



Estimating groundwater recharge uncertainty from joint application of an aquifer test and the water-table fluctuation method

H. Delottier¹ · A. Pryet¹ · J. M. Lemieux² · A. Dupuy¹

Received: 29 August 2017 / Accepted: 26 April 2018 / Published online: 17 May 2018
© Springer-Verlag GmbH Germany, part of Springer Nature 2018

Abstract

Specific yield and groundwater recharge of unconfined aquifers are both essential parameters for groundwater modeling and sustainable groundwater development, yet the collection of reliable estimates of these parameters remains challenging. Here, a joint approach combining an aquifer test with application of the water-table fluctuation (WTF) method is presented to estimate these parameters and quantify their uncertainty. The approach requires two wells: an observation well instrumented with a pressure probe for long-term monitoring and a pumping well, located in the vicinity, for the aquifer test. The derivative of observed drawdown levels highlights the necessity to represent delayed drainage from the unsaturated zone when interpreting the aquifer test results. Groundwater recharge is estimated with an event-based WTF method in order to minimize the transient effects of flow dynamics in the unsaturated zone. The uncertainty on groundwater recharge is obtained by the propagation of the uncertainties on specific yield (Bayesian inference) and groundwater recession dynamics (regression analysis) through the WTF equation. A major portion of the uncertainty on groundwater recharge originates from the uncertainty on the specific yield. The approach was applied to a site in Bordeaux (France). Groundwater recharge was estimated to be 335 mm with an associated uncertainty of 86.6 mm at 2σ . By the use of cost-effective instrumentation and parsimonious methods of interpretation, the replication of such a joint approach should be encouraged to provide reliable estimates of specific yield and groundwater recharge over a region of interest. This is necessary to reduce the predictive uncertainty of groundwater management models.

Keywords Hydraulic properties · Hydraulic testing · Groundwater recharge · Specific yield · Uncertainty analysis

Introduction

Reliable estimates of aquifer parameters such as hydraulic conductivity, storage and groundwater recharge are essential for the sustainable management of groundwater resources (Doble and Crosbie 2016; Knowling and Werner 2016), especially when groundwater models are used. Groundwater model parameters are traditionally estimated through calibration using nonlinear regression, which generally requires the use of regularization to address the issue of nonuniqueness and instability (Carrera et al. 2005; Aster et al. 2013; Zhou et al.

2014; Doherty 2015). One of the most traditionally applied regularization processes includes prior knowledge on model parameters (Cooley 1982; Tikhonov and Arsenin 1977; Carrera and Neuman 1986). The incorporation of prior information is also necessary for uncertainty analysis based on statistical inference (Fienen et al. 2010; Cui and Ward 2012); therefore, it is of utmost importance to collect reliable information on model parameters prior to model calibration or uncertainty analysis (Hunt et al. 2007).

Numerous methods exist for estimating groundwater recharge (Healy and Scanlon 2010; Scanlon et al. 2002), which can be classified according to the location of field measurements (unsaturated zone, saturated zone) as well as depending on the utilized approach (water budget, physically based flow model, chemical methods, etc.). The water-table fluctuation (WTF) method is one of the most attractive ways to estimate groundwater recharge because of the generally good availability of groundwater level records and the simplicity of its application (Healy and Cook 2002; Lucas et al. 2015; Ordens et al. 2012). The WTF method is based on the analysis of

✉ H. Delottier
delottierhugo@gmail.com

¹ EA 4592 Georessources & Environment, Bordeaux INP and Univ. Bordeaux Montaigne, ENSEGID, 1 allée Fernand Daguin, 33607 Pessac cedex, France

² Département de géologie et de génie géologique, Université Laval, 1065 avenue de la Médecine, Quebec City G1V 0A6, Canada

water level rises in response to individual precipitation events. Groundwater recharge is obtained from the product of the specific yield (S_y) with the *effective* water-table rise Δh^* (Healy and Cook 2002):

$$R = S_y \times \Delta h^* \quad (1)$$

where Δh^* , the effective water-table rise, corrects the observed water-table rise to take into account the water-table recession due to regional groundwater flow and the effect of air entrapment in the upper fringe of the rising water table (Crosbie et al. 2005; Cuthbert 2010).

As can be seen in Eq. (1), the estimation of recharge with the WTF method requires a storage parameter, the specific yield (S_y). The specific yield of an unconfined aquifer corresponds to the pore volume of water that is removed by gravity (i.e. the difference between the saturated and residual water content) when the water table drops (Hilberts et al. 2005; Healy and Scanlon 2010). This definition relies on the hydrostatic condition when the water content profile is at its equilibrium state (de Marsily 1986). Prior values on specific yield can be obtained from the literature, core sample analysis and aquifer tests. While literature data may be unavailable or uncertain for a given study site, the extraction and analysis of core samples is costly and may be poorly representative of macroscopic specific yield (Zhang et al. 2011). As an alternative, aquifer tests provide a relatively cost-effective and reliable approach for the estimation of specific yield at the macroscopic scale (de Marsily 1986). When conducting an aquifer test in an unconfined aquifer, the time-drawdown behavior is characterized by an S-shape curve with three successive steps (Boulton 1954). The first step (early time) transcribes the release of water from the elastic storage (confined aquifer behavior) and the third step (late time curve) corresponds to the equilibrium where the water is instantaneously released (drained aquifer). The middle step (intermediate time) represents the transition between the early and late times where the water of the unsaturated zone is not instantaneously released. During that period, which can take from a few hours up to several days, the slope of the time-drawdown behavior decreases due to the gradual drainage of the unsaturated zone, leading to an S-shape curve.

The main limitation of the WTF method is linked to the estimation of the storage parameter in Eq. (1), which has a strong impact on recharge estimates, but is not easy to evaluate precisely (Healy and Cook 2002; Cuthbert et al. 2016). However, the WTF method is seldom associated to a joint integrated estimation of the specific yield, which is most often obtained from the literature (Yin et al. 2011; King et al. 2017; Crosbie et al. 2015; Rawling and Newton 2016). The use of the WTF method to delineate groundwater recharge may become more robust with a careful estimate of the relevant storage parameter. In this context, the objective of this paper is to promote a joint cost-

effective approach where an aquifer test and the WTF method are used in conjunction for the estimation of groundwater recharge. This is of interest for parameter estimation and uncertainty analysis of groundwater management models.

Knowledge of hydraulic conductivity is also essential in order to cope with the well-known correlation with groundwater recharge when using solely groundwater heads for groundwater-model parameters estimation (Hill and Tiedeman 2006; Anderson et al. 2015; Delottier et al. 2017). However, reliable independent estimates of unconfined aquifer hydraulic conductivity can be derived by the use of classical aquifer test analytical solutions (Halford et al. 2006). In contrast, the estimation of specific yield is much more challenging and requires consistency with the WTF method application. For these reasons, the estimation of hydraulic conductivity is out of the scope of this paper.

