



Energy propagation through dense granular systems

Lou Kondic¹

Received: 28 February 2019 / Published online: 22 August 2019
© Springer-Verlag GmbH Germany, part of Springer Nature 2019

Abstract

We discuss affine and non-affine components of particle dynamics in the context of energy propagation through dense granular matter as a consequence of externally imposed boundary perturbation. Earlier work (Kondic et al. in Phys Rev E 79:041304, 2009) has shown that the frequency and the wavenumber of the imposed perturbations strongly influence propagation, and in particular that the frequencies and wavenumbers that lead to well-defined propagation are limited from above. The present work shows that strong non-affine component of particle dynamics is associated with dispersion and loss of coherence.

Keywords Dense granular matter · Energy propagation · DEM simulations

1 Introduction

One critical issue in the physics of granular materials is the basic physical mechanism of energy and/or stress propagation. One can roughly distinguish two groups of efforts here. The first one concentrates on forces and energy fluctuations inherent in granular systems, including force transmission. The second one discusses signal (such as sound) propagation and is often based on a continuum description. One important issue is how to link these approaches. More generally, the question is how to connect micro and meso scales, in addition to the relevant time scales.

In the first direction, there has been much recent debate regarding static force propagation. A substantial range of models has been proposed, with dramatically different properties. Some of these approaches, based on probabilistic models for force propagation, predict diffusive (parabolic) propagation of forces [5]. Diffusive behavior has been also reported in experiments [26], although for very small systems. Other approaches lead to a response which may be either wave-like or elastic [2, 3]. Related conclusions have

been reached in other works [4, 22]. Assemblies of frictionless particles are shown to lead to wave-like signal propagation [27]. Recent experiments, however, seem to be consistent with an elastic response to an applied point force [8, 9, 23]. It is puzzling that in different experiments and models one can observe such a variety of propagation mechanisms. One wonders whether different behavior is triggered as the parameters of a granular system vary [11, 20].

Continuum models used in engineering literature (e.g. [10, 24]) commonly assume elastic or elasto-plastic response, which does not seem to be consistent with the propagating, hyperbolic (or parabolic) type of response often obtained under microscopic picture. There is a question whether the models based on elasticity theory can be used to explain signal propagation through granular systems [21]. Some progress in connecting discrete and continuum descriptions has been reached in recent works that show that the system's response may change depending on the scale: one can see wave-like response on short (meso) scales, but elastic response on larger ones [11, 12]. These works also point out that friction and anisotropy can have an important effect. Still, there are numerous questions to be answered, even regarding relatively simple configuration of a static granular material exposed to a localized perturbation.

An issue of signal, such as sound, propagation through granular systems is closely connected to force and energy propagation although it is often treated differently. Within continuum theory, this issue is typically approached via effective medium theory [10, 28]. Most of the works explore

This article is part of the Topical Collection: In Memoriam of Robert P. Behringer.

✉ Lou Kondic
kondic@njit.edu

¹ Department of Mathematical Sciences and Center for Applied Mathematics and Statistics, New Jersey Institute of Technology, Newark, NJ 07102, USA

theoretically and experimentally the response of a granular system to a spatially independent perturbation which is large compared to a particle size, such as piston moving in a granular system. It has also been pointed out that force chains may be important in sound propagation [1, 13, 19]. Some recent papers try to reconcile the apparent differences between earlier works [14, 15] by pointing out that the direct and strongly scattered signal components should be treated separately. However, this issue is still not completely understood, in particular regarding propagation through fragile systems close to the rigidity threshold.

Some years back [16], we considered computationally a dense granular system exposed to perturbations that were space and time dependent. The computations were carried out using a discrete element method (DEM) simulations. The main finding of that work was that within certain regime, the process of energy propagation could be described reasonably well via linear wave equation with an additional damping term. Therefore, in [16] we showed that DEM simulations could be used to connect to the macro-scale and relate to continuum models for energy propagation. In the present work, we go back to the particle scale, and relate the results presented in [16] to the details of particle structure and dynamics, with particular focus on affine and non-affine particle motion. We will show that the origins of some macroscopic features relevant to energy propagation may be understood better by considering the details of particle dynamics.

