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RESEARCH ARTICLE

Why regulators adopt voluntary programs: a theoretical analysis of voluntary pollutant reduction programs

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Abstract To explain why regulators continue to implement voluntary emission reduction programs (VP), this study presents a model with multiple polluting firms, a trade association, a regulator, and a legislator who sets a mandatory standard and is politically influenced by the trade association, a representative of the polluting firms. We show that the regulator can implement a voluntary program, which generates less social cost and more aggregate abatement than a mandatory standard. We also find that assigning the greatest importance to the abatement rates of individual firms generates the highest level of social welfare if the damage due to individual firms' emissions does not depend on other firms' emissions. However, the importance of the participation rate will increase relatively to the abatement rate as the damage due to individual firms' emissions becomes more sensitive to other firms' emissions.

Keywords Environmental policy · Lobbying · Voluntary programs · Cooperation under threat · free riding

JEL Classification Q58 · D72 · D78

1 Introduction

Over the last four decades, environmental policy tools have been developed and applied in response to growing environmental concerns. Among these tools, voluntary approaches to environmental protection have become prominent in

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developed countries since the late 1980s. In the European Union (EU), the number of new voluntary agreements increased from 6 in 1981 to more than 45 in 1995 (OECD 1999). There are over 300 negotiated agreements between governments and polluting industries or firms in Europe. In the United States (US), Brouhle, Griffiths and Wolverton (2005) identified over 50 voluntary programs at the federal level alone since 1991, when the 33/50 program,[1] the first voluntary program, was launched. In Japan, over 30,000 negotiated agreements between local governments and (in most cases, individual) polluters are in effect.[2]

One of the main motives for voluntary emission abatement is legislative threats (or regulatory threats). Firms might take voluntary environmental actions (without the direct involvement of governments), make voluntary environmental agreements with governments or participate in voluntary environmental programs designed by governments to avoid a costly mandatory regulation. Numerous voluntary approaches have been adopted in response to threats of mandatory regulation. However, the low participation rates of voluntary programs (VPs) suggest that these threats are typically weak; for example, the participation rates of the US 33/50 program and the Canadian Accelerated Reduction/Elimination of Toxics (ARET) program, the goal of which is to reduce the release and/or transfer of chemicals, are 17.0 and 13.4 %, respectively.[3]

These voluntary programs have been led by governments. This prompts the following question: why do the governments continue to implement VPs rather than

[1] The US Environmental Protection Agency (EPA) launched this program to reduce the aggregate emissions of 17 chemicals by 33 % in 1992 and by 50 % in 1995, relative to the 1988 baseline. See Khanna (2007) for a more detailed review of the 33/50 program, for example. Empirical studies have been conducted by Khanna and Damon (1999), Gamper-Rabindran (2006), Innes and Sam (2008) and Bi and Khanna (2012).

[2] The reason that over 30,000 agreements have been implemented in Japan is that most agreements are between a firm and a municipality. When a firm constructs (or extends) its facility, it concludes an agreement with the municipality in which the facility is (will be) located. This is the typical setting in which agreements are made. Please see Welch and Hibiki (2003) for details. In contrast to Japan, many agreements occur at the federal government or industrial level in Europe and the US. This is why the number of agreements or voluntary policies differs so substantially between Japan and other developed countries.

[3] Regarding the 33/50 program, the total release and transfer of participating firms in 1988 (baseline year) was high (62.5 %) relative to its participation rate. However, the program did not cover approximately 40 % of total releases and transfers. In contrast, the Canadian ARET program was launched in 1994 to reduce the release of 117 toxic substances. Many of the ARET-listed substances were not required to be reported to the National Pollutant Release Inventory (NPRI), which legally mandates public reporting. Therefore, data on these substances are not available. See Antweiler and Harrison (2007) for NPRI-recorded emissions of ARET-listed substances and ARET-participating firms' shares of these emissions. They also provide a more detailed review of the ARET program with policy evaluation by regression analyses.

It is worth noting that the ARET program has some of the characteristics of a negotiated voluntary agreement because a committee of governments and nine industry associations issued the ARET Challenge in March 1994. In contrast to the ARET program, without the involvement of industry associations, the US Environmental Protection Agency asked the "top" 600 companies that accounted for 66 % of the total release of these chemicals in 1988 to join the program in January 1988 and asked 5400 other companies to join it in July 1991. However, in both programs, firms had the option to participate. This is a distinctive characteristic of voluntary programs relative to voluntary agreements that called on all firms to reduce their emissions.

mandatory policies given the former's low participation rates? Weak legislative threats primarily result from the political difficulty of establishing stringent mandatory regulations. Therefore, governments have implemented VPs due to the fear of weak mandatory regulations (or the lack thereof) even though their participation rates were low. In addition to political difficulty, it is possible that governments prefer a VP with a high abatement rate and low participation rate to one with a low abatement rate and high participation rate because the former generates greater social welfare than the latter. Because participation in VPs is not mandatory, there is a trade-off between the participation rate and the abatement rates of individual participating firms. By taking this trade-off into account, governments might place more weight on abatement rates of individual participating firms than the participation rate.

This paper addresses the following two questions; first, why have governments implemented VPs despite their low participation rates, and second, how should governments design VPs given the trade-off between the participation rate and the abatement rate. Although many researchers have studied voluntary approaches to environmental protection, we are unaware of any study that resolves both of these questions. Lutz, Lyon and Maxwell (2000) and Maxwell, Lyon and Hackett (2000) analyzed voluntary environmental actions by firms rather than voluntary programs designed by governments or voluntary agreements between governments and firms. Most studies on voluntary programs and voluntary agreements have ignored individual firms' participation decisions regarding voluntary programs or agreements because they focused on the case of a single polluter (Segerson and Miceli 1998, 1999; Hansen 1999; Glachant 2007; Fleckinger and Glachant 2011) or assumed that all polluters participate in voluntary approaches (Manzini and Mariotti 2003).

Lyon and Maxwell (2003), Dawson and Segerson (2008) and Brau and Carraro (2011) developed models that incorporate an individual firm's participation decision. However, at most, these studies only answered one of our two research questions. In the model of Lyon and Maxwell (2003), there is no trade-off between the participation rate and the abatement rate of individual participating firms, as the voluntary program concerns the adoption of a new technology that induces some fixed amount of emissions abatement. Therefore, they provided no information on how voluntary programs should be designed when there is a trade-off between firms' participation and abatement rates, although they provided a rationale for implementing a VP that differs from the one analyzed in this paper. Dawson and Segerson (2008) developed a model where a government seeks to achieve an exogenous industry-wide emissions abatement target through a VP or an emission tax. In their model, the government implements the VP if it achieves the abatement target, despite the government being able to impose the first-best tax. Thus, Dawson and Segerson (2008) analyzed the VP that is very similar to ours but did not provide a rationale for implementing such a VP. Brau and Carraro (2011) also analyzed voluntary emission abatement actions of firms within the same industry in a case with spillover effects in which firms' production costs decrease more as more firms commit to emissions abatement and as committing firms' levels of emissions abatement increase. Their main focus was on firms' behavior, in particularly the relationship between firms' commitment and spillover effects. As a result, their analysis did not provide the government's motivation for implementing low participation rate VPs and the implications of VP design when the government faces a trade-off between abatement and participation rates.

In this paper, we develop a multiple-firm (multiple-polluter) model of a voluntary program in the presence of a weak legislative threat to resolve the above two questions. The weak legislative threat in our model is inspired by Glachant (2007), who developed a model of a non-enforceable voluntary agreement without a trade-off between the participation and abatement rates (an agreement between a benevolent regulator and a single polluter). The weak legislative threat particularly results from a legislative mandatory standard in which a representative of the polluters lobbies the congress. Due to the weak legislative threat or laxness of the mandatory standard, a benevolent regulator might have an incentive to offer the VP to achieve greater social welfare, although the regulator cannot force the firms to participate. In addition to the regulator, the individual firms might have an incentive to participate in the VP to save lobbying costs.

We find that if the legislative threat is very weak, a low participation rate VP can generate higher social welfare and greater aggregate abatement than the mandatory standard. In addition, the highest social welfare and the greatest aggregate abatement are generated by the highest abatement rate of individual firms such that the participating firms' abatement costs under the VP are less than under the mandatory policy if the damage resulting from individual firms' emissions does not depend on other firms' emissions. The participation rate under this abatement rate is the lowest among the abatement rates of VPs that generate higher social welfare. Thus, the abatement rate of the participating firm is important when the damage due to individual firms' emissions is independent of other firms' emissions. However, as damage due to individual firms' emissions becomes more sensitive to other firms' emissions, the importance of the participation rate will increase relative to that of the abatement rate.

This paper is organized as follows. In next section, we presents a model and analyzes equilibrium conditions. Section 3 discusses the effects of settings other than the basic setting. Section 4 concludes this paper.

2 The model

Our model includes two types of public agencies, a legislative sector and a regulator (administrative sector or public environmental agency), to capture the political process of voluntary environmental programs in the same manner as Glachant (2007). In addition, we describe the political barriers faced by the legislative sector when it attempts to implement mandatory policies that are produced through political pressure (lobbying) from a trade association. As a representative of the polluting firms, the trade association lobbies the legislative sector to set a lax mandatory standard. Thus, the following four types of players are involved in our model: a regulator, a legislator, N polluting firms and a trade association.

