Fractal analysis of the Oyo River, cave systems, and topography of the Gunungsewu karst area, central Java, Indonesia

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Abstract Gunungsewu, the southernmost subzone of the Southern Mountains, central Java, is a classic karst terrain, bounded by the Oyo River on the north and the Indian Ocean on the south. In this study, the Oyo River, the main drainage in the area, is divided into 14 segments, and the fractal dimension of each segment is determined by the box-counting method. The 14 segments show various values of fractal dimensions, which are controlled by lithology and geologic structures of the area that is being dissected by the river. Easily eroded lithologies have larger fractal dimensions, and the value changes abruptly when the river crosses faults. Fractal dimensions of six underground rivers in the Bribin, Sodong, Semuluh, Jomblang, Soga, and Sumurup caves, and the surface topography above the caves were also determined. The caves have fractal dimensions that range from 1.043 ± 0.01 to 1.08 ± 0.01; the surface topography has fractal dimensions that range from 1.49 ± 0.01 to 1.732 ± 0.01. The fractal dimension of an underground river is proportional to the fractal dimension of the surface topography over the passage. Larger fractal dimensions of underground rivers are associated with smaller flow rates of the rivers.

Résumé Gunungsewu, la zone la plus méridionale des Monts du Sud, dans le centre de Java, est une région karstique classique, limitée par la rivière Oyo au nord et par l'Océan Indien au sud. Dans cette étude, la rivière Oyo, principal axe de drainage de la région, est divisée en 14 segments; la dimension fractale de chacun de ces segments est déterminée par la méthode du "comptage de boîtes" (box-counting). Les 14 segments présentent des valeurs variées de dimensions fractales, contrôlées par la lithologie et les structures géologiques de la région traversée par la rivière. Les lithologies facilement érodées présentent des dimensions fractales plus grandes; les valeurs changent brutalement quand la rivière recoupe des failles. Les dimensions fractales de six rivières souterraines, dans les grottes de Bribin, Sodong, Semuluh, Jomblang, Soga, et Sumurup et la topographie de la surface au-dessus des grottes ont aussi été calculées. Les grottes possèdent des dimensions fractales comprises entre 1.043 ± 0.01 et 1.08 ± 0.01; la topographie de surface a des dimensions fractales entre 1.49 ± 0.01 et 1.732 ± 0.01. La dimension fractale d'une rivière souterraine est proportionnelle à la dimension fractale de la topographie de la surface située au-dessus. Les plus grandes dimensions fractales des rivières souterraines sont associées à des débits plus faibles des rivières.

Resumen Gunungsewu es un terreno típicamente kárstico, situado en la zona meridional de las Montañas del Sur, en el centro de Java,. Está limitado al norte por el río Oyo y al sur por el Océano Índico. Para este estudio, se dividió el río Oyo en 14 segmentos, y se determinó la dimensión fractal de cada uno de ellos mediante el método "box-counting" (de volumen fijo o masa fija). Los 14 segmentos tienen dimensiones fractales diferentes que están condicionados por la litología y las estructuras geológicas drenadas por el río. Las capas fácilmente erosionables presentan dimensiones fractales mayores, y éstas varían abruptamente cuando el río intersecta fallas. También se determinó las dimensiones fractales de seis ríos subterráneos en las cuevas de Bribin, Sodong, Semuluh, Jomblang, Soga y Sumurup, así como la topografía, en superficie, de dichas cuevas. Las cuevas tienen dimensiones fractales comprendidas entre 1.043 ± 0.01 y 1.08 ± 0.01; la topografía en superficie, entre 1.49 ± 0.01 y 1.732 ± 0.01. La dimensión fractal de un río subterráneo es proporcional a la dimensión fractal correspondiente de la topografía en superficie. Además, cuanto más bajo es el caudal de un río subterráneo mayor es su dimensión fractal.

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Introduction

The Gunungsewu area is a subzone of the Southern Mountains, central Java, Indonesia, and is about 50 km southeast of Yogyakarta. The area is occupied by about 200,000 people and is well known for its water scarcity during the dry season, because most rainwater infiltrates rather than runs off. The Gunungsewu area is underlain by limestones that form a classic conical karst topography, bounded by the Oyo River on the north and the Indian Ocean on the south. The hydrogeology of the karst terrain is generally controlled by the secondary-porosity network and subterranean drainage system. Locations are shown in *Figure 1.*

The surface and subsurface stream flows are complex and are likely to be fractal geometry (Korvin 1992; Bunde and Havlin 1994). The objectives of this study were to identify the relationship between the patterns of surface streams and geologic factors; to correlate the pattern of topography and subsurface streams, using fractal analysis, and to compare the rate of water that is flowing in the subsurface streams. This work was done to identify an alternative method for predicting the water potential of subsurface flow based on its surface performance.