One important aspect related to joint application of the aquifer test and the WTF method is related to the transient concepts associated with unsaturated-zone hydrodynamics. In transient conditions, a distinction must be made according to water-table dynamics. For a water-table decline (e.g. aquifer test), the storage parameter is the drainable porosity. Time-dependence of the drainable porosity stems from delayed drainage of the unsaturated zone with a falling water table (e.g. Nachabe 2002). The WTF method, which deals with water-level rise, should be applied with the fillable porosity. Time-dependence of the fillable porosity stems from nonhydrostatic conditions of the unsaturated zone above a rising water table (e.g. Park 2012). However, when neglecting hysteretic effects and air entrapment, it has been shown that drainable and fillable porosities tend to the same value (the specific yield), when approaching hydrostatic conditions in the unsaturated zone (Acharya et al. 2012; de Marsily 1986). Consequently, the interest of a joint application of the WTF method with an aquifer test is to look for consistency in the storage parameter value.

The paper is organized as follows. General guidelines to design an appropriate monitoring station for the proposed method are first presented. Then, the analytical solution for the aquifer test and the implementation of WTF method are described with a focus on parametric uncertainty. Finally, a demonstration of the method is presented at a site located near Bordeaux (France) where specific yield and groundwater recharge are estimated with the associated uncertainty. The relevance and limitations of the approach are eventually discussed.

The monitoring station

The monitoring station described in this section aims at collecting field observations for a reliable estimate of both the specific yield and groundwater recharge (Fig. 1). The

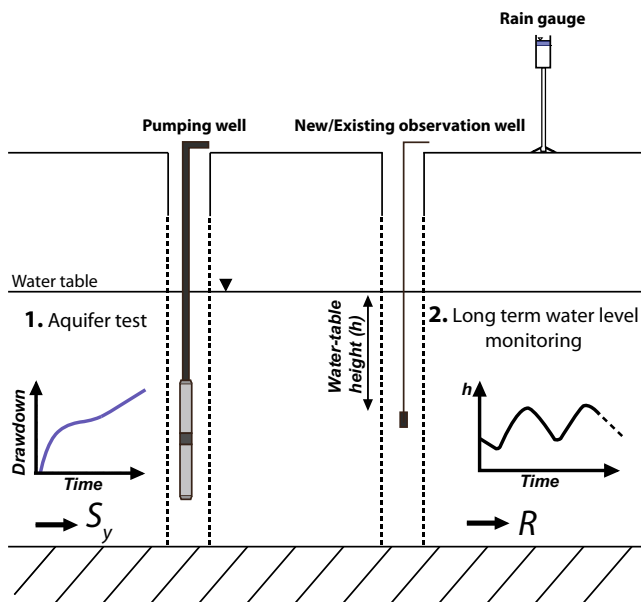


Fig. 1 The proposed monitoring station. Two wells are necessary to estimate the specific yield (S_y) with the aquifer test (1). The observation well is used to record long-term water-table fluctuations (2). The pumping well and the aquifer test may be completed close to an existing observation well where long-term records are already available. The rain gauge is suggested but no mandatory

monitoring station should therefore be designed to (1) conduct an aquifer test and (2) monitor long-term water-table fluctuations. Two wells are necessary for the estimation of the specific yield (Halford et al. 2006). One observation well should be located in the vicinity of the pumping well. The observation well can be an existing observation well for which long-term water-table fluctuation data are already available. Additionally, it can be convenient to set up a rain gauge, for two reasons: (1) the identification of dry periods where the aquifer cannot be influenced by precipitation and (2) to express recharge as a ratio of precipitation (RPR).

The station design should also remain sufficiently simple and cost-effective for the experiment to be replicated at several locations over a basin of interest. Water-table fluctuations should be mainly associated with meteoric precipitation so that external stresses such as streams or pumping stations should be avoided in the vicinity of the monitoring station.

Methods

The unconfined aquifer test

Here, the purpose of the aquifer test is to estimate the aquifer specific yield at a macroscopic scale from the interpretation of observed drawdowns. The principle of an aquifer test is to pump at a known constant discharge rate (Q) from a pumping well and record observed drawdown (s) from an observation well at a known distance (r) from the pumping well

(Kruseman and de Ridder 1990). If the interpretation can be made from a groundwater flow model, the use of a simple analytic solution is preferred because of (1) the availability of such a solution for aquifer test interpretation, (2) the simplicity of the application and (3) the computationally frugal aspect of analytical solutions (Renard 2005; Neuman and Mishra 2012).

The classical analytical solution proposed by Theis (1935) and its approximation (Cooper and Jacob 1946) constitute the basis of aquifer test interpretation. These solutions were initially developed for confined aquifers, but they may also be applicable to unconfined aquifers for the estimation of specific yield as long as the drainage of the unsaturated zone has a small influence on the observed drawdown.

Neuman’s analytical solution (Neuman 1972) reproduces the three parts of the theoretical S-shape drawdown curve generally observed for unconfined aquifers. The solution assumes that the drainage of the unsaturated zone is instantaneous. The effects of transient flow dynamics in the unsaturated zone are not considered.

Assuming instantaneous drainage, the upper-boundary condition for flow to a well in an unconfined aquifer is written in dimensionless form as follows (Mishra and Kuhlman 2013):

$$\frac{\partial^2 S_D}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial S_D}{\partial r_D} + K_D \frac{\partial^2 S_D}{\partial z_D^2} = \frac{\partial S_D}{\partial t_D} \tag{2}$$

where $S_D = \frac{s}{Q/(4\pi T)}$, $r_D = \frac{r}{b}$, $t_D = \frac{t}{(S_y b^2)/T}$ and $K_D = \frac{K_z}{K_r}$. K_z and K_r are the vertical and horizontal components of the saturated hydraulic conductivity, respectively. T is the transmissivity, b the aquifer thickness and r the distance of the observation well from the pumping well.

Neuman’s upper-boundary condition assumes that the shape of the water content profile above the dropping water table does not change but simply follows the water-table decline. Neuman’s solution is used by many hydrogeologists as the preferred model mainly because of the perception that neglecting the effects of gradual drainage from the unsaturated zone is reasonable for the estimate of aquifer parameters (Neuman 1972).

Nevertheless, in a majority of aquifer tests, only a small portion of water from the unsaturated zone is released during the early times of the aquifer test (Nwankwor et al. 1992; Moench 2004). Consequently, the aquifer test interpretation without consideration of delayed drainage of the unsaturated zone leads to an underestimation of the specific yield (Nwankwor et al. 1984). A solution to this problem is to conduct a long-term aquifer test until the influence of the delayed drainage of the unsaturated zone becomes negligible. However, such a long-term aquifer test is often difficult to implement because of three major constraints. The first is related to the cost and time for an aquifer test, which most of the time cannot exceed one day. The second is related to the

aquifer boundaries, which can be reached during long-term aquifer tests; reaching the boundaries invalidates the infinite aquifer assumption of the theoretical S-shape curve. The third is that meteoric precipitation events are likely to occur within the duration of a long-term aquifer test, which is likely to ruin the interpretation. Because of these reasons, for most of the time, the aquifer test should be interpreted with the influence of the delayed drainage of the unsaturated zone. An analytical solution considering delayed drainage should therefore be used to obtain a reliable estimation of the specific yield.