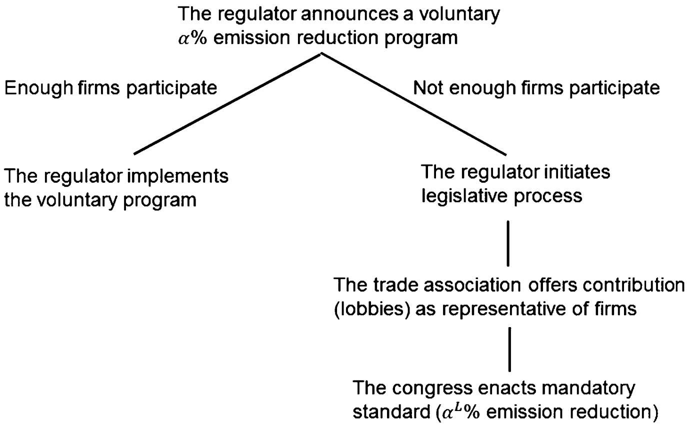

Fig. 1 Decision tree of game

A regulator is benevolent in that the regulator aims to minimize social costs, the total damage created by pollution and aggregate abatement costs. In the first stage, the regulator announces a voluntary α (%) emission reduction program (VP). In the second stage, polluting firms decide whether to participate in the VP. Thus, the firms do not have to participate in the VP. However, if an insufficient number of firms participate in the VP, the regulator initiates a legislative process to establish a mandatory standard. After the legislative process begins, a trade association, as the representative of the polluting firms, will lobby against it. Finally, the legislator will set the mandatory standard. The mandatory standard in this model can be interpreted as a regulatory threat, although the threat may be weak or the standard may be lax. Conversely, the VP is implemented by the regulator if a sufficient number of firms participate in the VP in the second stage. The timing of this model is described in Fig. 1.

It is possible that the polluting firms will take voluntary actions before the regulator announces the VP. However, if many firms are involved, it is difficult for trade associations to coordinate voluntary emission reduction efforts among their firms. As a result, voluntary actions and agreements are unlikely to be implemented. This paper considers such a case. In contrast to voluntary emission reduction efforts, polluting firms typically unite in opposition to mandatory regulations, or their trade associations lobby against the regulations as representatives of the polluting firms. Such lobbying activities are also likely to suffer from free riding in the case where the coordinating power of the trade association is weak. Therefore, we consider a case where an existing trade association influences mandatory policy through political contributions financed by contributions from the polluting firms. However, the association cannot force its firms to contribute to lobbying efforts against the mandatory policy, in other words, each firm decides how much it will contribute to the association due to the weak coordination power of the association.[4]

[4] It is also possible that the association could obtain money for political contributions from the firms as a membership fee (no free riding in lobbying) despite its weak coordinating power (it cannot coordinate VP participation among its firms). We consider such a case in the next section.

This paper does not address the possibility that a legislator implements a mandatory policy even if a VP is already in place. However, to create a law that enforces the mandatory policy, various types of information, such as scientific knowledge about pollution, must be obtained and integrated. In many cases, it is likely to be logistically difficult and/or costly to obtain such technical information and expertise without the cooperation of a government-run environmental organization, which is represented by the regulator in this paper. Therefore, it is not unreasonable to assume that the regulator would initiate both policies.

Figure 1 indicates that the regulator has two options: implement the voluntary program or (ask the legislator to) implement the mandatory policy to minimize social costs, which are the sum of the damage generated by pollution and aggregate abatement costs. We assume that all firms have the same abatement cost that is quadratic in emissions, $\frac{1}{2}c(\alpha\bar{e})^2$ where $\bar{e}$, α and c are the polluting emissions of each firm prior to regulation, the abatement rate and the slope of the marginal abatement cost, respectively. In this section, we assume that the damage induced by individual firms' emissions does not depend on other firms' emission or the damage is a linear function of emissions, $dN(1-\alpha)\bar{e}$ (under mandatory regulation), where d is the marginal damage.[5] By this assumption, in this section, we implicitly focus on nationwide environmental policy for local pollution problems, with a main target of plants in areas with a small number of plants. Because most impacts of toxic chemicals are local and the US and Canada have large land areas, nationwide policies for toxic chemicals in these countries such as the 33/50 program and the ARET program are likely to be typical cases in this section. We consider a case where the damage induced by individual firms' emissions depends on other firms' emissions in the next section.

Because the regulator is benevolent, the regulator's cost under the mandatory policy is the sum of the aggregate abatement costs and damages as follows;

$$R^{\mathrm{M}}(\alpha) = dN((1-\alpha)\bar{e}) + N\frac{1}{2}c(\alpha\bar{e})^2. \qquad \text{(Madantory policy)}$$

If the regulator could set a mandatory standard or a abatement rate, he/she would set the abatement rate as follows;

$$\alpha^* = \frac{d}{c\bar{e}}. \tag{1}$$

However, the legislator sets the abatement rate for the mandatory program. When the congress enacts a mandatory standard, the trade association, as the representative of the polluting firms influences it by offering political contributions to the legislator, who is a representative or median legislator. As Glachant (2007) described, the legislator's payoff function is assumed to be a weighted sum of political contributions, Ω, and social welfare (negative social costs) as follows;

[5] We obtain the same results as below if the damage is given by $Nf((1-\alpha)\bar{e})$ where f is convex with $f' > 0$ and $f'' \geq 0$.

$$L(\alpha, \Omega) = \lambda\Omega - (1-\lambda)\left[dN(1-\alpha)\bar{e} + N\frac{1}{2}c(\alpha\bar{e})^2\right]$$

where $0 < \lambda < 1$. Because only polluting firms have a lobby group, the parameter λ can be interpreted as the responsiveness of the legislator to lobbying or the political difficulty of setting an efficient mandatory abatement rate. Political difficulties may arise when a pollution problem is not an important issue on the political agenda. A high value of λ may reflect that it is not an important issue on the agenda.

Political contributions from the trade association are financed by voluntary contributions from individual firms. We assume that each firm decides how much it contributes to the association, taking other firms' contributions as given, as in voluntary public good provision games. Firm i' s cost is the sum of the abatement costs, $\frac{1}{2}c(\alpha\bar{e})^2$, and contributions to the lobbying group, Ω^i, $\frac{1}{2}c(\alpha\bar{e})^2 + \Omega^i$. Thus, taking other firms' contributions as given, each firm individually chooses its contribution to minimize its cost.

In contrast to the mandatory policy, the regulator can set the abatement rate of the VP, but it cannot force firms to participate and non-participating firms would not abate their emissions at all. Thus, if the regulator implements the VP, the objective is as follows;

$$\min_{\alpha} R^{\mathrm{V}}(\alpha) = d\{N_{\mathrm{P}}(1-\alpha)\bar{e} + (N - N_{\mathrm{P}})\bar{e}\} + N_{\mathrm{P}}\frac{1}{2}c(\alpha\bar{e})^2 \qquad \text{(VP)}$$

where N_{P} is the number of firms participating in the VP. As mentioned above, individual firms can decide whether they participate in the VP. Firms have no incentive to join the VP if their costs under the VP are greater than they would be under the mandatory policy. Therefore, N_{P} and the VP participation rate (N_{P}/N) depend on the abatement rate of the VP and on each firm's costs or abatement rate under the mandatory standard. In the next subsection, we analyze how the mandatory policy is set. Then, we examine how the regulator sets the VP.

2.1 The legislative subgame

First, we analyze the case in which the regulator uses the legislator to set the mandatory standard. In the final stage, the legislator sets the mandatory abatement rate and firms reduce their emissions. Prior to the final stage, the trade association offers political contributions that depend on the abatement rate, α. Because the legislator can reject the offer from the trade association and choose a socially optimal abatement rate, the potential political contribution must satisfy $L(\alpha, \Omega(\alpha)) \geq L(\alpha^*, 0)$. This constraint must hold with equality because the total cost to the firms increases with increases in the abatement rate, and the association moves first. Therefore, political contributions and the abatement rate chosen by the legislator have the following relationship

$$\Omega(\alpha) = \frac{1-\lambda}{\lambda}\left[dN(1-\alpha)\bar{e} + N\frac{1}{2}c(\alpha\bar{e})^2 - \left[dN(1-\alpha^*)\bar{e} + N\frac{1}{2}c(\alpha^*\bar{e})^2\right]\right]. \quad (2)$$

Let $\hat{\alpha}(\Omega)$ be the abatement rate satisfying (2) when the political contribution is Ω. Using $\hat{\alpha}(\Omega)$, we can describe an individual firm's problem. Their contributions affect political contributions directly and abatement rates indirectly. Therefore, taking it as given that other firms have also made contributions, firm i chooses its contribution to minimize its total cost as follows;

$$\frac{1}{2}c\left(\hat{\alpha}\left(\Omega^i + \Omega^{-i}\right)\bar{e}\right)^2 + \Omega^i \quad (3)$$

where $\Omega^{-i} = \sum_{j \neq i} \Omega^j$. Because $\Omega^i = -\Omega^{-i} + \Omega$, we enter (2) into (3) and then obtain

$$\begin{aligned}&\frac{1}{2}c\left(\hat{\alpha}(\Omega^i + \Omega^{-i})\bar{e}\right)^2 - \Omega^{-i} + \frac{1-\lambda}{\lambda}\left\{dN(1-\hat{\alpha}(\Omega^i + \Omega^{-i}))\bar{e}\right.\\ &\left.+N\frac{1}{2}c\left(\hat{\alpha}(\Omega^i + \Omega^{-i})\bar{e}\right)^2 - \left[dN(1-\alpha^*)\bar{e} + N\frac{1}{2}c(\alpha^*\bar{e})^2\right]\right\}.\end{aligned} \quad (4)$$

From the F.O.C for the minimization of (4),

$$c\hat{\alpha}\bar{e}^2\frac{\partial\hat{\alpha}}{\partial\Omega^i} = \frac{1-\lambda}{\lambda}\left[dN\bar{e} - Nc\hat{\alpha}\bar{e}^2\right]\frac{\partial\hat{\alpha}}{\partial\Omega^i}. \quad (5)$$

Therefore, the abatement rate chosen via the legislative process is

$$\alpha_{\mathrm{L}} = \frac{(1-\lambda)dN}{c\bar{e}[\lambda + (1-\lambda)N]}. \quad (6)$$

Unlike the socially optimal abatement rate, the abatement rate under the mandatory policy depends on the number of polluting firms due to free riding on other firms' lobbying contributions. If firms are unanimous in opposing the mandatory policy or the trade association minimizes the aggregate cost of the firms, $N\frac{1}{2}c(\alpha\bar{e})^2 + \Omega$, and can force the firms to make a contribution, then the abatement rate under the mandatory policy is $\alpha_{\mathrm{L}}^{\mathrm{WO}} = \frac{(1-\lambda)dN}{c\bar{e}[\lambda N + (1-\lambda)N]} = \frac{(1-\lambda)d}{c\bar{e}}$. Thus, the abatement rate does not depend on the number of polluting firms in such a case.[6] However, when all the polluting firms do not contribute to lobbying, they have stronger incentives to free ride as the number of firms increases. Because the lobbying efforts of individual firms decrease in the number of firms, the gap between α^* and α_{L} is smaller if the number of firms is larger.

