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# Numerical evaluation of pumping well transmissivity estimates in laterally heterogeneous formations

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**Abstract** Transmissivity estimates derived from non-steady-state, single well, constant discharge, aquifer tests in laterally heterogeneous environments generally are questioned relative to their representativeness of aquifer conditions. Drawdown in pumping wells reflects the removal of water from storage in the aquifer and transient refraction of groundwater path-lines during the evolution of a non-symmetrical cone of depression. Simulations of single-well aquifer tests in aquifers with simple, arbitrary distributions of block heterogeneities suggest that transmissivity ( $T$ ) values derived by the Cooper–Jacob (1946) method generally reflect volumetric, weighted mean  $T$  values of all of the heterogeneities contacted by the cone of depression at a particular time. This finding suggests that early-time drawdown data for single well aquifer tests reflect rapidly changing, volumetric, weighted mean  $T$  values proximal to the pumping well while late-time drawdown data reflect stabilized conditions and spatially averaged, volumetric weighted mean  $T$  out to a considerable distance from the pumping well.

**Resumé** Les estimés de transmissivité dérivés d'essais de pompage en régime transitoire dans un seul puits contenu dans une formation latéralement hétérogène sont généralement questionable en ce qui concerne la représentativité des conditions de l'aquifère. Le rabattement dans les puits de pompage reflète l'extraction d'eau provenant de l'emmagasinement de l'aquifère et la réfraction en régime transitoire des lignes d'écoulement durant l'évolution d'un cône de dépression non symétrique. Des simulations d'essai de pompage dans un seul puits contenu dans un domaine avec des blocs hétérogènes distribués arbitrairement suggèrent que les valeurs de transmissivité obtenus avec la méthode d'interprétation de Cooper–Jacob (1946) sont généralement représentatifs du volume moyen

pondéré de toutes les hétérogénéités en contact avec le cône de dépression à un temps particulier. Cette découverte suggère que les données de rabattement au début d'un essai pour un seul puits reflètent un changement rapide des valeurs moyennes proximales pondérées tandis que le rabattement à un temps tardif reflète les conditions stabilisées et la moyenne spatiale jusqu'à une distance considérable du puits de pompage.

**Resume** Los estimados de transmisividad derivados de estados no estables, pozos individuales, descarga constante, pruebas de acuíferos en ambientes heterogéneos laterales generalmente se cuestionan en cuanto a su representatividad de las condiciones de los acuíferos. La extracción de agua en pozos de bombeo refleja la extracción de agua del depósito en el acuífero y la refracción transitoria de líneas de trayectoria del agua subterránea durante la evolución de un cono no simétrico de depresión. Las simulaciones de pozos individuales, pruebas de acuíferos en acuíferos con distribuciones simples, arbitrarias de heterogeneidades de bloque sugieren que los valores de transmisividad ( $T$ ) derivados por el método Cooper–Jacob (1946) generalmente reflejan valores medios volumétricos potenciales (weighted) de todas las heterogeneidades en contacto con el cono de la depresión en un momento particular. Este descubrimiento sugiere que los datos de extracción tempranos de pruebas de pozos individuales reflejan valores medios potenciales volumétricos que cambian rápidamente en la proximidad del pozo de bombeo mientras que los valores de extracción tardíos reflejan condiciones estabilizadas y medias potenciales volumétricas a una distancia considerable del pozo de bombeo.

**Keywords** Aquifer testing · Heterogeneity · Modeling · Single-well tests · Volumetric weighted mean transmissivity

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## Introduction

The actual meaning of transmissivity ( $T$ ) estimates derived from drawdown data collected in pumping wells in heterogeneous aquifers is often questioned. Unlike slug tests, which measure the hydraulic conductivity of a relatively small volume of aquifer in the immediate vicinity

of the well bore, single-well, constant discharge, aquifer tests yield integrated drawdown curves for much larger, undefined, volumes of the aquifer. Analysis of these integrated drawdown curves provides estimates for aquifer  $T$  that have uncertain meaning with respect to the actual properties of the aquifer stressed during an aquifer test. The uncertainty typically manifests itself as some type of average  $T$  within the changing volume of aquifer stressed by pumping as time increases. The uncertainty is increased when one considers whether drawdown data for a pumping well adequately represent some type of localized average aquifer response to spatially distributed heterogeneities within the cone of depression. The meaning of derived aquifer coefficients often is considered to vary between early-time, intermediate-time, and late-time as different volumes of the aquifer are represented. The representativeness of  $T$  estimates derived from “good” straight-line plots of pumping well drawdown data is discussed in this paper. Semi-log plots of drawdown data produced by simulated aquifer tests in laterally heterogeneous environments with known distributions of heterogeneities are evaluated relative to the volumetric growth rates of specific portions of the cones of depression within individual heterogeneities.

## Previous Investigations

Much research has been conducted over past decades with respect to the development of methods to estimate the hydraulic properties of heterogeneous porous media. Many research efforts have focused on stochastic descriptions of synthetic heterogeneous media to define “average”, “effective”, or “equivalent” hydraulic properties. Comparatively few research efforts have focused on the plight of the practicing groundwater hydrologist who must interpret real aquifer drawdown responses for site characterization or regulatory purposes. Based on an analysis of core samples, Cardwell and Parsons (1945) suggested that the equivalent transmissive capacity of randomly distributed block heterogeneities lies between the harmonic and arithmetic means of the actual transmissive capacities of the heterogeneities. In addition, they suggested that for heterogeneities of different sizes, averages must be weighted for volume, and for radial flow the averages must be weighted by distance. Based on three-dimensional models composed of homogeneous blocks with different permeabilities, Warren and Price (1961) demonstrated that the permeability derived from pressure build-up tests reflected the equivalent permeability of the drainage volume. They concluded that the geometric mean permeability of the block heterogeneities within the drainage volume represented a reasonable estimate of the effective permeability, and that variations in geometry, anisotropy, and partial penetration had only a small effect on the equivalent permeability. Toth (1966) presented one of the first published accounts of the use of log-log and semi-log plots of late-time drawdown data (field data) to evaluate long-term aquifer yield for a het-