Boulton's analytical solution (Boulton 1954) assumes that the drainage of the unsaturated zone occurs gradually rather than instantaneously. The solution is based on an implicit representation of the unsaturated zone drainage that can be expressed as follows (Mishra and Kuhlman 2013):

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r} = \frac{S}{T} \frac{\partial s}{\partial t} + \alpha S_y \int_0^t \frac{\partial s}{\partial t} \exp^{-\alpha(t-\tau)} d\tau \quad (3)$$

where the first term on the right-hand side is the instantaneous confined storage and the second term accounts for the drainage from the zone above the water table that is assumed to decline exponentially with time since the beginning of the aquifer test. S is related to both aquifer compaction and the compressibility of water.

Moench's analytical solution (Moench 2004) combines Boulton's and Neuman's approaches. Moench's analytical solution can be written as follows (Mishra and Kuhlman 2013):

$$\int_0^t \frac{\partial s}{\partial \tau} \sum_{m=1}^M \alpha_m \exp^{-\alpha_m(t-\tau)} d\tau = -\frac{K_z}{S_y} \frac{\partial s}{\partial z} \quad (4)$$

where the solution can include several delayed drainage parameters α_m (m as the running index for the total number of delayed drainage parameters, M) in order to improve the fit between simulated and observed drawdowns comparing to the Boulton solution. When $M=1$, Moench's solution is identical to the Boulton equation.

When the empirical parameters (α_m) of Moench's solution are large, drawdown dynamics tend to an instantaneous drainage behavior (e.g. thick unconfined aquifer), whereas in contrast, low values of these empirical parameters lead to a nondrainage behavior (e.g. confined aquifer). A reasonably gradual drainage can be reached between these two extreme behaviors. Moench (2004) shows that a better fit to the observed drawdowns is obtained with $M=3$, while Trivedi and Kashyap (2015) state that the second-order ($M=2$) Moench's model may be considered as parsimoniously optimal.

The use of Moench's analytical model allows a better fit to the observed drawdown and more reliable specific yield estimation than Neuman's solution when the intermediate time curve is dominated by gradual drainage. Nevertheless, the classical Theis (1935) and Neuman (1972) analytical solutions are still used for practical purposes, mainly because of the

simplicity and the accessibility of these methods. For these reasons, Moench's analytical model along with the Theis (1935) and Neuman (1972) solutions were used thereafter for the observed drawdown interpretation to delineate the specific yield. Neuman's and Moench's solutions are implemented in the WTAQ Fortran-based package (Barlow and Moench 2011).

The event-based WTF method

The principle of the event-based WTF method is to estimate groundwater recharge (R) from the product of the effective water-table rise (Δh^*) with the aquifer specific yield (S_y). Because of the consideration of the specific yield as the storage parameter estimated from the aquifer test, the unsaturated zone hydrodynamics must be as close as possible to a steady-state condition. When considering groundwater recharge events (Nimmo et al. 2015), as opposed to discrete time implementation of the WTF method, the WTF method is applied over relatively long periods such that the influence of the unsaturated zone hydrodynamics can be neglected. In the meantime, additional factors may affect water-table fluctuations such as air entrapment and water-table recession due to regional groundwater flow. Indeed, when the time lag for infiltrating water to reach the water table is not negligible, the water-table recession should not be neglected at the risk of underestimating the effective water-table rise (Healy and Cook 2002).

These compensations are embedded within Eq. (1) by the use of an effective water-table rise (Δh^*) rather than the direct use of observed water-table rise. The effective water-table rise can be delineated by the use of a master recession curve (MRC), which predicts the characteristic rate of change of water level as a function of the current aquifer water level (Heppner and Nimmo 2005; Crosbie et al. 2005; Cuthbert 2010). For an individual recharge event, the associated effective water-table rise (Δh_i^*) is computed by the extrapolation of the MRC in both forward and backward directions in time (Fig. 2). Consequently, the event-based WTF method can be expressed as follows:

$$\Delta h_i^* = \Delta h_i + D_i - O_i \quad (5)$$

where Δh_i is the observed water-table rise, D_i is the natural recession due to regional flow, and O_i the overshoot due to effects such as air trapping (Nimmo et al. 2015). Finally, the groundwater recharge (R_i) associated with the i -th event is computed by the product of Δh_i^* by the specific yield (Eq. 1).

The classical application of the event-based WTF method is a graphical approach where the effective water-table rise is estimated after a manual extrapolation of the MRC for each groundwater recharge event (Rasmussen and Andreasen 1959). Because the graphical approach requires manual

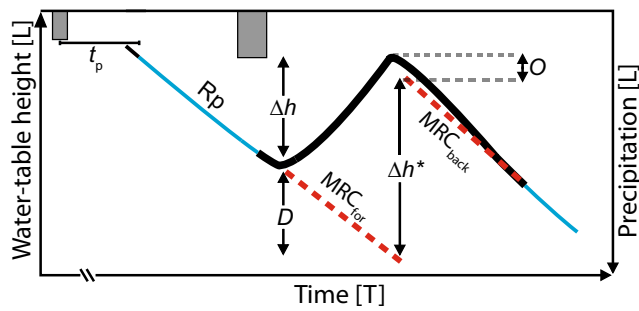


Fig. 2 Application of the event-based water-table fluctuation method. The recession period (R_p) is delineated after a delay since the latest precipitation event (t_p). The analysis of recession periods yields the master recession curve (MRC). Δh is the observed water-table rise for a given precipitation event. The MRC is extrapolated both in forward (MRC_{for}) and backward (MRC_{back}) directions to obtain the effective water-table fluctuation (Δh^*) which accounts for the effects of water-table recession (D) and overshoot (O)

attention for each episode, its application may become time-consuming and be associated with a certain degree of subjectivity.

Given the inherent subjectivity and the monotonous characteristics of the graphical event-based approach, the applied event-based WTF approach relies on the automatic identification of recharge events by the WTF rates inspection. Elements of subjectivity of the graphical-based approach linked to the recession curve are encapsulated into three parameters: the storm recovery time (t_p), the fluctuation tolerance (δ_T), and the precipitation time lag (t_l) (Nimmo et al. 2015). The first parameter, the storm recovery time, is used to identify recession periods. It corresponds to the minimum time interval between a precipitation and recession event, allowing storm-generated accretion to become negligible (Fig. 2). The MRC is delineated from the analysis of the form of the relation between water-table elevation and water-table decline during recession periods (without precipitation events). For the current study, a linear MRC is adjusted to characterize the relation between water-table height and decline rate (Heppner and Nimmo 2005):

$$D_i = ah(t)_i + b \tag{6}$$

where h is the water-table height [L] from the pressure transducer in the observation well (Fig. 1), a is the slope [T^{-1}] and b is the intercept on the decline rate axis [L/T]. The parameters a and b are the recession parameters of the MRC. The second parameter, the fluctuation tolerance (δ_T), is a measurement noise criterion used to discard insignificant water-table fluctuations. The third parameter, the precipitation time lag (t_l), reflects the response time for recharge caused by a given precipitation event. While t_p is used to define the MRC for the natural water-table recession characterization, δ_T and t_l are used for the delineation of the discrete recharge episodes. The event-based approach is implemented within a package

proposed by Nimmo et al. (2015) which is based on the software R (R Core Team 2013).