If we focus on a symmetric equilibrium,[7] then firm i' s contribution is

[6] In this case, the lobby minimizes $\sum_i \frac{1}{2}c(\hat{\alpha}(\Omega)\bar{e})^2 + \Omega$. From the F.O.C, $Nc\hat{\alpha}\bar{e}^2\frac{\partial\hat{\alpha}}{\partial\Omega} = \frac{1-\lambda}{\lambda}\left[dN\bar{e} - Nc\hat{\alpha}\bar{e}^2\right]\frac{\partial\hat{\alpha}}{\partial\Omega^i}$.

[7] In the next section, we consider asymmetric equilibria or cases where different firms may pay different amounts of political contributions.

$$\begin{aligned}\Omega^i &= \Omega(\alpha_{\rm L})/N \\ &= \frac{1-\lambda}{\lambda}\left[dN(1-\alpha_{\rm L})\bar{e} + N\frac{1}{2}c(\alpha_{\rm L}\bar{e})^2 - \left[dN(1-\alpha^*)\bar{e} + N\frac{1}{2}c(\alpha^*\bar{e})^2\right]\right]\Big/ N \\ &= \frac{\lambda(1-\lambda)d^2}{2c[\lambda+(1-\lambda)N]^2}\end{aligned} \tag{7}$$

Finally, an individual firm's cost under the mandated standard is

$$\begin{aligned}\bar{C} = \frac{1}{2}c(\alpha^{\rm L}\bar{e})^2 + \Omega^i &= \frac{\lambda(1-\lambda)d^2}{2c[\lambda+(1-\lambda)N]^2} + \frac{1}{2c}\left[\frac{(1-\lambda)dN}{\lambda+(1-\lambda)N}\right]^2 \\ &= \frac{(1-\lambda)d^2[\lambda+(1-\lambda)N^2]}{2c[\lambda+(1-\lambda)N]^2}.\end{aligned} \tag{8}$$

2.2 The VP subgame

The regulator implements the VP if the VP generates lower social costs than the mandatory policy. Otherwise, it initiates the legislative process to establish the mandatory policy. Therefore, if the VP is implemented, the following inequality must hold;

$$d[(N - N_{\rm P}\alpha)\bar{e}] + N_{\rm P}\frac{1}{2}c(\alpha\bar{e})^2 \leq R^{\rm M}(\alpha_{\rm L}). \tag{9}$$

For polluting firms to participate in the VP, their costs must be smaller than they would be under the mandatory standard. Thus, the abatement rate under the VP, α, must satisfy

$$\frac{1}{2}c(\alpha\bar{e})^2 \leq \bar{C}. \tag{10}$$

In the equilibrium, individual firms must not unilaterally change their participation decision (e.g., no "not participating" firms change their decision from "not participate" to "participate"). Equation (10) is not enough to characterize equilibrium. Actually, there are cases when VP-participating firms have an incentive not to participate in a VP even though costs of the participating firms are smaller than those under the mandatory standard (even though (10) holds). Notice that costs of firms "not participating" are zero when the VP is implemented. Therefore, participating firms have an incentive not to participate in the VP when the VP is implemented regardless of whether the number of participating firms decreases.

However, no participating firms have an incentive to change their participation decision when the VP would not be implemented due to a potential decrease in the number of participating firms (of course, this is true only if costs of participating firms are lower than those under the MP). Note that the number of participating firms is N_P and that the VP is not implemented if social costs under the VP are greater than social costs under the MP. Then, the condition that no participating firms have an incentive to change their participation decision is equivalent when social costs under the VP with $N_P - 1$ participating firms are greater than social costs under the MP. This condition is technically expressed by

$$d[(N - (N_P - 1)\alpha)\bar{e}] + (N_P - 1)\frac{1}{2}c(\alpha\bar{e})^2 \geq R^M(\alpha_L). \tag{11}$$

In an equilibrium where the VP is implemented, all three conditions, (9), (10) and (11), must hold.[8] Otherwise, the mandatory policy is implemented in equilibrium.

Subject to (10) and (11), the regulator attempts to maximize its payoff or minimize social costs. Let α_V be such that $\frac{1}{2}c(\alpha_V\bar{e})^2 = \bar{C}$. When the abatement rate under the VP is α_V, the cost of the VP for the participating firms is the same as the costs of the mandatory policy. If abatement rate under the VP is greater than α_V, the cost of the VP for the participating firms is greater than the costs of the mandatory policy or all firms prefer the mandatory policy over the VP. Thus, α_V is the highest abatement rate of the VP that firms accept. The following proposition characterizes the combination of abatement and participation rates that generates the lowest social cost and shows that it is possible for the VP to better perform than the mandatory policy.

Proposition 1 *The lowest social cost and largest aggregate abatement under the VP is generated by the abatement rate α_V and the participation rate, N_P^V/N,*

$$N_P^V/N = \begin{cases} \dfrac{2d\alpha_L\bar{e} - c\alpha_L^2\bar{e}^2}{2d\alpha_V\bar{e} - c\alpha_V^2\bar{e}^2} + \dfrac{1}{N} & \text{If N} > \dfrac{2d\alpha_V e - c\alpha_V^2 e^2}{2d\alpha_V e - c\alpha_V^2 e^2 - (2d\alpha_L e - c\alpha_L^2 e^2)} \\ 1 & \text{Otherwise.} \end{cases} \tag{12}$$

In addition, the abatement rate α_V with participation rate N_P^V/N generates higher aggregate abatement and lower social cost than a mandatory standard. However, there exist equilibria where fewer than $(N_P^V - 1)$ firms participate in the VP and the mandatory policy is implemented.

Proof See Appendix. □

The first part of Proposition 1, the combination of the abatement and participation rates generating the lowest social cost and largest aggregate abatement, is derived from (11). The regulator prefers high abatement and participation rates, but a trade-off exists between the abatement rate and the participation rate due to the constraint (11), which is a type of constraint on social costs. Condition (11) proposes that the social cost with $N_P - 1$ participating firms must not be lower than the social cost under the mandatory policy regardless of the participation rate.

[8] We do not have to check whether non-participating firms have an incentive to change their participation decision. Dawson and Segerson (2008) show that no non-participating firms have an incentive to change their participation decision in equilibria where the VP is implemented. This is because in such equilibria, costs of non-participating firms are zero, while costs of participating firms greater than zero.

Therefore, the "minimization" of social costs under the VP is equal to the "maximization" of the decrease in social costs due to emissions abatement by the "N_Pth" participating firm. The regulator can maximize the decrease in social costs by requiring the "N_Pth" participating firm to reduce its emissions to the greatest extent possible because emissions by participating firms are excessive (insufficient abatement) and other firms' emissions do not affect the marginal benefit from emissions abatement by the "N_Pth" participating firm.

The second part of Proposition 1 implies that the highest VP abatement rate accepted by the firms can generate a higher aggregate abatement and a lower social cost than the mandatory standard as well as other feasible VPs. The VP with abatement rate α_L and participation rate $1(=N/N)$ is implementable ($(\alpha_L, 1)$ satisfies (10) and (11)) and generates the same social cost as the mandatory policy. However, it is also possible that the mandatory policy is implemented rather than the VP. For example, if $N_P^V/N < 1/N$, then no firms have an incentive to change their participation decision from "not participating" to "participating" given that all other firms do not participate in a VP.[9] Thus, an equilibrium always exists where the mandatory standard is implemented.

In general, the proposition suggests that a combination of high abatement and low participation rates (α_V and N_P^V/N) is better than a combination of low abatement and high participation rates (α_L and 1). More formally, the VP is the most environmentally effective and efficient if the regulator chooses the highest abatement rate, such that the participating firms' abatement costs under the VP are less than under the mandatory policy. If the regulator selects the highest abatement rate, some firms generally do not participate in the VP. Thus, symmetric firms may take asymmetric actions (i.e., some of them participate but others do not).[10] In addition, the proposition implies that the regulator might implement the VP even if the participation rate is low (e.g., when the legislator is not concerned about the social costs related to pollution or when it is politically difficult to implement the mandatory policy). In the first subsection of the next section (Sect. 3.1), we discuss how changes in the parameters affect the VP (abatement rate and participation rate) when it is effective or the combination of the abatement rate and participation rate is that characterized in Proposition 1.