erogeneous aquifer. He showed that log-log and semi-log plots of late-time drawdown data for multiple observation wells converged on single curves or straight lines, respectively, which represented large-scale average conditions. Toth (1966) showed that early-time drawdown data generally produced higher and much more variable estimates for  $T$  than late-time data. He noted also that a wide range of calculated  $S$  values was produced during multiple well, aquifer tests. Freeze (1975) furthered the work of Warren and Price (1961), and questioned whether aquifer coefficients derived by aquifer testing are at all representative of the stochastic properties of a non-uniform formation. Vandenberg (1977) directed his investigation toward the practicing groundwater hydrologist when he duplicated and extended the work of Warren and Price (1961); he found average  $T$  values for the model nodes to be closer to the arithmetic mean than the geometric mean. Bibby (1977) evaluated 122 drawdown curves for pumping wells in heterogeneous, clastic sediments. He described four basic drawdown curve shapes, and used the Cooper and Jacob (1946) method to ascertain the “short-term transmissive capacity” near each well. Bibby (1979) discussed the meaning of aquifer coefficients derived from early and late-time data with respect to the estimation of local and regional averages; based on the work of Cardwell and Parsons (1945), he defined weighted arithmetic, harmonic, and geometric means for the long-term transmissive capacity of a drainage volume divided into concentric rings centered on the pumping well. Bibby (1979) developed weights based on block heterogeneities of equal volume, and the mean radial distance between the pumping well and the center of each block. Barker and Herbert (1982) simulated a real aquifer test conducted in a “patchy” aquifer to aid interpretation of the aquifer drawdown response. Streltsova (1988) suggested that calculated  $T$  and  $S$  values are averages of the between well properties and the properties surrounding the wells; however, heterogeneities near the pumping well were suggested to exert much more influence on the drawdown response than those located farther from the well. Streltsova (1988) suggested also that the type of averaging that occurs is dependent upon the areal distribution of the heterogeneities. Butler (1986, 1988, 1990) and Butler and McElwee (1990) presented insights into the interpretation of aquifer response data for aquifers composed of radially symmetrical disk shaped heterogeneities. According to Butler (1988), the Theis (1935) method will yield estimates of  $T$  and  $S$  that are weighted averages of near-well and far-field properties for heterogeneities, which are distributed radially symmetrical about the pumping well. Butler (1991) extended his earlier work by incorporating the effects of lateral heterogeneity into his analysis of aquifer-drawdown response. He concluded that  $T$  values derived from observation well-drawdown curves provide reasonable estimates for most practical applications. Butler and Liu (1993) analyzed the effects of the presence of a disk-shaped heterogeneity on observation well drawdown. The effects were evaluated for different radial and angular locations from the pumping well.

They showed that measurable effects on observation well drawdown were primarily a function of distance between the disk and the pumping well. Butler and Liu (1993) concluded that constant-rate aquifer tests are not very effective for the characterization of lateral variations in flow properties. Schad and Teutsch (1994) compared  $T$  and  $S$  values derived from several small-scale and large-scale aquifer tests in a braided stream environment. They suggested that the effective length scale of the heterogeneity structure can be estimated from their aquifer test data, but not “true” effective  $T$  and  $S$  values. Meier et al. (1998) suggested that the straight-line method of Cooper and Jacob (1946) will provide a good approximation of the effective  $T$  in multi-lognormal and non-multi-lognormal  $T$  fields when constrained to late-time data. Meier et al. (1999) suggested that  $T$  values derived from single well tests change over the duration of the tests from approximately the geometric mean of the  $T$  values close to the well at early times to the effective  $T$  of the heterogeneous system at late times. Sánchez-Vila et al. (1999) found that estimated transmissivity values for different observation points tended to converge on a single value, which in a multi-log-Gaussian field corresponded to the geometric mean of the point  $T$  values. In addition, they noted that  $T$  values derived from long-term pumping tests reflected weighted averages of the  $T$  values throughout the entire domain. Osiensky et al. (2000) showed that representative  $T$  values can be derived by the Theis (1935) method for multiple well aquifer tests when log-log plots of drawdown versus  $t/r^2$  are constrained by drawdown data collected in observation wells located close to the pumping well. However, Toth (1966), Meier et al. (1998, 1999), and Osiensky et al. (2000) noted that estimates of  $S$  ranged widely.

Naff (1991) used a perturbation solution for three-dimensional radial flow in heterogeneous porous media. He concluded that the effective hydraulic conductivity will be essentially constant at a distance greater than two to three length scales from the well bore and will have a value dependent upon the statistical anisotropy of the medium. Desbarats (1992) investigated steady-state flow in a heterogeneous medium using a combined numerical-empirical approach. His findings were consistent with those of Cardwell and Parsons (1945) and suggested that the effective transmissivity of a single-well radial system could be estimated as a spatial geometric average of point transmissivities weighted by their inverse squared distance from the well bore axis. Desbarats (1993) extended his investigation to steady-state flow between an injection well and a pumping well. He concluded that inter-well transmissivity could be represented by the harmonic mean of transmissivities averaged over circular regions centered at each well. Oliver (1993) used a perturbation approach to evaluate the effects of two-dimensional areal variations in  $T$  and  $S$  on observation well drawdown. He suggested that the area influencing observation well drawdown is bounded by an ellipse that encloses the pumping well and the observation well.