The reliability of the estimated groundwater recharge is linked to the uncertainty on specific yield on the one hand and to the recession parameters on the other hand. According to Hughes and Hase (2010), the rule for the propagation of errors for the event-based WTF method multi-variable function (Eq. 1) can be written as follows:

$$\sigma_R = R \left(\sqrt{\left(\frac{\sigma_{S_y}}{S_y}\right)^2} + \sqrt{\left(\frac{\sigma_{\Delta h^*}}{\Delta h^*}\right)^2} \right) \tag{7}$$

where σ_{S_y} and $\sigma_{\Delta h^*}$ are the uncertainties related to the estimation of the specific yield and to the effective water-table rise for the whole period (i.e. the arithmetic sum of recharge events).

Model fitting and uncertainty analysis

The estimation of specific yield is based on fitting the theoretical analytical solution (Moench, Theis, or Neuman) to the observed drawdown. Due to parameters correlation and potential insensitivities to drawdown data, the estimation of specific yield can be nonunique (Carrera et al. 2005). Given the short computation time of the aquifer test analytical solutions, a stochastic analysis seems the most appropriate method to fit the model and define specific yield uncertainty at the same time (Renard 2005; Yustres et al. 2012).

Bayes’ theorem states that the posterior probability of the parameters, given the data, is proportional to the product of the likelihood function of the parameters given the data and the prior probability of the parameters (Aster et al. 2013):

$$f(\mathbf{k}|\mathbf{s}) \propto L(\mathbf{k}|\mathbf{s}) f(\mathbf{k}) \tag{8}$$

where \mathbf{k} is the vector containing the unknown parameters, \mathbf{s} the vector containing drawdown observations, $L(\mathbf{k}|\mathbf{s})$ the likelihood function and $f(\mathbf{k})$ the prior distribution of the parameters. Due to the unawareness to a prior mean of parameters, the simplest assumption is to define a uniform distribution where large parameter space boundaries are used to define a noninformative prior distribution (Cui and Ward 2012). A normal likelihood function is used with the defined uniform prior distributions for all parameters.

For the application of the Bayesian inference, the Markov chain Monte Carlo (MCMC) techniques are used, specifically the Metropolis-Hasting algorithm which, after convergence, is able to draw samples from the posterior distribution (Metropolis et al. 1953; Hastings 1970). The Metropolis-Hasting algorithm is applied through a Python based package (Patil et al. 2010).

For the estimation of $\sigma_{\Delta h^*}$, given that a linear model is fitted to the extracted recession data to delineate the recession parameters (a and b), the standard error associated to these two recession parameters can be derived by linear regression analysis. Then, for each groundwater recharge event, the associated standard errors for the recession parameters are used to delineate an upper limit for the estimation of Δh^* at 1σ , after which, $\sigma_{\Delta h^*}$ can be estimated as the difference between the upper limit and the mean.

Illustration of the method

Study area

An unconfined aquifer located in the region of Bordeaux (France) is used to illustrate the proposed approach. The geological units consist of, from top to bottom, a 1-m unsaturated zone primarily composed of coarse sand with heterogeneous gravels and a 5-m aquifer layer composed of fine sand deposits overlying a thick clay layer. The natural dynamics of the phreatic aquifer are primarily controlled by precipitation and evapotranspiration. There are no surface-water bodies or pumping wells in the vicinity of the investigated area. The groundwater level is monitored with one observation well. A pumping well was completed for the purpose of this study at a distance of 6.8 m from the observation well. The two wells have similar characteristics with a totally penetrating casing screened from 1 m below ground surface to the bottom of the aquifer. The mean saturated aquifer thickness is taken to be a known parameter, equal to 5 m. A weather station, located close to the wells, recorded precipitation and classic climatic variables. For this study, the observation period was October 2015 to January 2017 (445 days). Observations were recorded with a 6-min time step. Over this period, a total of 970-mm precipitation was recorded.

An aquifer test was conducted at a constant discharge rate of $6 \text{ m}^3/\text{h}$ over 22 h (Fig. 3). Drawdown values were recorded with a fixed 3-s time step and thereafter resampled with a logarithmic progression. Observed drawdown levels are presented with their associated logarithmic derivatives in Fig. 3. Drawdown derivatives rise continuously during the aquifer test period. Such a behavior can be explained by the influence of the overlying unsaturated zone during the aquifer test, resulting in a conceptual error of aquifer test interpretation when using methods such as Theis and Neuman. Nevertheless, these two widely used methods are not discarded from the interpretation in order to illustrate the necessary compensation of the gradual drainage when the specific yield is estimated.

Estimation of specific yield

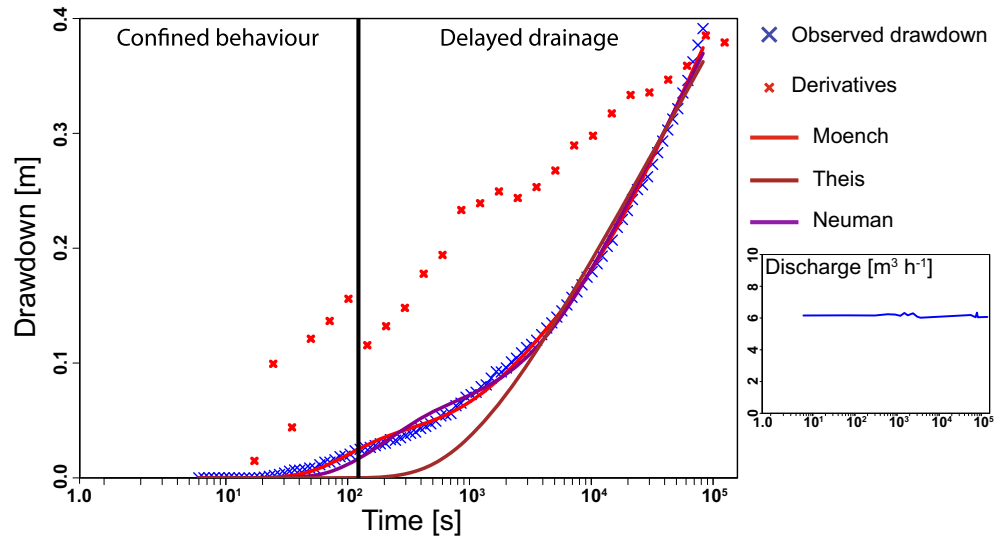
Observed drawdown values are used for the estimation of six unknown parameters (S , S_y , K_z , K_h , α_1 and α_2) for Moench's model, four unknown parameters (S , S_y , K_z , and K_h) for Neuman's model, and two unknown parameters (S_y and T) for Theis' model. A prior uniform distribution is assumed for all the parameters involved in the aforementioned theoretical models. Despite the subjective judgment required to define a prior distribution, the boundaries were chosen large enough to minimize the effect of a priori assumptions. From these prior distributions, the Markov chain Monte Carlo (MCMC) method is used to get 40,000 samples in the posterior distribution. The samples are retained when the values converge to their joint stationary posterior distribution. The first 2,000 values are used to initialize the posterior distribution, while the remaining 38,000 are kept. The MCMC often results in strong autocorrelation among samples that can result in imprecise posterior inference (van der Spek and Bakker 2017). To circumvent this artifact, the posterior distribution is thinned in order to retain only one sample for every 100 samples for each parameter. The resulting posterior distribution for the estimation of specific yield is shown for the three analytical models in Fig. 4.