Figure 1 Location and physiographic map of the study area in Java, Indonesia

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The Oyo River is the main surface flow draining the Southern Mountains, central Java. It flows from east to west, originating from springs of Mount Kukusan, joining the Opak River, and discharging to the Indian Ocean. The fractal length of this river, based upon a topographic map at 1 : 50,000 scale, is 135 km (Kusumayudha et al. 1997). Based on the pattern and the general flow direction, the Oyo River was divided into 14 segments. The fractal dimension of each segment was determined by the box-counting method, utilizing air photos and topographic maps.

Method of Study: Fractal Analysis

Mandelbrot (1983) used the word "fractal" to describe objects that are scale invariant. In a fractal, simple shapes grow more complex as the shape is repeated in miniature around the edges of the first shape of (Xie 1993). Smaller versions of the shape grow out these smaller shapes, and so on to infinitesimal scale. The end result is infinite, swirling, and complex.

Many natural objects and phenomena have fractal characteristics. They include, for example, clouds, waves, coastlines, river flows, mountain chains, joint patterns, and crystal growth. In a fractal, parts are reminiscent of the whole (Sahimi and Yortsos 1990). Other important fractal characteristics are self-similarity, self-affinity, self-inverse, and self-squaring (Peitgen et al. 1992). Because of these properties, fractals can unravel the geometry of a natural object into its basic elements. The fractal scaling system is specified by a non-integer number called the fractal dimension (Mandelbrot 1983), which can be used to quantify the degree of fractal irregularity (Sukmono 1996). Determination of the fractal dimension is useful because it can provide insights regarding the origin or process-generation of the fractal object.

In hydrogeology, fractal concepts are applicable to problems concerning the spatial distribution (heterogeneity) of properties such as porosity. Sahimi and Yortsos (1990) demonstrate that typical porosity logs are fractional Gaussian noises (fGn), whereas areal distributions of permeability are fractional Brownian motions (fBm). Fractal analysis also can be utilized to help delineate the distribution of carbonate aquifers on a map, based on the fractal characteristic of the limestone porosity (Kusumayudha et al. 1998).

Several methods exist to determine a fractal dimension, e.g., similarity method, cantor-dust method, balls-covering method, sandbox method, and box-counting method (Mandelbrot 1983). When the fractal dimension is derived by the box-counting method, it is called box dimension. The conventional box-counting method can be used for statistical self-similar fractals or statistical self-affinity fractals. The method is done by drawing grids with certain lengths (r) over the fractal object. Then the fractal dimension (D) is determined using the equation:

$$D = \lim_{r \to \infty} \frac{\log Nr(F)}{-\log r} \quad (1)$$

where $Nr(F)$ is the number of boxes that cover the fractal set (F), and r is the length of the box side. The computation of $Nr(F)$ is repeated by changing the length of the box side (r), so that r approaches zero. $Nr(F)$ values and r are plotted on a log-log graph to derive the fractal dimension, e.g., the slope of the plot (Tricot 1996).

Geology and Hydrogeology

The Southern Mountains of central Java consist of three physiographic subzones, i.e., the Baturagung Range, the Panggung Massive, and the Plopoh Range in the north; the Wonosari Plateau in the central area; and the Gunungsewu subzone in the south (Van Bemmelen 1949; *Figure 1*). The geology of the study area is shown in *Figure 2.*

The study area is principally underlain by the Gunungsewu Group, which includes the Oyo Formation, the Wonosari Formation, and the Kepek Formation. These units consist predominantly of calcareous sandstones, tuffaceous sandstones, reefs, bedded limestones, and marl of middle to late Miocene age (Suyoto 1994). This carbonate group is underlain by a unit of volcanic sediments called the Besole Group (Suyoto 1994). The Besole Group consists of the Semilir Formation of tuff, tuffaceous sandstones, pumice, and breccia; and the Nglanggran Formation of breccia, agglomerates, lava deposits, and sandstones of Oligocene to early Miocene age. The Sambipitu Formation, which occurs in places between the Gunungsewu Group and Besole Group, comprises siltstone, marl, and tuff of middle to late Miocene age.