The purpose of this paper is to illustrate some of the major factors that affect the slope of semi-log plots of drawdown versus time for pumping wells in laterally heterogeneous aquifers, and the meaning of aquifer transmissivity derived from those plots. Aquifer tests in confined aquifers with different, known, hypothetical distributions of heterogeneities were simulated with MODFLOW (McDonald and Harbaugh 1988) based on the grid convention introduced by Barrash and Dougherty (1997). Simulated drawdown data for pumping wells were analyzed by the Cooper and Jacob (1946) method to estimate transmissivity.

## Drawdown Due to Pumping

Groundwater produced by a well in an extensive confined aquifer is derived from storage within the aquifer. Discharge ( $Q$ ) from the well must be equal to the product of the aquifer storativity ( $S$ ) and the rate of decline in head ( $h$ ) integrated over the area affected by pumping (Davis and DeWiest 1966).

If the extensive aquifer is homogeneous and isotropic, the distribution of head about the pumping well is radially symmetrical and

$$Q = -2\pi S \int_{r_w}^{\infty} r \frac{\partial h(r,t)}{\partial t} dr \quad (1)$$

where  $r$  is distance from the pumping well [L];  $r_w$  is the radius of the well [L]; and  $t$  is time.

Cooper and Jacob (1946) presented a modified non-equilibrium equation to describe drawdown for large values of  $t$  and/or small values for  $r$

$$s = \frac{2.3Q}{4\pi T} \log \frac{2.25Tt}{r^2 S} \quad (2)$$

where  $s$  is the drawdown due to pumping [L];  $Q$  is the constant pumping rate [ $L^3/t$ ];  $T$  is the aquifer transmissivity [ $L^2/t$ ]; and  $S$  is the aquifer storativity [dimensionless].

Based on Eq. (2), a plot of drawdown versus the logarithm of time will form a straight line, the slope of which represents the aquifer transmissivity.

Application of Eq. (2) to heterogeneous aquifers is problematic because it no longer can be considered to describe drawdown due to pumping uniquely in any radial direction from a pumping well. Drawdown generally is not radially symmetrical in heterogeneous environments because the refracted path-lines followed by fluid particles in route to the pumping well change with position as the cone of depression grows.

## Volume Computations Within a Cone of Depression

The volume of a cone of depression in the potentiometric surface of a confined aquifer is defined uniquely in an aquifer of constant storativity by

$$V = \frac{Q}{S}t \quad (3)$$

where  $V$  is the volume of the cone of depression [ $L^3$ ].

Whereas the  $S$  controls the volume of the cone of depression for a given volume of pumping,  $T$  exerts a greater control over the ultimate shape and extent of the cone. This fact is well known. However, in a heterogeneous aquifer,  $S$  and  $T$ , by definition, vary in space, and the combined effects of variability in either one or both of the aquifer coefficients are reflected in the area and shape of the cone of depression expressed at the potentiometric surface. Therefore, it is proposed that analysis of spatial, volumetric variations in the cone of depression in a heterogeneous aquifer is appropriate and offers a more general solution to evaluate the meaning of  $T$  values derived from single well aquifer tests. In heterogeneous aquifers, the volumetric rate of growth of a cone of depression will vary in space. Therefore, the  $S$ ,  $T$  and physical volume of an individual heterogeneity control the “volume portion” of the total cone of depression that is due to that heterogeneity. A volume portion is defined in this paper as the percentage of the measurable total volume of a cone of depression that exists within an individual heterogeneity of constant  $T$  and  $S$ . The term “measurable” implies the practical field measurement of drawdown at an observation point to the nearest 0.003 m with the technology available currently (2004). Use of Eq. (2) involves fitting an ideal, theoretical straight line to real, measurable, pumping well drawdown that, in reality, often incorporate spatially and temporally variable effects of path-line refraction through three-dimensional heterogeneities.

Drawdown stabilizes as a function of radial distance ( $r$ ) from the pumping well and time ( $t$ ). Bibby (1979) suggested that the early-time portion of a drawdown curve probably reflects the average or effective local conditions near the well whereas the late-time portion of a drawdown curve reflects more regional average or effective conditions. Bibby (1979) and Butler (1988) suggested that the late portions of drawdown curves can be used to estimate sustainable aquifer yield.

To evaluate the volume portions during the simulated aquifer tests, the percentage of the total volume of the cone of depression contained within each of the heterogeneities was estimated for the 25 time steps of each simulation. Volumes were estimated by the trapezoidal rule with SURFER 7 (Golden Software, Inc. 1999). The estimated volumes were used to derive arithmetic, harmonic, and geometric weighted mean  $T$  values for all of the aquifer materials contacted by each cone of depression for every time step as follows (Osiensky et al. 2000):

$$AWMT = \frac{\sum_{i=1}^n w_i T_{H_i}}{\sum_{i=1}^n w_i} \quad (4)$$

$$HWMT = \frac{1}{\frac{\sum_{i=1}^n \frac{1}{T_{H_i}^{w_i}}}{\sum_{i=1}^n w_i}} \quad (5)$$

$$GWMT = \left( \prod_{i=1}^n T_{H_i}^{w_i} \right) \frac{1}{\sum_{i=1}^n w_i} \quad (6)$$

where  $AWMT$  is the arithmetic weighted mean  $T$  [ $L^2/t$ ];  $HWMT$  is the harmonic weighted mean  $T$  [ $L^2/t$ ];  $GWMT$  is the geometric weighted mean  $T$  [ $L^2/t$ ];  $w_i$  is the weighting factor equal to the ratio ( $V_{H_i}/V_T$ ) which is independent of pumping rate;  $V_{H_i}$  is the volume of the cone of depression within the  $i^{\text{th}}$  heterogeneity at time  $t$  [ $L^3$ ];  $V_T$  is the total volume of the cone of depression at time  $t$  [ $L^3$ ];  $T_{H_i}$  is the actual  $T$  of the  $i^{\text{th}}$  heterogeneity [ $L^2/t$ ].