The maximum likelihood values of specific yield are 8.5, 9.5 and 14.2% for Theis', Neuman's and Moench's solutions, respectively. It appears that the estimated specific yield with the Theis and Neuman models clearly differ from the value obtained with the Moench model. As already shown by Nwankwor et al. (1992), neglecting the delayed drainage process leads to the underestimation of the specific yield. Indeed, Theis' and Neuman's models present a conceptual error as they both disregard the delayed drainage, as shown by the derivative behavior of the drawdown curve (Fig. 3). Moench's model, which accounts for this process, is therefore more realistic; nevertheless, the additional parameters necessary to account for delayed drainage lead to a greater level of uncertainty.

Recharge estimation

The records of water-table height are first resampled at a daily time step to identify the significant features and facilitate groundwater-recharge event delineation. According to an autocorrelation analysis applied for the whole period, it is found that the water table does not respond to precipitation events below a threshold of 0.5 mm. Moreover, a partial cross-correlation between the rainfall records and the resulting water-table fluctuation data reveals that the precipitation lag time, which accounts for the transit through the unsaturated zone, can be fixed at 1 day. The recession curve was determined from identified recession periods on the basis of a storm recovery time ($t_p = 6$ days) adjusted by trial and error to minimize the variability of the water-table rate of change while keeping a significant

Fig. 3 Observed drawdowns and their logarithmic derivatives together with the Theis, Neuman and Moench analytical solutions. The discharge rate at the pumping well remained relatively stable ($6 \text{ m}^3/\text{h}$)



number of data points to adjust the recession curve. Following examination of the recession data (Fig. 5), the decision to use a linear function to relate the rate of change of the water-table height to the water-table height seems coherent. The recession line parameters are $a = 0.0238 \text{ m}^{-1}$ and $b = -0.0109 \text{ m/day}$, determined with a regression coefficient $R^2 = 0.62$. Given the manufacturer’s data of the pressure probe accuracy, the fluctuation tolerance (δ_T) is fixed to 0.01 m. A total of 12 individual recharge events were identified over the study period (Fig. 6). The effective water-table rise is estimated for each of them and summed in order to get a total effective water-table rise for the overall period as 2.358 m with a standard deviation of 1.5 mm (1σ). The uncertainty on the determination of the total effective water-table rise is related to regression of the MRC parameters.

Finally, the mean groundwater recharge is estimated with Eq. (1) for the whole period to be 335 mm with an associated uncertainty of 86.6 mm at 2σ . The corresponding uncertainty is related to the propagation of the specific yield and effective water-table rise uncertainties (Fig. 7). In terms of the recharge-to-precipitation ratio (RPR), the mean groundwater recharge corresponds to 25–43% of the recorded rainfall at 2σ .

Discussion

An approach has been detailed to evaluate groundwater recharge and uncertainty from the joint application of an aquifer test with the WTF method. In the context of the case study, the storage parameter (S_y) ranges between 0.10 and 0.17 at 2σ . The effective water-table rise (Δh^*) is estimated to 2.35 m with a negligible uncertainty (3 mm at 2σ). These values have been used to obtain the estimate of groundwater recharge, i.e. 335 mm with an associated uncertainty of 86.6 mm at 2σ . The uncertainty on the storage parameter is responsible for a major portion of the uncertainty on groundwater recharge. Specific care should therefore be taken for the interpretation of the aquifer test so as to obtain a reliable estimate for the storage parameter. As a solution to reduce the uncertainty on recharge, Moench’s analytical model parameterization should be linked to the degree of complexity of the observed time-drawdown curve. A parsimonious model must be encouraged when compatible with the observed data.

Though it comes at a cost, the use of an aquifer test to obtain an estimate of the storage parameter appears as more relevant than a value taken from the literature, as is often the case. Compared to core sample analysis, a value provided by an aquifer test has greater spatial representativeness; however, aquifer test interpretations are most often based on the

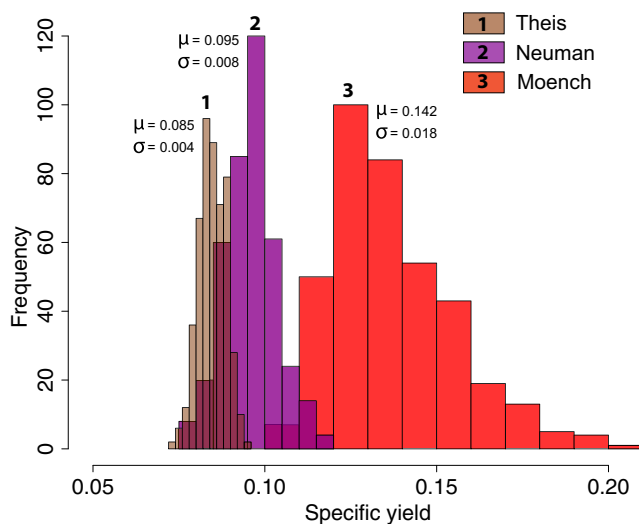
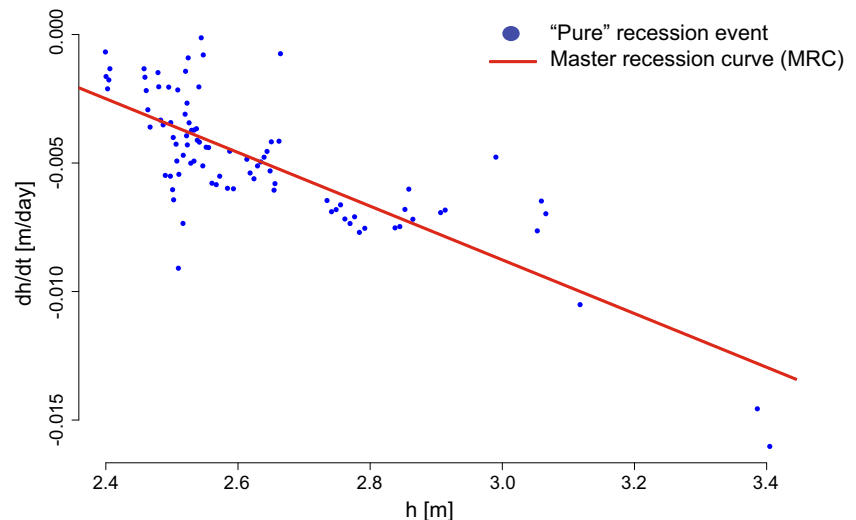