The geologic structure of the Gunungsewu area is homoclinal, regionally dipping southward. The region is also dissected by faults that strike northwest and northeast. A syncline is in the center part of the Southern Mountains with a northeast-trending axis (*Figure 2*). Limestones in the Gunungsewu area form two types of aquifers, karstic and non-karstic. The karstic aquifer is composed of karstified limestones, whereas the non-karstic aquifer is composed of calichified limestones (caliche).

In the northern part of the Gunungsewu area, where bioclastic limestones occur, the water table is 5–10 m deep. This depth increases abruptly and is 150 m or more in the south, which is underlain by reef limestones. The presence of caliche commonly results in the occurrence of perched water. Areas of shallow groundwater and areas of deep groundwater in the study area are in general separated by faults, which act as seals.

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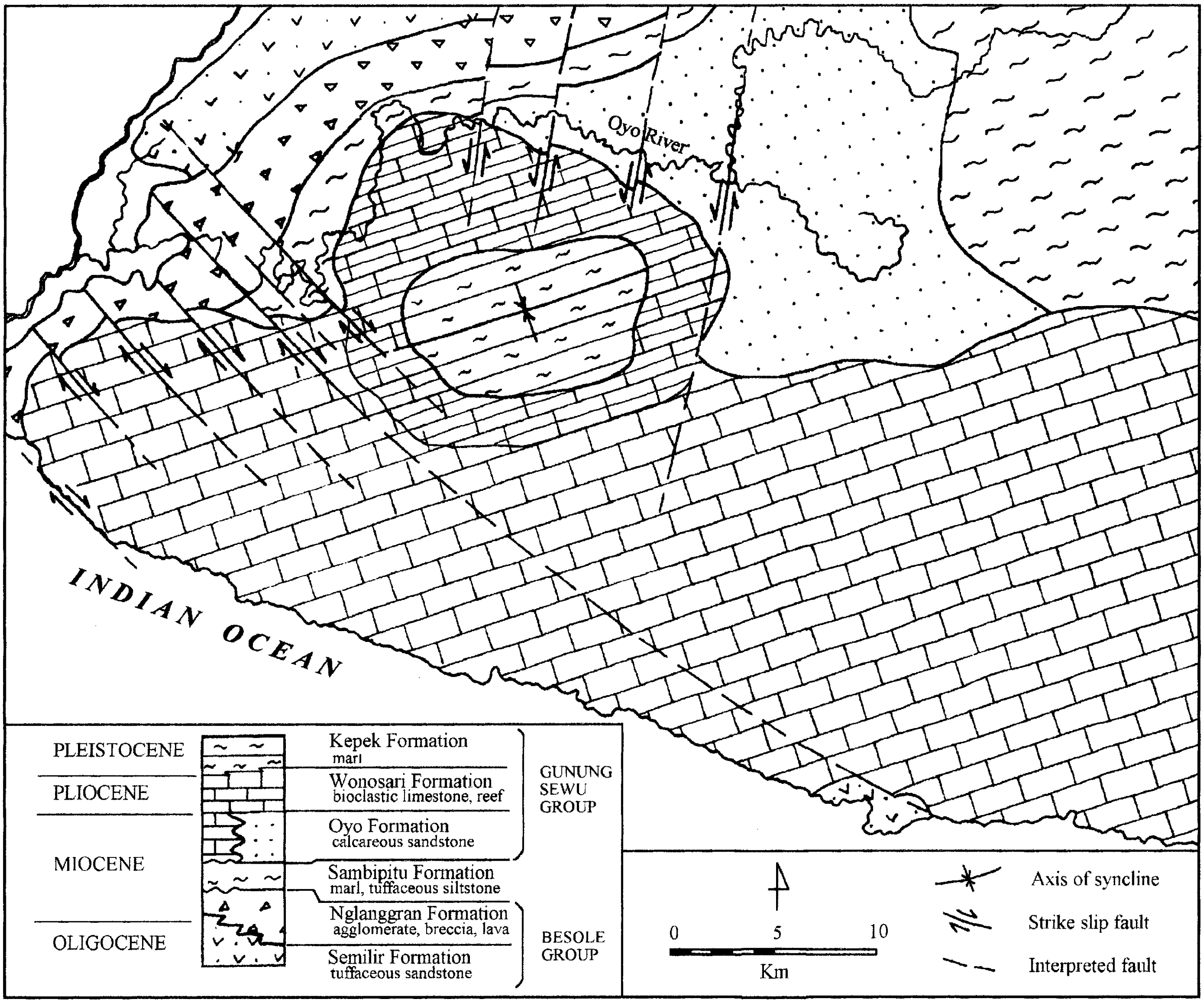

