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Watermark location via back-lighting and recto removal

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Abstract We consider the problem of locating a watermark in pages of archaic documents that have been both scanned and back-lit: the problem is of interest to codicologists in identifying and tracking paper materials. Commonly, documents of interest are worn or damaged, and all information is victim to very unfavourable signal-to-noise ratios—this is especially true of 'hidden' data such as watermarks and chain lines. We present an approach to recto removal, followed by highlighting of such 'hidden' data. The result is still of very low signal quality, and we also present a statistical approach to locate watermarks from a known lexicon of fragments. Results are presented from a comprehensively scanned nineteenth century copy of the Qur'ān. The approach has lent itself to immediate exploitation in improving known watermarks, and distinguishing between twin copies.

Keywords Watermark · Recto removal · Back-lighting

1 Introduction

The study of watermarks is a seductive if somewhat esoteric pastime. While it is normally the beauty and aesthetic quality of watermarks that initially attract the researcher, they are more than just pretty affectations and can shed light on historic trends and events [46].

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This comment by Pavelka acquires more veracity as time passes. Watermarks are indeed compelling viewing in their own right, especially when extracted from ancient and unusual documents—Fig. 1 illustrates just one example. Modern opportunities for high quantity and high quality digital repositories suddenly make available many documents hitherto only selectively available (our own prototype is just one small example [30]); at the same time, image analysis and understanding techniques continue to grow in sophistication and the modern codicologist has new horizons in studying these beautiful and informative patterns.

Paper watermarks, which are changes in paper thickness, have been in use for over 700 years, with the oldest known watermarked paper produced in 1282 in Fabriano [9,10,57]. They have been used as trademarks of paper-makers, as identification marks for sizes of moulds used in manufacture, as symbols of religious groups called 'Albigenses', as an aid to illiterate workmen, and as an exercise in imagination by paper-makers, just to show their artistic skills. Their use spread and came to be used to trademark paper, a proof of the date of manufacture, and an indication of paper size, culminating in use as a mark against counterfeiting on money and other documents [33].

Two main types of paper watermarks exist: *line* (typically known as wire), and *shadow* (light and shade). *Combined* watermarks have both. Further types are described in [34,39]. Wire watermarks are made using lines to form various patterns, such as letters, numbers, portraits, or other designs; they appear lighter than surrounding paper area. Light and shade watermarks have patterns resulting from relief sculptures on the mould; these designs give the watermark further variations to support more features, and may appear lighter or darker than the surrounding area. They offer more details compared to wire watermarks, but are relatively expensive, depending on the size and the quality of the

Fig. 1 A double-headed eagle watermark, extracted from challenging material which explains the poor contrast. The central horizontal line is due to two sheets being cut from one larger one. Note the faint ‘A G’ countermark just legible at the bottom

mould model [33]. In many mills, paper making was often accelerated by making pairs of moulds with two very similar but not identical watermark designs; watermarks are generally twins [57,60].

Watermarks in paper have attracted a wide range of interest from researchers for centuries; their study is helpful in tracing and studying old documents and artefacts to provide plausible historical relationships and background information. Even when they do not bear explicit dates, the temporal and other evidence they provide can be significant—see, for example, [6,26,50,57,59,62,63]. However, watermark designs are available not only in several different forms, but also dynamically change over time. This has introduced some complications that have hindered more systematic study of the artefacts.

We present here an approach for identifying and locating watermarks in ‘difficult’ artefacts, characterised by thick uneven paper and significant quantities of thick-stroke recto and verso inscription. One side effect of the approach is to reveal other details of the original mould such as chain lines, which have been found to be as useful as the watermark in some examples of paper identification [61,69]. We seek to do this with equipment of modest cost with an approach that minimises hazard to the original artefact while generating good resolution, easily distributable digital images.

This paper proceeds as: Sect. 2 presents recent and related work in watermark study. Section 3 describes the data used in our approach, with the capture system used. Section 4 presents our approach in removing recto information, while Sect. 5 focuses on localisation of watermarks. Section 6 provides results from the approach, and Sect. 7 develops these further. We conclude in Sect. 8.

2 Background

Watermarks only become visible to the eye when faced against light, and are usually obstructed by writing ink and other noise in paper. Many approaches have been developed in order to reproduce and exploit them; a survey appears in [29].

Image collection techniques fall under four broad heads:

Manual: The most primitive techniques would either place a document on a light table and a user would copy the watermark onto tracing paper laid on top, or alternatively a clean sheet is placed over the document and a pencil is rubbed over it with long diagonal strokes. The former is simple and easy but is time consuming and highly subjective—well-known catalogues of traced watermarks include [9,13,28]. The latter is quick, easy, and does not require special equipment, but it does not produce good results, and may damage the paper [2]. Examples of watermark reproduction by rubbing can be found in [27].

Back-lighting: This requires a high resolution digital CCD camera and a light source. The camera captures reflected (with normal light) and transmitted (with back-light from slim light or light box) images of the paper [2,5,12,58,64,70]. It is quick and relatively low cost, and produces good image quality without darkroom conditions. It differs from other techniques in that it is digital, making it very attractive to scholars [58]. However, it captures all the details of paper, including the watermark and any other designs that may interfere with it.

Radiographic techniques: The advantage of radiography is in the ability to display changes of paper thickness, no matter what is printed on it [58]. X-rays are useful because of not being absorbed by the writing ink (usually Carbon) on paper [1].

1. Beta-radiography uses beta-isotopes (Carbon-14) to record variations in paper thickness (watermark, countermark, chain and laid lines, and sewing dots) on an X-ray film [2,54]. It gives an accurate image of the watermark with minimum interference, and films produced can be duplicated easily, but unfortunately is time consuming and expensive [55,68]. For this reason, only large institutes and museums use it [58], and it requires darkroom conditions [54]. There are also some concerns regarding radiation safety [55]. Results of watermark images of radiographic techniques may be blurred depending on the paper thickness [49], and the imperfect contact of the

watermarked paper, the beta-isotope plate and the X-ray film [38].

