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Rheology of a wet, fragmenting granular flow and the riddle of the anomalous friction of large rock avalanches

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Abstract The effective friction coefficient of rock avalanches diminishes gradually as a function of the avalanche volume. Large rock avalanches can reach run-out distances as long as ten times the fall height, despite the fact that the physics of friction would indicate a run-out only a little greater than the fall height. Numerous suggestions have been put forward to explain this remarkable departure from the predictions of both small-scale experiments and basic theory. It is shown here that accounting for rock fragmentation within the avalanche in combination with the presence of water, leads to results in line with the data.

1 Introduction

Rock avalanches are fast-moving, hazardous granular flows taking origin from the collapse and disintegration of a thick rock layer [1]. A known problem emerges comparing the deposits of rock avalanches with the results of small-scale experimental granular flows or rigid objects [2]. The theory of sliding friction predicts that the ratio between the fall height and the horizontal displacement of a granular avalanche is independent of its mass. The friction force, which is directed parallel to the slope and opposite to the velocity, has the form $F_{\mathrm{F}} = -\mathrm{g}\,\mathrm{M}\cos\beta\,\mu$ [3] where g, M, β are respectively the gravity acceleration, the avalanche mass, the local slope angle, and $\mu = \tan\phi$ is the friction coefficient expressed in terms of the internal friction angle ϕ, an experimental property that for rocks is of the order $\sim 38° - 40°$. The avalanche comes to rest down slope where the gradient is smaller. Because the kinetic energy is zero in the initial and final states, the overall change in the potential energy from start to stop must be equal to the work performed by the friction force. If dl is an infinitesimal displacement along the path, we have $\mathrm{Mg}H = -\int_{\mathrm{Initial}}^{\mathrm{Final}} \mathrm{F}_F \mathrm{dl}$ where H is the total fall height in the gravity field. Upon substitution of the expression for F_{F} it is found that the ratio between the vertical to horizontal displacement of the rocky mass H/R is just equal to the friction coefficient of the rock involved: $H/R = \tan\phi \equiv \mu$, which is a material constant. Thus, the theory predicts that independent of the path form and of the mass, the distance reached by the centre of mass of the avalanche equals the fall height divided by the friction coefficient, $R = H/\mu$. With typical values for the friction coefficient, $\mu \sim \tan(38°) - \tan(40°) \approx 0.78$–$0.84$ one finds that the runout length should be slightly greater than the height of fall in the gravity field. Small-scale experiments with granular flows confirm that the ratio H/R is indeed constant and independent of the mass [2].

However, a very different trend is obtained when similar data are gathered for rock avalanches [3,4]. Figure 1 shows a plot of data from many different events, demonstrating that the ratio H/R decreases with the volume (it is usual to use the volume rather than the mass, a reasonable approximation considering the limited variation of the rock densities). A similar trend is exhibited by Martian landslides [5,6], a fact that reveals a generality of the phenomenon. The data for the Earth can be fitted by a power law [3,4]

$$H/R \equiv \mu' \equiv \tan\varphi' = aV^{-\gamma} \tag{1}$$

where the primed quantities μ' and ϕ' can be called the effective friction coefficient and the effective friction angle respectively; the exponent is of the order $\gamma \approx 0.15$.

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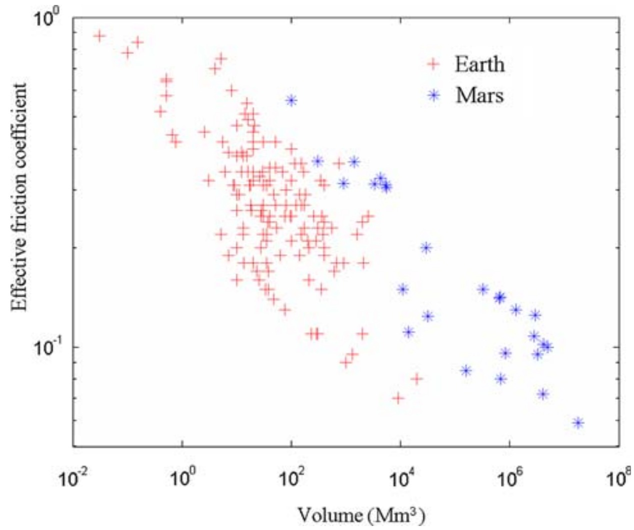