In contrast with Dawson and Segerson's (2008) model where the regulator always prefers the mandatory policy over the VP, the regulator in this model might prefer the VP over the mandatory policy if the participation rate is high enough, which depends on the firms' beliefs about other firms' participation decisions. This result occurs due to the difference in the regulator' s objective and the mandatory policy-making process. In Dawson and Segerson (2008), the regulator' s objective is to achieve some aggregate emission level, and in our model, it is to minimize social costs. Because some firms might not join the VP, the mandatory standard is better

[9] See Section 2 of Dawson and Segerson (2008) for a more intuitive explanation. It provides simple examples that explain the existence of equilibria where the mandatory policy is implemented.

[10] The simple examples of Dawson and Segerson (2008) also well explain why symmetric firms may take asymmetric actions. It is also possible that all of the firms participate if the legislator is significantly concerned about social welfare or the damage caused by the pollution is serious.

than the VP for achieving some aggregate abatement level. Without political difficulties in the mandatory policy-making process as in Dawson and Segerson (2008), the regulator can implement the mandatory policy, which achieves a socially efficient outcome. Thus, the difference between this paper and Dawson and Segerson's paper is crucial to the implementation of a VP by the government.

To sum up, by incorporating political difficulties in the mandatory policy-making process, we provide a rationale for implementing a VP in contrast with Dawson and Segerson (2008) and show that the low participation rate VP might generate the highest social welfare. In this paper, we incorporate political difficulties by introducing the polluting industry's lobbying. Even though we incorporate the difficulties by introducing uncertainty of the mandatory standard adoption as in Segerson and Miceli (1998) and Fleckinger and Glachant (2011), we obtain the same results (Proposition 1).

3 Results under different settings

In the last section, we assume that (1) lobbying by the polluting industry against the mandatory standard and participation in the voluntary program suffer from the free-rider problem, (2) the damage induced by individual firms' emissions does not depend on other firms' emissions, and (3) all firms equally share costs of political contributions.

Even if the trade association cannot coordinate its firms' participation in the VP, the association might be able to force its firms to pay its membership fee to support political contributions or the cost of a political campaign. Thus, it is also possible that there are free riders in the VP but no free riders in lobbying against the mandatory standard. In this section, we first determine whether Proposition 1 holds in a case without free riding in lobbying and examine the effects of changes in industry size (the number of polluting firms) with and without free riding in lobbying.

The previous assumption regarding damage is rather strong. If the damage induced by individual firms' emissions depends on other firms' emissions, how should the regulator design the VP? In some cases, the regulator might set the abatement rate low to ensure a high participation rate and minimize social costs, whereas, in other cases, the regulator might select the highest possible abatement rate as in the last section. We examine this following the analysis of a case without free riding in lobbying.

Finally, we discuss a case of asymmetric political contributions between firms. If political contributions are different between firms, then costs under the mandatory standard are also different. Consequently, the highest VP abatement rates that firms accept are different between firms. We examine how different highest VP abatement rates affect impacts of VP.

3.1 Comparison of cases with and without free riding in lobbying

If we assume that all firms cooperate in lobbying and no firms free-ride, then the lobbying group minimizes $C(\Omega) = N\frac{1}{2}c(\alpha(\Omega)\bar{e})^2 + \Omega$ subject to $\frac{\partial L}{\partial \Omega} = 0$ and $L(\alpha, \Omega) = L(\alpha^*, 0)$. From the F.O.C and $L(\alpha, \Omega) = L(\alpha^*, 0)$, we obtain $\alpha_L^{WO} = \frac{d}{c\bar{e}}(1-\lambda) < \alpha_L < \alpha^*$ and $\Omega^{WO} = \frac{\lambda(1-\lambda)d^2}{2c}N$. If we assume that political contributions are shared equally by all firms as in the last section, then $\alpha_V^{WO} = \frac{d}{c\bar{e}}(1-\lambda)^{\frac{1}{2}} < \alpha_V < \alpha^*$ [11] such that the abatement cost of individual participating firms under the VP is equal to their cost under the mandatory standard, in other words, the following inequality satisfies with equality;

$$\frac{1}{2}c(\alpha\bar{e})^2 \leq \bar{C}^{WO} = \frac{1}{2}c(\alpha_L^{WO}\bar{e})^2 + \Omega^{WO}/N. \tag{13}$$

In equilibrium, the condition that no participating firm has an incentive to unilaterally become a non-participating firm must also hold. This condition is given by

$$d[(N-(N_P-1)\alpha)\bar{e}] + (N_P-1)\frac{1}{2}c(\alpha\bar{e})^2 \geq R^M(\alpha_L^{WO}). \tag{14}$$

Because $\alpha_L^{WO} < \alpha_V^{WO} < \alpha^*$ and (14) implies that the "minimization" of social costs is equal to the "maximization" of the decrease in social costs due to emissions abatement by the "N_Pth" participating firm, as in the case with free riding in lobbying, Proposition 1 hold in the case without free riding in lobbying. The next proposition formalizes this.

Proposition 2 *The lowest social cost and largest aggregate abatement under the VP is generated by the abatement rate α_V^{WO} and the participation rate, $N_P^{V,WO}/N$,*

$$N_P^{V,WO}/N = \begin{cases} \dfrac{2d\alpha_L^{WO}\bar{e} - c(\alpha_L^{WO}\bar{e})^2}{2d\alpha_V^{WO}\bar{e} - c(\alpha_V^{WO}\bar{e})^2} + \dfrac{1}{N} & \text{If} N > \dfrac{2d\alpha_V^{WO}e - c(\alpha_V^{WO}e)^2}{2d\alpha_V^{WO}e - c(\alpha_V^{WO}e)^2 - [2d\alpha_L^{WO}e - c(\alpha_L^{WO}e)^2]} \\ 1 & \text{Otherwise.} \end{cases} \tag{15}$$

The combination of α_V^{WO} and $N_P^{V,WO}/N$ generates higher aggregate abatement and lower social cost than a mandatory standard. However, there also exist equilibria where fewer than $(N_P^{V,WO}-1)$ firms participate in the VP and the mandatory policy is implemented.

Proof The conditions that must hold in all equilibria where the VP is implemented change from (10) and (11) to (13) and (14). However, the structure of the equilibrium conditions and the regulator's objective function in cases with and without free riding in lobbying are the same. Therefore, proofs of Proposition 1 can apply to that of Proposition 2. □

[11] This is because $\alpha_V = \frac{d}{c\bar{e}}(1-\lambda)^{\frac{1}{2}}[\lambda + (1-\lambda)N^2]^{\frac{1}{2}}/[\lambda + (1-\lambda)N]$ and $\lambda + (1-\lambda)N^2 > [\lambda + (1-\lambda)N]^2$

Thus, presence of free riding in lobbying does not affect characteristics of equilibrium that generate the highest social welfare and the largest aggregate abatement and existence of an equilibrium where the mandatory standard is implemented. However, impact of change in industry size or the number of firms on abatement rate under the VP and mandatory standard ($\alpha_{\mathrm{V}}^{\mathrm{WO}}$ and $\alpha_{\mathrm{L}}^{\mathrm{WO}}$) and participation rate ($N_{\mathrm{P}}^{\mathrm{V,WO}}/N$) are different under cases with and without free riding in lobbying. The next proposition states this.

Proposition 3 *Industry size has no impact on abatement rates under the VP or mandatory policy in the case without free riding in lobbying* ($\frac{\partial\alpha_{\mathrm{L}}^{\mathrm{WO}}}{\partial N} = \frac{\partial\alpha_{\mathrm{V}}^{\mathrm{WO}}}{\partial N} = 0$), whereas the effect of a change in industry size on the participation rate is negative ($\frac{\partial(N_{\mathrm{P}}^{\mathrm{V,WO}}/N)}{\partial N} < 0$). In the case with free riding in lobbying, the abatement rates under the VP and the mandatory policy are higher if industry size is larger. Its effect on abatement rates under the VP is greater than under the mandatory policy ($\frac{\partial\alpha_{\mathrm{L}}}{\partial N} > \frac{\partial\alpha_{\mathrm{V}}}{\partial N} > 0$). However, the effect on the participation rate is ambiguous (but is likely to be negative ($\frac{\partial(N_{\mathrm{P}}^{\mathrm{V}}/N)}{\partial N} < 0$) in most cases).

Proof Because $\alpha_{\mathrm{L}}^{\mathrm{WO}}$ and $\alpha_{\mathrm{V}}^{\mathrm{WO}}$ are not a function of N, a change in N has no impact on α^{L} and α^{V}. From a simple calculation, we have $\partial(\frac{N_{\mathrm{P}}^{\mathrm{V,WO}}}{N})/\partial N = -1/N^2 < 0$ because $N_{\mathrm{P}}^{\mathrm{V,WO}}/N = \frac{1-\lambda^2}{[2-(2\lambda^2-2\lambda+1)^{\frac{1}{2}}](2\lambda^2-2\lambda+1)^{\frac{1}{2}}} + 1/N$.

See 'Proof of Proposition 3' in Appendix for $\frac{\partial\alpha_{\mathrm{L}}}{\partial N} > \frac{\partial\alpha_{\mathrm{V}}}{\partial N} > 0$ and $\frac{\partial(N_{\mathrm{P}}^{\mathrm{V}}/N)}{\partial N}$. □

The results in the case without free riding in lobbying seem quite intuitive. As industry size does not affect the socially optimal abatement rate when the damage induced by the firms' emissions is independent of those of other firms, the industry size does not affect the abatement rate under the mandatory standard or the VP. However, the participation rate decreases in industry size. In other words, the larger the industry size the more serious the free riding.