2. Soft (or low voltage) X-radiography uses a low voltage energy radiated from the X-ray source through the paper to a phosphor plate; the plate is then read by a laser reader and the watermark image is generated digitally some time later [1,7,32,58]. Low voltage radiation, which produces very long wavelengths, is used because it gives high contrast. This method gives very good images and is cheaper, faster and relatively safer than beta-radiography [68]. It has been used in portable conditions, but is still expensive [1,68].
3. Electron-radiography uses high energy X-rays to penetrate the paper to a photographic film—the film will hold an image of the watermark with minimum interference [7,8,52,53,74]. This technique produces very good watermark images and is faster than other radiographic techniques [68]), and does not require darkroom conditions. It has the advantage over other radiographic methods in that metallic ink will not absorb the X-rays and so will not interfere with the final image [7]. However, it is very expensive, and requires safe (radiation shield) conditions.

Special purpose techniques: A range of other, specialist approaches is or has been in use. These include:

1. The 'Dylux' method which uses paper differently susceptible to visible and UV light [19,25] to betray thin [watermark] regions. It is low cost and swift but also captures any design that interferes with the watermark, and its effectiveness depends on paper thickness, ink opacity and light source types [54]. It is not permitted in many libraries and museums because of the use of UV and some hazard to users [24,26].
2. The 'Ilkley' method requires two glass plates, a light source with photographic timer, and a Kodak Precision Line film LPD4. It is simple and quick, and the film produced can be duplicated quickly and easily [54]. However, it requires dark room conditions, and will capture any details in the paper in addition to the watermark. Hence, it is only useful for reproducing watermarks in clean paper without interference.
3. Phosphorescence watermark imaging requires ultraviolet and infrared light, a phosphorescent pigment plate, a glass plate, and photographic film. This method is quick, but image quality depends on the distance between pigment plate and light sources, and also on paper thickness and ink opacity. This method also captures image interference in addition to the watermark design.
4. Thermography, or thermal photography, is a technique that exploits the fact that writing ink on paper is transparent under thermal radiation. It works by placing a thermal source behind the watermarked paper, and using an IR camera in front. The camera records the changes of the watermark density in paper, and generates a digital watermark image. This method is fast, and produces good images: the limitation is concerned with the safety of the watermarked paper, which is safe as long as it is kept at a distance from the warm plate, and the exposed time is only one second [41].

A number of web archives of watermarks exists [5,15,37, 72,73].

To date, most work on watermark extraction has been in pursuit of the compilation of databases. Clearly, manual techniques represent an end in themselves, but the more sophisticated approaches have been subject to further computer-based manipulation. Commonly, combinations of edge detection, region extraction and morphology are used to try to isolate clean watermark representations: examples of such approaches using back-lighting [14,16,20,43,64,71], X-rays [1,3,4,36,40], Dylux [21], and thermography [41,45], may be found.

Enhanced approaches deploy reflectance models [64], or seek identifiable properties of paper such as [regular] chain lines (where Fourier techniques are of use), for example [22, 67,71]. Work is also in hand on matching extracted watermarks to existing databases [47,48].

Common to most work to date have been problems with noise in images, attributable to paper quality and interference from recto and verso inscription. Self evidently, this will hamper work on many of the most interesting artefacts to be found in libraries. We also observe in many solutions to date a reliance on certain parameter settings in a range of algorithms, which hinders their broader applicability.

Our work is interested in addressing these, and related, problems in seeking watermarks. Specifically:

- We mean to develop techniques that will work on materials seen to date as very challenging.
- We mean to attempt to extract complete, or near-complete, watermarks from collections of documents in which only fragments are easily accessible.
- We mean to develop tools that may be useful in distinguishing between 'twins' [57,60].

This work should be done in a context of parameter selection that is, as far as possible, automatic or adaptive. As a side effect, we will be able to make available via the WWW some materials that may be of benchmark value in future study, and are of immediate interest to scholars of Arabic [30].

3 Data and digitisation

We have chosen to use back-lighting, because it is simple, relatively quick, and cheap, generating output that is natively digital. The system we use is mounted using a stand with lights by Kaiser Fototechnik [35] and a FUJIFILM FinePix S1 Pro camera [18]. It uses a slim light sheet for back-lighting to provide even homogeneous illumination behind the paper. Each sheet is captured three times, reflected images of front and back, and a transmitted image; the recto and the back-lit scans are co-registered. In these experiments a resolution of 258dpi was used.

The data used has been selected from the Arabic holdings of the University of Leeds, which include a number of rare, unusual and little-known texts. Those chosen all carry wire watermarks, represented by a relative thinning of the paper. Specifically:

- The 'Mahdiyya' Qur'ān [11]: written in 1881 in Sudan. This exhibits thick writing strokes and very thick, uneven paper, and is a very significant challenge to processing. It bears a double-headed eagle watermark of the Austro-Hungarian Empire with the countermark 'Andrea Galvani Pordenone' with a shield containing a moonface. No pages contain a complete watermark or countermark as the paper was cut before use. One page (only) turned out to originate from a different source.
- The 'West African' Qur'ān: This is a complete copy of the Qur'ān containing the tre lune watermark, which appears in different variations. The countermark used is the letter pair 'C L', with two variations. Part of the manuscript also has the 'tre lune with three moonfaces' watermark with the 'Andrea Galvani Pordenone' countermark. The manuscript is not dated, but watermark and countermark evidence leads to an estimate of between 1836-80 [11].
- A long Islamic Prayer: *Kitāb Durrat 'iqd al-naḥr fī 'asrār ḥizb al-baḥr*. No date is given but it is believed to be 18th century. The watermark used is tre lune, with a letter 'C' countermark. Each pair of pages is bound together, which permits a complete watermark to appear clearly.

Of these, the most challenging by far is the first and it is on this document that we have done most development: an example is at Fig. 2. The others provide regular verification that 'easier' scans are indeed more easily accessible.

Each of these volumes carries different watermarks and in aggregate represent over 800 sheets. We see merit in establishing benchmark datasets for the work we have done; since creation of such data may be non-trivial in time and library

Fig. 2 Sample input scanned and back-lit images—part of a page. The back-lit image shows verso inscription and details within the paper—watermark features are in the right hand margin

permissions, we have made these scans publicly available online [30].

4 Recto removal

In earlier work [31] we have located watermarks using an image processing bottom-up approach that deploys background estimation and morphology, and is comparable in power to others used in the literature, with the benefit of automatic parameter selection. On the very difficult data of the 'Mahdiyya' Qur'ān we found these approaches less than successful and in some cases wholly unproductive as a result of the noise levels and very faint watermark evidence [29]. We chose instead to build a model of back-lighting to take a top-down view.