Fig. 1 The effective friction coefficient as a function of the volume for terrestrial and Martian rock avalanches. The effective friction coefficient is obtained dividing the drop height of the displaced mass by the horizontal distance reached

Consequently, while small rock avalanches typically rest at the toe of the main gradient sloping at 30–40°, giant landslides (volume $> 10^9 \mathrm{m}^3$) are capable of travelling up to ten times the fall height.

Several hypotheses have been put forward to explain the enigmatic decrease of frictional resistance for large rock avalanches, also called the volume effect. Suggested mechanisms can be divided into two categories: those based on the standard physics of granular materials [7–14]; and the ones invoking more exotic mechanisms such as air lubrication, acoustic fluidization due to high-frequency acoustic waves travelling through the granular medium, vaporized pore water, dispersive forces exerted by powder-sized grains, the energetic disintegration of the avalanche, the formation of a molten layer, or the presence of water [15–19].

In this work, it is argued that the anomalous mobility of large rock avalanches could be partly explained invoking the joint action of two basic processes: (i) the disintegration of the rocky material, combined with (ii) the presence of water. In short, the state of high stress at the base of the rock avalanche causes a strong fragmentation. Because the water table most usually lies above the detachment level of a landslide, it is reasonable to assume that some water is present above this level [19]. Thus, the crushing of the granular aggregate occurs in the presence of water. As fragmentation proceeds, the pore space decreases because fragmentation, by producing very fine material, causes an overall compaction. The pore water pressure increases, contributing to support the weight of the avalanche and as a consequence the effective friction diminishes. At this point, the rheology of the fluid-granular mixture switches from frictional to that of a fluid with non-Newtonian properties. It it shown here that this transformation causes a marked dependence of the mobility on the volume of the failed mass, and hence leads naturally to a volume effect.

Both fragmentation and the effect of water have been separately invoked to explain the anomalous mobility or rock avalanches and the volume effect. Davis and McSaveney, for example [20] propose disintegration as a possible mechanism for improved run-out. However, their suggested mechanism can only augment the stretching of the avalanche, whereas momentum conservation requires the movement of the centre of mass to remains unaffected. Some researchers and in particular Legros [21,22] has emphasized the role of water in enhancing the mobility of giant rock avalanches. Nevertheless, the mechanical processes involved in water lubrication are still obscure.

2 From frictional to fluid-dominated rheological behaviour of a wet granular medium

Let us consider the model of a granular medium saturated with water, and subject to intense normal pressure causing compaction and grain crushing. We seek an expression for the compaction of the granular aggregate as a function of the applied normal stress. The question is complex because the compression depends on the elastic grain deformation, on grain rearrangement, and on the process of crushing. Following the semi empirical approach or Ref. [23] based on comminution theory, it is possible to write the void fraction ϕ in the absence of the impregnating water as $\phi = \phi_0 \exp\{-[P/P_C]^n\}$ where P_C and n are materials constants, P is the normal stress and ϕ_0 is the void fraction at zero normal stress. In the presence of pore water at pressure P_W, the equation simply changes to

$$\phi = \phi_0 \exp\left\{-[(P - P_W)/P_C]^n\right\}. \tag{2}$$

An independent equation for the pore water pressure as a function of the void fraction ϕ is $P_W = P_{W0} - \beta_S^{-1} \ln(\phi/\phi_0)$ where P_{W0}, ϕ_0 are the values at some reference point, and $\beta_S = -V^{-1}(\partial V/\partial P) \approx 5 \times 10^{-10} Pa^{-1}$ is water incompressibility. The expression for P_W can be used in conjunction with (1) to determine the pressure acquired by water when the impregnated granular aggregate is subject to a total external pressure P, as well as the corresponding void fraction. Table 1 reports a series of values calculated at different pressures and shows that water pressure remains very close to the external pressure.