This weighting scheme, modified after the development of Bibby (1979), weights  $T$  as a function of the volume of the cone of depression established within each heterogeneity, and yields  $AWMT \geq GWMT \geq HWMT$ .

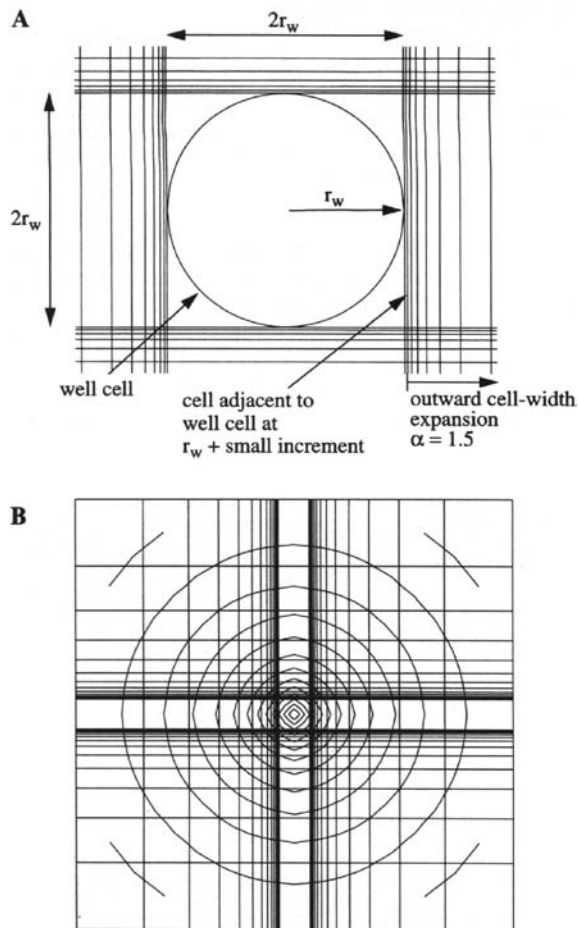
### Simulated Experimental Aquifer Conditions

MODFLOW was used to simulate two-dimensional transient groundwater flow to a pumping well in a confined aquifer with simple, lateral heterogeneities. Two models were developed and run using Visual MODFLOW (Waterloo Hydrogeologic, Inc. 2000). The zonation approach was used whereby the region of groundwater flow for each simulation was divided into distinct zones (i.e., block heterogeneities as rectangular parallelepipeds) with constant  $T$  and  $S$  values assigned to each zone. The distribution of heterogeneities was chosen so that calculation of the volumetric distribution of the cone of depression within the heterogeneities would be a tractable problem. In addition, the distribution was chosen to illustrate clearly that aquifer transmissivity values derived from pumping well drawdown data may not reflect actual values or expected average values near the well.

The grids for the two models were developed based on the discretization scheme of Barrash and Dougherty (1997) for simulation of pumping well drawdown (Fig. 1). A fully penetrating, pumping well with a diameter of 0.154167 m (6 inches) was placed at the geometric center of each grid.

The conditions of a confined aquifer with a constant thickness of 30 m and a uniform storativity ( $S$ ) of 0.005 were simulated for both models. Each model domain consisted of a square grid of dimensions 2,450×2,450 m with variable cell spacings. Impermeable boundaries were placed around the perimeter of the model grids (Modflow default condition) beyond the influence of pumping. Model input values for  $T$  and  $S$ , and the pumping rate were chosen to control drawdown within the cone of depression so that no significant drawdown occurred





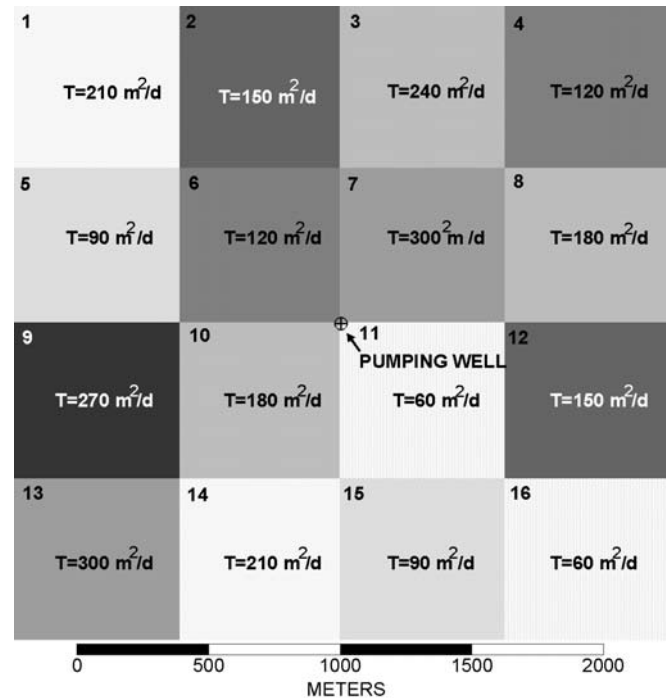
**Fig. 1** **A** Discretization scheme: width of well cell is well diameter; width of cell adjacent to well cell is given a small width relative to the well cell. Cell widths outward from this small-width cell increase progressively by a factor of  $\alpha = 1.5$ . **B** Drawdown at the well is accurately simulated because the grid discretization scheme (1) locates nodes essentially at the edge of the well, and (2) has dense discretization both outward from the well cell and around the curvature of the well-cell corner (from Barrash and Dougherty 1997)

along the perimeter of the model grids. The initial drawdown everywhere in the aquifer was zero for all simulations. Each aquifer test consisted of a 24-h (1,440 min) stress period divided into 25 time steps with a time-step multiplier of 1.2. Drawdown along the perimeter of the model grids was  $<0.0001$  m for the duration of both simulated aquifer tests.