This model is illustrated in simplified form in Fig. 3. The RGB vector detected at a particular pixel is dependent on the paper properties (absence or presence of watermark or other manufactured feature), recto features and verso features. In an ideal world, blank featureless paper (labelled 'A' in the figure) would always produce the same output, but we do not have to assume that the same is true of inked regions (e.g., 'B'), paper features, or combinations thereof.

For clarity, we shall define at this point a feature to be *visible* if it is visible on the recto—thus, recto writing and other paper features visible to the reader. Other features betrayed in the back-lit image (watermark, verso writing, dirt on the verso face etc.) we shall collectively call *hidden*. Back-lit pixels at which no hidden data are evident we shall call *uncorrupted*.

In fact, the noise and damage that we experience produce significant variations across all regions that we might wish to be internally homogeneous, as may be clear from Fig. 2. This however is not critical—what we can exploit is the difference between pixels that represent just blank paper or recto features, and those representing verso or other features, such as internal ones.

Consider momentarily a blank, featureless page which we scan as image S and back-light as image B, and define an image D in which pixels are given by the difference between

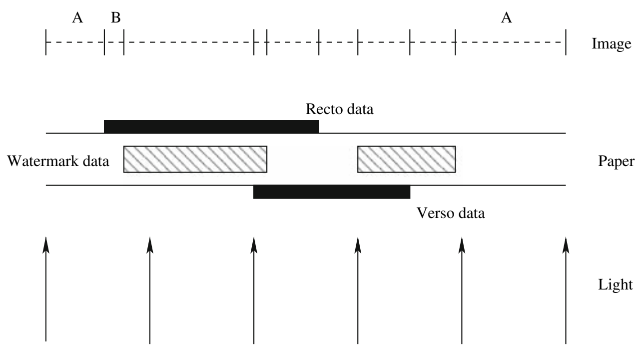

Fig. 3 The model of back-lighting. The paper is lit from below (*up-arrows*) and the image (*dotted line*) sensed above; data may be received from blank paper, or some combination of recto, verso, or 'interior' features. The *vertical lines* along the image indicate points at which the received signal *may* change: at 'A', we are detecting blank featureless paper, at 'B' recto data inscribed on it. Of course, recto and verso inscriptions need not be uniform, nor need watermark features, and there may be many other influences as well, including dirt and noise

their detected back-lit intensity (in B), and the intensity we might *expect* given the corresponding location in S. In the ideal case this page will be of uniform intensity (r, g, b) in S and, say, (ρ, γ, β) in B. We hypothesise some transform T which describes the back-lighting, and subtract $T(r, g, b)$ from the corresponding (ρ, γ, β) in B. We should see $(0, 0, 0)$ at all locations. If there are paper or verso features (invisible in S), these will be revealed by this differencing process.

In fact, of course, regions are not uniform in intensity and blank paper will scan and back-light as a range of (r, g, b), (ρ, γ, β) vectors—these may, however, be expected to cluster reasonably tightly, and to be related to each other. If we define

$$\begin{aligned}(\mu_r, \mu_g, \mu_b) &= \text{mean}(r_p, g_p, b_p) : p \in S \\ (\mu_\rho, \mu_\gamma, \mu_\beta) &= \text{mean}(\rho_p, \gamma_p, \beta_p) : p \in B\end{aligned} \tag{1}$$

then a simple approach is to seek a linear relationship

$$\begin{aligned}(\rho_p, \gamma_p, \beta_p) \approx\ & A((r_p, g_p, b_p) - (\mu_r, \mu_g, \mu_b)) \\ & + (\mu_\rho, \mu_\gamma, \mu_\beta)\end{aligned} \tag{2}$$

for some 3×3 matrix A that models the back-lighting. Lighting effects are often subtle and it is most unlikely that the effect we observe will indeed be linear, but we proceed with this simplification on the understanding that it is applied only to pixels that are 'similar', and in the ideal case identical.

In the event that there are no internal or verso features, we can derive an optimal A by considering Eq. 2 for all pixels p as an over-determined system and 'inverting'[1]

$$\begin{aligned}A = &\left[(\rho_p, \gamma_p, \beta_p) - (\mu_\rho, \mu_\gamma, \mu_\beta)\right] \\ &\times \left[(r_p, g_p, b_p) - (\mu_r, \mu_g, \mu_b)\right]^{-1}\end{aligned} \tag{3}$$

Then, for the simple case of a blank page,

$$\begin{aligned}D = &(\rho_p, \gamma_p, \beta_p) \\ &- A((r_p, g_p, b_p) - (\mu_r, \mu_g, \mu_b)) - (\mu_\rho, \mu_\gamma, \mu_\beta)\end{aligned} \tag{4}$$

and we will expect significant differences from (0, 0, 0) to betray hidden information.

In the event that the image does contain hidden features, this approach lends itself to an immediate improvement. Assuming that there exist uncorrupted pixels in B and the relative number of hidden features is small, we shall expect the (wire) watermark to exhibit a high magnitude response in D, and the uncorrupted areas to be low (ideally 0). Therefore, we may recompute A by reducing the set of pixels from which it is derived to those we expect to be featureless; thus, Eq. 3 may be re-employed;

$$\begin{aligned}&\hat{D} = \{p : |D_p| < t\} \\ &\text{For } p \in \hat{D},\ A_{\text{new}} = [(\rho_p, \gamma_p, \beta_p) \\ &\quad -(\mu_\rho, \mu_\gamma, \mu_\beta)][(r_p, g_p, b_p) - (\mu_r, \mu_g, \mu_b)]^{-1}\end{aligned} \tag{5}$$

where $|D_p|$ is a measure of the magnitude of the difference vector at p—Euclidean length is an obvious choice. A choice for the threshold t is given in Sect. 6. This procedure is open, of course, to iteration in attempting only to compute A from pixels which are uncorrupted.

In the general case we shall expect scans to carry recto material and so the preceding assumptions about a 'blank piece of paper' are invalid. Nevertheless, the approach is sound if we can apply it to pixels of S that are similar in intensity. This is straightforwardly achieved by clustering the data of S in RGB space, and deriving a matrix A for each such cluster. Formally;

1. Using K-means [56] or similar, cluster the RGB data of S into a partition of K_1 clusters $C_1, C_2, \ldots, C_{K_1}$. These clusters may have spatial coherence, and may not.
2. For each cluster C_i derive a matrix A_i according to Eq. 3, where p is restricted to C_i (not the whole image).