In the presence of water at high pressure, the friction resistance at the base of the rock avalanche becomes $F_{\mathrm{F}} = -\mathrm{g}\,\mathrm{M}\cos\beta\ \mu[1 - P/P_{\mathrm{W}}]\,\mathrm{sgn(U)}$. The ratio $1 - P_W/P$, which is also shown in Table 1, indicates that the frictional resistance can be practically obliterated in the regions that are saturated prior to the landslip. The total resistance will

Table 1 Calculated values of the pore water pressure and of friction reduction with $P_{W0} = 0$

P (MPa)	P_W (MPa)	$1 - P_W/P$
0.1	0.0992	8×10^{-3}
1	0.987	1.29×10^{-2}
10	9.80	1.99×10^{-2}
100	96.96	3.04×10^{-2}

The values of the constants (fitted from Furstenau [23] for the case of quartz) are $P_C = 1.11 \times 10^2$ MPa; $n = 0.841$. Expected values for P beneath large rock avalanches are in the range 3–30 MPa

not necessarily drop, however, because the fines mixed with water produce a viscous medium. Therefore, it can be conjectured that a viscous-like resistance term sets in, which overtakes the frictional resistance.

Before examining this viscous term, we notice that the previous calculations assume a completely saturated granular medium during the disintegration process. However, the permeability diminishes during fragmentation, and it may be questioned whether water has enough time to percolate inside the medium as fragmentation takes place [20]. If water seeps too slowly, it will form pockets of wet material in the medium rather than an uniform distribution. Let us consider the simple model of spherical, monosized grains initially saturated with water at a certain pressure P_w. For simplicity we assume that the material disintegrates during discrete steps $n-1, n, \ldots$. In correspondence of each step, particles of diameter D_{n-1} decreases in size to a diameter $D_n = D_{n-1} m^{-1/3}$ where m is the number of new particles produced at each step per original particle. At step n, the flow velocity of water in the granular medium is about $\alpha_n \approx (C D_n^2)(P_w/D_{n-1})/\eta$ where η is the water viscosity. The first and second parentheses represent the permeability and pressure gradient, respectively. The dimensionless parameter C is gives as $C \approx \varepsilon^3/\left[180(1-\varepsilon^2)\right]$, where the void fraction ε is regarded as constant in this simple estimate [24].

The time needed for water to percolate in the medium at step n is then $\tau_n \approx D_{n-1}/\alpha_n = \eta m^{2/3}/(C P_w)$ for each step, provided that the number of newly produced particles m is constant during the process. With $\varepsilon = 0.35$, $m = 10$ and $P_w = 10^5$ Pa (corresponding to only 10 m of water column), it follows that $\tau_n \approx 10^{-4}$s. Such small values of time, much shorter than the few minutes necessary for the flowage of a rock avalanche, indicate that the wetting of the material occurs fast; therefore, water will be more or less evenly distributed during the process. The process of homogeneization, however, will probably become less efficient as the amount of fines increases the viscosity of the fluid. Rock avalanche deposits exhibit a wide variety of grain size, spanning from clay-size materials up to large boulders. It is only recently that the size spectrum has been measured with accuracy [25–27]. Crosta et al. [25,26] have shown that the landslide deposit of the 1987 Val Pola landslide contains 20–30% of clay and fine silt-sized materials at a depth of 50–70 m. Furthermore, the percentage of fines increases with depth, albeit not dramatically.

To summarize, it can be expected that the fines produced by the rock avalanche disintegration may form an interstitial fluid when mixed with water [28,29]. In switching between frictional to viscous–cohesive, the resistive forces change radically the dynamics of the avalanche. This is because in contrast to a frictional material, the resistance in a purely viscous–cohesive material is independent of the load and depends instead on the velocity.