2 Methods

The simulation techniques are described in [16] and in more detail in [7]; here we just provide a brief summary. We choose a relatively simple granular geometry in two spatial dimensions with the granular particles constrained between two rough walls (up–down) with periodic boundary conditions (left–right). We use constant volume protocol such that the volume fraction, ρ , is prescribed by the walls' positions; constant pressure protocol is discussed in [7]. The particles are polydisperse discs, with the radii varying randomly in some range r about the mean. For simplicity, we put gravity to zero. The particle–particle and particle–wall collisions are modeled using linear springs describing the normal forces between the particles, and Cundall–Strack model for tangential forces. The wall particles are typically chosen as strongly inelastic and frictional so to reduce the reflection phenomena. The simulations are prepared by very slow compression without applied perturbation until the required ρ is reached. After this initial stage, the system is relaxed. The upper boundary is fixed and the lower boundary is perturbed. The perturbations are of the form $z(x) = z_0 + A \sin(\omega t) \sin(kx)$,

where A , ω , k are the amplitude, angular frequency, and the wave number, respectively.

Here we list the parameters that are used in the simulations that follow; the choice of parameters has emerged through the years from a number of works, such as [6, 17, 18], involving simulations of photoelastic particles from Behringer's lab. The additional simulations, reported in [7], have shown that the qualitative features of the results that follow rather weakly depend on the parameters specifying the particles and walls. *Particle properties*: normal spring constant is given the value of $k_n = k_f mg/d$, $k_f = 4 \times 10^3$, where g is the acceleration of gravity used here for convenience, although gravitational effects are not considered in the simulations), and m , d are average mass and diameter of a particle. Note that based on k_n , one can specify the binary particle collision time, $\tau_c = 2\pi \sqrt{d/(2gk_f)}$ that we will use as the time scale. The tangential spring constant, entering Cundall–Stark model, is given the same strength as the normal one, so $k_t = k_n$. The dissipation is specified by γ_n and γ_t which are given the values leading to the coefficient of restitution $e_n = 0.5$; polydispersity is introduced by specifying the particles sizes in the range $(1 \pm r)d$ with $r = 0.1$; the coefficient of friction is $\mu = 0.5$. The wall particles are characterized by $e_n = 0.1$, $\mu = 0.9$, and are monodisperse. *System properties*: the perturbation is applied by perturbing the lower wall which defines the x axis; typically 40,000 particles are used, with the x dimension of the system being $250d$. The volume fraction is $\rho = 0.9$. Perturbation properties: Amplitude $A = 0.6d$; $f = 30$ Hz, $\lambda = 250d$. These values are used in all simulations except if specified differently. The particular numerical values used for the perturbation are chosen so that wave propagation can be clearly observed; it should be noted that for (relatively soft) particles used, rather large volume fractions and perturbation amplitudes are needed; some results regarding the influence of these and other parameters can be found in [7]. Even for rather soft particles considered here, we note that for all considered simulations $1/f \gg \tau_c$, so that the dynamics imposed by the perturbation is much slower than the natural time scale describing binary particle interaction.

3 Results

The main focus of [16] was on relating the energy propagation results to the ones that would be expected from a simple continuous method of the following form:

$$\nabla^2 E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} - \frac{1}{D} \frac{\partial E}{\partial t} = 0, \quad (1)$$

where E is the measure of energy propagation (such as elastic energy stored in the compressive contacts between

particles), c is the speed of energy propagation, and D measures diffusive damping. As $D \rightarrow \infty$, the model reduces to linear wave equation, and as $c \rightarrow \infty$ to diffusion equation. Assuming that the boundary condition is specified by a perturbation of the form $E(x, 0, t) = E_0 e^{i\omega t} e^{ikx}$ (corresponding to the perturbation imposed in the DEM simulations), where $\omega = 2\pi f$, $k = 2\pi/\lambda$ and f and λ are the frequency and wavelength of imposed perturbations, one can find a solution of Eq. (1) in the form of a damped wave whose properties are described by a simple dispersion relation [16],

$$q = -|q|e^{i\phi/2}, |q|^2 = \mathcal{X}^2 + (\omega/D)^2, \tan \phi = -\frac{\omega}{(D\mathcal{X})}, \quad (2)$$

