in our experiments were intrinsically single mode AlGaInP MQW diode lasers (Mitsubishi ML1412R) that oscillated at a wavelength of 690 nm and a maximum power of 30 mW. The cavity length of the laser was 650 μm. Since the laser was a high power visible laser, the intensity reflectivity of the laser front facet was as low as 10% and that of the rear facet was 80%. The two lasers were mutually coupled through a neutral density filter NDF to control the injection ratios, thus the laser LD1 was injected by laser LD2 and LD2

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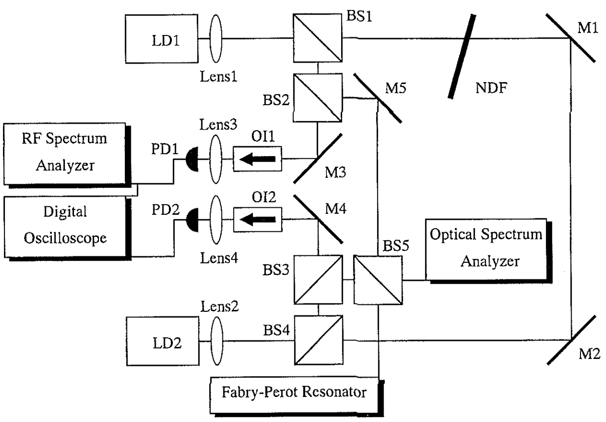

Fig. 1. Experimental setup of mutually injected system. LD: laser diode, BS: beam splitter, M: mirror, OI: optical isolator, PD: photodetector, NDF: neutral density filter.

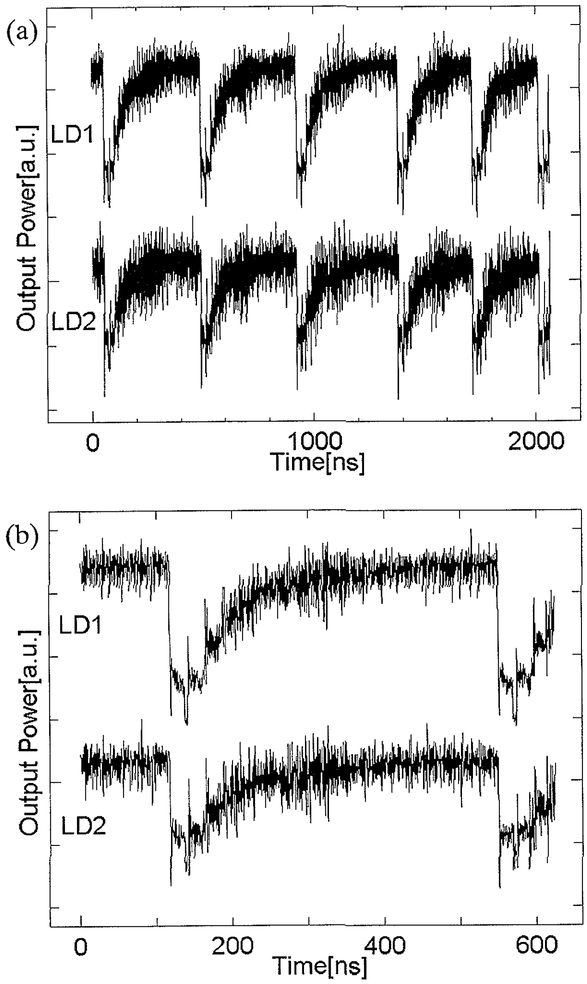

Fig. 2. Synchronized chaotic oscillations of two lasers in a LFF regime at $I_1 = 29.3$ and $I_2 = 28.4$ mA. (a) Chaotic oscillations and (b) one-shot of enlarged LFFs. Occurrence of power drops in LD2 is delayed from that in LD1. (The time delay is compensated in the figure.)