Figure 2 Geology of the Gunungsewu area

Results

Fractal of the Oyo River

The Oyo River was divided into 14 segments, based on (1) the meandering pattern, expressed as the ratio between amplitude (A) and wavelength (L) of the tortuosity; and (2) the general direction of flow. The fractal dimension of each segment is given in *Table 1* and displayed in *Figure 3.*

All segments of the Oyo River have about the same gradient of less than 1%. Therefore, the influence of gradient factor in this analysis is ignored. Segments 1 and 2 flow on volcanic rocks of Nglang-

Table 1 Fractal analysis of the Oyo River

Lithology	Formation	Geologic structure	Fractal dimension of flow pattern (D)
Breccia, volcanic sandstone	Nglanggran	–	1.02 ± 0.01
		Fault (parallel)	1.18 ± 0.02
Massive and bedded limestones	Wonosari	–	1.07 ± 0.01 to 1.10 ± 0.01
		Fault	1.27 ± 0.01
Sandy, marly, tuffaceous limestone	Oyo	–	1.04 ± 0.01 to 1.07 ± 0.01
		Fault zone (parallel)	1.33 ± 0.01
Marl	Sambipitu	–	1.13 ± 0.01 to 1.16 ± 0.01
		Joints/fault	1.51 ± 0.01

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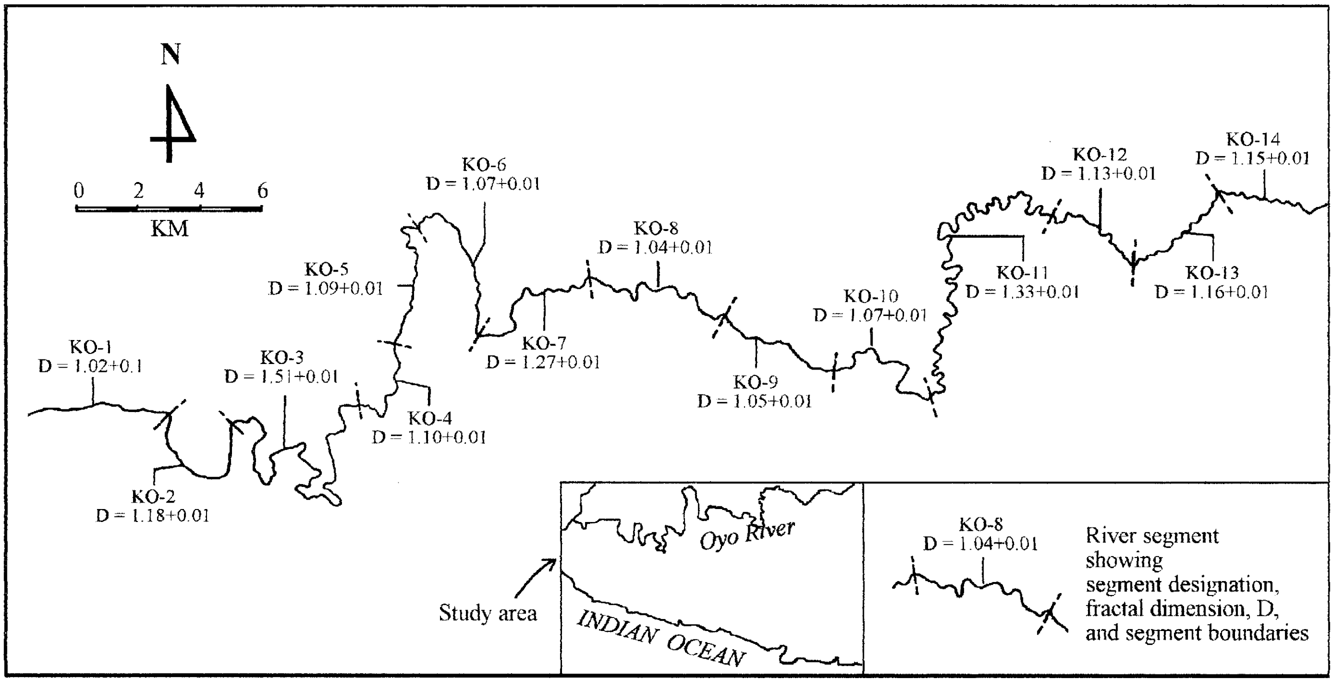