where $\mathcal{X} = (\omega/c)^2 - k^2$, and q is the wave number corresponding to the z direction. Detailed discussion of the consequences of this dispersion relation could be found in [16]. The DEM simulations [16] have shown that there is a rather narrow range of f, λ leading to a well defined weakly-dispersive energy propagation; Fig. 1 shows four snapshots of elastic energy during one period of the boundary motion for the perturbation parameters that lead to a well defined wave. The existence of such a range was rationalized based on the existing results for c resulting from the elasticity theory, and for D based on the estimate $D \approx v_e \ell/3$, where it was assumed that $v_e = c$, and ℓ was some typical length-scale over which coherent properties of a propagating wave are lost [14, 25]. The DEM simulations and the model specified by Eq. (1) suggest large values for $\ell \approx 30 - 40d$. While such large values of a typical length scale are still not well understood, in what follows we will show that they are consistent with the length-scales describing non-affine particle motion.

Computations of affine (conforming) and non-affine (non-conforming) components of particles' motion was carried

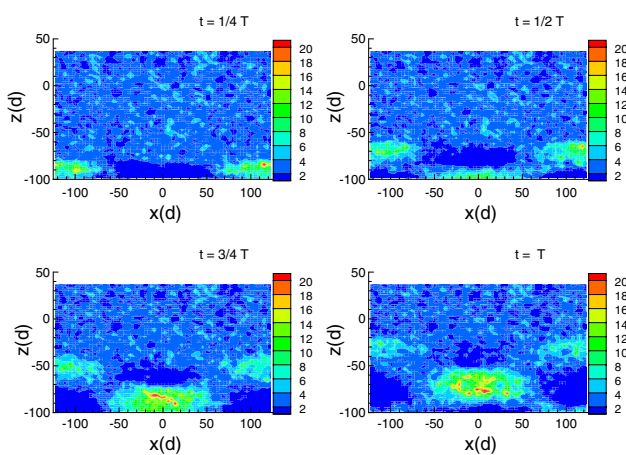


Fig. 1 Elastic energy at four instances during one period of the boundary oscillation; The upper wall is static, and the lower one performs standing type of motion. In this and following figures we use $r = 0.1$, $f = 30$ Hz, $\lambda = 250d$, the amplitude of wall perturbation, $A = 0.6d$, and 40, 000 particles except if specified differently

out using the approach outlined in [17]. The algorithm implemented computes first affine component by considering the motion of a particle relative to the surrounding particles (in a small circle whose radius is equal to $2.5d$). The non-affine component is then found by subtracting the affine component from the particle dynamics computed in simulations. Before considering non-affine component of particle motion, we discuss the affine one. For brevity, here we focus only on the reference case discussed in [16] ($f = 30$ Hz, $\lambda = 250d$, $r = 0.1$, and $\rho = 0.9$ in 2D) and on limited set of other cases where either f or λ are varied.

Figure 2 shows the affine component for the reference system at a given time (for the ease of comparison all the similar figures will show the results at the same (nondimensional) time). This figure suggests well defined affine motion for this choice of parameters. However, Fig. 3, where either the frequency (part a) or the wavelength (part b) are modified, shows that, in both cases, the affine component decreases very quickly away from the oscillating boundary.

Figure 4 shows the time-averaged dominant mode amplitude of the Fourier transform (Fourier expanded in the x direction), for few systems, including those shown in Figs. 2 and 3. Similarly as in analysis of 'raw' DEM results [16], we find wave-like propagation for the reference case (part a), while a change of the frequency or wavelength leads to a quick decay of the affine component as one moves ahead from the oscillating boundary. To

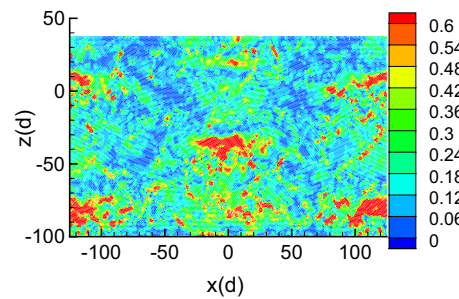
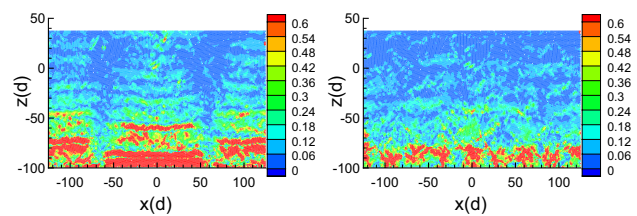


Fig. 2 Affine component for the reference system at $t = 240$ (in units of τ_c)



(a) $f = 100$ Hz, $\lambda = 250d$. **(b)** $f = 30$ Hz, $\lambda = 62.5d$.