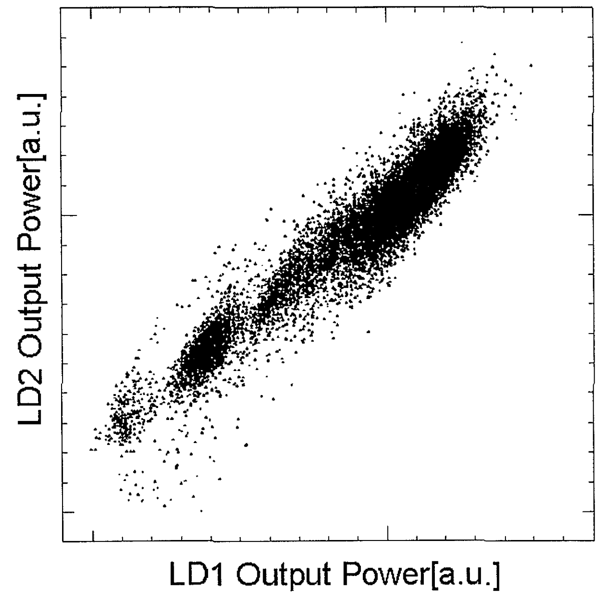

Fig. 3. Correlation plot for Fig. 2.

was also injected by LD1. The free-running threshold currents of LD1 and LD2 were 27.0 and 27.1 mA, respectively. The bias injection currents of the two lasers were controlled by stabilized current source drivers and laser temperature was stabilized at 25.0 and 24.9°C, respectively, by automatic temperature control circuits. At and near those temperatures, no internal mode hopping originating from temperature fluctuations was observed. The two lasers used in our experiments had device parameters with very similar characteristics. The lasers could be easily tuned to the same oscillation frequencies by changing the injection currents and the difference between the injection currents at the tuned frequency was very small. The slope efficiencies were 0.70 and 0.69 W/A for LD1 and LD2, respectively, and the relaxation oscillation frequencies of the two lasers were also almost the same, ~ 1 GHz at the threshold.

The coupling length between the two lasers was 3.48 m which corresponds to a time delay of 11.5 ns. Within certain bias injection currents and optical injection levels the output powers from the two lasers showed chaotic oscillations. In the experiments, the optical injection strengths were calculated from the transmittances and reflectances through optical components in the experimental system. However, the real fractions of the injections into the laser active layers were different from these values owing to the diffraction effect of collimating lenses and other losses of light. The actual levels of the optical injection fractions were roughly estimated to be less than one-tenth of the values. In actuality, the coupling strengths were finely tuned to obtain the best synchronization. The output intensities of both lasers were detected by high-speed photodetectors PD1 and PD2, respectively (New Focus 1537M-LF: bandwidth of 6.0 GHz). The chaotic waveforms were analyzed by a radio frequency (RF) spectrum analyzer (Hewlett-Packard 8595E: bandwidth of 6.5 GHz) and a fast digital oscilloscope (Hewlett-Packard 54845A: bandwidth of 1.5 GHz). The oscillation modes and optical spectra of the two laser outputs were also investigated by an optical spectrum analyzer and a Fabry-Perot interferometer (free spectral range of 10 GHz). The level of the optical injection to each laser was around 10% of averaged chaotic powers for LFF regimes, while it was 1% for high frequency fluctuation regions over the range of GHz. In both cases, synchronization of chaotic oscillations was observed. There was no optical feedback from BSs or PDs to the laser cavities. However, the couplings by the reflections from the laser facets were not negligible when the optical injection level was larger than 10%. The accuracy of synchronization in LFF regimes was thus more or less affected by the reflections.

We first demonstrate synchronization of chaotic oscillations in LFF regimes. Since LFFs are typically observed at low injection current of a semiconductor laser close to its threshold, the bias injection currents for LD1 and LD2 were chosen to be $I_1 = 29.3$ and $I_2 = 28.4$ mA, respectively. The optical injection ratio from LD1 to LD2 was 12.1% and that from LD2 to LD1 was 9.6%. The beam splitters used in the experiments were not exact half beam splitters at the wavelength used, therefore, they were non-symmetrically injected. Figure 2 shows an example of sets of synchronized chaotic waveforms for a LFF regime. From Fig. 2(a), we can see power drops of LFFs at an average frequency of about 0.4 MHz. The output power followed by a drop recovers in a stepwise manner. Actually, the occurrence of each power drop in LD2 is delayed from that in LD1 and the time delay is 11.6 ns. So laser LD2 was injected by laser LD1 and is forced to be synchronized with LD1. The delay time of 11.6 ns is almost equal to the trip time of light corresponding to the coupling length between the two lasers: laser LD1 is thus the master and LD2 becomes the slave under these conditions. This means that the synchronization is originated from so called generalized synchronization in a nonlinear sys-