Figure 3 Segments of the Oyo River and their fractal dimensions (D)

gran Formation and have fractal dimension (D) = 1.51 ± 0.01. This site is traversed by at least two faults. Segments 4, 5, 6, and 7 cross limestones of the Wonosari Formation; D = 1.10 ± 0.02, 1.07 ± 0.01, and 1.27 ± 0.01, respectively. A part of segment 7 crosses a fault zone (*Figure 2*). Segments 8, 9, 10, and 11 overlie calcarenites and sandy limestones of the Oyo Formation. Segment 11 also crosses a large fault zone. The fractal dimensions of segments 8, 9, and 10 are 1.04 ± 0.02, 1.05 ± 0.02, and 1.07 ± 0.01, respectively, whereas the fractal dimension of segment 11 is 1.33 ± 0.01. Segments 1, 13, and 14 overlie the Sambipitu Formation and have fractal dimensions 1.13 ± 0.01, 1.16 ± 0.01, and 1.15 ± 0.01, respectively.

Fractal of Subsurface Rivers

There are 253 underground rivers in the Gunungsewu area, as mapped by Sir Macdonald and Partners (1984). For this study, rivers in six caves were selected as the samples for analysis: the Bribin River, the Sodong River, the Semuluh River, the Jomblang River, the Soga River, and the Sumurup River. Locations are shown in *Figure 4.* Using speleologic maps, such as the example shown in *Figure 5,* the fractal dimensions of the flow patterns of the caves were determined. The plots of box-counting are shown in *Figure 6.* Fractal dimensions and average flow rates of the underground rivers are given in *Table 2.* The data in *Table 2* indicate that the order of increasing value of fractal dimension for the subsurface river in caves is as follows: Bribin, Sumurup, Semuluh, Soga, Sodong, and Jomblang.

Fractal of Surface Valleys

To analyze the topography of the Gunungsewu area, erosional valley patterns were traced from air-photo images, such as the one shown in *Figure 7.* The fractal dimensions of the valley patterns that overlie each cave being analyzed were also determined. Results are shown in *Table 3* and *Figure 8.* The data in *Table 3* indicate that the order of increasing value of fractal dimension for the valleys overlying the caves is as follows: Bribin, Sumurup, Semuluh, Soga, Sodong, and Jomblang.

Table 2 Fractal dimensions of subsurface flow patterns in the Gunungsewu area

Underground river (cave)	Length of cave (m)	Depth of cave (m)	Flow rate (L/s)	Fractal dimension (D)
Bribin	3900	33	1500	1.043 ± 0.01
Sumurup	1435	58	200	1.046 ± 0.01
Semuluh	1559	72	15	1.057 ± 0.01
Soga	2428	177	7	1.065 ± 0.02
Sodong	2070	90	8	1.070 ± 0.01
Jomblang	3326	134	2	1.080 ± 0.01

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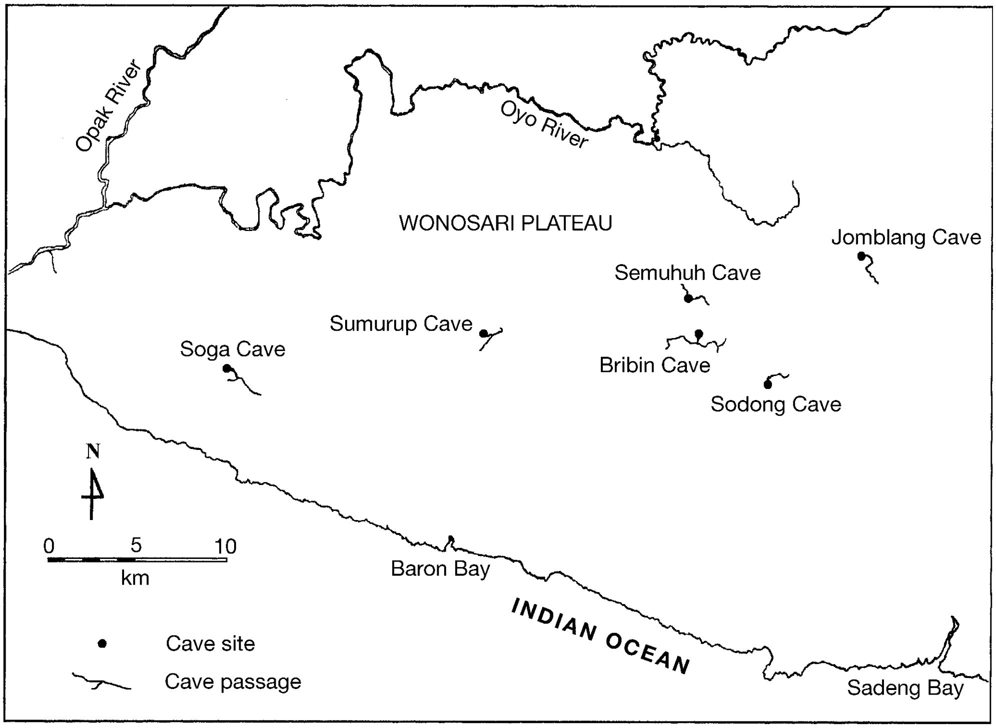