## Discussion of Modeling Results

### Model 1: Aquifer Test 1

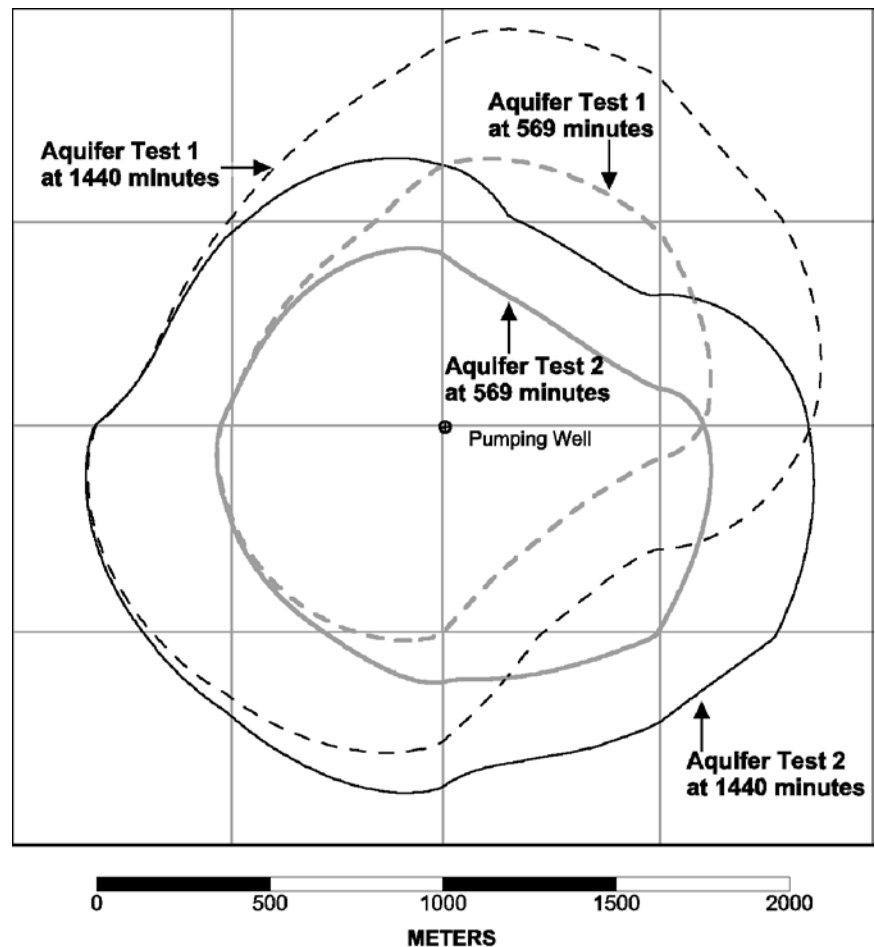
The entire model domain for Aquifer Test 1 consisted of 16 separate zones with nine different  $T$  values (Fig. 2). The pumping well was located within the upper left corner of zone 11 ( $T = 60$  m<sup>2</sup>/day), and not in the center of the zone.



**Fig. 2** Hydrogeologic map of the transmissivity distribution for Aquifer Test 1. Note: the transmissivity distribution for Aquifer Test 2 was the same as Aquifer Test 1, except that the transmissivities for zones 7 and 11 were interchanged; zone 11 was replaced with  $T = 300$  m<sup>2</sup>/day and zone 7 was replaced with  $T = 60$  m<sup>2</sup>/day. The outside perimeter of the figure represents the impermeable boundaries of the model grid

Simulation of Aquifer Test 1 was designed specifically to evaluate the volumetric effects that each of the 16 heterogeneities exerted on the growth of the cone of depression over time and the resulting pumping well drawdown data. Time varying,  $AWMT$ ,  $HWMT$ , and  $GWMT$  values were derived based on Eqs. (4), (5), and (6), respectively. The volumetric rate of growth of specific portions of the cone of depression was a function of the pumping rate, and the hydraulic diffusivity ( $T/S$ ) and physical volume of each heterogeneity contacted by the entire cone at a particular point in time. Most of the cone of depression was contained within zone 11 for the first few minutes of the aquifer test because of the pumping well location in the upper left corner of this zone. However, cone growth was non-axisymmetrical at later times as the cone of depression grew preferentially in the higher diffusivity zones at the expense of further cone expansion in zone 11 (Fig. 3). This resulted in continually increasing  $AWMT$ ,  $HWMT$ , and  $GWMT$  values (Fig. 4) with time as higher diffusivity materials exerted greater influence over the spatially varying, volumetric growth rate of the cone of depression. The weighted mean  $T$  values increased most rapidly for early values of time ( $<400$  min) when the cone of depression was being established in the aquifer materials close to the pumping well (within about 500 m). This finding is consistent with the work of Streltsova (1988), Butler (1988,1991), and Butler and Liu (1993) who indicated that the aquifer properties close to the

**Fig. 3** Cones of depression formed during Aquifer Test 1 and Aquifer Test 2. The *dashed lines* represent 99.3% of the total cone volume at time = 569 min (*dashed gray line*) and 1,440 min (*dashed black line*) of pumping for Aquifer Test 1. The *solid lines* represent 99.3% of the cone volume at time = 569 min (*gray line*) and 1,440 min (*black line*) of pumping for Aquifer Test 2. The outside perimeter of the figure represents the impermeable boundaries of the model grid



pumping well exert a much greater influence on observation well drawdown than aquifer materials at other locations in the aquifer. At later times ( $>400$  min), the *AWMT*, *HWMT*, and *GWMT* values increased log-linearly as the cone of depression developed toward steady shape. The effects of spatial variation in  $T$  propagated throughout the entire cone of depression as a function of time.