3 Change in rheological behaviour

The rheology of highly-concentrated solid suspensions is still not completely understood even in the laboratory conditions. In the absence of particles in the size range of sand, a water-fines fluid typically exhibits a non-Newtonian behaviour, namely: (i) the shear stress must exceed a finite value (yield stress) in order for the material to shear, and (ii) the viscosity may depend on the shear rate. If sand or larger particles are also present, the resulting fluid can be envisaged as an ensemble of non-colloidal grains immersed in the fluid of the water-fines suspension [28]. Recent analysis (see for example [28]) substantiates the classic equation of Krieger and Dougherty for the viscosity η of a suspension of non-colloidal particles with solid fraction $1-\phi$

$$\eta = \eta_0 \left[1 - \frac{(1-\phi)}{(1-\phi_{\mathrm{lock}})}\right]^{-2.5(1-\phi_{\mathrm{lock}})} \tag{3}$$

where η_0 should be interpreted as the viscosity of the muddy interstitial fluid (fines and water) and ϕ_{lock} is the void fraction in correspondence of sand particles interlocking [30–32]. The viscosity of the fines-water slurry η_0 is typically much greater than that of pure water. Equation (3) then shows that the viscosity can further increase by several orders of magnitude with respect to the muddy matrix whenever $\phi \approx \phi_{\mathrm{lock}}$. Measurements have shown that also the yield stress exhibits a sharp increase as a function of solid concentration [32]. A fit to a set of experiments has led Mahaut et al. [32] to an interpolating law of the form

$$\tau_y = \tau_y 0 \sqrt{\phi \left[1 - \frac{(1-\phi)}{(1-\phi_{\mathrm{lock}})}\right]^{-2.5(1-\phi_{\mathrm{lock}})}} \tag{4}$$

where $\tau_y 0$ is the yield stress of the muddy matrix in the absence of the large particles. In conclusion, Eqs. (3, 4) predict a dramatic increase of the viscosity and yield stress with addition of particles. Unfortunately, the uncertainties in the state of the material underneath the rock avalanche prevent a predictive use of the equations (3, 4), which merely indicate the possible mechanisms of increased stiffness of the material. In addition, the rheology will vary from case to case according to the initial type of rock, the energy of disintegration, and the water amount. Thus, in the following we have to consider the rheological constants as essentially free. A similar uncertainty also affects most computer simulations of landslides and debris flows, where the rheological properties are not identified from first principles, but are usually back-calculated based on the distance of reach.

4 Simulating the volume effect

The suggestion to model a rock avalanche as a non-Newtonian fluid is not new (see, e.g. [33]). A partial justification is that the transition between the compacted ("solid state") to the flowing state ("liquid state") in a rapid granular flow mimics the effect of a yield stress [34].

The underlying principle for the use of a non-Newtonian fluid model is different in the present work, and stems from the assumption examined earlier that the disintegrated material in mixture with water will produce a mud of non-Newtonian properties. For simplicity, calculations are performed using the simplest model for a non-Newtonian fluid, the Bingham model. Similar to a Newonian fluid, a Bingham fluid is characterized by a linear relationship between shear stress τ and shear rate $\dot{\gamma}$ in the form $\tau = \tau_y + \eta_B\dot{\gamma}$ where the proportionality constant η_B is the Bingham viscosity and τ_y is the yield stress, which is the minimum value of the shear stress necessary for the flow to take place.