Figure 4 Locations of selected caves of the Gunungsewu area

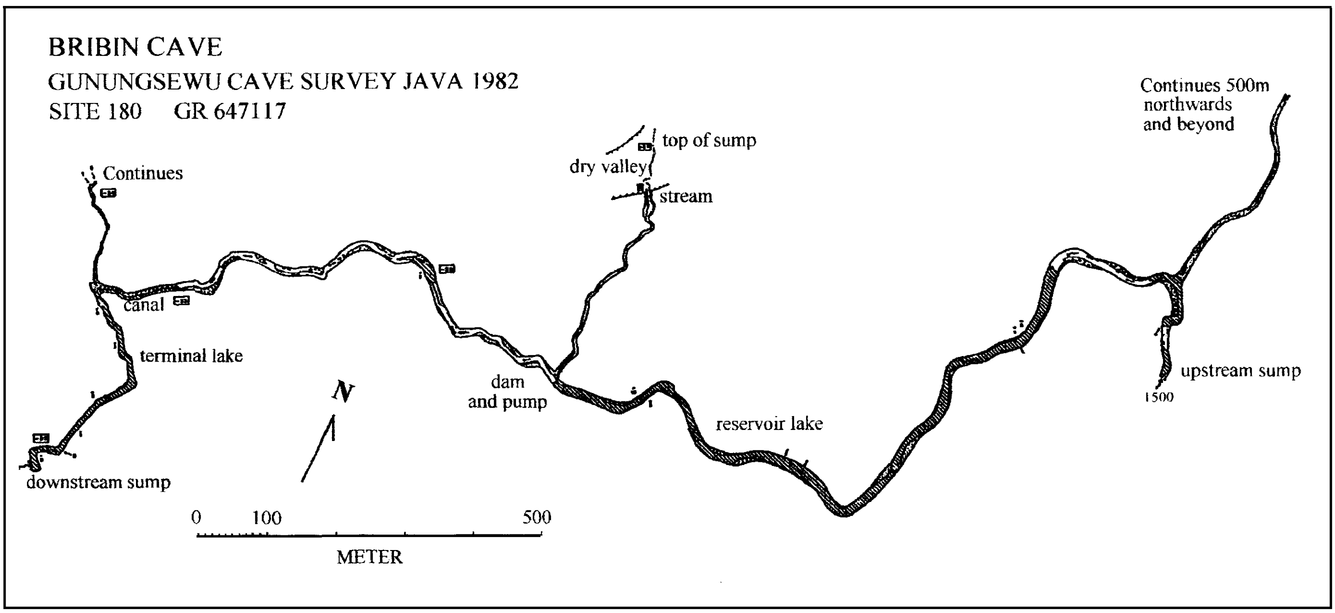

Figure 5 Geologic map of the Bribin Cave (Sir Macdonald and Partners 1984)

Discussion

The fractal dimensions of the Oyo River are smaller in volcanic rocks than in non-volcanic sediments. However, where volcanic rock is transected by a fault, a large fractal dimension results. Marl has fractal dimensions larger than volcanic rocks and limestones (1.13–1.16), and pure limestones have fractal dimensions larger than sandy limestones.