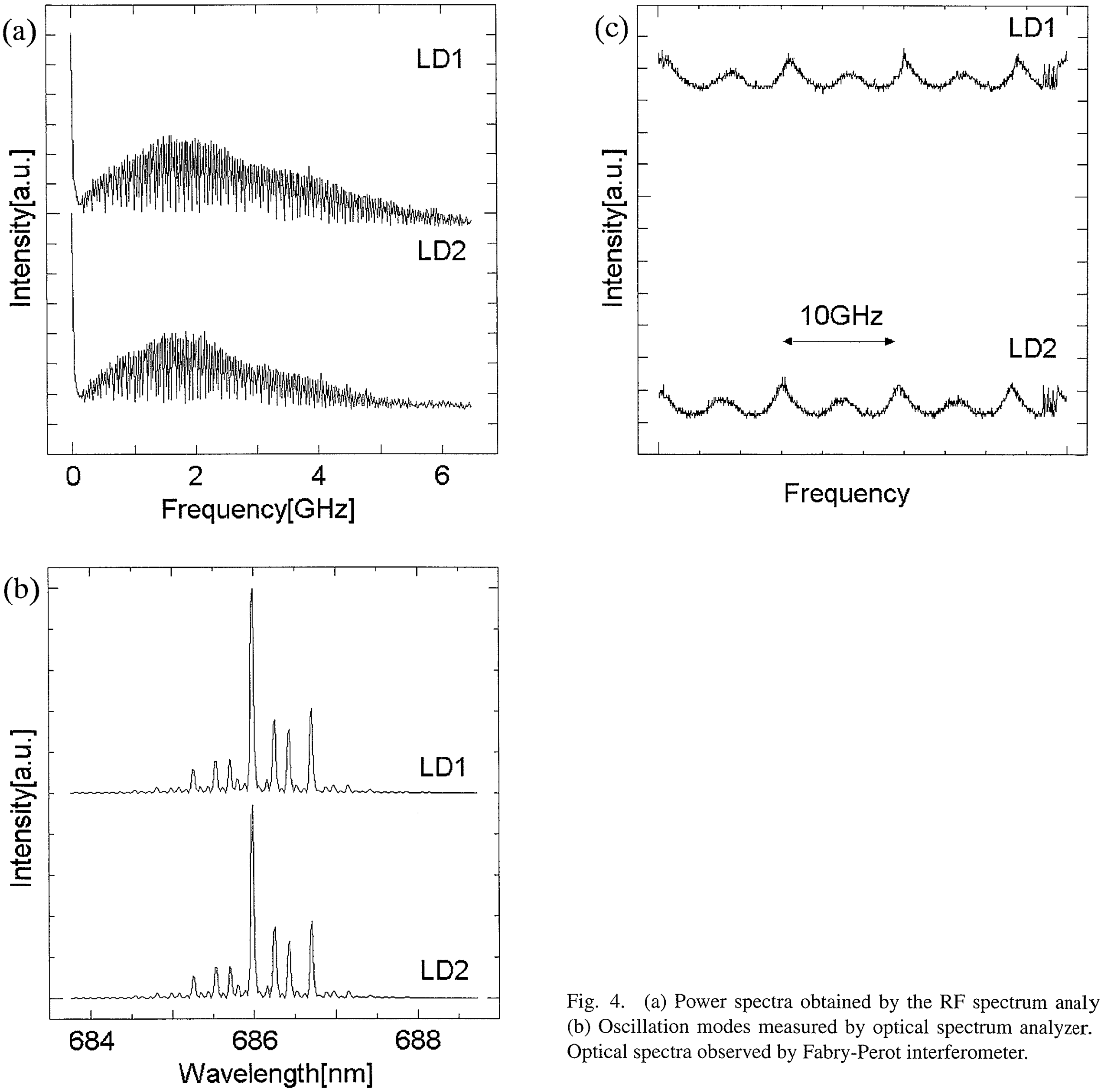

Fig. 4. (a) Power spectra obtained by the RF spectrum analyzer. (b) Oscillation modes measured by optical spectrum analyzer. (c) Optical spectra observed by Fabry-Perot interferometer.