In contrast to the case without free riding in lobbying, industry size affects the abatement rate under the VP and the mandatory standard in the case with free riding. Polluting firms have greater incentives to free ride on other firms' lobbying contributions and make fewer contributions to the trade association if the industry size is larger. As a result, the abatement rate under the mandatory standard is higher. Because a threat, the mandatory standard, is more serious, the VP is also more stringent in the sense that the abatement rate is higher. However, when the number of polluting firms changes, the abatement rate under the VP changes less than under the mandatory standard. We can derive this result from equations (7) and (10) that characterize the political contributions of individual firms and costs of firms participating in VP. If industry size increases, the abatement cost each firm faces under the mandatory standard increases, but political contributions decrease from (7). Because each participating firm's abatement cost under the VP is equal to the sum of its abatement cost under the mandatory standard and political contributions from (10) (with equality), the abatement cost of each participating firm under the VP must change less than its abatement cost under the mandatory standard. Thus, the impact of a change in industry size on abatement rates under the VP is smaller than under the mandatory standard.

In the case with free riding in lobbying it is possible that participation rate increases when industry size increases slightly although the participation rate generally decreases as industry size increases. Equation (12) implies that the participation rate is determined by the inverse of industry size (the second term, 1 / N) and the ratio of the welfare impact of emissions abatement by each firm under the mandatory standard to that under the VP (the first term, $\frac{2d\alpha_L\bar{e}-c\alpha_L\bar{e}^2}{2d\alpha_V\bar{e}-c\alpha_V\bar{e}^2}$). Lobbying effectiveness or free riding in lobbying against the mandatory standard indirectly affects the abatement rate under the VP, which is determined by the sum of the abatement cost under the mandatory standard and political contributions because free riding in lobbying affects the abatement rate under the mandatory standard. Thus, the first term of (12) implies that free riding in lobbying indirectly affects the participation rate.

If the industry size increases, free riding in lobbying becomes more serious and lobbying is less effective. As a consequence, abatement rates under the mandatory standard and the VP increase. The first term of the RHS of (12) is the ratio of the value of an increasing concave function at α_L to the one at α_V [12] and $0 < \alpha_L < \alpha_V < \alpha^*$. Therefore, the first term of the participation rate increases due to an increase in industry size.[13] Thus, the indirect effect of free riding in lobbying on the participation rate is always positive.

In contrast to the indirect effect of free riding in lobbying, the direct effect of industry size is always negative. Whether the impact of a change in industry size on the participation rate is positive depends on which impact is larger. Generally, the direct effect dominates the indirect effect. However, the indirect effect dominates the direct effect when λ is very large but is not close to one.[14] In such a case, the difference between the impact of a change in industry size on abatement rates under the mandatory standard and the VP is large. As a result, the indirect effect becomes larger than the direct effect that does not depend on λ.

3.2 The case where the damage induced by individual firms' emissions depends on other firms' emissions

When impacts of emissions are local and polluting facilities are low agglomerated or when marginal damage is constant, the damage induced by individual firms' emissions is very likely to be independent of other firms' emissions. Therefore, up to this point, we have considered such cases or nationwide environmental policies whose primary target is to have plants in areas with a small number of plants. In this subsection, we relax the assumption that the damage induced by individual firms'

[12] Let $f(\alpha) = 2d\alpha\bar{e} - c\alpha^2\bar{e}^2$. Then, $N_P^V/N = f(\alpha^L)/f(\alpha^V) + 1/N$, $f'(\alpha) > 0$ and $f''(\alpha) < 0$ if $\alpha \in (0, \alpha^*)$.

[13] $\partial(f(\alpha^L)/f(\alpha^V))/\partial N = [f(\alpha^V)]^{-2}([f'(\alpha^L)\frac{\partial\alpha^L}{\partial N}f(\alpha^V) - f'(\alpha^V)\frac{\partial\alpha^V}{\partial N}f(\alpha^L)] > 0$ because $\frac{\partial\alpha^L}{\partial N} > \frac{\partial\alpha^V}{\partial N} > 0$.

[14] The range of λ that causes the indirect effect to dominate the direct effect depends on industry size. See 'Proof of Proposition 3' in Appendix for additional details.

emissions does not depend on other firms' emissions. Polices for reductions in greenhouse gases are likely typical examples of this subsection. Concretely, the damage function is assumed to be $\frac{1}{2}d(\sum_i^N e_i)^2$. This assumption of the damage function simplifies the notations without altering any main results in this subsection. Then, the social cost under the VP is given by

$$\mathrm{SC_V}(\alpha, N_\mathrm{P}) = \frac{1}{2}d(N\bar{e} - N_\mathrm{P}\alpha\bar{e})^2 + N_\mathrm{P}\frac{1}{2}c(\alpha\bar{e})^2.$$

In equilibrium, no participating firm has an incentive to unilaterally become a non-participating firm and, technically

$$\mathrm{SC_L} \leq \mathrm{SC_V}(\alpha, N_\mathrm{P} - 1) \tag{16}$$

where

$$\mathrm{SC_V}(\alpha, N_\mathrm{P} - 1) = \frac{1}{2}d(N\bar{e} - (N_\mathrm{P} - 1)\alpha\bar{e})^2 + (N_\mathrm{P} - 1)\frac{1}{2}c(\alpha\bar{e})^2$$

$$\text{and} \quad \mathrm{SC_L} = \frac{1}{2}d(N\bar{e} - N\alpha_\mathrm{L}\bar{e})^2 + N\frac{1}{2}c(\alpha_\mathrm{L}\bar{e})^2.$$

We focus on a set of the abatement rate and number of participating firms that generates the smallest social cost. The set must satisfy (16) with equality and must maximize the difference between social cost with $N_\mathrm{P}(\alpha)$ participating firms and $N_\mathrm{P}(\alpha) - 1$ participating firms, $\Delta = \mathrm{SC_V}(\alpha, N_\mathrm{P}(\alpha)) - \mathrm{SC_V}(\alpha, N_\mathrm{P}(\alpha) - 1)$ where $N_\mathrm{P}(\alpha)$ is the number of participating firms that satisfies (16) with equality when abatement rate under the VP is α. This difference is given by

$$\begin{aligned}\Delta &= \frac{1}{2}d\left(2N\alpha\bar{e}^2 - (2N_\mathrm{P}(\alpha) - 1)\alpha^2\bar{e}^2\right) - \frac{1}{2}c(\alpha\bar{e})^2 \\ &= \frac{1}{2}\alpha\bar{e}^2(2dN - 2dN_\mathrm{P}(\alpha)\alpha + d\alpha - c\alpha).\end{aligned}$$

We assume that the lobbying is not subject to free riding. The absence of free riding in the lobbying simplifies the notation without altering the structure of the combination of abatement and participation rates that generates the lowest social cost, as we demonstrated in the previous subsection. Under this assumption, the abatement rate under the mandatory policy is $\alpha_\mathrm{L} = \frac{(1-\lambda)dN}{c+(1-\lambda)dN}$ and the highest abatement rate under the VP (such that abatement cost under the VP is equal to cost under the mandatory standard) is

$$\alpha_\mathrm{V} = \frac{(1-\lambda)dN}{c + (1-\lambda)dN}\left[\frac{c + dN}{c(1-\lambda)\lambda + (c + dN)(1-\lambda)^2}\right]^{\frac{1}{2}}.$$

In an equilibrium where the VP is implemented, the abatement rate under the VP must not be greater than α_V. Let α_F be the abatement rate that satisfies $\mathrm{SC_L} = \mathrm{SC_V}(\alpha_\mathrm{F}, N - 1)$ or the highest abatement rate such that a full participation VP equilibrium exists and α_V^* be the abatement rate when $(\alpha_\mathrm{V}^*, N_\mathrm{P}(\alpha_\mathrm{V}^*))$ generates the lowest social cost (and is feasible). In contrast to the case where the damage induced by individual firms' emissions is independent of other firms' emissions, $(\alpha_V, N_P(\alpha_V)) \neq (\alpha_V^*, N_P(\alpha_V^*))$ or $(\alpha_V, N_P(\alpha_V))$ generally does not generate the lowest social cost. Table 1 and Fig. 2 provide numerical examples where $(\alpha_V, N_P(\alpha_V))$ does not generate the lowest social cost.

Comparing Cases 1 and 2 of Table 1 and Fig. 2, we find that the participation rate is weighted higher than the abatement rate when the marginal abatement cost (MAC) curve is flatter than the marginal damage (MD) curve. Due to (16), $(\alpha_V^*, N_P(\alpha_V^*))$ maximizes net benefits from emissions abatement by the pivotal participating firm as in the previous section. The benefit (and marginal benefit) from emissions abatement by the pivotal participating firm depends on the extent to which participating firms other than the pivotal one abate. If aggregate abatement by the participating firms other than the pivotal one is larger (higher abatement rate and lower participation rate if the LHS of (16) does not change), the marginal benefit from the pivotal participating firm's abatement becomes smaller. In particular, the marginal benefit is more sensitive to (or sharply decreases in) aggregate abatement by the firms already participating when the MD curve is steep. Therefore, the participation rate is weighted higher than the abatement rate in Case 2 (when the MAC curve is flatter than the MD curve) relative to Case 1.