Fig. 3 Affine component of particle dynamics for the specified systems at the same time as shown in Fig. 2

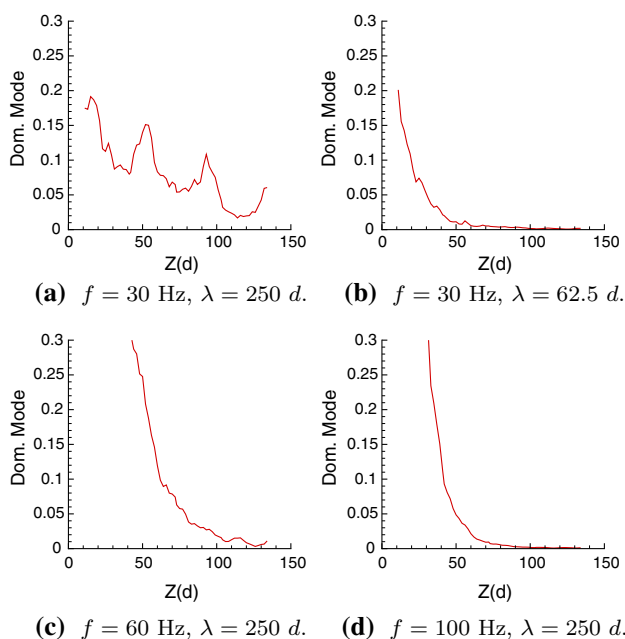


Fig. 4 Dominant mode amplitude of the Fourier transform of the affine component

conclude, we find that the intensity of affine component of particle dynamics is strongly correlated with the wave-like propagation of energy.

We proceed by discussing non-affine component of particle motion. This component is described by the quantity D_{min} , computed by subtracting affine component from particle dynamics and averaging appropriately [7, 17]. Large D_{min} corresponds to large non-affine part of the particles' dynamics.

Figure 5 shows the results for the reference case, at the same time as the affine motion shown in Fig. 2. The part (a) shows the contact number of the particles, and part (b) shows non-affine motion. We note that while the parts of the computational domain corresponding to stronger affine component are characterized by smaller number of contacts, the non-affine motion does not follow this trend and does not show any obvious spatial dependence.

Next we discuss the influence of the frequency and wavelength of perturbation on non-affine component of particle dynamics. Figure 6 shows two typical examples illustrating prominently strong non-affine component close to the oscillating boundary. To confirm that this is a general feature of the results, Fig. 7 shows the non-affine component averaged over the x direction and over time. We see large D_{min} for short wavelengths and high frequencies of perturbations, consistently with the snapshots shown in Fig. 6. These results suggest strongly that non-affine part of the particle motion is responsible for removal of wave-like propagation of energy. Note also that the length scale characterizing

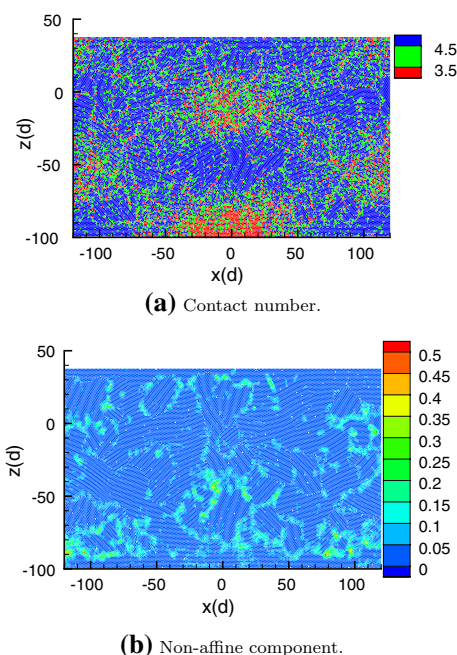


Fig. 5 Reference case results at the same time as in Fig. 2

non-affine dynamics is similar (20–30 d) to the quantity ℓ entering the diffusion equation in Eq. (1).