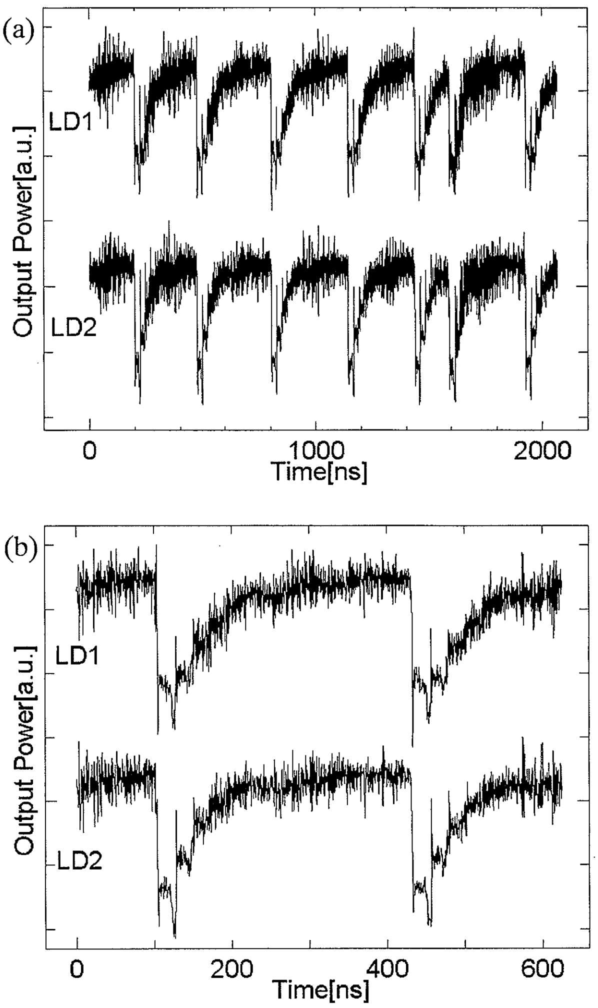

Fig. 5. Synchronized chaotic oscillations of two lasers in a low frequency fluctuation regime at $I_1 = 29.9$ and $I_2 = 28.4\,\mathrm{mA}$. (a) Chaotic oscillations and (b) one-shot of enlarged LFFs. Occurrence of power drops in LD1 is delayed from that in LD2 (Master-slave configuration is reversed compared with that in Fig. 2).

tem (synchronization induced by optical injection or amplification) and not from complete chaos synchronization (chaos synchronization in the mathematical sense which is discussed later). In the figure, the delay is compensated to enhance the resemblance of the output waveforms. One-shot of LFFs is enlarged and displayed in Fig. 2(b). A sudden power drop and the following stepwise power recovery are clearly seen and the fine structures of steps in the recovery process are also identical with each other. Thus, they are synchronized, not only from the occurrences of power drop events but also from the fine structures of the power recovery process between the two laser outputs. The duration of each step in the power recovery process is 23 ns which corresponds to the round trip time of light between the two lasers. LFFs are observed not only in optically injected semiconductor lasers but also in external cavity semiconductor lasers. For LFFs induced by external optical feedback in semiconductor lasers, the duration of steps in the power recovery process is ruled by the round trip time in the external cavity. It is therefore noted that the duration of the power recovery steps in mutually injected semiconductor lasers is the same as that in external cavity semiconductor lasers. Figure 3 shows the correlation plot of the two laser output powers corresponding to Fig. 2(a). Though not perfect, a linear relation between the two laser outputs is well established in the correlation plane.