Fig. 4 Bayesian inference for the estimation of specific yield with the Theis, Neuman and Moench analytical models. The mean (μ) and standard deviation (σ) are given for each distribution

Fig. 5 Water-table rate of change (dh/dt) as a function of water level height (h) for selected recession periods (points) where the influence of unsaturated zone drainage can be negligible. A linear master recession curve (MRC) can be fitted with an adjusted R^2 of 0.62



assumption of a homogeneous medium. As noted by Meier et al. (1998), an effective estimation of the specific yield is challenging when the aquifer is heterogeneous. In heterogeneous media, the estimated specific yield can span over a broad range of values according to the location of the observed well with respect to both the horizontal and vertical directions. In such contexts, the estimated effective specific yield values are dominated by heterogeneities between the pumping and the observation well (Wu et al. 2005; Mao et al. 2011). When possible, it can be recommended to use data from multiple observation wells around the pumping well to address this issue. When the geological medium presents marked vertical contrast over the domain of water-table fluctuations, a depth-dependent value of specific yield may be used for the application of the WTF method (Crosbie et al. 2005). It should also be verified that groundwater levels during the aquifer test remain sufficiently close to that of natural groundwater levels subject to seasonal fluctuations. To this

effect, it appears more relevant to conduct the aquifer test during a relatively wet period.

The application of an aquifer test to an unconfined aquifer also involves unsaturated flow dynamics. The use of drawdown derivatives is advised in order to identify whether or not the vertical flux component from the unsaturated zone can be neglected (Renard et al. 2009). When the assumption of instantaneous drainage cannot be honored, as is the case for the study site, drawdown derivatives do not stabilize and analytical solutions such as Moench (2004), Tartakovsky and Neuman (2007), Mathias and Butler (2006) and Mishra and Neuman (2010) should be employed to account for delayed drainage. As illustrated in the case study, the inappropriate application of the classic Theis or Neuman analytical solutions may lead to an important underestimate of the specific yield, and in turn, of groundwater recharge.

While the aquifer test provides an estimate of the drainable porosity, what is of actual interest for the WTF method is the fillable porosity (Crosbie 2005; Park 2012; Sophocleous 1991). Both of these parameters theoretically converge to the same value, the specific yield (S_y), with increasing time after a perturbation (i.e. equilibrium state) (Nachabe 2002; Acharya et al. 2012). Close to this equilibrium state, the specific yield constitutes a relevant storage parameter to be included within the event-based WTF method. However, as detailed by de Marsily (1986), the water content profile almost never reaches the equilibrium and air entrapment is one of the causes of the hysteresis effect (e.g. spatial connectivity of pores, variations in the liquid-solid contact angle). It should therefore be noted that the presented approach is only applicable to relatively shallow water tables and permeable formations (i.e. for the unsaturated zone to reach relatively quickly the equilibrium state). Such considerations advocate for the use of event-based implementations of the WTF method, which are based on greater time intervals than discrete-time

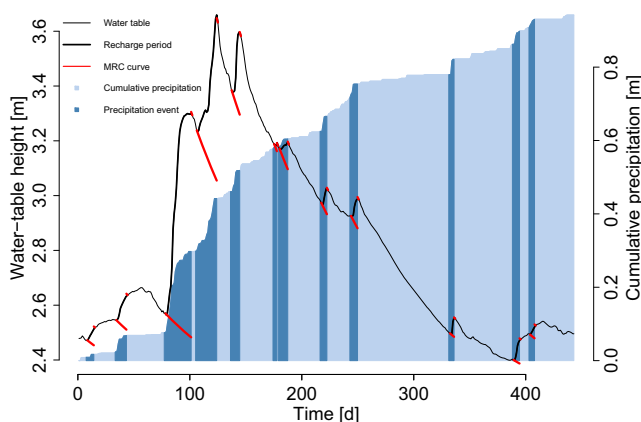


Fig. 6 Groundwater-level fluctuations and identified recharge events with associated recession curves

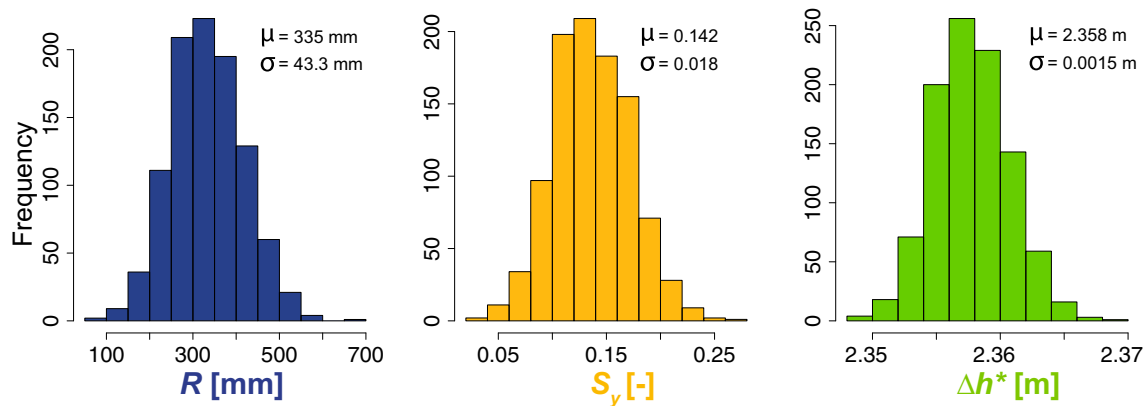


Fig. 7 Probability distributions of groundwater recharge ($R = S_y \times \Delta h^*$) based on uncertainty propagation of specific yield (S_y) and effective water-table rise (Δh^*). The mean (μ) and standard deviation (σ) are given for each distribution

approaches such as the episodic master recession (EMR) method proposed by Nimmo et al. (2015). This latter method, which has been used in this study, presents the advantage to correct for water-table overshoots with backward extrapolation. However, even with the presented approach, hysteretic and nonequilibrium processes in the unsaturated zone may alter the estimate of groundwater recharge.

The event-based WTF method implemented in this study is based on MRC analysis. One important aspect inherent to this approach is the choice of the functional relationship between the water-table decline rate and the water-table height (Heppner and Nimmo 2005). In the present study, a linear model was fit to the experimental data, which theoretically leads to an exponential recession (Rorabaugh 1960). In different contexts, other types of functional relationships can become more relevant such as constant-rate, bin-average, or power-law functions (Cuthbert 2014; Heppner and Nimmo 2005). It should be noted that for the present case, the uncertainty originating from the regression of the experimental MRC curve was small with respect to the uncertainty originating from the specific yield. Recharge processes are likely to evolve throughout the year with the seasons so that the parameters of the WTF method (lag time, fluctuation tolerance and MRC characteristics) may take different values depending on the season (Jeong and Park 2017).