Figure 2 shows an example of time evolution of a simulated two-dimensional rock avalanche moving down a slope. The avalanche collapses at a steep angle and then comes to rest at a gentler slope. The effective friction coefficient is obtained dividing the fall height by the run-out distance for the various simulated volumes. The different points in Fig. 3 are calculated varying the initial thickness, and hence the volume, of the avalanche. Because the simulation is two-dimensional, the volume per unit width is used as a proxy for the avalanche volume. The left panel shows the results with low viscosity of 100 Pa s and yield stress of 100 kPa and 2 MPa, a situation corresponding to high Bingham numbers (the Bingham number is the ratio between the yield stress and the viscous stress). The three sets of data (crosses, squares, and circles) represent the runout ratio of the simulated avalanche obtained from the displacements of the front (crosses) and of the centre of mass (squares). Circles show the Fahrböschung [4], a measure for mobility frequently utilized by field geologists (see Fig. 2). As a general trend, it is found that the effective friction coefficient decreases as a function of the volume per unit width, which is in line with the empirical equation (1) and the tendency exhibited by data in Fig. 1. The volume effect is very modest for the highest

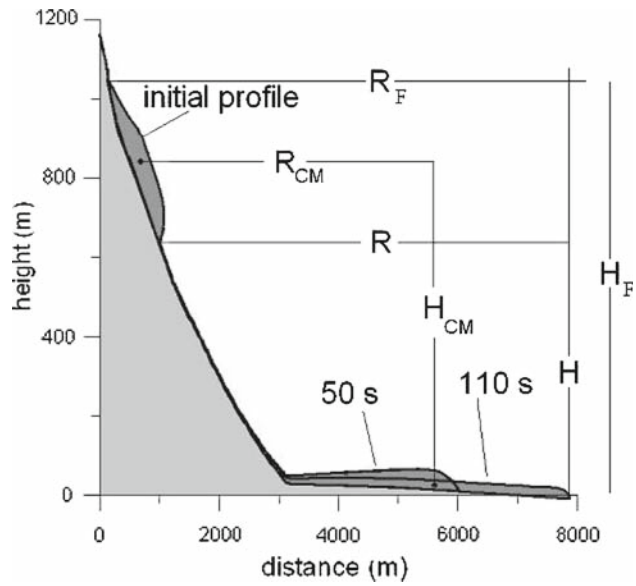

Fig. 2 Simulation of a rock avalanche with the Bingham fluid model. The 2D profiles are shown for the initial profile, and after 50 and 110 s when the simulated velocity becomes very small. The yield stress and viscosity are, respectively $\tau_y = 2\,\mathrm{MPa}$; $\eta_B = 100\,\mathrm{Pa\ s}$. The computer program used in these calculations is described in Ref. [37]. The slope profile is artificial but realistic for a rock avalanche scar. The measures of the effective friction coefficient are: (1) relative to the centre of mass, H_{CM}/R_{CM}; (2) relative to the front point, H/R; (3) the Fahrböschung used by field geologists is defined as H_F/R_F

values of the yield stress and small volumes, because in this case the slides stop early where the inclination angle changes little with distance. The volume effect depends very much on the values of $\mu \quad \eta_B$ and τ_y chosen in the simulations. To verify the robustness of the conclusions, different values for the rheological parameters can be utilized. It is found that even though the exact value of the slope and intercept depend on the rheological parameters, the trend in a decrease of the effective friction coefficient is common to all simulations. As an example, Fig. 3b reports simulations with low Bingham numbers, showing a similar decrease of the effective friction coefficient as a function of the volume per unit width.

It should also be noticed that the values of the effective friction coefficient relative to the landslide front, to the centre of mass and the Fahrböschung are not dramatically different. This indicates that an analysis of the Fahrböschung , for which data are more easily gained, is valuable. This conclusion should not discourage field geologists from carrying out a complete analysis of the centre of mass displacement, which is the one bearing physical relevance.

These results can also be qualitatively appreciated with simple arguments. The equation of motion of a Bingham fluid element is approximately [35]

$$\frac{\mathrm{dU}}{\mathrm{dt}} \approx \mathrm{g} \sin\beta - \frac{\tau_y}{\rho\,\mathrm{D}} - \frac{\eta_B \mathrm{U}}{\rho\,\mathrm{D}^2} \tag{5}$$