The iterative refinement approach of Eq. 5 is applicable to each such cluster.

The choice of K_1 is interesting: in many applications it is desirable to minimise the number of clusters chosen, leading to a more compact data encoding. Here, the problem is somewhat different: the more clusters we define, the better the subtraction process is likely to perform, provided the matrices A_i are approximating uncorrupted pixels. This issue is considered further in Sect. 6. Figure 4 illustrates this procedure.

[1] A linear algebraic operation straightforwardly available in libraries provided by, e.g., MATLAB [66].

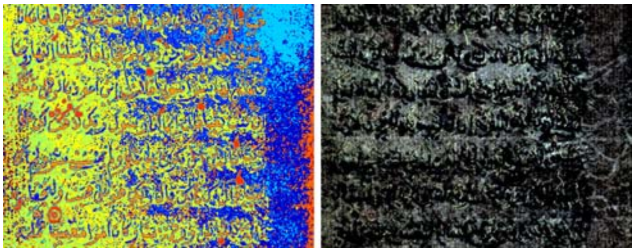

Fig. 4 On the left, an image colour coded according to the cluster that the pixel belongs to in S; on the *right*, the difference image D generated (contrast stretched for display). Watermark data are visible in the right hand margin

Fig. 5 Three clusters derived from the difference image shown in Fig. 4. Some of these clusters contain valuable information of the watermark design

Fig. 6 Two fragments of the double-headed watermark shown in Fig. 1 (these are rotated 90^o in the page images)

5 Watermark location

Recto removal is robust and successful (see Sect. 6). In pursuit of specific features we make two further assumptions:

1. We might expect verso inscription to be dark relative to paper and so the components of relevant pixels in D to be negative: we shall set all such components to null.
2. We assume we know a set of possible or likely watermarks *or parts thereof*, and seek their occurrence. This is not unreasonable as a task;

   – In what follows, neither a precise nor complete representation of the watermark is necessary. For example, a rough template of a watermark fragment could be user-outlined from a small number of trial pages (or just one).
   – For a given document, foreknowledge may well provide a set of plausible paper manufacturers and dates, and thence a set of candidate watermarks from a known database.
   – Watermarks often occur as *near*-identical twins [60]: our approach will find such twins and allow a later refinement to determine which of the pair is actually seen.

The output of the differencing phase contains very significant noise in addition to information of value; Fig. 4 illustrates this. The presence of fragments of value is clear, but the information of interest is not among the strongest responses, and simple thresholding approaches are unlikely to assist. On the other hand, pixels of the watermark are similar in RGB intensity, and to exploit this we re-cluster the D image.

We generate K_2 binary images $D_1, D_2, \ldots, D_{K_2}$ by partitioning D—the choice of K_2 is discussed in Sect. 6. Figure 5 illustrates some of these for the example of Fig. 4. Some of these clusters will represent binary images that include good representations of fragments of the watermark, while others may not. In particular the 'background'—including the nulled pixels—will. We proceed by selecting informative fragments of the watermark and seeking a binary match in each of these partitions of D. Figure 6 illustrates two such fragments from the watermark of Fig. 1.

'Matching' here is a binary templating task. We proceed for a given template (watermark fragment) W_i by assuming it contains N pixels, of which w_i are 1's. When the template is offered at a particular offset in the image D_j, we count the number of pixels that match (both 1's or both 0's) and interpret this 'score' in the light of what may be expected in noise. If at this offset in D_j there are d 1's, and these are chosen randomly, we have an instance of sampling without replacement to which the hyper-geometric distribution is applicable [42]. If at template offset p we write $u(p) = \{$Number of pixels at which *both* template and image are 0 or 1$\}$, then (see Appendix)

$$\mu(u(p)) = N + 2\frac{w_i d}{N} - (w_i + d)$$

$$\sigma^2(u(p)) = \frac{4w_i d(N - w_i)(N - d)}{N^2(N - 1)}$$

—both mean and variance depending on the properties of the template fragment and the position in the image.

In seeking plausible locations for the fragment, we are interested in significant deviations from the mean expected in noise $\mu(u)$. 'Significance' might be measured with respect to the standard deviation $\sigma(u)$. Thus at pixel position p in image D_j we will compute

$$m(p) = \frac{u(p) - \mu(u(p))}{\sigma(u(p))} \tag{6}$$

Herein, high positive responses will represent plausible match positions. The exception is the binary image D_0 representing 'background' (zero pixels) which might be expected to generate a strong negative response at matching positions. For the background data, we thus negate $m(p)$.

An example result $M_i = m(p)$ is illustrated in Fig. 7.

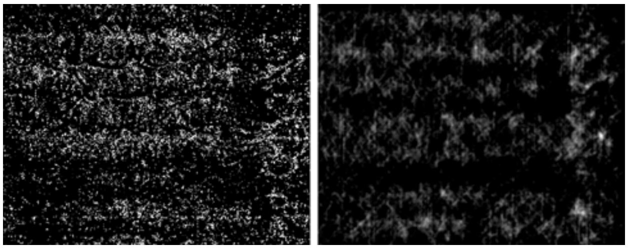

Fig. 7 On the left an image D_i in which the watermark fragment shown in Fig. 6 (*left*) is sought. On the *right*, the response m, given in Eq. 6—the *brightest spot* shows the best response (here, correctly)

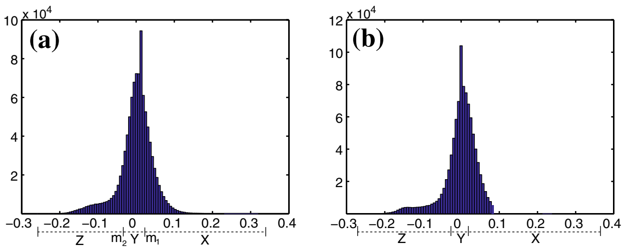

Fig. 8 Histogram distribution of image D, **a** before, and **b** after improving transform A

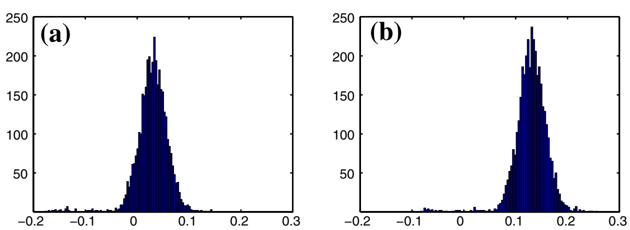

Fig. 9 Histogram distribution of watermark features in D, **a** before, and **b** after improving transform A

At this stage we can accumulate the M_i;

$$M(p) = \sum_{i=1}^{K_2} M_i(p) \tag{7}$$

Significant peaks in this array represent evidence for the fragment in the original image. In fact, we have valuable additional evidence from second, or further, fragments of the watermark: applying this procedure for each such fragment we can exploit their known geometric relationship in inspecting peaks in the M array; this is explored in Sect. 6.