A relationship exists between fractal dimension of flow and either mineral composition or grain size of the formation being dissected by the river, as shown in *Table 4.* The lower the hardness and the finer the grain size of the rock-forming minerals, the easier it is for the rock to be eroded. The easier it is for the rock to be eroded, the larger the fractal dimension being developed by the flow.

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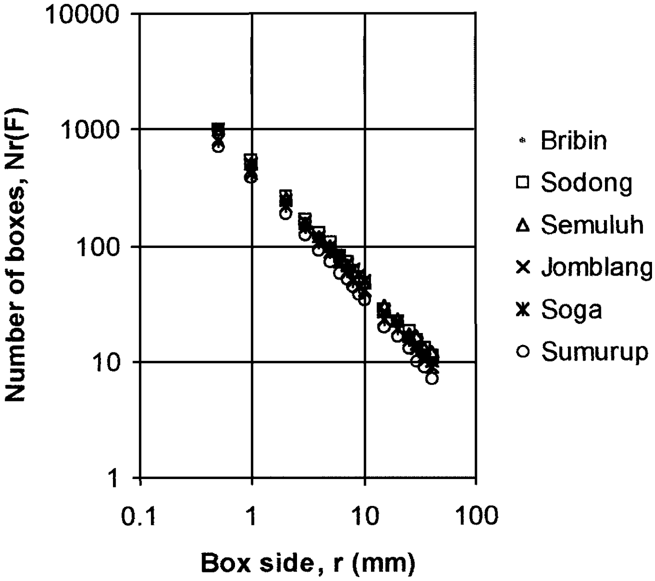

Figure 6 Box-counting plots of underground river-flow pattern

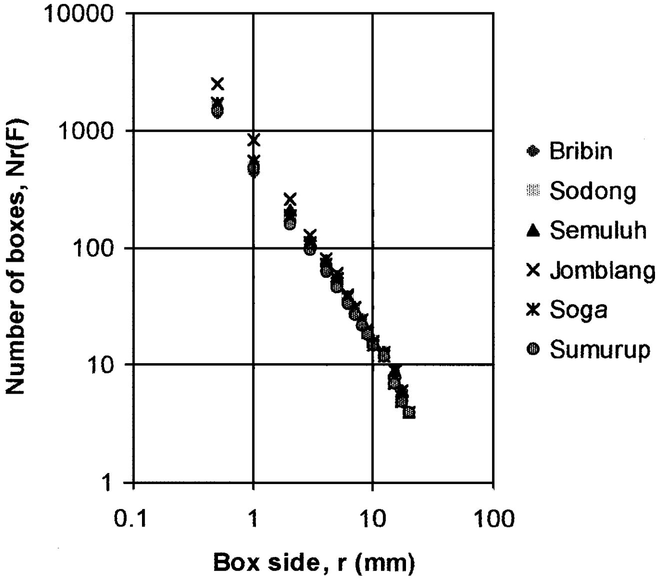

Figure 8 Box-counting plots of surface valleys overlying caves

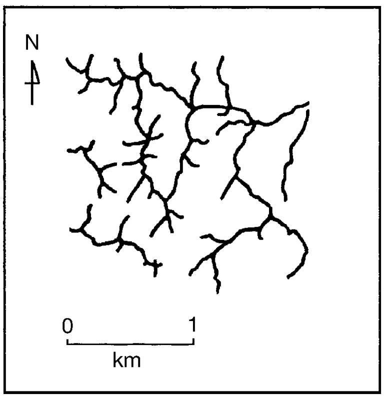

Figure 7 Valley pattern overlying the Bribin Cave

Faults produce brecciation, phyllonitization, or mylonitization that make the rock more susceptible to the formation of irregularities. Thus, areas transected by faults have higher fractal dimensions than areas without faults. When the flow orthogonally crosses a fault zone, the fractal dimension is less than if the flow parallels the fault, as shown by segment 8. The complete results are given in *Table 3.*

Table 3 Fractal dimensions of surface valley patterns

Location (village)	Fractal dimension (D)
Bribin	1.494 ± 0.01
Sumurup	1.505 ± 0.01
Semuluh	1.511 ± 0.01
Soga	1.540 ± 0.01
Sodong	1.552 ± 0.01
Jomblang	1.732 ± 0.01