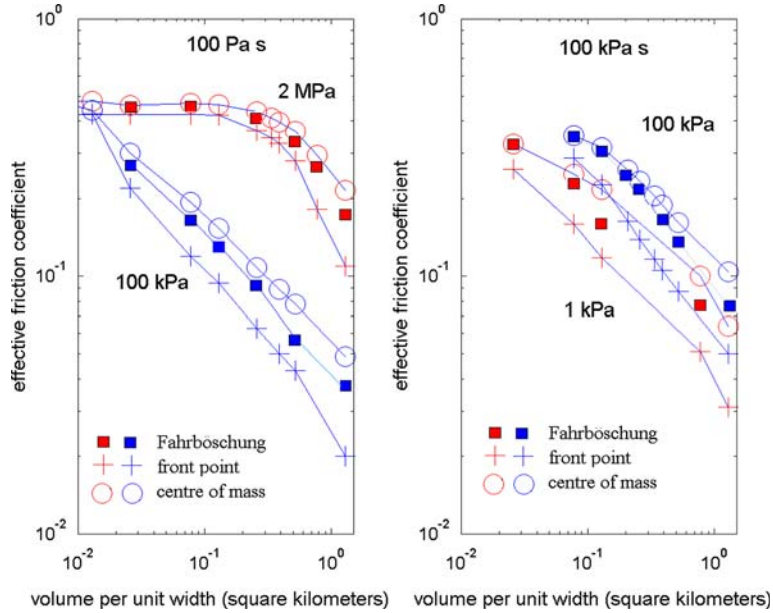

Fig. 3 Simulated effective friction coefficient. *Left* Representative cases for high Bingham numbers, with viscosity of 100 *Pa s* for all the simulations. Yield stresses are 2 and 0.1 MPa. *Right* results with viscosity of 10^5 Pa s corresponding to low Bingham numbers. Values of the yield stress are 1 kPa and 0.1 MPa. Data are shown for the centre of mass of the distribution (*circles*), for the front point (*pluses*) and for the Fahrböschung (*squares*)

where ρ is the density of the material. It is assumed that the profile of the fluid is sufficiently smooth to neglect the earth-pressure force arising from variations in the height of the material [35]. In the regime of high Bingham numbers [the last term on the right hand side of Eq. (5) is negligible compared to the second] the Bingham fluid decelerates when its thickness reduces to a value

$$\mathrm{D} \approx \frac{\tau_y}{\rho\ \mathrm{g} \sin\beta} \tag{6}$$

Depending on the value of the velocity, the material may continue moving initially for some distance; we neglect this aspect in the present estimate, which is however accounted for in the simulations of Fig. 3. Equation (6) is essentially the criterion of Johnson [36] relating the thickness of a Bigham fluid deposit to the value of the yield stress. It is shown in the Appendix that this sole criterion coupled to a realistic slope path (slope fading to zero at large distances) results in a volume effect.

5 Conclusions

It has been suggested that the intense stresses on a rock avalanche produce a large amount of fines embedding coarser material. Mixed with water, the rheology of the slurry shifts from a Coulomb-frictional toward non-Newtonian. This transition results in a load-independent shear resistance and hence in a volume effect comparable to the one observed in the field.

Due to the uncertainties of the model, the results presented are not meant to provide a final answer to the long-standing problem of the anomalous frictional properties of granular avalanches. First, the processes of comminution underneath a rock avalanche, that affect very much the conclusions of this paper, are complex and poorly understood. Second, the rheology resulting from crushed rock in water is basically unknown. As a consequence, the parameters adopted for the Bingham model were arbitrary. The results do not exclude the possibility of other mechanisms for increased mobility of a rock avalanche. More complete modelling and quantitative field observations will be necessary to substantiate or reject the explanation explored in this work.

Finally, it is interesting to refer again to Fig. 1 where the decrease in frictional resistance is displayed also for Mars [5]. If the present explanation is correct, it would validate the important role played by water on the red planet [21,22]. Water might have resulted from the transformation of buried ice as a consequence of the intense friction during the fall and flow of the rock avalanche. Even though the present arguments become even more speculative when applied to Mars, they also put the riddle of the anomalous friction of large rock avalanches in an even more thrilling perspective.