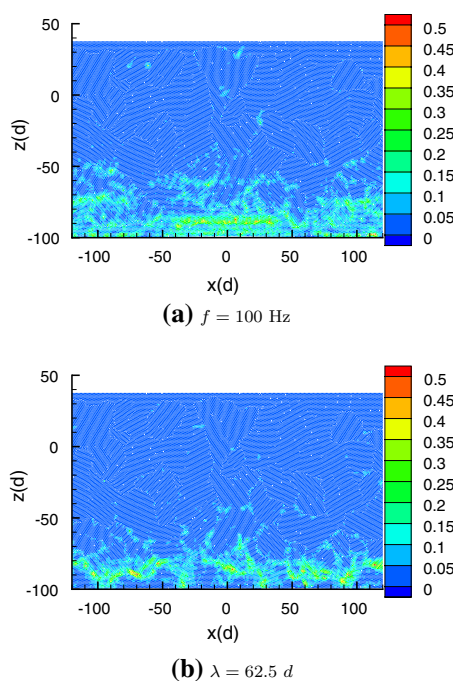


Fig. 6 Non-affine component at the same time as in Fig. 2. All the parameters that are not listed are the same as in the reference case

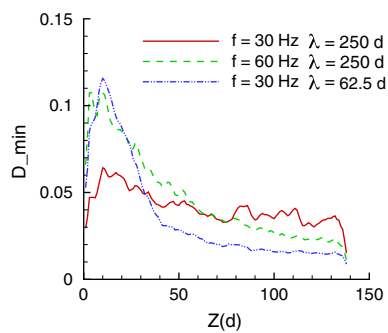


Fig. 7 Non-affine component as a function of the distance from the oscillating boundary

4 Conclusions

In this paper, we discuss particle-scale aspects of energy propagation through dense granular system. We focus in particular on affine and non-affine components of particle dynamics. The parameters that lead to a well defined coherent energy wave lead to strong affine and rather weak non-affine dynamics. However, the parameters for which we observe strong dissipation and quick loss of coherence are found to lead to stronger non-affine component of particle dynamics, and to fast attenuation of the affine component, as one moves away from the oscillating boundary. We conjecture that this strong non-affine component leads to a significant energy loss and that it is responsible for the loss of wave-like properties of propagating energy wave. We expect that this finding will serve as a basis for the future work discussing in more detail mechanisms determining the length scale on which non-affine part of particle dynamics becomes significant.

Acknowledgements This contribution to the Special Issue of Granular Matter is a follow up to the original work carried out in collaboration with Bob Behringer [16]. Thanks are due to Oleh Dybenko, our co-author on [16], and to Xiaoni Fang, PhD, who carried out the reported simulations as a part of her thesis research. The original research project focusing on energy propagation was supported by NSF Grants No. DMS 0605857, 0835611, and DMR No. 0555431, and more recent work by NSF Grant No. DMS 1521717 and ARO Grant No. W911NF1810184.

Compliance with ethical standards

Conflict of interest The author declares that he has no conflict of interest.