6 Results

We have tested this approach with 346 pages of data from the 'Mahdiyya' copy of the Qur'ān. An evaluative measure is necessary in judging levels of success, and we have chosen to use the signal-to-noise ratio (SNR) [56] of known data in a small number of samples. If a watermark and its position are known, we split image pixels into two groups: watermark features W, and all others which we regard as noise N. Then SNR may be calculated as

$$SNR = \frac{\sum_{i \in W} x_i^2}{\sum_{j \in N} x_j^2} \tag{8}$$

(where x denotes the mean RGB value of each pixel). This considers the watermark to be a binary feature; this is based on all the watermarks here considered to be wire.

6.1 Recto removal

As discussed, we compute a transform A that approximates the intensity effect of back-lighting; this is then used to remove all recto information in a differencing operation. Using the simple computation of A (Eq. 3), Fig. 8a illustrates the distribution of differences (computed as the average of the RGB channels) for a sample image pair. We expect high differences to correspond to hidden, bright features in the back-lit image B (region X on the horizontal axis), small differences (region Y) to be due to uncorrupted pixels, and dark features in B, such as verso writing, to be negative differences (region Z). This distribution is asymmetric, with verso features appearing prominently as negative; low magnitude pixels are modal, suggesting that the transform was good enough to model the back-lighting. High magnitude pixels in this distribution are relatively small in number, and represent the watermark and some other hidden features.

Adopting the approach outlined in Sect. 4, we have iteratively refined A by recomputing the pixels from which it is derived. We have selected pixels between the means of positive and negative observations in the differences. This is a simple way of trying to restrict the computation to uncorrupted areas of the image in the light of the distribution being asymmetric. Figure 8b illustrates the distribution after this iteration has been conducted; observe that region Y in this new distribution is narrowed, while regions X and Z (which hold verso and hidden features) were pushed to right and left respectively. With foreknowledge of the watermark, we can draw its distribution before and after improving A. Figure 9a, b illustrate such distributions; pixel intensities were increased after iterating A—the watermark signal has been strengthened.

It is not clear in the general case whether the iteration will converge or when it should be halted, but we can demonstrate its beneficial effect from data with known ground truth. Figure 10 shows the SNR as the matrix A is iterated, showing that—as anticipated—the signal improves. In this case, the watermark signal improves monotonically until there is convergence: we have seen this effect in all examples we have studied.

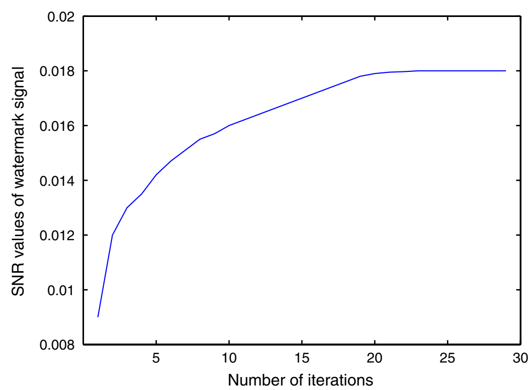

Fig. 10 Evolution of SNR as transform A is iterated

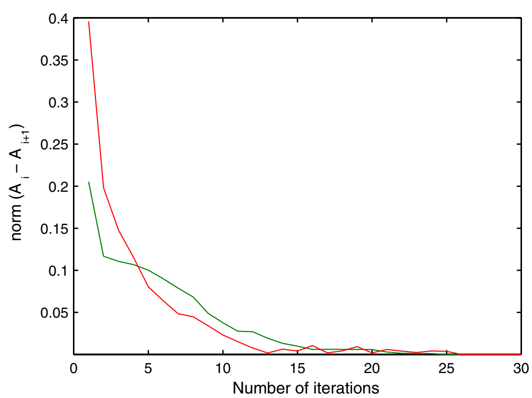

Fig. 11 Frobenius norm of the differences in iterated values of A

In the unknown case, SNR cannot be measured. Figure 11 plots the Frobenius norm [23] of the difference between successive iterations of A for just two examples (others are similar) suggesting that this adequately mirrors the signal improvement we wish to see.

We therefore adopt a convergence criterion that iterates until the matrix A stabilises, so the norm of the difference between successive iterations becomes 0. In the event this is not observed, we may halt the iteration either when the norm reaches a minimum, or at the 'elbow' in the plot of Fig. 11, calculated for example by the L-method [51].

To observe the effect of recomputing the transform, the initial matrix A, and after 30 iterations, for a specific cluster, are

$$\begin{bmatrix} 0.315 & 0.513 & -0.419 \\ 0.208 & 1.113 & -0.796 \\ -0.013 & 0.213 & 0.084 \end{bmatrix} \begin{bmatrix} 0.396 & 1.006 & -0.639 \\ 0.323 & 2.025 & -1.19 \\ 0.048 & 0.357 & 0.027 \end{bmatrix}$$

In this example, values in the first and second columns (red and green) have increased, while the third column (blue) has decreased. These observations vary among different clusters.

In selecting K_1, most literature, e.g. [17,51], seeks a trade off that seeks the lowest value which is simultaneously high enough to capture the nature of the data. Plotting clustering cost (summed distances from data to centroids) against K (see Fig. 12), one seeks the point of diminishing returns where the cost starts to decrease slowly: the L-method of Salvador [51] is a well-known approach.