Under the synchronized state, RF and optical spectra were investigated. Figures 4(a), (b), and (c) show RF spectra, optical spectra detected by the optical spectrum analyzer, and optical spectra measured by the Fabry-Perot interferometer, respectively. The optical frequencies of the two lasers are not exactly equal and they have a detuning of about 2 GHz at isolated oscillations. Figure 4(a) shows RF spectra at the synchronized state but the detuned frequency component is embedded into the noise floor centered at the laser relaxation frequency. Under the mutual coupling, we could not measure the exact detuning between the two laser frequencies due to the broadening of the optical spectra. However, it can be said from the observations of RF and optical spectra that frequency pulling to the master laser occurs as a result of the optical injection. In a LFF region, a semiconductor laser usually oscillates in multi-mode. In Fig. 4(b), the two lasers also oscillated in multi-mode and the two optical spectra were almost equal. At this state, no clear spectral peak was visible in the optical spectra within the range of GHz observed by the Fabry-Perot interferometer and the coherence of the lasers was greatly deteriorated as shown in Fig. 4(c). All spectra showed that the two lasers were synchronized by the mutual injections under a LFF regime.

Figure 5 shows another instance of synchronization of chaotic oscillations. The bias injection current of LD1 was increased to $I_1 = 29.9\,\mathrm{mA}$, while that for LD2 remained unchanged at $I_2 = 28.4\,\mathrm{mA}$. The conversion coefficient of the frequency to the injection current (equivalently, the optical power) for the used lasers was about 2 GHz/mA, so that the increased fraction of the detuning from that in Fig. 2 was roughly estimated to be 1.2 GHz. The injection ratio from LD1 to LD2 was increased to 13.8%, while, that from LD2 to LD1 was decreased to 7.3%. We also observed synchronization of chaotic oscillations between the two laser outputs as shown in Fig. 5(a), but the occurrence of each power drop in LD2 gained 11.6 ns from that in LD1 in this case. Therefore, the master-slave configuration was reversed compared with the case of Fig. 2. The time offset was also compensated in this figure. The reason for the switching can be attributed to the change in the degree of instability for the two lasers. Namely, laser LD1 became rather stable oscillation due to increase of the bias injection current, however the unstable oscillation of laser LD2 remained unchanged. Laser LD1 thus became the slave and the chaotic power from LD2 was injected into LD1. To understand this phenomenon, however, may require. Figure 5(b) shows enlarged waveforms for one-shot of LFFs. Each step of the power recoveries also corresponds to the round trip time of the coupling length. Steps in the power recovery process are well copied from LD2 to LD1. Figure 6 is the correlation plot of the synchronized waveforms corresponding to Fig. 5(a). Except for lower output power regions, a linear relation between the two laser outputs is obtained. A careful look at the waveforms shows that the first steps in the recovery processes followed by the sud-

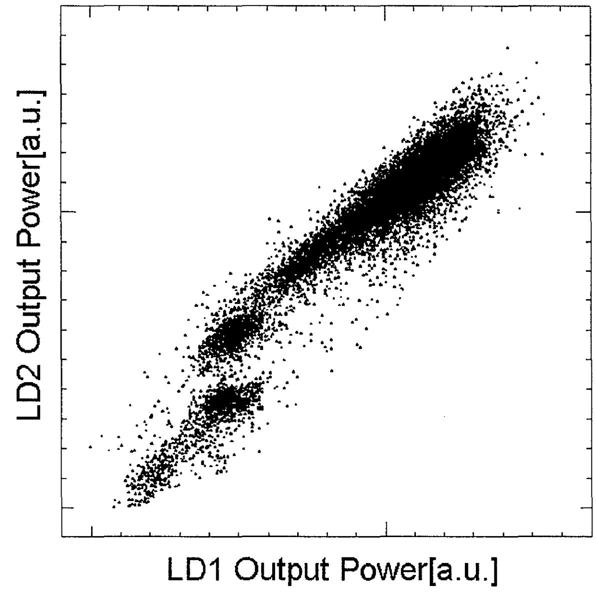

Fig. 6. Correlation plot for Fig. 5.