Acknowledgments This is the International Centre for Geohazards contribution nr. 225.

Appendix: A Bingham fluid model with a realistic slope path reproduces a volume effect

We consider an element of Bingham fluid moving down slope from an initial to a final position, like in Fig. 2, and restrict our considerations to two dimensions. We assume that the fluid stops when the shear stress at the base equals the yield stress, which yields the Johnson's criterion for the local thickness of the deposit [36]

$$D \approx \frac{\tau_y}{\rho\, g \sin \beta}$$

Integrating the thickness along the slope path gives the volume per unit width

$$\bar{V} = \int_0^R \frac{D}{\cos \beta} dx = \frac{\tau_y}{\rho g} \int_0^R \frac{1}{\sin \beta \cos \beta} dx$$

whereas the height of fall H of the last point is by definition

$$H = \int_0^R \tan \beta dx$$

Hence, if we calculate

$$\frac{d(H/R)}{d\bar{V}} = \frac{d(H/R)}{dR}\frac{dR}{d\bar{V}} = \frac{\rho\, g \sin \beta(R) \cos \beta(R)}{\tau_y} \frac{1}{R}\left[\tan \beta(R) - \frac{H}{R}\right]$$

it is found that $d(H/R)/d\bar{V} < 0$ if $\frac{H}{R} > \tan \beta(R)$, which is always the case if the slope angle decreases with the distance, i.e., if the topography is concave upward. The result $d(H/R)/d\bar{V} < 0$ implies that the effective friction coefficient decreases with the avalanche volume.