The instrumentation necessary for the application of the presented approach consists of two wells: a pumping well for the aquifer test and an observation well equipped with a pressure probe for groundwater-level monitoring. A weather station is preferred, but not required. The experimental station is relatively cost effective and the use of analytical solutions is relatively simple, making the approach practical from an operational perspective and potentially replicable over a basin of interest. When dealing with groundwater management of large aquifer systems, it would be of interest to replicate this approach over the major units of land cover and geological formations so as to investigate the spatial variability of recharge.

Conclusion

A practical approach consisting of joint implementation of an aquifer test with the WTF method has been detailed to obtain estimates of specific yield and groundwater recharge. The experimental station involves one pumping well and one observation well equipped with a pressure probe for long-term water-table monitoring. The specific yield is estimated with the Moench model, which accounts for unsaturated zone drainage. This value is subsequently used for the application of an event-based WTF method, assuming unsaturated zone conditions to return close to the equilibrium state between recharge events. The preferred contexts for the application of the joint method are therefore shallow aquifers in relatively permeable formations. The uncertainty regarding groundwater recharge mainly originates from the uncertainty related to the specific yield. The presented approach, replicated over a basin of interest, can constitute prior information and improve the predictive capabilities of models used for groundwater management.

Acknowledgements The authors are thankful to Professor Julio Goncalves for his relevant comments on the original version of the manuscript. We also thank the associate editor, an anonymous reviewer and Professor Eungyu Park for providing useful comments on the manuscript. The OPURES Climate project and the Aquitaine Region provided funding for the experimental set up.

References

- Acharya S, Jawitz JW, Mylavarapu RS (2012) Analytical expressions for drainable and fillable porosity of phreatic aquifers under vertical fluxes from evapotranspiration and recharge. *Water Resour Res* 48(11)
- Anderson MP, Woessner WW, Hunt RJ (2015) *Applied groundwater modeling: simulation of flow and advective transport*. Academic, Cambridge, MA
- Aster RC, Borchers B, Thurber CH (2013) *Parameter estimation and inverse problems*. Academic, Cambridge, MA
- Barlow PM, Moench AF (2011) WTAQ version 2: a computer program for analysis of aquifer tests in confined and water table aquifers with

- alternative representations of drainage from the unsaturated zone. *US Geol Surv Techniques and Methods* 3-B9, 41 pp
- Boulton NS (1954) The drawdown of the water table under non-steady conditions near a pumped well in an unconfined formation. *Proc Inst Civ Eng* 3(4):564–579
- Carrera J, Neuman SP (1986) Estimation of aquifer parameters under transient and steady state conditions: 2. uniqueness, stability, and solution algorithms. *Water Resour Res* 22(2):211–227
- Carrera J, Alcolea A, Medina A, Hidalgo J, Slooten LJ (2005) Inverse problem in hydrogeology. *Hydrogeol J* 13(1):206–222
- Cooley RL (1982) Incorporation of prior information on parameters into nonlinear regression ground-water flow models: 1. theory. *Water Resour Res* 18(4):965–976
- Cooper HH, Jacob C (1946) A generalized graphical method for evaluating formation constants and summarizing well-field history. *EOS Trans Am Geophys Union* 27(4):526–534
- Crosbie RS, Binning P, Kalma JD (2005) A time series approach to inferring groundwater recharge using the water table fluctuation method. *Water Resour Res* 41(1)
- Crosbie RS, Davies P, Harrington N, Lamontagne S (2015) Ground truthing groundwater-recharge estimates derived from remotely sensed evapotranspiration: a case in South Australia. *Hydrogeol J* 23(2):335
- Cui T, Ward ND (2012) Uncertainty quantification for stream depletion tests. *J Hydrol Eng* 18(12):1581–1590
- Cuthbert MO (2010) An improved time series approach for estimating groundwater recharge from groundwater level fluctuations. *Water Resour Res* 46(9)
- Cuthbert MO (2014) Straight thinking about groundwater recession. *Water Resour Res* 50(3):2407–2424
- Cuthbert MO, Acworth RI, Andersen MS, Larsen JR, McCallum AM, Rau GC, Tellam JH (2016) Understanding and quantifying focused, indirect groundwater recharge from ephemeral streams using water table fluctuations. *Water Resour Res* 52(2):857–840
- de Marsily G (1986) *Quantitative hydrogeology: groundwater hydrology for engineers*. Academic, New York
- Delottier H, Pryet A, Dupuy A (2017) Why should practitioners be concerned about predictive uncertainty of groundwater management models? *Water Resour Manag* 31:61–73
- Doble R, Crosbie RS (2016) Current and emerging methods for catchment-scale modelling of recharge and evapotranspiration from shallow groundwater. *Hydrogeol J* 1(25):3–23
- Doherty J (2015) Calibration and uncertainty analysis for complex environmental models - PEST: complete theory and what it means for modelling the real world. Watermark, Brisbane, Australia
- Fienen MN, Doherty JE, Hunt RJ, Reeves HW (2010) Using prediction uncertainty analysis to design hydrologic monitoring networks: example applications from the Great Lakes Water Availability Pilot Project - Appendix 1. Tech. Rep., US Geological Survey, Reston, VA
- Halford KJ, Weight WD, Schreiber RP (2006) Interpretation of transmissivity estimates from single-well pumping aquifer tests. *Ground Water* 44(3):467–471
- Hastings WK (1970) Monte Carlo sampling methods using Markov chains and their applications. *Biometrika* 57(1):97–109
- Healy RW, Cook PG (2002) Using groundwater levels to estimate recharge. *Hydrogeol J* 10(1):91–109
- Healy RW, Scanlon BR (2010) *Estimating groundwater recharge*, vol 237. Cambridge University Press, Cambridge
- Heppner CS, Nimmo JR (2005) A computer program for predicting recharge with a master recession curve. *US Geol Surv Sci Invest Rep* 2005-5172
- Hilberts AGJ, Troch PA, Paniconi C (2005) Storage-dependent drainable porosity for complex hillslopes. *Water Resour Res* 41(6)
- Hill MC, Tiedeman CR (2006) *Effective groundwater model calibration: with analysis of data, sensitivities, predictions, and uncertainty*. Wiley, Hoboken, NJ
- Hughes I, Hase T (2010) *Measurements and their uncertainties: a practical guide to modern error analysis*. Oxford University Press, Oxford
- Hunt RJ, Doherty J, Tonkin MJ (2007) Are models too simple? Arguments for increased parameterization. *Ground Water* 45(3):254–262
- Jeong J, Park E (2017) A shallow water table fluctuation model in response to precipitation with consideration of unsaturated gravitational flow. *Water Resour Res* 53(4):3505–3512. <https://doi.org/10.1002/2016WR020177>
- King AC, Adam C, Raiber M, Cox ME, Cendon DI (2017) Comparison of groundwater recharge estimation techniques in an alluvial aquifer system with an intermittent/ephemeral stream (Queensland, Australia). *Hydrogeol J* 25(6):1759–1777
- Knowling MJ, Werner AD (2016) Estimability of recharge through groundwater model calibration: insights from a field-scale steady-state example. *J Hydrol* 540:973–987
- Kruseman GP, de Ridder NA (1990) *Analysis and evaluation of pumping test data*. Pub. 47, ILRI, Wageningen, The Netherlands
- Lucas M, Paulo T, Oliveira S, Davi C, Melo D, Wendland E (2015) Evaluation of remotely sensed data for estimating recharge to an outcrop zone of the Guarani aquifer system (South America). *Hydrogeol J* 23(5):961
- Mao D, Wan L, Yeh T-C, Lee, Hsu K-C, Wen J-C, Lu W (2011) A revisit of drawdown behavior during pumping in unconfined aquifers. *Water Resour Res* 47(5)
- Mathias S, Butler A (2006) Linearized Richards' equation approach to pumping test analysis in compressible aquifers. *Water Resour Res* 42(6)
- Meier PM, Carrera J, Sánchez-Vila X (1998) An evaluation of Jacob's method for the interpretation of pumping tests in heterogeneous formations. *Water Resour Res* 34(5):1011–1025
- Metropolis N, Rosenbluth AW, Rosenbluth MN, Teller AH, Teller E (1953) Equation of state calculations by fast computing machines. *J Chem Phys* 21(6):1087–1092
- Mishra PK, Kuhlman KL (2013) Unconfined aquifer flow theory: from Dupuit to present, chap 9. In: *Advances in hydrogeology*. Springer, Heidelberg, Germany, pp 185–202
- Mishra PK, Neuman SP (2010) Improved forward and inverse analyses of saturated-unsaturated flow toward a well in a compressible unconfined aquifer. *Water Resour Res* 46(7)
- Moench AF (2004) Importance of the vadose zone in analyses of unconfined aquifer tests. *Ground Water* 42(2):223
- Nachabe MH (2002) Analytical expressions for transient specific yield and shallow water table drainage. *Water Resour Res* 38(10)
- Neuman SP (1972) Theory flow in unconfined aquifers considering delayed response of the water table. *Water Resour Res* 8:1031–1045
- Neuman SP, Mishra PK (2012) Comments on “a revisit of drawdown behavior during pumping in unconfined aquifers” by D. Mao, I. Wan, T.-C. Yeh, C.-H. Lee, K.-C. Hsu, J.-C. Wen, and W. Lu. *Water Resour Res* 48(2)
- Nimmo JR, Horowitz C, Mitchell L (2015) Discrete-storm water table fluctuation method to estimate episodic recharge. *Groundwater* 53(2):282–292
- Nwankwor G, Cherry J, Gillham R (1984) A comparative study of specific yield determinations for a shallow sand aquifer. *Ground Water* 22(6):764–772
- Nwankwor G, Gillham R, Kamp G, Akindunni F (1992) Unsaturated and saturated flow in response to pumping of an unconfined aquifer: field evidence of delayed drainage. *Ground Water* 30(5):690–700
- Ordens CM, Werner AD, Adrian D, Post VEA, Hutson J, Simmons CT, Irvine BM (2012) Groundwater recharge to a sedimentary aquifer in the topographically closed Uley South Basin, South Australia. *Hydrogeol J* 20(1):61–72
- Park E (2012) Delineation of recharge rate from a hybrid water table fluctuation method. *Water Resour Res* 48(7)