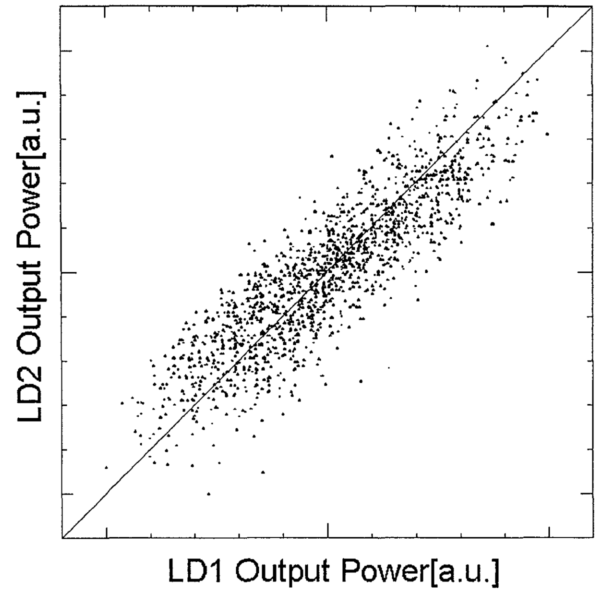

Fig. 8. Correlation plot for Fig. 7.

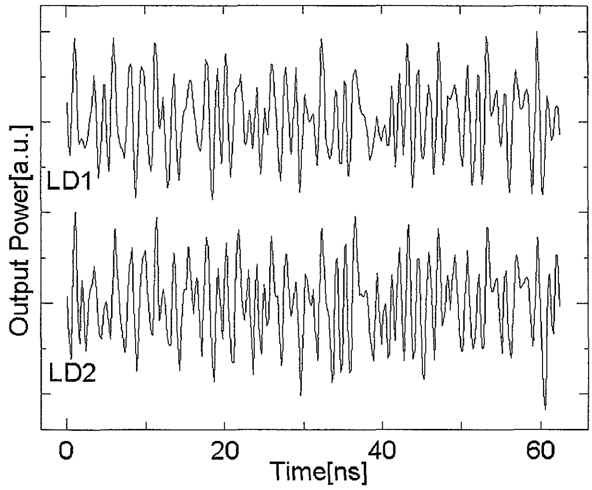

Fig. 7. Synchronized chaotic oscillations of two lasers at high frequency fluctuations over the range of GHz at $I_1 = 32.8$ and $I_2 = 32.0$ mA.

den power drops are not exactly alike. This part corresponds to the transition of the state from the solitary laser level to one of the modes related to the coupling distance between the two lasers. Overall structures of LFFs between the two lasers are almost equal, but the reason for this discontinuity is not clear at present. We have also examined spectra of the two laser outputs and results similar to those in Fig. 2 were obtained. In the RF spectra, we observed spectral peak for the detuning between the two laser oscillations and it was 2.7 GHz. The observed synchronization of chaotic oscillations is also considered as a generalized synchronization of chaos. Chaos synchronization in frequency-detuned external-cavity VCSEL's has been analyzed theoretically by Spencer and Mirasso.[21] They observed synchronization in the discrete points of frequency-detuning, although we could not observe such points in our experiment. Since the synchronization of our results is for frequency-detuned lasers, it is considered to be a generalized synchronization of chaos.

Phenomena involved in LFFs are very convenient and useful to observe a time delay of chaotic waveforms at a synchronized state. However, synchronization of chaotic oscillations in high frequency regions is still important from an application point of view. We demonstrate the synchronization of chaotic oscillations over the range of GHz. To observe fast chaotic oscillations, the bias injection currents were chosen to be rather higher values at $I_1 = 32.8$ and $I_2 = 32.0$ mA for LD1 and LD2, respectively. The detuning between the two lasers was about 2 GHz. The injection ratio from LD1 to LD2 was 1.2% and that from LD2 to LD1 was 1.3%. The other experimental conditions were the same as the previous ones. Figure 7 shows an example of sets of time series for the two laser outputs at a synchronized state. The time offset of the waveforms for laser LD2 is 11.6 ns (the delay is compensated in the figure), so that laser LD2 was injected by LD1. This scheme is also a generalized synchronization of chaotic oscillations. Figure 8 shows the correlation plot for the waveforms of Fig. 7, and we can see a linear relation between the two laser outputs.