From the numerical examples, we also find that the participation rate is weighed more as the legislator places more weight on social welfare. The difference between the marginal benefit from emission abatement by the pivotal participating firm in a low participation rate VP and a high participation rate VP that satisfy (16) with equality become larger as λ becomes smaller. If λ is not very large or the legislator considers social welfare to some extent, then a mandatory standard generates modest aggregate abatement and the efficiency loss required to achieve the same social welfare as such a mandatory standard by the low participation rate VP will be large. The substantial efficiency loss will be compensated by high aggregate abatement. Therefore, the difference in aggregate abatement under the low and high participation rate VPs becomes large as λ become small. Because the difference in the marginal benefit is proportional to the difference in the aggregate abatement, the

Table 1 Abatement and participation rates under different sets of parameters

	α_V^* (%)	$\frac{N_P(\alpha_V^*)}{N}$ (%)	α_V (%)	$\frac{N_P(\alpha_V)}{N}$ (%)	α_F (%)
Case 1: $d = 1,\ c = 400,\ N = 200,\ \bar{e} = 1,\ \alpha^* = 1/3$					
(a) $\lambda = 0.99$	1.01	99.44	6.08	17.28	1.00
(b) $\lambda = 0.999$	0.22	44.92	1.94	5.25	0.10
(c) $\lambda = 0.9999$	0.25	4.04	0.61	1.64	0.01
Case 2: $d = 2,\ c = 200,\ N = 200,\ \bar{e} = 1,\ \alpha^* = 2/3$					
(a) $\lambda = 0.99$	3.95	99.87	33.63	12.58	3.94
(b) $\lambda = 0.999$	0.57	70.68	10.92	3.73	0.40
(c) $\lambda = 0.9999$	0.97	4.14	3.46	1.16	0.04

α_V^* is the abatement rate such that $(\alpha_V^*, \frac{N_P(\alpha_V^*)}{N})$ generates the lowest social cost

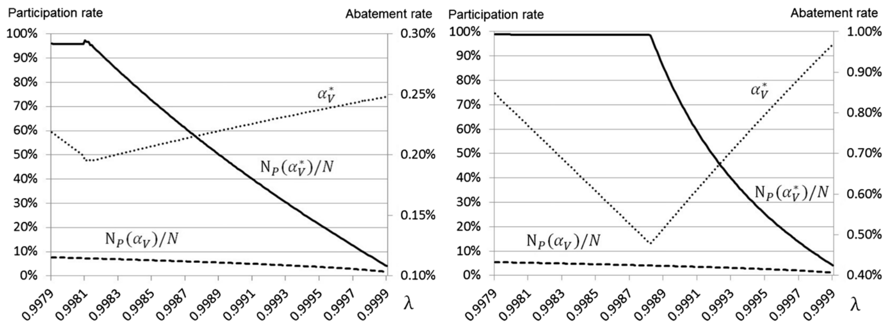

Fig. 2 Abatement and participation rates under $0.9979<\lambda<0.9999$. Case 1: $d=1,\ c=400,\ N=200,\ \bar{e}=1$ (*Left*). Case 2: $d=2,\ c=200,\ N=200,\ \bar{e}=1$ (*Right*). Note: α_{V}^* is the abatement rate such that $(\alpha_{\mathrm{V}}^*, \frac{N_{\mathrm{P}}(\alpha_{\mathrm{V}}^*)}{N})$ generates the lowest social cost

difference between the marginal benefit from emission abatement by the pivotal participating firm in the low and high participation rate VPs also becomes large. Due to this mechanism, the participation rate is weighed more as the legislator relatively weighs social welfare more or λ becomes smaller.

3.3 The asymmetric political contributions case

We focused on a case where all firms pay the same amount of political contribution in the legislative subgame. In this subsection, we consider what happens if amounts of political contributions paid by individual firms are different. Even though amounts of political contribution paid by individual firms are different, the abatement rate under the mandatory standard, α_{L}, is the same as that under the symmetric political contributions case. Therefore, the equilibrium condition that no participating firm has an incentive to unilaterally become a non-participating firm ((11), (14) or (16)) is the same as that under the symmetric political contributions case. However, another equilibrium condition that individual firms' costs under the VP must be smaller than those under the mandatory standard changes from the condition under the symmetric political contributions case ((10) or (13)) to the following inequalities; for all i,

$$\frac{1}{2}c(\alpha\bar{e})^2 \le \bar{C}_i = \frac{1}{2}c(\alpha_{\mathrm{L}}\bar{e})^2 + \Omega_i \tag{17}$$

$$0 \le \Omega_i \le \frac{1}{2}c(\alpha^*\bar{e})^2 - \frac{1}{2}c(\alpha_{\mathrm{L}}\bar{e})^2 \tag{18}$$

and $\sum_i \Omega_i = \Omega$. Although (17) changes from (10) or (13) due only to heterogeneity of political contribution, (18) implies that individual firms' costs under the mandatory standard with lobbying must be smaller than those without lobbying. Let $\alpha_{\mathrm{V}}^{N_{\mathrm{P}}}$ be the abatement rate that satisfies (17) with equality for the firm that pays the N_{P}th largest amount of political contribution. First, we consider a case where the damage induced by individual firms' emissions does not depend on other firms' emissions. The lowest social cost is generated by $(\alpha_V^{N_P}, N_P^A)$ where N_P^A is the smallest number such that $(N_P^A, \alpha_V^{N_P})$ satisfies $N_P^A \leq N_P$ and

$$d[(N-(N_P^A-1)\alpha_V^{N_P})\bar{e}]+(N_P^A-1)\frac{1}{2}c(\alpha_V^{N_P}\bar{e})^2=R^M(\alpha_L). \qquad (19)$$

From the argument in Proposition 1, we can say that the VP with $(\alpha_V^{N_P}, N_P^A/N)$ generates lower social cost than does the mandatory standard if $\alpha_V^{N_P} > \alpha_L$. Moreover, the VP with $(\alpha_V^{N_P}, N_P^A/N)$ generates lower social cost than the VP with $(\alpha_V^{N_P}, N_P^V/N)$ does if $\alpha_V^{N_P} > \alpha_V$. Thus, heterogeneity of individual firms' political contributions can improve VP performance. However, the heterogeneity can deteriorate the performance of VP in the sense that there exists $\hat{\Omega} = (\hat{\Omega}_1, \hat{\Omega}_2, \ldots, \hat{\Omega}_N)$ for any set of $(c, d, \bar{e}, N, \lambda)$ such that $(\alpha_V^{N_P}, N_P^A/N) = (\alpha_L, 1)$. The existence of $\hat{\Omega}$ can be proved by showing that the smallest number of firms that must pay political contribution to have the abatement rate under the mandatory standard α_L $(N^\Omega = \Omega/[\frac{1}{2}c(\alpha^*\bar{e})^2 - \frac{1}{2}c(\alpha_L\bar{e})^2])$ is strictly smaller than the smallest number of firms that must participate in a VP to generate the same social cost as the mandatory standard (N^{SC} such that $d[(N-N^{SC}\alpha^*)\bar{e}] + N^{SC}\frac{1}{2}c(\alpha^*\bar{e})^2 = R^M(\alpha_L)$) or by showing $N^\Omega < N^{SC}$. Let $\tilde{\Omega}$ such that $\tilde{\Omega}_1 = \tilde{\Omega}_2 = \cdots = \tilde{\Omega}_{N^\Omega} = [\frac{1}{2}c(\alpha^*\bar{e})^2 - \frac{1}{2}c(\alpha_L\bar{e})^2]$ and $\tilde{\Omega}_{N^\Omega+1} = \cdots = \tilde{\Omega}_N = 0$. Then, $\tilde{\Omega}$ is an example of $\hat{\Omega}$ if $N^\Omega < N^{SC}$.

Here, we show that $\tilde{\Omega}$ is an example of $\hat{\Omega}$ if $N^\Omega < N^{SC}$. Suppose $N^\Omega < N^{SC}$ and the allocation of political contributions is $\tilde{\Omega}$. On the one hand, it is feasible to implement α_L as the mandatory standard because $\sum_i \tilde{\Omega}_i = N^\Omega[\frac{1}{2}c(\alpha^*\bar{e})^2 - \frac{1}{2}c(\alpha_L\bar{e})^2] = \Omega$. On the other hand, it is also feasible to implement a VP with $(\alpha^*, N^\Omega/N)$ but the regulator would not because $d[(N-N^\Omega\alpha^*)\bar{e}] + N^\Omega\frac{1}{2}c(\alpha^*\bar{e})^2 > R^M(\alpha_L)$ due to $N^\Omega < N^{SC}$. Because $(N-N^\Omega)$ firms accept only α^* as the VP abatement rate, a VP with $(\alpha_L, 1)$ or the mandatory standard is implemented. Thus, if $N^\Omega < N^{SC}$, $\tilde{\Omega}$ is an allocation of political contributions such that the VP cannot generate strictly smaller social costs than the mandatory standard. In 'Proof of $N^\Omega < N^{SC}$' of Appendix, we prove that $N^\Omega < N^{SC}$ always holds.

Existence and characteristics of $\tilde{\Omega}$ indicate that a VP generates the same social cost as a mandatory standard if costs of political contributions are shared by payment of too few firms. However, the VP can generate strictly smaller social costs than the mandatory standard if all firms pay political contributions. Moreover, if costs of political contributions are equally shared by too few but not all firms (for example, N_P^V firms), this kinds of heterogeneity of individual firms' political contributions can improve the performance of VP relative to a case under which all firms equally share costs of political contributions.

We can infer the impact of heterogeneity of political contributions under a case where the damage induced by individual firms' emissions depends on other firms' emissions from the results of the last subsection. In this case, the participation rate is important relative to a case where the damage induced by individual firms' emissions does not depends on other firms' emission. Because higher participation rate implies that cost sharing is important, the heterogeneity of political contributions paid by individual firms is likely to have adverse effects on social welfare relative to the case where the damage induced by individual firms' emissions does not depend on other firms'.

4 Conclusion

We built a model with multiple polluting firms, a trade association, a regulator, and a legislator who sets a mandatory standard and is politically influenced by the trade association, which is a representative of the polluting firms. We then used the model to explain why the regulator implements a VP. In this model, the regulator can implement the VP, which generates lower social costs and higher aggregate emissions abatement than the mandatory standard, in contrast to the model of Dawson and Segerson (2008).