- Patil A, Huard D, Fonnesbeck CJ (2010) PyMC: Bayesian stochastic modelling in Python. *J Stat Softw* 35(4):1
- R Core Team (2013). R: a language and environment for statistical computing. R Foundation for Statistical X, Vienna. <http://www.R-project.org/>
- Rasmussen WC, Andreasen GE (1959) Hydrologic budget of the Beaverdam Creek basin, Maryland. US Geol Surv Water Suppl Pap 1472
- Rawling G, Newton BT (2016) Quantity and location of groundwater recharge in the Sacramento Mountains, south-central New Mexico (USA), and their relation to the adjacent Roswell Artesian Basin. *Hydrogeol J* 24(4):757
- Renard P (2005) The future of hydraulic tests. *Hydrogeol J* 13(1):259–262
- Renard P, Glenz D, Mejias M (2009) Understanding diagnostic plots for well-test interpretation. *Hydrogeol J* 17(3):589–600
- Rorabaugh MI (1960) Use of water levels in estimating aquifer constants in a finite aquifer. IAHS Publ. 52, IAHS, Wallingford, UK, pp 314–323
- Scanlon BR, Healy RW, Cook PG (2002) Choosing appropriate techniques for quantifying groundwater recharge. *Hydrogeol J* 10(1): 18–39
- Sophocleous MA (1991) Combining the soilwater balance and water-level fluctuation methods to estimate natural groundwater recharge: practical aspects. *J Hydrol* 124(3–4):229–241
- Tartakovsky GD, Neuman SP (2007) Three-dimensional saturated-unsaturated flow with axial symmetry to a partially penetrating well in a compressible unconfined aquifer. *Water Resour Res* 43(1)
- Theis CV (1935) The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using ground-water storage. *EOS Trans Am Geophys Union* 16(2):519–524
- Tikhonov A, Arsenin V (1977) Solutions of ill-posed problems. Halsted, New York
- Trivedi NM, Kashyap D (2015) Modeling of variably saturated flow in response to pumping from an unconfined aquifer: numerical evidence of gravity-delayed drainage and its parameterization. *J Hydrol Eng* 21(2):06015,014
- van der Spek JE, Bakker M (2017) The influence of the length of the calibration period and observation frequency on predictive uncertainty in time series modeling of groundwater dynamics. *Water Resour Res* 53(3)
- Wu C-M, Yeh T-CJ, Zhu J, Lee TH, Hsu N-S, Chen C-H, Sancho AF (2005) Traditional analysis of aquifer tests: comparing apples to oranges? *Water Resour Res* 41(9)
- Yin L, Hu G, Huang J, Wen D, Dong J, Wang X, Li H (2011) Groundwater-recharge estimation in the Ordos plateau, China: comparison of methods. *Hydrogeol J* 19(8):1563–1575
- Yustres Á, Asensio L, Alonso J, Navarro V (2012) A review of Markov chain Monte Carlo and information theory tools for inverse problems in subsurface flow. *Comput Geosci* 16(1):1–20
- Zhang J, van Heyden J, Bendel D, Barthel R (2011) Combination of soil-water balance models and water table fluctuation methods for evaluation and improvement of groundwater recharge calculations. *Hydrogeol J* 19(8):1487–1502
- Zhou H, Gómez-Hernández JJ, Li L (2014) Inverse methods in hydrogeology: evolution and recent trends. *Adv Water Resour* 63:22–37