References

- Bassett, D.S., Owens, E.T., Daniels, K.E., Porter, M.A.: Influence of network topology on sound propagation in granular materials. *Phys. Rev. E* **86**, 041306 (2012)
- Bouchaud, J.P., Cates, M.E., Claudin, P.: Stress distribution in granular media and nonlinear wave equation. *J. Phys. I* **5**, 639–656 (1995)
- Bouchaud, J.P., Claudin, P., Cates, M.E., Wittmer, J.P.: Models of stress propagation in granular media. In: Herrmann, H.J., Hovi, J.P., Luding, S. (eds.) *Physics of Dry Granular Media*, p. 97. Kluwer Academic Publishers, Dordrecht (1998)
- Bouchaud, J.P., Claudin, P., Levine, D., Otto, M.: Force chain splitting in granular materials: a mechanism for large-scale pseudo-elastic behavior. *Eur. Phys. J. E* **4**, 451 (2001)
- Coppersmith, S.N., Liu, C., Majumdar, S., Narayan, O., Witten, T.A.: Model for force fluctuations in bead packs. *Phys. Rev. E* **53**, 4673 (1996)
- Dijksman, J.A., Kovalcinova, L., Ren, J., Behringer, R.P., Kramár, M., Mischaikow, K., Kondic, L.: Characterizing granular networks using topological metrics. *Phys. Rev. E* **97**, 042903 (2018)
- Fang X.: Energy propagation in jammed granular matter. PhD thesis, New Jersey Institute of Technology, Newark, NJ (2011)
- Geng, J., Howell, D., Longhi, E., Behringer, R.P., Reydellet, G., Vanel, L., Clément, E., Luding, S.: Footprints in sand: the response of a granular material to local perturbations. *Phys. Rev. Lett.* **87**, 035506 (2001)
- Geng, J., Behringer, R.P., Reydellet, G.: Green's function measurements of force transmission in 2d granular materials. *Phys. D* **182**, 274–303 (2003)
- Goddard, J.D.: Nonlinear elasticity and pressure-dependent wave speeds in granular media. *Proc. R. Soc. Lond. A* **430**, 105 (1990)
- Goldenberg, C., Goldhirsch, I.: Force chains, microelasticity, and macroelasticity. *Phys. Rev. Lett.* **89**, 084302 (2003)
- Goldenberg, C., Goldhirsch, I.: Friction enhances elasticity in granular solids. *Nature* **435**, 188 (2005)
- Hostler, S.R., Brennen, C.E.: Pressure wave propagation in a granular bed. *Phys. Rev. E* **72**, 031303 (2005)
- Jia, X.: Codalike multiple scattering of elastic waves in dense granular media. *Phys. Rev. Lett.* **93**, 154303 (2004)
- Jia, X., Caroli, C., Velicky, B.: Ultrasound propagation in externally stressed granular media. *Phys. Rev. Lett.* **82**, 1863 (1999)
- Kondic, L., Dybenko, O., Behringer, R.P.: Probing dense granular materials by space–time dependent perturbations. *Phys. Rev. E* **79**, 041304 (2009)
- Kondic, L., Fang, X., Losert, W., O'Hern, C., Behringer, R.: Microstructure evolution during impact on granular matter. *Phys. Rev. E* **85**, 011305 (2012a)
- Kondic, L., Goulet, A., O'Hern, C., Kramar, M., Mischaikow, K., Behringer, R.: Topology of force networks in compressed granular media. *Europhys. Lett.* **97**, 54001 (2012b)
- Liu, C., Nagel, S.R.: Sound in sand. *Phys. Rev. Lett.* **68**, 2301–2304 (1992)
- Luding, S.: Information propagation. *Nature* **435**, 159 (2005)
- Makse, H.A., Gland, N., Johnson, D.J., Schwartz, L.M.: Granular packings: nonlinear elasticity, sound propagation, and collective relaxation dynamics. *Phys. Rev. E* **70**, 061302 (2004)
- Otto, M., Bouchaud, J.P., Claudin, P., Socolar, J.E.S.: Anisotropy in granular media: classical elasticity and directed-force chain network. *Phys. Rev. E* **67**, 0313023 (2003)
- Reydellet, G., Clément, E.: Green's function probe of a static granular piling. *Phys. Rev. Lett.* **86**, 3308 (2001)
- Savage, S.B.: Analyses of slow high-concentration flows of granular materials. *J. Fluid. Mech.* **377**, 1–26 (1998)
- Sheng, P.: *Introduction to Wave Scattering, Localization, and Mesoscopic Phenomena*. Academic Press, San Diego (1995)
- Silva, M.D., Rajchenbach, J.: Stress transmission through a model system of cohesionless elastic grains. *Nature* **406**, 708–710 (2000)
- Tkachenko, A.V., Witten, T.A.: Stress propagation through frictionless granular material. *Phys. Rev. E* **60**, 687 (1999)

28. Velicky, B., Caroli, C.: Pressure dependence of the sound velocity in a two-dimensional lattice of Hertz–Mindlin balls: mean-field description. *Phys. Rev. E* **65**, 021307 (2002)

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.