This difference between the models occurs because we introduced an element of political economy into a mandatory policy-making process, and the regulator in their model had a different objective from the regulator in this paper's model. The regulator's objective in Dawson and Segerson (2008) was to achieve some aggregate emission level, but in this paper, it was used to minimize social costs. Because the VP is subject to free riding, the mandatory standard better achieves an aggregate abatement level than does the VP. Without political difficulties in the mandatory policy-making process, the regulator could implement the mandatory policy, which would achieve a socially efficient outcome. Thus, the differences between this paper's model and Dawson and Segerson's shed important light on the ways a government should implement VPs.

We found that the regulator should set the abatement rate under the VP at the highest possible level and should not set the abatement rate to maximize the participation rate if the damage due to individual firms' emissions does not depend on other firms' emissions. However, the importance of the abatement rate relative to the participation rate declines as the damage due to individual firms' emissions becomes more sensitive to those of other firms. This is because a high abatement rate reduces the marginal benefit from emissions abatement by the pivotal participating firm and the marginal benefit decreases more sharply in the abatement rate as the damage caused by individual firms' emissions becomes more sensitive to other firms' emissions. We also found that the importance of abatement rate relative to the participation rate becomes smaller as the legislator weighs social welfare more.

This study assumed that the emission abatement functions of all firms are the same. However, one way to extend this paper's model is to introduce heterogeneity of abatement technology among polluting firms. Given that most voluntary approaches cannot enforce a firm's commitment, it would be interesting to consider the case in which voluntary programs are not enforceable. Legislators have the right to make laws, and it is thus possible that legislators would set a mandatory standard even if a government or environmental organization decides to implement a VP. Exploring these types of extensions remains an endeavor for future research.

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Appendix

Proof of Proposition 1

If $d[(N-(N-1)\alpha_{\mathrm{V}})\bar{e}]+(N-1)\frac{1}{2}c(\alpha_{\mathrm{V}}\bar{e})^2 \leq d[(N-N\alpha_{\mathrm{L}})\bar{e}]+N\frac{1}{2}c(\alpha_{\mathrm{L}}\bar{e})^2$ or $N \leq \frac{2d\alpha_{\mathrm{V}}\bar{e}-c\alpha_{\mathrm{V}}^2\bar{e}^2}{2d\alpha_{\mathrm{V}}\bar{e}-c\alpha_{\mathrm{V}}^2\bar{e}^2-(2d\alpha_{\mathrm{L}}\bar{e}-c\alpha_{\mathrm{L}}^2\bar{e}^2)}$, then all firms have an incentive to join the VP because (10) and (11) hold for any $N_{\mathrm{P}} \leq N$. Because α_{V} is the highest abatement rate satisfying (10) but is smaller than the socially optimal abatement rate, α^*, α_{V} with $N_{\mathrm{P}}^{\mathrm{V}}/N=1$ generates the smallest social cost and largest aggregate abatement.

If $N > \frac{2d\alpha_{\mathrm{V}}\bar{e}-c\alpha_{\mathrm{V}}^2\bar{e}^2}{2d\alpha_{\mathrm{V}}\bar{e}-c\alpha_{\mathrm{V}}^2\bar{e}^2-(2d\alpha_{\mathrm{L}}\bar{e}-c\alpha_{\mathrm{L}}^2\bar{e}^2)}$, (11) must hold with equality. By substituting $d[(N-(N_{\mathrm{P}}-1)\alpha)\bar{e}]+(N_{\mathrm{P}}-1)\frac{1}{2}c(\alpha\bar{e})^2 = R^{\mathrm{M}}(\alpha_{\mathrm{L}})$ into $R^{\mathrm{V}}(\alpha)$, we have $R^{\mathrm{V}}(\alpha)=R^{\mathrm{M}}(\alpha_{\mathrm{L}})+d\alpha\bar{e}-\frac{1}{2}c(\alpha\bar{e})^2$. Because $d\alpha\bar{e}-\frac{1}{2}c(\alpha\bar{e})^2=-\frac{1}{2}c\bar{e}^2(\alpha-\frac{d}{c\bar{e}})^2+\frac{d^2}{2c}$ and $\alpha_{\mathrm{V}}<\alpha^*=\frac{d}{c\bar{e}}$, α_{V} with $N_{\mathrm{P}}^{\mathrm{V}}/N$ generates the smallest social cost.

From (11), $d[(N-(N_{\mathrm{P}}^{\mathrm{V}}-1)\alpha_{\mathrm{V}})\bar{e}]+(N_{\mathrm{P}}^{\mathrm{V}}-1)\frac{1}{2}c(\alpha_{\mathrm{V}}\bar{e})^2 \leq d[(N-(N_{\mathrm{P}}-1)\alpha)\bar{e}]+(N_{\mathrm{P}}-1)\frac{1}{2}c(\alpha\bar{e})^2$ for any combination of α and N_{P}/N that satisfy both (10) and (11). By rearranging this inequality,

$$-d(N_{\mathrm{P}}^{\mathrm{V}}-1)\alpha_{\mathrm{V}}\bar{e}+(N_{\mathrm{P}}^{\mathrm{V}}-1)\frac{1}{2}c(\alpha_{\mathrm{V}}\bar{e})^2 \leq -d(N_{\mathrm{P}}-1)\alpha\bar{e}+(N_{\mathrm{P}}-1)\frac{1}{2}c(\alpha\bar{e})^2$$

$$(N_{\mathrm{P}}^{\mathrm{V}}-1)[-d\alpha_{\mathrm{V}}\bar{e}+\frac{1}{2}c(\alpha_{\mathrm{V}}\bar{e})^2] \leq (N_{\mathrm{P}}-1)[-d\alpha\bar{e}+\frac{1}{2}c(\alpha\bar{e})^2].$$

Because $-d\alpha_{\mathrm{V}}\bar{e}+\frac{1}{2}c(\alpha_{\mathrm{V}}\bar{e})^2=\frac{1}{2}c\alpha_{\mathrm{V}}\bar{e}^2(\alpha_{\mathrm{V}}-\frac{2d}{c\bar{e}})<0$ from $\alpha_{\mathrm{V}}<\alpha^*=\frac{d}{c\bar{e}}$

$$N_{\mathrm{P}}^{\mathrm{V}} \geq (N_{\mathrm{P}}-1)\frac{-d\alpha\bar{e}+\frac{1}{2}c(\alpha\bar{e})^2}{-d\alpha_{\mathrm{V}}\bar{e}+\frac{1}{2}c(\alpha_{\mathrm{V}}\bar{e})^2}+1.$$

Therefore,

$$\begin{aligned} N_{\mathrm{P}}^{\mathrm{V}}\alpha_{\mathrm{V}}\bar{e} &\geq (N_{\mathrm{P}}-1)\frac{-d\alpha\bar{e}+\frac{1}{2}c(\alpha\bar{e})^2}{-d\alpha_{\mathrm{V}}\bar{e}+\frac{1}{2}c(\alpha_{\mathrm{V}}\bar{e})^2}\alpha_{\mathrm{V}}\bar{e}+\alpha_{\mathrm{V}}\bar{e}. \\ &= \frac{(N_{\mathrm{P}}-1)}{-d\alpha_{\mathrm{V}}\bar{e}+\frac{1}{2}c(\alpha_{\mathrm{V}}\bar{e})^2}\alpha_{\mathrm{V}}\bar{e}\left(-d\alpha\bar{e}+\frac{1}{2}c(\alpha\bar{e})^2\right)+\alpha_{\mathrm{V}}\bar{e} \\ &\geq \frac{(N_{\mathrm{P}}-1)}{-d\alpha_{\mathrm{V}}\bar{e}+\frac{1}{2}c(\alpha_{\mathrm{V}}\bar{e})^2}\alpha\bar{e}\left(-d\alpha_{\mathrm{V}}\bar{e}+\frac{1}{2}c(\alpha_{\mathrm{V}}\bar{e})^2\right)+\alpha_{\mathrm{V}}\bar{e} \\ &= (N_{\mathrm{P}}-1)\alpha\bar{e}+\alpha_{\mathrm{V}}\bar{e} \\ &\geq N_{\mathrm{P}}\alpha\bar{e}. \end{aligned} \tag{20}$$

The second inequality holds because $\|\alpha_{\mathrm{V}}[-d\alpha\bar{e}+\frac{1}{2}c(\alpha\bar{e})^2]\| \geq \|\alpha[-d\alpha_{\mathrm{V}}\bar{e}+\frac{1}{2}c(\alpha_{\mathrm{V}}\bar{e})^2]\|$ due to $-d\alpha\bar{e}+\frac{1}{2}c(\alpha\bar{e})^2 \leq -d\alpha_{\mathrm{V}}\bar{e}+\frac{1}{2}c(\alpha_{\mathrm{V}}\bar{e})^2 \leq 0$ and $\alpha_{\mathrm{V}} \geq \alpha \geq 0$ for any α that satisfies (11). Thus, a combination of α_{V} and $N_{\mathrm{P}}^{\mathrm{V}}/N$ generates the largest aggregate abatement.

Next, we prove that the abatement rate α_{V} with participation rate $N_{\mathrm{P}}^{\mathrm{V}}/N$ generates higher aggregate abatement and lower social cost than a mandatory standard. Consider the VP with abatement rate α_{L} and participation rate $1(=N/N)$. Because the combination of α_{L} and 1 satisfies (10) and (11), this VP can be implemented and generates the same social cost as the mandatory policy does. The abatement rate α_{V} with participation rate $N_{\mathrm{P}}^{\mathrm{V}}/N$ generates higher aggregate abatement and lower social cost than the abatement rate α_{L} with a participation rate of 1 or a mandatory standard.