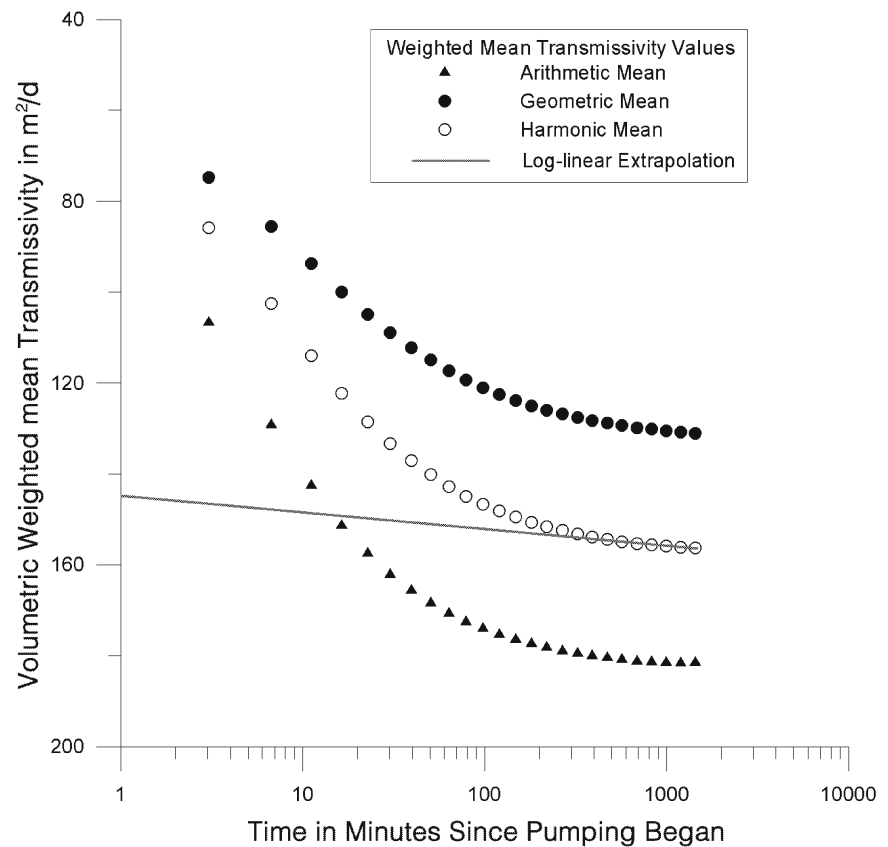
The volumetric rate of growth of the cone of depression was related directly to the uniform  $S$  of the aquifer for Aquifer Test 1. However, distortion of the cone of depression (Fig. 3) was a function of the  $T$  distribution only. McElwee and Yukler (1978), and Serrano (1997) showed that drawdown predicted by the Theis equation is more sensitive to  $T$  than to  $S$  near the pumping well, and that the sensitivity decreases with distance from the pumping well. McElwee and Yukler (1978) also showed that the effect due to a change in  $S$  is significant over a larger area than that due to a change in  $T$ . However, the specific effects of spatial variability in  $T$  and/or  $S$  on measured drawdown data generally are not discernible in a semi-log plot. Therefore, average  $T$  values are derived (rather than unique values) based on the slope of a straight line that reflects the net effects of heterogeneities contacted by the cone of influence.

The advent of electronic, pressure transducers and submersible data loggers has provided a means for the

collection of frequent pumping well drawdown measurements on the time frame of seconds. However, because of the critical time at which the Cooper–Jacob method becomes valid for pumping wells [i.e., Eq. (2)], and other complicating factors such as potential borehole storage and skin effects, it generally is not recommended to use early-time, drawdown data to derive aquifer properties for real wells.

Figure 5 shows a semi-log plot of drawdown versus time for the pumping well for Aquifer Test 1. A non-linear least-squares straight-line match to the late-time data (last ten measurements) yields a  $T$  value of  $155.6 \text{ m}^2/\text{day}$ , which is nearly identical to the *GWMT* value of  $156.2 \text{ m}^2/\text{day}$  within the cone of depression at  $t=1,440$  min of pumping (Fig. 3). Early-time deviations from the straight line may reflect a combination of model error and the rapidly changing volumetric weighted mean  $T$  values; however, comparison of Figs. 4 with 5 suggests that early-time drawdown data for Aquifer Test 1 probably were affected by a period of transition between arithmetic averaging during early times, and geometric averaging after about  $t=200$  min as the cone of depression began to stabilize. Figure 4 also suggests that a short-term aquifer test of less than about 200 min in duration will not yield predictable values for average  $T$  for the conditions simulated in Aquifer Test 1. The late-time  $T$  value derived

**Fig. 4** Semi-log plots of time-varying arithmetic, harmonic, and geometric, weighted mean, transmissivity values within the cone of depression during Aquifer Test 1



from the pumping well drawdown data clearly represents the *GWMT* conditions within the entire cone of depression, and not just the conditions immediate to the pumping well.