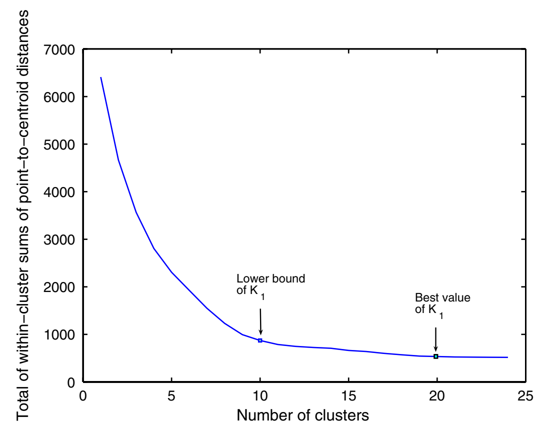

Fig. 12 Clustering 'cost' in image S

The problem here is different: the more clusters we define, the better the subtraction process is likely to perform provided we do not develop clusters in which hidden features are numerically dominant.

To avoid this, we choose a lower bound for K_1 using the L-method and iterate it until reaching an unacceptability criterion. Having knowledge of the mean of image B (Eq. 1), we can similarly compute a mean from B for each cluster $C_1, \ldots, C_{K_1}$

$$(\mu_\rho^i, \mu_\gamma^i, \mu_\beta^i) = \text{mean}(\rho_p, \gamma_p, \beta_p) : p \in C_i, i = 1, \ldots, K_1$$

By experiment we discover that the condition

$$\mu_\rho^i > \mu_\rho \text{ AND } \mu_\gamma^i > \mu_\gamma \text{ AND } \mu_\beta^i > \mu_\beta$$

is sufficiently strict—should a cluster channel mean exceed the global one on all three colour channels, we decrement K_1 and accept it as the value with which to proceed.

Having foreknowledge of the watermark design and its position, we can verify the applicability of the preceding algorithm. At each iteration, we consider the pixel locations of each cluster in B, and compare them with the location of the known watermark. If most pixels of a single cluster represent watermark features, we can compare K_1 with the best value obtained from the algorithm. This verification was successful with 30 randomly chosen test pages.

Characteristically, for the difficult data of the 'Mahdiyya' Qur'ān, starting values of K_1 chosen by the L-method were in the range 9–11, and the final values using our algorithm were in the range 20–25 clusters. An example of a sample input S and a transform of it are shown in Fig. 13. It is clear that background features vary from one region to another, but the transform has compensated for this.

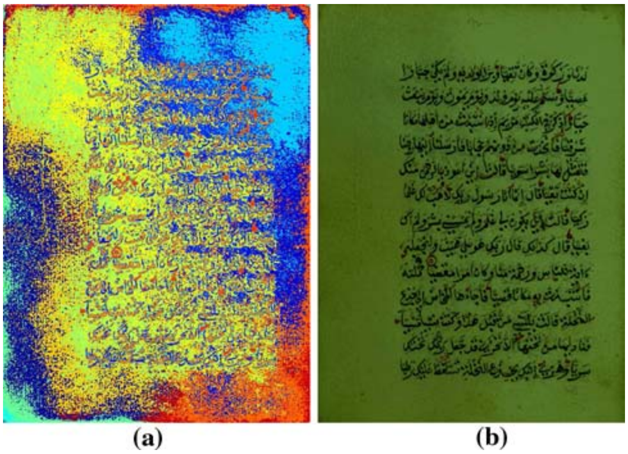

Fig. 13 **a** Cluster distribution of image S shown in Fig. 2a, using $K_1 = 20$, **b** transformed image of S. Figure 4 (*left*) shows a sub-window of **a** (this image is most favourably viewed in colour)

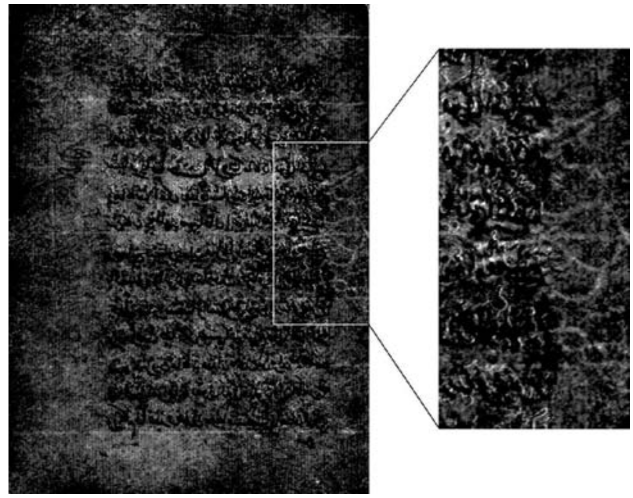

Fig. 14 Differenced image D (contrast stretched for display)—a watermark fragment is visible in the right hand margin. This is the full page of Fig. 4 (*right*)

6.2 Watermark location

The differenced image D is improved by setting negative pixel values to 0—we set a pixel value to 0 if any of its RGB channels is negative. Figure 14 shows an example resulting D. While the watermark features are partially evident here, noise is still very considerable. We find a partial segmentation by clustering to K_2 centroids the RGB data in D; this time the L-method [51] is a suitable approach as we seek to minimise computational load while maximising information retention. In all experiments we have performed, on data of a range of qualities, the number of clusters so determined has been of the order of 10. Figure 15 illustrates the cluster distribution of D: the zoomed window shows that these clusters do successfully pick out watermark features, in addition to many noise and other artefacts.

We now construct the array M (Eq. 7) which aggregates the evidence of fit. With well-chosen templates, we find a thresholding approach successful at this stage, but it is sensitive to threshold choice in the event of significant noise. Figures 16 and 17 illustrate this response M for two watermark fragments, where dots denote significant peaks, and squares their centroids (zoomed for better viewing).

A simple remedy is to exploit the fragments' known geometric relationship (offset from one another) in inspecting these peaks. In other words, we seek co-occurrences of peaks in accumulated M arrays that match the known geometric relationship of the fragments.

After locating the centroids of significant peaks for each fragment, we find the geometric relations (offsets) between each pair. Known geometric relations are inspected between significant peaks in a generalised Hough transform-like approach [56], and the match with the highest combined non-zero response accepted as best possible. If no suitably offset peaks are found, a negative ('no fit') result is generated.