However, the mandatory policy generates a (weakly) lower level of social welfare if the abatement rate is α_{V} and if the number of participating firms is below $(N_{\mathrm{P}}^{\mathrm{V}}-1)$ because $d[(N-(N_{\mathrm{P}}^{\mathrm{V}}-1)\alpha_{\mathrm{V}})\bar{e}]+(N_{\mathrm{P}}^{\mathrm{V}}-1)\frac{1}{2}c(\alpha_{\mathrm{V}}\bar{e})^2 = R^{\mathrm{M}}(\alpha_{\mathrm{L}})$. Therefore, the mandatory policy is implemented under such a case. Suppose that N_{P} (fewer than $N_{\mathrm{P}}^{\mathrm{V}}-1$) firms participate in the VP. As long as $N_{\mathrm{P}} < N_{\mathrm{P}}^{\mathrm{V}}$, the mandatory policy is implemented. Even if the VP is implemented, participating firms obtain the same payoffs as they do when the mandatory policy is implemented. Therefore, if $N_{\mathrm{P}} < N_{\mathrm{P}}^{\mathrm{V}}$, no participating firms change their decisions (from "participate" to "not participate") In addition, all "not participating" firms do not change their decisions (from "not participate" to "participate"). Thus, there exists equilibria where the mandatory policy will be implemented.

Proof of Proposition 3 (part on $\partial\alpha_{\mathrm{V}}/\partial N$, $\partial\alpha_{\mathrm{L}}/\partial N$ and $\partial(N_{\mathrm{P}}^{\mathrm{V}}/N)/\partial N$)

By taking the derivatives of α_{V} and α_{L} with respect to N,

$$\begin{aligned} \frac{\partial\alpha_{\mathrm{V}}}{\partial N} = &\Big\{d(1-\lambda)^{\frac{1}{2}}[\lambda+(1-\lambda)N^2]^{-\frac{1}{2}}\cdot(1-\lambda)N[\lambda+(1-\lambda)N] \\ &\times -d(1-\lambda)^{\frac{1}{2}}[\lambda+(1-\lambda)N^2]^{\frac{1}{2}}\cdot(1-\lambda)\Big\}\cdot\frac{1}{c\bar{e}[\lambda+(1-\lambda)N]^2} \end{aligned}$$

$$= \frac{d(1-\lambda)^{\frac{3}{2}}[\lambda N + (1-\lambda)N^2] - d(1-\lambda)^{\frac{3}{2}}[\lambda + (1-\lambda)N^2]}{c\bar{e}[\lambda + (1-\lambda)N]^2[\lambda + (1-\lambda)N^2]^{\frac{1}{2}}}$$
$$= \frac{d\lambda(1-\lambda)^{\frac{3}{2}}(N-1)}{c\bar{e}[\lambda + (1-\lambda)N]^2[\lambda + (1-\lambda)N^2]^{\frac{1}{2}}} > 0$$

and

$$\frac{\partial\alpha_L}{\partial N} = \frac{(1-\lambda)d[\lambda + (1-\lambda)N] - (1-\lambda)dN(1-\lambda)}{c\bar{e}[\lambda + (1-\lambda)N]^2}$$
$$= \frac{(1-\lambda)\lambda d}{c\bar{e}[\lambda + (1-\lambda)N]^2} > 0.$$

Because $\lambda + (1-\lambda)N^2 > (1-\lambda)(N-1)^2$, $[\lambda + (1-\lambda)N^2]^{\frac{1}{2}} > (1-\lambda)^{\frac{1}{2}}(N-1)$ or $\frac{(1-\lambda)^{\frac{1}{2}}(N-1)}{[\lambda+(1-\lambda)N^2]^{\frac{1}{2}}} < 1$. Therefore, $\frac{\partial\alpha_L}{\partial N} > \frac{\partial\alpha_V}{\partial N} > 0$.

Let $F = (2\lambda + (1-\lambda)N)(1-\lambda)N$ and

$$G = (f-g)g$$
$$= [2(\lambda + (1-\lambda)N) - (1-\lambda)^{\frac{1}{2}}(\lambda + (1-\lambda)N^2)^{\frac{1}{2}}](1-\lambda)^{\frac{1}{2}}(\lambda + (1-\lambda)N^2)^{\frac{1}{2}}.$$

Then, $\frac{N_P^V}{N} = \frac{NF+G}{NG}$ and its derivative with respect to N is

$$\partial\left(\frac{N_P^V}{N}\right)/\partial N = \frac{(NF' + G' + F)NG - (NF+G)(G+NG')}{(NG)^2}$$
$$= \frac{F'}{G} - \frac{FG'}{G^2} - \frac{1}{N^2}.$$

Because $F' = 2(1-\lambda)(\lambda + (1-\lambda)N)$, $G' = 4(1-\lambda)g - 2(1-\lambda)^2N + \frac{2\lambda(1-\lambda)(N-1)}{\lambda+(1-\lambda)N^2}g$,

$$\frac{F'}{G} - \frac{FG'}{G^2} - \frac{1}{N^2} \to \frac{2N}{N} - \frac{N\cdot(4N-2N-0)}{N^2} - \frac{1}{N^2} = -\frac{1}{N^2} \quad \text{as}\lambda \to 0.$$

Thus, $\partial(\frac{N_P^V}{N})/\partial N < 0$ as $\lambda \to 0$. As λ approaches 1, $G \to 0, f-g \to 2$, $G' \to 0$ and

$$\frac{F'}{G} - \frac{FG'}{G^2} - \frac{1}{N^2} = \frac{2[\lambda + (1-\lambda)N](1-\lambda)^{\frac{1}{2}}}{(f-g)(\lambda + (1-\lambda)N^2)^{\frac{1}{2}}} - \frac{(2\lambda + (1-\lambda)N)NG'}{(f-g)^2(\lambda + (1-\lambda)N^2)} - \frac{1}{N^2} \to -\frac{1}{N^2}. \tag{21}$$

Thus, $\partial(\frac{N_P^V}{N})/\partial N < 0$ if $\lambda \to 1$. However, $\partial(\frac{N_P^V}{N})/\partial N$ might be positive in other cases because the first term of (21) is positive. In fact, when $N = 10$, $\partial(\frac{N_P^V}{N})/\partial N$ is positive if $0.9356154 < \lambda < 0.9998979$ (When $N = 100$, $\partial(\frac{N_P^V}{N})/\partial N$ is positive if

$0.9920507 < \lambda < 0.9999999$. When $N = 1000$, $\partial(\frac{N_P^V}{N})/\partial N$ is positive if $0.9991883 < \lambda < 0.9999999$.).

Proof of $N^{\Omega} < N^{SC}$

Let $x = \alpha_L/\alpha^* = \frac{(1-\lambda)N}{\lambda+(1-\lambda)N} > (1-\lambda)$ (because $0 < \lambda < 1$ and $N > 1$). Then,

$$\Omega = \frac{1-\lambda}{\lambda}[R^M(\alpha_L) - R^M(\alpha^*)] = \frac{1-\lambda}{\lambda}\frac{1}{2}N(1-x)^2\frac{d^2}{c}$$
$$\tilde{\Omega}_1 = \frac{1}{2}c[(\alpha^*\bar{e})^2 - (x\alpha^*\bar{e})^2] = \frac{1}{2}(1-x^2)\frac{d^2}{c} \quad (22)$$
$$\frac{N^{\Omega}}{N} = \frac{\Omega}{\tilde{\Omega}_1 N} = \frac{(1-\lambda)(1-x)}{\lambda(1+x)}$$

$$\frac{N^{SC}}{N} = \frac{-dx\alpha^*\bar{e} + \frac{1}{2}c(x\alpha^*\bar{e})^2}{-d\alpha^*\bar{e} + \frac{1}{2}c(\alpha^*\bar{e})^2} = \frac{-\frac{d^2}{2c}x(2-x)}{-\frac{d^2}{2c}} = x(2-x) \quad (23)$$

where (23) is derived from $d(N - N^{SC}\alpha^*)\bar{e} + N^{SC}\frac{1}{2}c(\alpha^*\bar{e})^2 = R^M(\alpha_L)$. Because $\partial(\frac{N^{\Omega}}{N})/\partial x = \frac{(1-\lambda)}{\lambda(1+x)^2}\cdot(-(1+x)-(1-x)) = \frac{-2(1-\lambda)}{\lambda(1+x)^2} < 0$ and $\partial(\frac{N^{SC}}{N})/\partial x = (2-x) - x > 0$, $\frac{N^{\Omega}}{N}$ is decreasing with x and $\frac{N^{SC}}{N}$ is increasing with x. Therefore, $\frac{N^{\Omega}}{N} < \frac{N^{SC}}{N}$ for any x if $\frac{N^{\Omega}}{N} < \frac{N^{SC}}{N}$ at the greatest lower bound of $x(= 1-\lambda)$. By plugging $x = 1-\lambda$ into (22) and (23),

$$\frac{N^{\Omega}}{N} = \frac{(1-\lambda)\lambda}{\lambda(2-\lambda)} = \frac{(1-\lambda)}{(2-\lambda)} < (1-\lambda)$$
$$\frac{N^{SC}}{N} = (1-\lambda)(1+\lambda) > (1-\lambda).$$

Thus, $\frac{N^{\Omega}}{N} < \frac{N^{SC}}{N}$ or $N^{\Omega} < N^{SC}$. In addition to a case where lobbying suffers from free riding, $N^{\Omega} < N^{SC}$ also holds when lobbying does not suffer from free riding because $\alpha_L^{WO}/\alpha^* = 1-\lambda$.

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