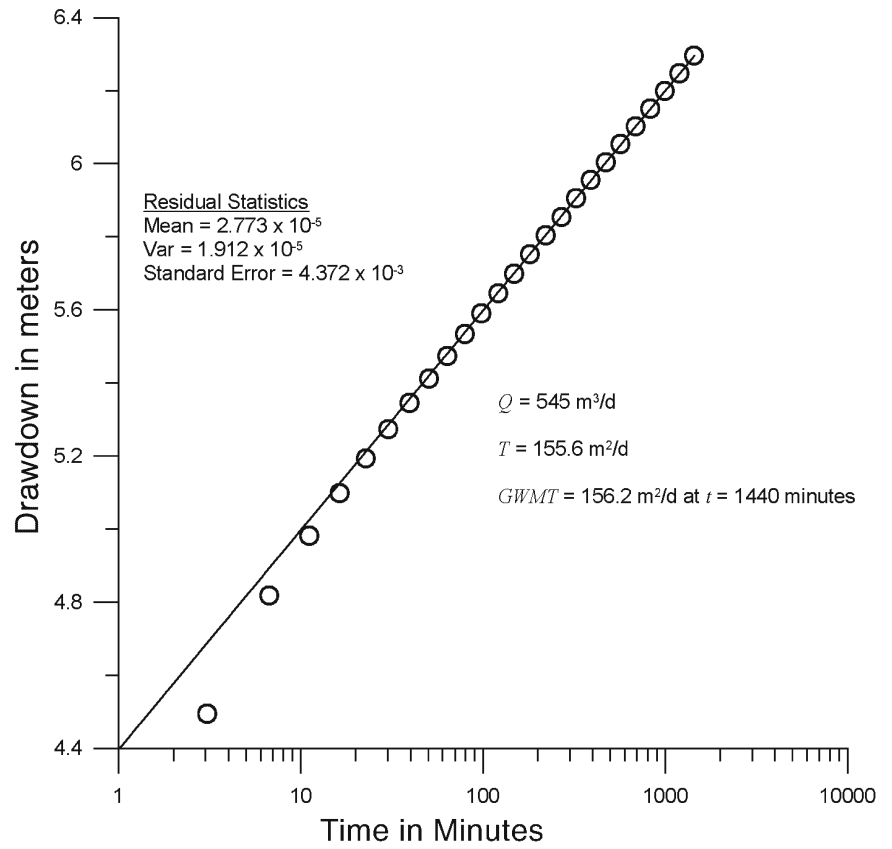
### Model 2: Aquifer Test 2

The entire model domain for Aquifer Test 2 consisted of 16 separate zones with nine different *T* values (Fig. 2). The pumping well again was located within the upper left corner of zone 11, and not in the center of the zone. However, for Aquifer Test 2, the material properties for zones 7 and 11 were interchanged. Zone 11 was replaced with  $T=300$  m<sup>2</sup>/day and zone 7 was replaced with  $T=60$  m<sup>2</sup>/day. The material properties of all other zones were the same as for Aquifer Test 1. Simulation of Aquifer Test 2 was designed to evaluate the effects that placement of the pumping well in high *T* material would have on the volumetric rate of growth of the cone of depression over time and the resulting drawdown plot. The percentage of the total volume of the cone of depression contained within each of the 16 heterogeneities was calculated for the 25 time steps of the simulation in the same manner as for Aquifer Test 1. Time varying, *AWMT*, *HWMT*, and *GWMT* values were derived based on Eqs. (4), (5), and (6), respectively (Fig. 6). The weighted mean *T* values decreased over time contrary to Aquifer Test 1 because the cone of depression grew from high *T* into lower *T* materials. At later times, the *AWMT*, *HWMT*,

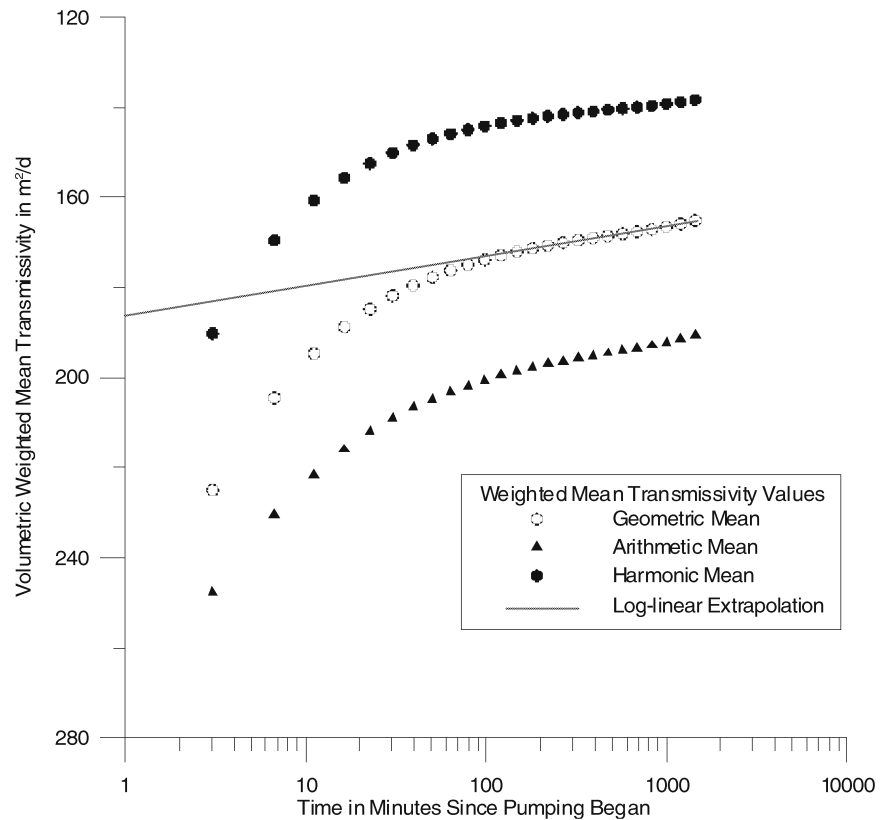
and *GWMT* values decreased log-linearly as the cone of depression developed toward steady shape. Comparison of Fig. 4 with Fig. 6 suggests that plots of the *AWMT*, *HWMT*, and *GWMT* values for Aquifer Test 1 and Aquifer Test 2, respectively, reflect nearly the same steady-shape conditions. However, spatial differences in the volumetric growth rate of the cone of depression for Aquifer Test 2 relative to Aquifer Test 1 resulted in significantly different 1,440-min cone shapes (Fig. 3).