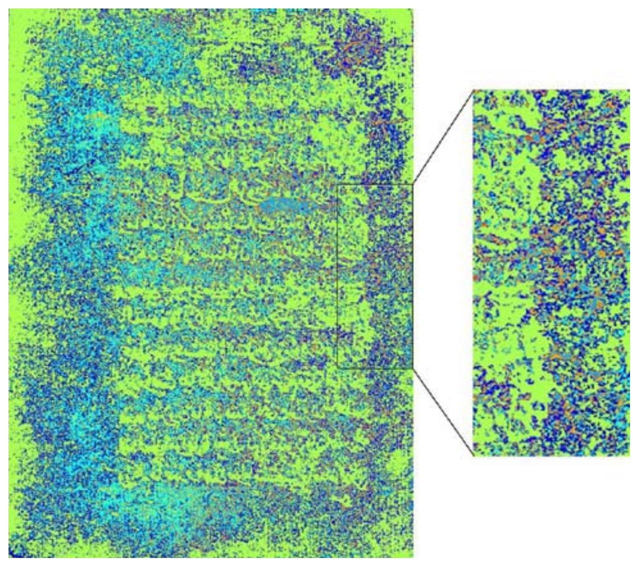

Fig. 15 Cluster distribution of image D presented in Fig. 14, using $K_2 = 10$, with watermark area enlarged on the right (this image is most favourably viewed in colour)

It will be clear that the existence of three or more fragments from a watermark would improve the potential of this approach further, and we have demonstrated this with three fragments in a few cases [29]. It turns out that two are sufficient for nearly all the data we have processed; further, it is often the case that not all three are discernible, and a pairwise search is more productive.

Our classifier works very well in recognising watermarks, even those of weak signal, with a very high percentage of true positives and no false positives. Table 1 shows retrieval

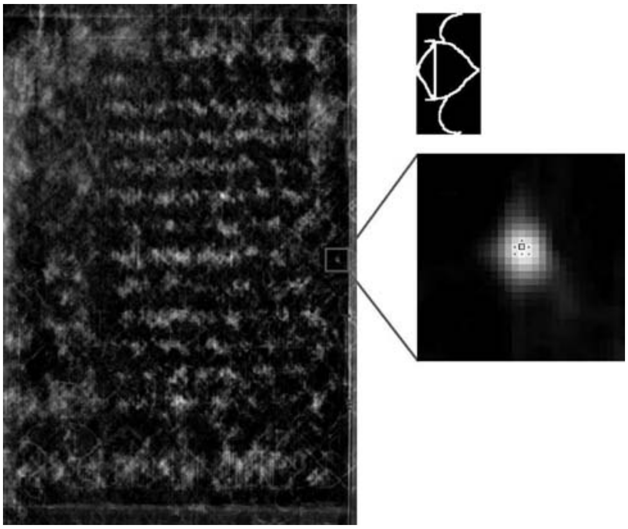

Fig. 16 The accumulator M, with positions of significant peaks of 1st fragment ($s = 6$), and its selected centroids, square-marked

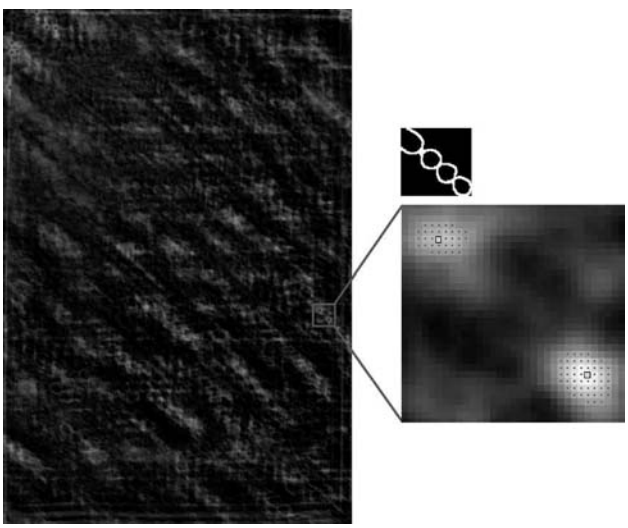

Fig. 17 The accumulator M, with positions of significant peaks of second fragment ($s = 6$), and its selected centroids, square-marked

Table 1 Matching results for different watermark shapes (%)

Watermark	M upper	M lower	E upper	E lower
True positive	98.8	97.7	96.5	94.3
True negative	100	100	100	100

Each shape is divided into two by the cutting of large paper sheets

results for four design parts: the double-headed eagle watermark 'E', and a moonface-within-shield countermark 'M' used in the 'Mahdiyya' Qur'ān.

There remain a few false negatives when the signal is very weak, due to the very poor signal evident in the original scans. Various approaches allow boosting of this signal in manipulations of the D image, but these usually have a side effect of generating many false positives—even deploying the known geometric relationship of fragments leaves this problem. Nevertheless, the results we have to hand are most encouraging since they are extracted with negligible interaction. The 'correct' answer does always show evidence, leaving open the opportunity for a swift interactive confirmation in very difficult cases.

We proceeded to test the approach with the other data sets and it gave perfect scores for both. This is doubly encouraging since—despite being simpler—these data are of significant scholarly interest and have not to date been studied. It is thus our belief that libraries hold a wealth of material that will be susceptible to the algorithms we present here, implying that the publication of systematic backlit scanning is a worthwhile exercise.

7 Watermark aggregation and 'twins'

The output of Sect. 6.2 provides many examples of the same watermark. Individually, they are incomplete, often seriously so, and we have experimented with aggregating them to improve the signal: this has then been trialled for known watermarks using the SNR measure discussed in Sect. 6. A simple statistical model suggests that this measure will either improve or deteriorate as the reciprocal of the number of images used—see Appendix.

The value and interest of this procedure is well demonstrated by example, since it has revealed details of watermarks that we could not observe before. Figure 1 illustrates the superimpositions of the double-headed eagle: we could not detect the 'A G' countermark below it in single sheets before applying this process. Many other details of the design also become clear that cannot be detected in individual sheets. We also observe chain lines have developed high response in the aggregated image.

The more superimpositions, the clearer the watermark details. Experiments confirm that adding more samples provides a better SNR than individual images—Fig. 18 shows SNR values of superimposing 2 and more differenced images D_k of the double-headed watermark.

It is clear that some parts of the superimposed watermarks in Fig. 1 are brighter than others; lower quality areas are attributable to the (removed) presence of recto features, and the nulling of pixels associated with verso features.