Figures 9(a), (b), and (c) show RF spectra, optical spectra detected by the optical spectrum analyzer, and optical spectra measured by the Fabry-Perot interferometer, respectively. In Fig. 9(a), the two spectra are almost equal and the original detuned frequency component is also embedded in the noise floor centered at the laser relaxation frequency. At high frequency chaotic oscillations over GHz range, the lasers also oscillated with multi-mode as shown in Fig. 9(b). At the synchronized state, no clear spectral peak is visible in the optical spectrum in Fig. 9(c) and the coherence of the lasers has been completely destroyed. All spectra in Fig. 9 are almost identical, which also supports the synchronization between the two lasers.

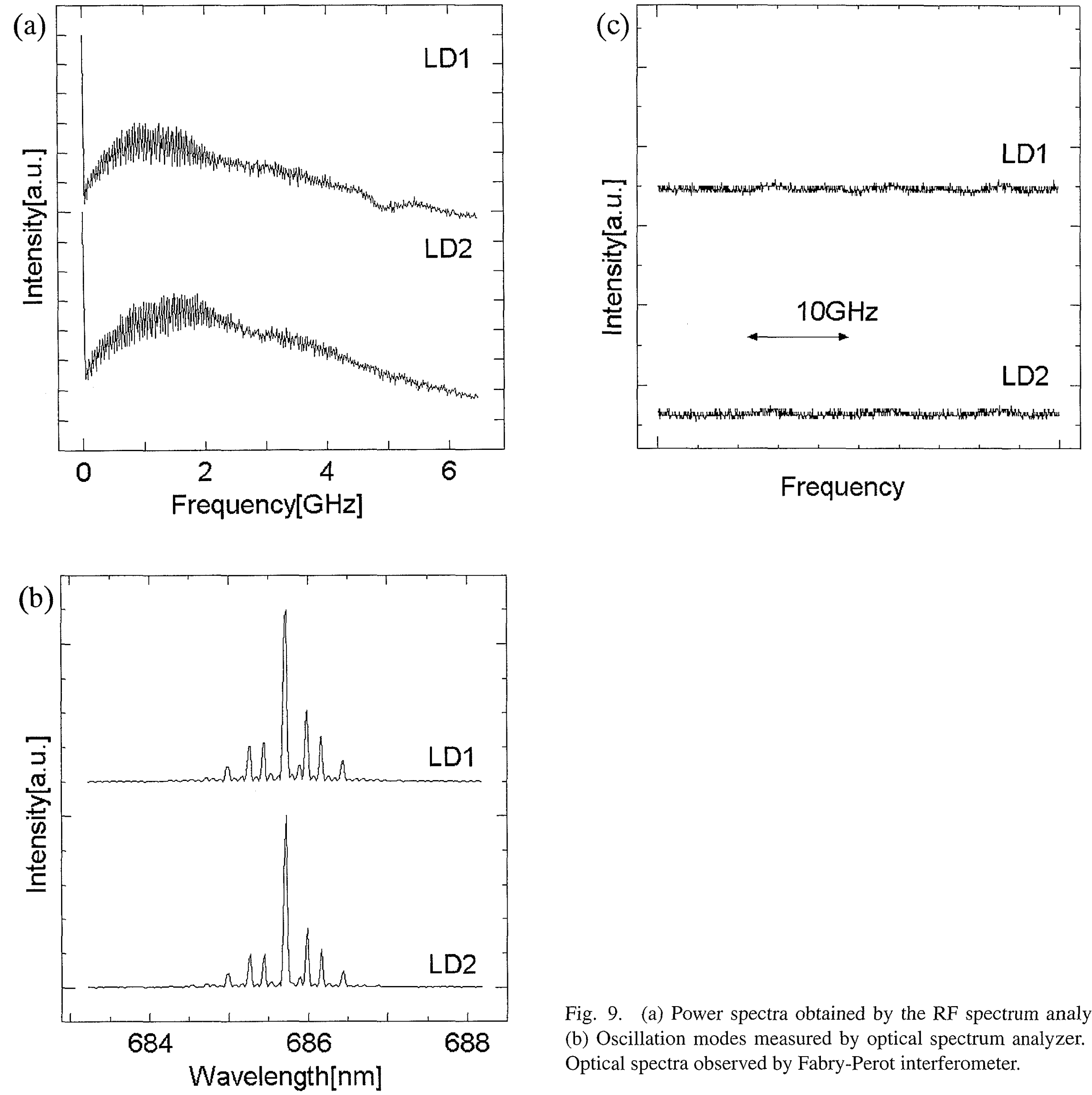

Fig. 9. (a) Power spectra obtained by the RF spectrum analyzer. (b) Oscillation modes measured by optical spectrum analyzer. (c) Optical spectra observed by Fabry-Perot interferometer.

3. Discussion

Here, we briefly discuss the principle of chaotic synchronization in mutually injected semiconductor lasers. The theoretical model of mutually coupled lasers is described by the rate equations with mutually injected terms. One possible solution for chaos synchronization is easily derived when all device parameters including the bias injection currents and oscillation frequencies of the two lasers are identical and the optical injection ratios are equal. Under these conditions, the two laser rate equations become completely the same delayed differential equations and there exists a possibility that they have the same field amplitudes $E_1(t) = E_2(t)$ as a common seed of chaos, where E_1 and E_2 are the field amplitudes of the two lasers. The important point is that the two lasers show the same oscillations without time delay under these conditions. This situation is called "complete chaos synchronization." Complete chaos synchronization (perfect synchronization) in unidirectional coupled external-cavity semiconductor lasers has been reported theoretically by Ahlers *et al.*[13]

However, our results include a time delay that is equal to the transmission time τ_c of light from one laser to the other. Still, the parameters of the two lasers are not identical, for example, the injection currents are different and the two laser frequencies are detuned. The rate equations for such a model must be different from those for a complete case. Therefore, the results in our experiments are not for complete chaos synchronization but for "generalized synchronization of chaotic oscillations" by optical injections.