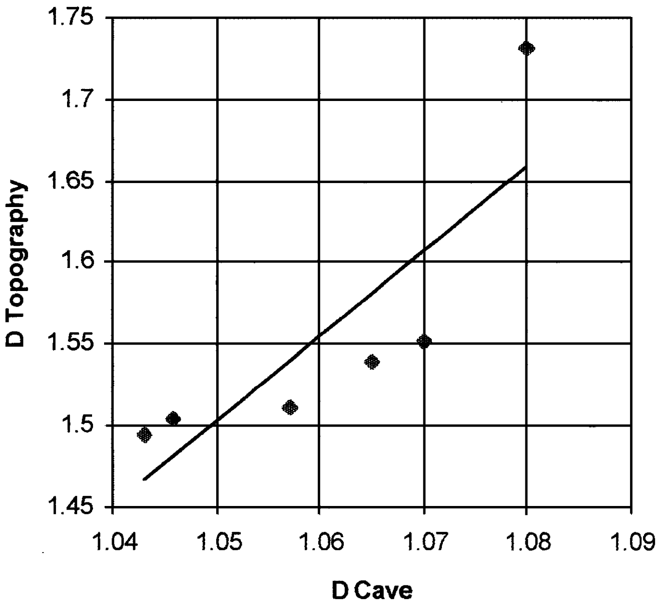

Figure 9 Relation between fractal dimensions of caves and fractal dimensions of overlying surface valleys

The order of increasing fractal dimension is the same for the six rivers in caves (*Table 2*) as it is for the overlying surface valleys (*Table 3*). The relationship is shown in *Figure 9,* which indicates that the fractal dimensions of the two types of features are directly proportional.

Caves with small fractal dimension have large river flows. This relationship is illustrated by two underground rivers: Sumurup (D = 1.046 ± 0.01 and rate = 200 L/s) and Bribin (D = 1.043 ± 0.01 and rate = 1500 L/s). On the other hand, caves with larger fractal dimension are generally occupied by rivers with small

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Table 4 Relationship between fractal dimension of surface flow and grain size and mineralogy

Formation	Grain-size range (mm)	Main mineralogy	Hardness (Mohs scale)	Fractal dimension of surface flow (D)
Nglanggran	1/16–2 to 256	Plagioclase, pyroxene	6–6.5	1.51 ± 0.01
Sambipitu	<1/256–2	Clay minerals, carbonates, tuffs	3–3.5	1.13 ± 0.01 1.16 ± 0.01 1.15 ± 0.01
Oyo	1/256–1/32 to 2	Carbonate minerals, plagioclase, quartz impurities	3–3.5	1.04 ± 0.01 1.05 ± 0.01 1.07 ± 0.01
Wonosari	<1/256 to 50	Calcite, dolomite	2.5–3.5	1.10 ± 0.01 1.07 ± 0.01 1.27 ± 0.01

flow rates. The Sodong Cave (D = 1.070 ± 0.01), the Semuluh Cave (D = 1.057 ± 0.01), and the Soga Cave (D = 1.065 ± 0.02) have flow rates of 8, 15, and 7 L/s, respectively. The cave with largest fractal dimension (Jomblang, D = 1.080 ± 0.01) has a flow rate of 2 L/s. One hypothesis is that this phenomenon resulted because the larger the rate of the river, the faster the obliteration of irregular shape of the channel by erosion. Therefore, the larger the flow rate, the smaller the fractal dimension of the river. However, to test this hypothesis, further study is needed.

Conclusions

1. The fractal dimension of the Oyo River ranges from 1.02 ± 0.01 to 1.51 ± 0.01. The fractal dimensions of segments of the Oyo River are controlled by mineral composition and grain size of the rock being dissected by the river and by the presence or absence of faults.
2. The lower the hardness of the mineral composition and the smaller the grain size of the rock, the easier for the rock to be eroded, and the larger the fractal dimension of the river.
3. Six underground rivers of the Gunungsewu (Bribin, Sodong, Semuluh, Soga, Sumurup, and Jomblang) have fractal dimensions that range from 1.043 ± 0.01 to 1.08 ± 0.01. Surface topography or valley patterns that overlie the underground rivers have fractal dimensions that range from 1.494 ± 0.01 to 1.732 ± 0.01
4. The value of fractal dimension of an underground-river pattern is directly proportional to the fractal dimension of its surface topography.
5. By knowing the fractal dimensions of erosional valley patterns in a karst area, the fractal dimensions of underground rivers beneath the topography can be predicted. By knowing the approximate fractal dimension of an underground flow pattern, the flow rate of the river can be estimated.

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