The aggregation operation may also be useful in the study of 'twins': when similar designs are superimposed, it could be easy to identify the differences between them. To illustrate this, we isolated 3 tre lune watermarks taken from different sheets of the Prayer. Figure 19 shows two different pairwise aggregations: in this example, the first two watermarks were observed as 'identical', where the third shape was 'twin'—this is obvious by looking into the slight changes of

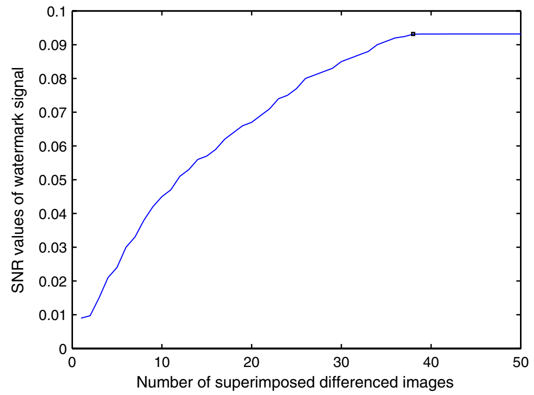

Fig. 18 SNR values of superimposed differenced images D_i

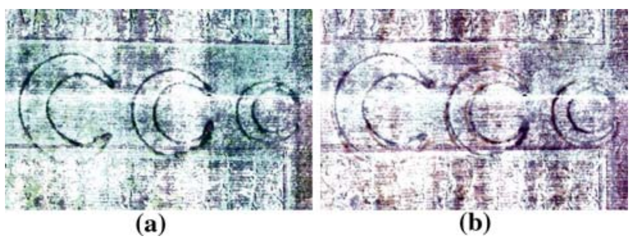

Fig. 19 Aggregated watermark designs. **a** The first and second designs, which are identical, **b** the first and third designs, which are 'twins'. Differences can be seen very clearly when viewed in colour

the crescents' edges. This figure is magnified for better visualisation.

8 Conclusion

This paper has considered the location and extraction of watermarks from paper. Its contributions are

- An approach aimed at challenging data: variable paper thickness, damage and noise, and heavy occlusion by inscription recto and verso.
- An approach to extracting representations of a full watermark given only a possibly imprecise fragment.
- Consequent on this, an approach to distinguishing between 'twins'.

Success has been demonstrated in these on a large and difficult data set bearing four different individual watermarks, and further demonstrated on two different volumes (with different marks) which—while less challenging—have not to date been analysed for watermark content. Algorithms require negligible interaction and most parameters are derived automatically. The same algorithms applied to easier data provide very good results where more traditional techniques [31] are often seen to fail.

We see opportunities for improving and streamlining the ideas presented here in several ways. Among these are:

- We have not to date used knowledge of the verso scan to improve the data of the difference image D (Eq. 5). There is clear scope for analysing this to reduce the incidence of interference.
- The cluster-by-cluster linearity assumption developed in Sect. 4 has been seen to work very well, but may possibly improve with a more sophisticated (quadratic or cubic) lighting model.
- We see scope for more intelligent extraction of the watermark pattern from D, and will be exploring the potential of Markov random fields [65] in pursuit of this.
- Of longer term interest is to explore the image D when the watermark (or fragments) are *not* known: in a large document, it is possible that a given fragment will recur (this is certainly true in the 'Mahdiyya' Qur'ān): if we could automate the identification of such fragments, we would have a basis for automatic location (via aggregation) of unknown watermarks.

These represent work in hand.

We are confident that the work we present here is easily applicable to a range of historically interesting documents: our current work means to exploit further some of the scarce holdings in the University of Leeds collection.

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Appendix A: Mean and variance of a match measure on two binary vectors of known 'tally'

Suppose we have two binary vectors of dimension N:

$$\mathbf{v_1} = (v_1^1, v_1^2, \ldots, v_1^N), \quad \mathbf{v_2} = (v_2^1, v_2^2, \ldots, v_2^N), \quad v_i^j \in \{0, 1\}$$

We are told that there are I 1's in $\mathbf{v_1}$ and J in $\mathbf{v_2}$:

$$\sum_{k=1}^{N} v_1^k = I, \quad \sum_{k=1}^{N} v_2^k = J$$

Count $w(\mathbf{v_1}, \mathbf{v_2})$ as the number of times corresponding vector components are both 1 or 0; then $0 \leq w(\mathbf{v_1}, \mathbf{v_2}) \leq N$:

$$w(\mathbf{v_1}, \mathbf{v_2}) = \sum_{k=1}^{N} (1 - XOR(v_1^k, v_2^k))$$

Given $\mathbf{v_1}$, suppose $\mathbf{v_2}$ is chosen randomly—we seek the mean and variance of w. Suppose

v_1^k	v_2^k	Occurrences
1	1	a
1	0	b
0	1	c
0	0	d

where then

$$I = a + b, \qquad N - I = c + d$$
$$J = a + c, \qquad N - J = b + d$$
$$N = a + b + c + d$$

Then we seek

$$\begin{aligned} w &= a + d \\ &= a + (N - a - b - c) \\ &= a + (N - a - (I - a) - (J - a)) \\ &= 2a + N - I - J \end{aligned}$$

Now the distribution of a is hyper-geometric (see, e.g., [42]) giving

$$\mu(a) = \frac{IJ}{N}$$
$$\sigma^2(a) = \frac{IJ(N-I)(N-J)}{N^2(N-1)}$$

So

$$\begin{aligned} \mu(w) &= 2\mu(a) + N - I - J \\ &= 2\frac{IJ}{N} + N - I - J \\ \sigma^2(w) &= 4\sigma^2(a) \\ &= \frac{4\,IJ(N-I)(N-J)}{N^2(N-1)} \end{aligned}$$

Appendix B: Expected SNR from aggregating responses

Suppose a window of N pixels contains S pixels of signal, distributed with mean μ_s and variance σ_s^2. The remaining $N - S$ pixels are noise with mean μ_n and variance σ_n^2.

Using Eq. 8 to calculate SNR, the expected value will be

$$\frac{S(\mu_s^2 + \sigma_s^2)}{(N-S)(\mu_n^2 + \sigma_n^2)}$$

and when k such images are superimposed, will be

$$\frac{S(k^2\mu_s^2 + k\sigma_s^2)}{(N-S)(k^2\mu_n^2 + k\sigma_n^2)} = \frac{S}{(N-S)}\left(\frac{\mu_s^2}{\mu_n^2} + \frac{\sigma_s^2 - \frac{\mu_s^2}{\mu_n^2}\sigma_n^2}{k\mu_n^2 + \sigma_n^2}\right)$$

This is thus reciprocal in k, with the sign provided by the relative magnitudes of the signal and noise parameters.

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