[13] For a generalized synchronization, the relation between the two laser fields is not the same as the complete case. For example, when LD1 leads LD2, $E_1(t-\tau_c) \propto E_2(t)$. In spite of parameter mismatches, the two lasers synchronized with each other even for small optical injection strengths around 1% (the actual injection ratio into the laser cavity may be less than 0.1% in optical intensity). The synchronization of chaotic oscillations in our case is somewhat different from usual injection or amplification phenomena in semiconductor lasers, although our system shares common features with the optical injection model, the delayed optical feedback model, and the evanescently coupled model.[20] Furthermore, it should be noted that we can achieve synchronization of chaotic oscillations in laser outputs in spite of a small amount of optical injections. The results described here are for semiconductor lasers with very similar characteristics. We have also tested a pair of other lasers of the same type but having slight device parameter mismatches. Though better synchronization was achieved for the lasers with similar characteristics, synchronization was still possible for those with different ones. This synchronization of chaotic oscillations can be called "generalized synchronization" and is distinguished from "complete synchronization" of chaos in the mathematical sense. Discussion of the difference between complete and generalized chaos synchronization can be found in Ref. 20). Generalized synchronization of chaotic oscillations in semiconductor lasers has already been reported for external optical feedback systems.[11,12,14,15]

4. Conclusions

We have done experimental studies of synchronization of chaotic oscillations for mutually injected semiconductor lasers. Synchronization of chaos in delay differential systems is an interesting subject from a basic research viewpoint. Synchronization of chaotic oscillations is classified into two categories, complete and generalized synchronization of chaos. We have experimentally demonstrated synchronization of chaotic oscillations both in LFF regimes on the order of MHz and high frequency fluctuation regions over the range of GHz. Our results were for generalized synchronization, since the delay between the two lasers was equal to the transmission time of light from one laser to the other. Investigations of the differences between complete and generalized chaos synchronization and interpretation of the phenomena are interesting issues. The study on the robustness of synchronization for the parameter mismatches is another important problem and is left for the future. The dynamics studied here also gives useful information on the applications of phase locking and control in mutually coupled semiconductor laser systems.

Acknowledgments

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