References

1. Voight, B., Pariseau, G.W.G.: In: Voight, B. (ed.) Rockslides and Avalanches. Elsevier, Amsterdam (1978)
2. Davies, T.R.H., McSaveney, M.J.: Runout of dry granular avalanches. Can. Geotech. J. **36**, 313–320 (1999)
3. Middleton, W.: Mechanics in the Earth and Environmental Sciences. Cambridge University Press, Cambridge, 459 pp (1994)
4. Scheidegger, A.E.: On the prediction of the release and velocity of catastrophic rockfalls. Rock Mech. **5**(4), 231–236 (1973)
5. Lucchitta, B.K.: Landslides in Vallis Marineris, Mars. J. Geophys. Res. **84**, 8097–8113 (1979)
6. McEwen, A.S.: Mobility of large rock avalanches: evidence from Valles Marineris, Mars. Geology, **17**, 1111–1114 (1989)
7. Erismann, T.H.: Mechanisms of large landslides. Rock Mech. Rock Eng. **12**, 5–46 (1979)
8. Goguel, J.: Scale-dependent rockslide mechanisms, with emphasis on the role of pore fluid vaporization. In: Voight, B. (ed.) Rockslides and Avalanches, pp. 693–706. Elsevier, Amsterdam (1978)
9. Habib, P.: Production of gaseous pore pressure during rock slides. Rock Mech. Rock Eng. **7**, 193–197 (1975)
10. Kent, P.E.: The transport mechanism in catastrophic rock falls. J. Geol. **74**, 79–83 (1966)
11. Melosh, H.J.: Acoustic fluidization: a new geological process?. J. Geophys. Res. **84**, 7513–7520 (1979)
12. Hsu, K.J.: Catastrophic debris streams (Sturzstroms) generated by rockfalls. Geol. Soc. Am. Bull. **86**, 129–140 (1975)
13. Shreve, R.L.: The Blackhawk landslide. Geol. Soc. Amer. Spec. Paper 198, 47 (1968)
14. De Blasio, F.V., Elverhøi, A.: A model for frictional melt production beneath large rock avalanches. J. Geophys. Res (2008, in press)
15. Davies, T.R.H.: Spreading of rock avalanche debris by mechanical fluidization. Rock Mech. Rock Eng. **15**, 9–24 (1982)
16. Hsü, K.J.: Catastrophic debris streams (Sturzstroms) generated by rockfalls. Geol. Soc. Am. Bull. **86**(1), 129–140 (1975)
17. Straub, S.: Bagnold revisited; implications for the rapid motion of high concentration sediment flows. In: McCaffrey, W.D., Kneller, B.C. (eds.) Particulate Gravity Currents, pp. 91–109. International Association of Sedimentologists Special Publication, 31 (2001)
18. Campbell, C.S., Cleary, P.W., Hopkins, M.: Large scale landslide simulations: global deformations, velocities and basal friction. J. Geoph. Res. **100**, 8267–8283 (1995)
19. De Blasio, F.V.: Production of frictional heat and hot vapour in a model of self-lubricating landslides. Rock Mech. Rock Eng. **41**, 219–226 (2008)
20. Davies, T.R., McSaveney, M.J., Hogdson, K.A.: A fragmentation-spreading model for long-runout rock avalanches. Can. Geotechn. J. **36**, 1096–1110 (1999)
21. Legros, F.: The mobility of long-runout landslides. Eng. Geol. **63**, 301–331 (2002)
22. Legros, F.: In: Evans, S.G., Scarascia Mugnozza, G., Strom, A., Hermanns, R.L. (eds.) Landslides from Massive Rock Slope Failure. NATO science series, Springer, Berlin (2006)
23. Furstenau, D.W., Gutsche, O., Kapur, P.C.: Int. J. Min. Process. **44**, 521 (1996)
24. Rhodes, M.: Introduction to Particle Technology. Wiley, New York (1998)
25. Crosta, G.B., Frattini, P., Fusi, N.: Fragmentation in the Val Pola rock avalanche, Italian Alps. JGR 112 (2007). doi:10.1029/2005JF000455
26. Crosta, G.B., Frattini, P., Fusi, N., Sosio, R.: Granulometria, segregazione e frammentazione negli accumuli di valanghe di roccia. Giornale di Geologia Applicata **4**, 31–40 (2006)
27. Locat, P., Couture, R., Leroueil, S., Locat, J., Jaboyedoff, M: Fragmentation energy in rock avalanches. Can. Geotech. J. **43**, 830–851 (2006)
28. Coussot, P.C.: Mudflow rheology and dynamics: IAHR-AIRH monographs, AA Balkema Publishers, Rotterdam, 272 p (1996)
29. Ancey, C.: Plasticity and geophysical flows: a review. J. J. Non-Newtonian Fluid Mech. **142**, 4–35 (2007)
30. Stickel, J., Powell, R.L.: Fluid mechanics and rheology of dense suspensions. Ann. Rev. Fluid Mech. **37**, 129–149 (2005)
31. Ovarlez, G., Bertrand, F., Rodts, S.: Local determination of the constitutive law of a dense suspension of non-colloidal particles through MRI. cond-mat/0509336 (2005)
32. Mahaut F., Chateau, X., Coussot, P., Ovarlez, G.: Yield stress and elastic modulus of suspensions of noncolloidal particles in yield stress fluids. J. Rheol. **52**(1), 287–313. doi:10.1122/1.2798234
33. Trunk, F.J., Dent, J.D., Lang, T.E.: Computer modelling of large rock slides. J. Geotech. Eng. **112**(3), 348–360 (1986)
34. Forterre, Y., Pouliquen, O.: Flows in dense granular media. Ann. Rev. Fluid Mech. **40**, 1–24 (2008)
35. De Blasio, F.V., Elverhøi, A., Issler, D., Harbitz, C.B., Bryn, P., Lien, R.: On the dynamics of subaqueous clay rich gravity mass flows—the giant Storegga slide, Norway. Mar. Petrol. Geol. **22**, 179–186 (2005)
36. Johnson, A.M.: Physical Processes in Geology. Freeman, San Francisco, 577 pp (1970)
37. Imran, J., Harff, P, Parker, G.: A numerical model of submarine debris flows with graphical user interface. Comp. Geosci. **27**(6), 721–733 (2001)