Figure 7 shows a semi-log plot of drawdown versus time for the pumping well for Aquifer Test 2. A non-linear least-squares straight-line match to the late-time data (last ten measurements) yields a *T* value of 165.7 m<sup>2</sup>/day, which is nearly identical to the *GWMT* value of 165.2 m<sup>2</sup>/day within the cone of depression at  $t=1,440$  min of pumping. Again, early-time deviations from the straight line may reflect a combination of model error and the rapidly changing volumetric weighted mean *T* values; however, the log-linear extrapolation shown in Fig. 6 suggests that early-time drawdown data for Aquifer Test 2 (Fig. 7) probably were affected by a period of transition between harmonic averaging during early times and geometric averaging after about  $t=100$  min. This suggests that a short-term aquifer test of less than about 100 min in duration will not yield predictable values for average *T* for the conditions simulated during Aquifer Test 2. The plot of volumetric, weighted mean *T* values within the evolving cone of depression became log-linear about 100 min earlier than during Aquifer Test 1 due to

**Fig. 5** Semi-log plot of pumping well drawdown versus time for Aquifer Test 1 showing Cooper–Jacob, non-linear least-squares straight-line match to the late-time data (last ten points)

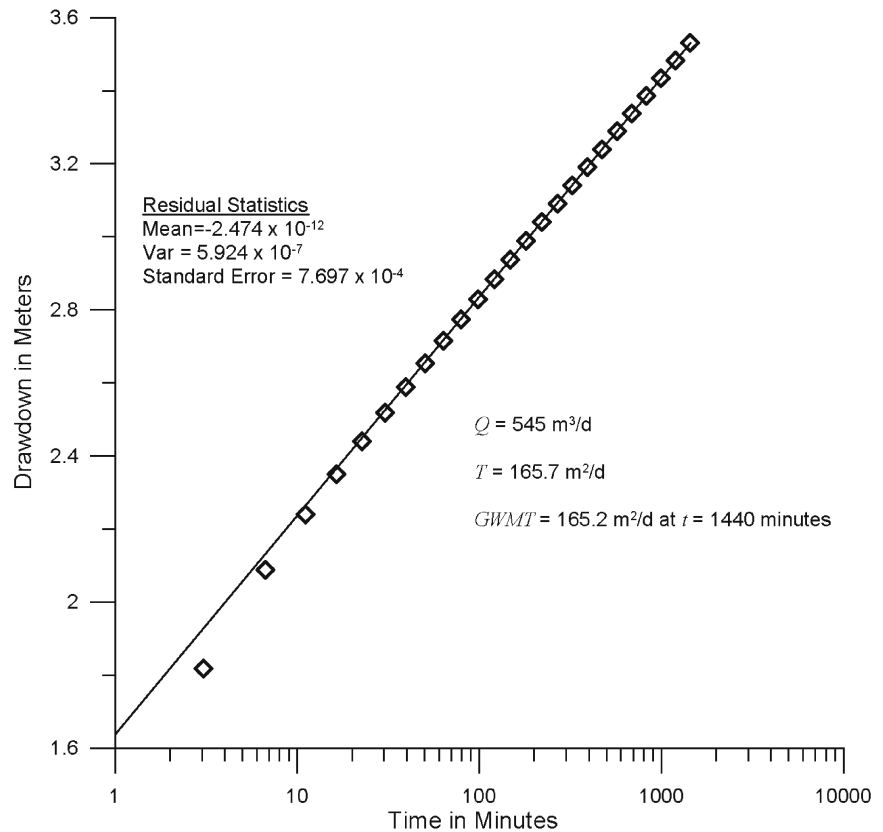


**Fig. 6** Semi-log plots of time-varying arithmetic, harmonic, and geometric, weighted mean, transmissivity values within the cone of depression during Aquifer Test 2





**Fig. 7** Semilog plot of pumping well drawdown versus time for Aquifer Test 2 showing Cooper–Jacob, non-linear least-squares straight-line match to the late-time data (last ten points)



pumping from the higher  $T$  materials. As expected, the late-time  $T$  value derived from the pumping well drawdown data accurately reflects the  $GWMT$  conditions within the entire cone of depression formed during Aquifer Test 2.

## Summary and Conclusions

Simulations of the aquifer tests produced heterogeneous drawdown data that reflected the net effects of water removed from storage in the aquifer heterogeneities, combined with spatially and temporally varying hydraulic gradients. Analysis of the late-time pumping well-drawdown data for the two aquifer tests by the method of Cooper and Jacob (1946) provided large-scale, average  $T$  values rather than  $T$  values proximal to the pumping well. Calculations of volumetric weighted mean  $T$  suggest that the nature of averaging changed during the course of each aquifer test. Between early time and intermediate time, averaging appeared to change from approximately arithmetic to geometric during Aquifer Test 1, and from approximately harmonic to geometric during Aquifer Test 2. These results are consistent with those of other investigators such as Cardwell and Parsons (1945), Warren and Price (1961), Bibby (1979), Vandenberg (1977), Streltsova (1988), and Meier et al. (1999) who reported variations in the nature of averaging for different times, and/or different orientations and distributions of heterogeneities.

Late-time pumping well-drawdown data for both aquifer tests accurately reflect somewhat different  $GWMT$  conditions out to several hundred meters from the pumping well. The results presented herein are in excellent agreement with the results presented by Meier et al. (1999), and are in good agreement with the results of Sánchez-Vila, et al. (1999). For these simulations, the average  $T$  values derived by non-linear least-squares straight-line matches to the late-time (last ten points) pumping well-drawdown data are essentially identical to the  $GWMT$  values calculated for each cone of influence at  $t=1,440$  min of pumping.

Further research is needed to test whether analysis of hydrogeologic coefficients by volumetric weighted means can be extended to the evaluation of three-dimensional distributions of heterogeneities, and partially penetrating wells. Further research also is needed to evaluate the combined effect that spatial variations in specific storage and transmissivity have on the analysis of storativity estimates, and early-time transmissivity estimates, derived from aquifer test data.

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